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Comment on 'The effect of sea tides on gravity tidal observations on the Antarctic Ekström ice shelf' by Bülent Tezkan and Ugur Yaramanci

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INTRODUCTION

In a recent paper, Tezkan & Yaramanci (1993; hereafter referred to as T&Y) present observations of sea-surface and ice-shelf surface-elevation variations from the Ekström Ice Shelf, Antarctica. The sea-surface observations come from a pressure sensor moored in the open ocean about 10 km from the ice front and it is assumed that these data closely resemble the vertical tidal motion of the ice front itself. The ice-shelf surface observations were obtained using a gravimeter at Georg von Neumayer station (GvN), situated about 10 km inland from the ice front. The quality of both data sets is very good and the analyses for tidal constituents and for the different components of the gravity reduction are rigorous and detailed. However, we disagree with a short section near the end of the paper where T&Y interpret the GvN observations using a simple cantilever, elastic-beam model for the ice shelf, in which one end of the beam is fixed (the hinge line), leaving the remainder to be influenced by tidal forcing. This problem was first studied by Robin (1958). T&Y have assumed that the whole beam is subjected to a uniformly distributed load due to the ice-shelf mass, but the supporting force of the underlying water is not included. Our doubts arise from this fundamental omission, as the physics on which the model is based is inadequate. Their conclusion regarding the position of the hinge line on Ekström Ice Shelf is based on this model and is, therefore, incorrect. T&Y place the hinge line about 55 km south of GvN (T&Y Fig. 2) which implies that, south of this line, tides will have no effect on the motion of the ice shelf. This line is up to 50 km north of the grounding line (the point at which the ice flowing off the land begins to float) identified from Landsat imagery. All the evidence from other ice shelves show tidal motion right up to the grounding line. It is highly unlikely that Ekström Ice Shelf will be different and T&Y present no evidence to that effect.

ELASTIC BEAM MODELLING OF ICE SHELVES

T&Y admit that their model is a very simplified approach to the problem. We believe, however, that it is incorrect and should not be used, despite the attractions of its simplicity. We present, in summary, a more appropriate model similar to that first proposed by Robin (1958) which has since been used by a number of authors (e.g. Holdsworth 1969, 1977; Hughes 1977; Lingle, Hughes & Kollmeyer 1981; Stephenson 1984; Kobarg 1988; Smith 1991) to model the tidal flexing of ice shelves. We represent an ice shelf floating on the sea as an elastic beam resting on an elastic foundation (see for example Fig. 7 of Smith 1991). The sea is considered to provide an elastic foundation since the restoring hydrostatic stress is proportional to the displacement. The differential equation for this problem is derived in many text books (e.g. Hetenyi 1946; Turcotte & Schubert 1982) and, ignoring longitudinal stress, is:

$$D\frac{\partial^4 w}{\partial x^4} = \rho_{\text{water}}\, g w(x),$$

where $w(x)$ is the vertical deflection of the ice shelf, ρ_{water} the sea-water density, g the gravitational acceleration and D is the flexural rigidity that is related to the material properties of the ice.

The tidal displacement is obtained by applying boundary conditions to the differential equation. The ice shelf suffers no tidal displacement at the hinge zone ($x = 0$) but follows the sea surface, $A(t)$ at $x = \infty$. We consider only the floating portion $x \geq 0$, assuming that the hinge zone is in the same place as the grounding line. The boundary conditions are thus: $w(x) = dw/dx = 0$ at $x = 0$; and $w(x) = A(t)$ at $x = \infty$. The solution may be shown to be:

$$w = A(t) - A(t)e^{-\beta x}(\cos\beta x + \sin\beta x), \quad (1)$$

where, once again β is related to the material properties of the ice (Smith 1991), and can be considered as the inverse of both the decay length and the spatial frequency of the flexural response. β can be calculated using values for the physical and elastic properties of the ice shelf and the underlying sea water (see for example Stephenson 1984; Smith 1991).

This solution (eq. 1) has already been derived by a number of authors and has been applied to observations from Antarctica and Greenland by, for example, Lingle *et al.* (1981), Stephenson (1984), Kobarg (1988) and Smith (1991). We would, therefore, replace T&Y's simple eq. (2), containing one parameter (L), with our eq. (1) that contains the parameter β.

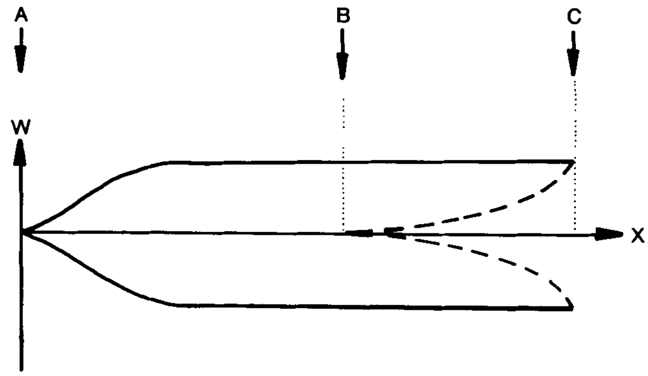

Figure 1. Schematic comparison of the two equations when applied to the Ekström Ice Shelf observations. Solid lines show the position of the ice-shelf centre line at high and low tides for the established model (our eq. 1); dashed lines show the same for T&Y eq. (2). Both equations follow the x axis at the mean tide position. A is the grounding line and also the hinge line for the established model; B is the hinge line determined by T&Y; and C is the ice front. For clarity, the tidal range is exaggerated and the figure is not to scale.

OBSERVATIONS IN SUPPORT OF THE ESTABLISHED MODEL

The cantilever model used by T&Y assumes that the form of the bending profile will be as shown by their Fig. 10 and given by their eq. (2). These imply that tidal variations in both surface displacement and surface tilt will continue to increase with distance from the hinge zone all the way to the ice front. In contrast, our solution implies that, beyond a certain distance from the hinge zone, the ice shelf rises and falls freely without tidally induced tilting. Variations of surface tilt will be restricted to a region relatively close (of the order of a few kilometres) to the hinge zone. Fig. 1 illustrates the differences between the two equations.

Observations from ice shelves in Antarctica support the predictions of eq. (1), rather than T&Y's eq. (2). Robin (1958) observed tidal flexing at ice-shelf grounding lines. Gravimeters have shown vertical displacements, both close to grounding lines and well away from them (e.g. Williams & Robinson 1980; Eckstaller & Miller 1984; Doake 1992). Amplitudes are seen to be independent of distance from the ice front. Tiltmeters installed close to grounding lines show results which are modelled successfully by eq. (1) (e.g. Stephenson, Doake & Horsfall 1979; Stephenson 1984; Smith 1991). Long-term records (e.g. 43 days; Smith 1991) confirm unequivocally that the observations are tidal in origin. Kinematic GPS profiling (Vaughan 1994) has shown the smooth transition from grounded ice, through the region of tilting and out onto ice rising and falling freely.

Available observations, therefore, agree with a model that includes the support of the underlying ocean and refute the predictions of T&Y's model, which does not.

APPLICATION OF THE MODELS TO THE DATA FROM THE EKSTRÖM ICE SHELF

Using typical and published values for the ice- and sea-water properties, it is possible to do a comparison of the Ekström Ice-Shelf data with our eq. (1), similar to that which is presented by T&Y for their eq. (2). However, in this case the numerical results of applying *either* model to the data are of limited use for the following reasons.

(a) It is known that in an area close to the ice front, about 10 km north of GvN, part of Ekström Ice Shelf is aground (Kobarg 1988). A similar grounded area exists to the east (Grosfeld *et al.* 1989). The observed differences in tidal amplitude between GvN and the ice front are likely to be influenced strongly by these areas, and certainly more so than by a proposed hinge line 55 km to the south.

(b) With such a complex ice shelf and sea-bed topography, the sea-surface observations from 20 km away cannot be used as a reliable tidal reference for modelling the observations at GvN.

(c) The gravity observations only provide one data point to compare with a model. Although T&Y present reasonable agreement between models for different tidal frequencies, this agreement is only to be expected from their earlier observation comparing the amplitudes at GvN with those at the ice front. The ratios of these two amplitudes, for all the tidal frequencies, are similar. In other words, Z/Z_0 in T&Y eq. (2). will be roughly constant for all frequencies, so the resulting values of L are bound to be similar.

Hence, we believe that T&Y's model should not be applied here because it is incorrect and, whilst ours can be applied, there are insufficient data to do so usefully.

SUMMARY

We claim that the equation used by T&Y to model the elastic bending of an ice shelf is incorrect because it does not include the support of the underlying sea water. The physical principles on which the model is based are flawed and the model should not be applied to the observed data.

We present a long-accepted, alternative equation that does include the effect of the underlying water. Without exception, available observations support the use of our model and contradict the predictions of that proposed by T&Y. Where data are sufficient, they are modelled successfully using our approach, despite assumed uncertainies in ice rheology at tidal frequencies.

Applying this model to T&Y's observations is straightforward but there are insufficient data to give meaningful results.

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