Relations between Electrical Conductivity of a Mantle and Fluctuating Magnetic Fields

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Summary

It is shown that the ratio of the vertical field to the horizontal field gradient outside a geological body can often be used to measure the electrical conductivity at depth within it. The in-phase part of this ratio, at a given frequency, gives the penetration depth of the magnetic field, the out-of-phase component yields approximately the conductivity at the penetration depth.

In this paper it will be shown that the electrical conductivity within an object can often be related to simple properties of fluctuating magnetic fields observed and generated outside of it. The specific question to which we address ourselves is the determination of the electrical conductivity as a function of depth into a geological body by observing the properties of fluctuating magnetic fields outside that body. In the case of the Earth, we are concerned with the fluctuating magnetic fields which are excited by electric currents flowing in the ionosphere, in the case of the Moon by magnetic fields which are convected by the solar wind. We shall mathematically analyse the interaction of an oscillating magnetic field which is slightly non-uniform in the horizontal plane with a laterally uniform body whose electrical conductivity increases with depth. It will be shown that, above and outside the body, the magnetic field has two simply interpretable properties whose importance for interpreting magnetic deep sounding data has not been realized heretofore. The ratio of the ' in phase' part of the vertical field to the 'gradient' of the oscillating horizontal field component is equal to an appropriately defined field penetration depth. The ratio of the out-of-phase part of the vertical field and the same horizontal field gradient is approximately equal to the field attenuation length associated with the electrical conductivity of the body evaluated at the field penetration depth.

Chapman (1919) pointed out that the fluctuating magnetic fields observed at the surface of the Earth could be resolved into normal modes and that the part of each normal mode which is due to eddy currents flowing inside the body could be separated from that due to currents flowing externally. By noting that the internal currents were coupled to and excited by the external currents, he obtained an approximate value for the Earth's conductivity. Various combinations of the 'complex' ratio of the internal current and the external current have subsequently been denoted as magnetic response functions. By studying these response functions as a function frequency means have been devised to generate conductivity profiles of the Earth and Moon from experimental data (Rikitake 1966).

Most analyses have seriously utilized only the real part of this response function (e.g. Banks 1969), i.e. they have considered only the magnitude of the horizontal

to vertical field ratio as a function of frequency. The inversion of experimental data, using this parameter to obtain a conductivity profile, has raised serious questions of uniqueness of fit. Backus & Gilbert (1970) devised a method to evaluate the likelihood of a given conductivity profile as its parameters are varied slightly. Parker (1970) reanalysed the conclusions of Banks, using these criteria, and showed that there was considerable latitude in the model derived. The field fluctuations on the lunar surface show a qualitatively different response than on Earth though similar questions of uniqueness arise. Sonnet *et al.* (1971) concluded that the Moon has an outer 220 km non-conducting shell, followed by a shell about 100 km thick whose conductivity peaks to 10^{-2} mho/m followed by conductivity minimum of somewhat less than 10^{-4} mho/m at a depth of 350 km. Kuckes (1971) showed that these data could equally well be understood on the basis of a Moon with an insulating mantle about 160 km thick and an inner core with a uniform conductivity of 6×10^{-4} mho/m. The geological conclusions, which the two conductivity models suggest, are completely different from each other.

Aside from questions of uniqueness, the use of the usual modal analyses has shown other serious limitations. The method assumes a lateral uniformity in the conductivity which is greater in extent than the lateral scale distance of the field, i.e. the wavelength of the exciting field. If the deeper parts of the Earth's mantle are being investigated, there is some experimental justification for this hypothesis. The studies of Schmucker (1959, 1964) and Porath *et al.* (1970, 1971a, b) show that, in the upper mantle, there are geologically interesting variations over distances of several hundred kilometres which are amenable to study by this method. Schmucker first did experiments with arrays of magnetometers specially deployed for identifying and studying localized conductivity anomalies. Analysis of these data proceeded using conformal mapping techniques which pointed out several conductivity anomalies both in Germany and the south-west of the U.S. More recently, Porath *et al.* have presented the results from arrays of magnetometers studying conductivity profiles in the South-west U.S. and well up into Canada. Porath, Oldenburg & Gough (1970) analysed the data of the Western United States using a technique, due to Siebert & Kertz (1957), to separate the internally and externally generated fields. Their analysis was greatly complicated by the fact that the scale dimensions of the exciting fields were large compared to the dimensions of their magnetometer array. For this reason, they devised an involved separation procedure which defined normal and anomalous fields. Conformal mapping analysis was applied to the anomalous part of the separated fields. In general, their studies indicate the great need for a more systematic method for directly deducing a conductivity profile from local properties of the fields (Porath & Dziewonski 1971).

The contemporary interest in studying the structure of the upper mantle to test theories of mantle dynamics gives considerable impetus to developing and making better use of these electromagnetic fluctuations.

It is the purpose of this paper to show that, because most geomagnetic disturbances have a lateral scale distance which is somewhat longer than the depth to which we often want to probe, a systematic technique for locally analysing data and subsequently making conductivity profile maps of the Earth can be developed. Our results require a conductivity uniformity over a lateral extent which is a few times the depth being probed rather than over a wavelength of the exciting field as modal analyses assume.

1. A heuristic analysis

Before proceeding with the more exact analysis of relating the vertical field and the longitudinal gradient of the horizontal field above a conductor, an approximate

analysis will be developed which brings out the physical ideas underlying the more exact considerations.

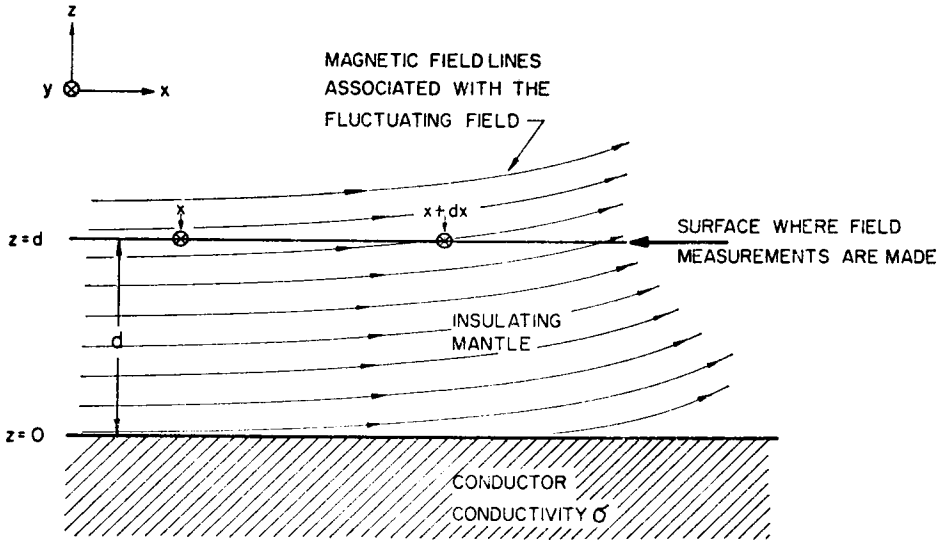Consider the fields above a perfectly conducting plate as shown in Fig. 1. If the

FIG. 1. Diagram showing the field lines of a slightly non-uniform fluctuating magnetic field above a conducting plane to indicate the relation of the theoretical quantities.

horizontal field varies slowly in the horizontal direction x, it is easy to compute the vertical field B_z above the plate by considering the continuity of magnetic flux, i.e. $\oiint \mathbf{B} \cdot d\mathbf{A} = 0$. If the depth d is small compared to the lateral scale length of the field, the horizontal field component B_x is approximately constant between $z = 0$ and $z = d$. Noting that no magnetic flux enters through the conductor at $z = 0$; simple flux continuity gives

$$d\, dy [B_x(x+dx) - B_x(x)] + B_z\, dy\, dx = 0$$

$$B_z = -d\, \frac{\partial B_x}{\partial x}. \qquad\qquad \left.\right\} \qquad (1)$$

For a perfectly conducting plane below a vacuum region, the above considerations indicate that the vertical field B_z and the horizontal field gradient $\partial B_x/\partial x$ will be in phase with one another if the field varies with time.

If the perfect conductor below $z = 0$ is replaced by one with conductivity σ, it is well known that an oscillating magnetic field will penetrate into it and be attenuated by $1/e$ in a skin depth δ $(\delta = \sqrt{(2/\omega\mu_0\, \sigma)}$ (MKS units)). If, at $z = 0$, $B_x = B_0 \cos \omega t$, at depth z in the conductor the horizontal field will be

$$B_x = B_0\, e^{z/\delta} \cos (\omega t + z/\delta). \qquad (2)$$

If the horizontal exciting field B_0 varies slowly with x as $B_0(x)$, field continuity requires that there be a vertical field in the conductor which will have one component in phase with $\partial B/\partial x$ and one $\pi/2$ out-of-phase, i.e. to a first approximation

$$B_z(x, z) = -\frac{\delta}{2}\, \frac{\partial B_0}{\partial x}\, e^{z/\delta} [\cos (\omega t + z/\delta) + \sin (\omega t + z/\delta)] \qquad (3)$$

and at $z = 0$

$$B_z(x, 0) = -\frac{\delta}{2}\frac{\partial B_0}{\partial x}[\cos \omega t + \sin \omega t]. \tag{3a}$$

This relation says that the vertical field leads $\partial B_0/\partial x$ by a phase shift of $\pi/4$ in time.

Assuming the horizontal field at $z = d$ is given by $B_x = B_0(x) \cos \omega t$, then the total vertical flux will be the sum of that generated in the vacuum region which lead to (1) and that coming out of the conducting plane which lead to (3a). We thus obtain

$$B_z(d) = -\frac{\partial B_x}{\partial x}\left[\left(d + \frac{\delta}{2}\right)\cos \omega t + \frac{\delta}{2}\sin \omega t\right]. \tag{4}$$

We note that the in-phase part of B_z is related to a field penetration depth p which we will define as the thickness of the vacuum region plus one-half the skin penetration depth into the conductor. The out-of-phase component of B_z is directly related to a thickness a of the attenuating region where the field is actually reduced to zero. Assuming $B_x' = B_x(x, z) e^{i\omega t}$ and $B_z' = B_z(x, z) e^{i\omega t}$ we would say that

$$p \equiv -\mathrm{Re}\left(\frac{B_z(d)}{\frac{\partial B_x(d)}{\partial x}}\right) = d + \frac{\delta}{2} \qquad a \equiv -\mathrm{Im}\left(\frac{B_z(d)}{\frac{\partial B_x(d)}{\partial x}}\right) = \frac{\delta}{2}. \tag{5}$$

The essence of the more careful computations which follow is to bear out that these interpretations hold also for more complicated and more realistic conductivity profiles and excitations.

The experimentally determined attenuation length a defines an electrical conductivity through the skin depth relation. We shall find that this conductivity is, in fact, the conductivity which is associated with the experimentally defined penetration depth p for conductivity profiles which increase monotonically with depth. In three dimensions we must replace $\partial B_x/\partial x$ by $(\partial B_x/\partial x + \partial B_y/\partial y)$ in (5).

2. Field above a conducting plane

To make the conclusion of Section 1 more quantitative and general, we consider an oscillating magnetic field above a conducting plane analytically. It is convenient to introduce a vector potential $\mathbf{A}'$ from which the magnetic field $\mathbf{B}$ is computed according to $\mathbf{B} = \nabla \times \mathbf{A}'$ with $\nabla \cdot \mathbf{A}' = 0$; using MKS units we have

$$\nabla \times (\nabla \times \mathbf{A}') = \mu_0 \mathbf{j}' = \mu_0 \sigma \mathbf{E}' = -\mu_0 \sigma \frac{\partial \mathbf{A}'}{\partial t}. \tag{6}$$

Assuming no variation of σ with x and y, it is sufficient to consider a vector potential with only x and y components which vary as $\mathbf{A}' = \mathbf{A}(z) \exp[i(k_x x + k_y y + \omega t)]$, i.e. in the conductor the differential equation for $\mathbf{A}(z)$ is

$$\delta^2 \frac{\partial^2 \mathbf{A}}{\partial z^2} = (2i + k^2 \delta^2)\mathbf{A}$$

$$\delta = +\sqrt{\left(\frac{2}{\omega \mu_0 \sigma}\right)} \qquad k^2 = k_x^2 + k_y^2 \tag{7}$$

and in the vacuum region

$$\frac{\partial^2 \mathbf{A}}{\partial z^2} = k^2 \mathbf{A}. \tag{8}$$

The relation $\nabla \cdot \mathbf{A}' = 0$ gives $k_x A_x + k_y A_y = 0$. At a distance z above the conducting plane, we obtain, solving (7) and (8), assuming the field goes to zero as $z \to -\infty$,

$$A_x = A_0 [(\alpha - k) e^{-kz} - (\alpha + k) e^{kz}] \exp [i(k_x x + k_y y - \omega t)] \qquad z > 0$$

$$\alpha = (1/\delta)\sqrt{(2i + k^2 \delta^2)} \qquad \qquad \text{Re } \alpha > 0.$$

$$(9)$$

After working out appropriate derivatives, the analagous expression to (5) can be written

$$\frac{B_z}{\dfrac{\partial B_x}{\partial x} + \dfrac{\partial B_y}{\partial y}} = \left(-\frac{1}{A_x} \frac{\partial A_x}{\partial z} \right)^{-1} = \frac{1 + (\alpha/k) \tan k\,kz}{\alpha(1 + (k/\alpha) \tan k\,kz)} . \qquad (10)$$

We are concerned with the specific case where the distance scale in the horizontal direction (i.e. $1/k$) is somewhat greater than that in the vertical (i.e. d or δ); thus, consider (10) in the limit of small kd and $k\delta$; evaluating (10) at $z = d$ and expanding in a power series in k we obtain

$$\frac{B_z(d)}{\dfrac{\partial B_x(d)}{\partial x} + \dfrac{\partial B_y(d)}{\partial y}} = -\left(d + \frac{\delta}{2} \right) \left[1 - \left(\frac{2d + 3\delta}{6d + 3\delta} \right) (kd)^2 ... \right]$$

$$- \frac{i\delta}{2} \left[1 - \left(\frac{d + \delta}{d} \right) (kd)^2 ... \right] \qquad (11)$$

which is a good approximation to the result obtained in (5) if kd and $k\delta$ are small.

When the excitation B_x and B_y are not pure modes but consist of a complex superposition of modes which all have long wavelengths in the x and y directions compared to d and δ, property (11) is true for each mode independently and (11) will also hold for the superposition.

Non-uniform conductivity

One purpose of doing geomagnetic sounding is to deduce the non-uniformity in the conductivity profile as a function of depth into the body. We will focus attention on a body whose conductivity increases monotonically with depth by analysing the response of a planar conductor in which the σ increases with depth as z^n (n, z are positive quantities).

Examination of (11) and (10) shows that we are concerned with computing $(1/A_x) (dA_x/dz)$ in the limit of k_x and k_y going to zero. From the previous section we are able to estimate the error associated with finite k_x and k_y, thus we choose these parameters equal to zero at the outset and obtain:

$$\frac{\partial^2 A_x}{\partial z^2} = 2i \frac{\omega \mu_0 \sigma_0}{2} z^{(2n-2)} A_x$$

$$\sigma = \sigma_0 z^{(2n-2)}$$

$$(12)$$

or rewritten in dimensionless form

$$\zeta^2 \frac{\partial^2 A_x}{\partial \zeta^2} = 2in^2 \zeta^{2n} A_x$$

$$\zeta = \frac{z}{\delta_w} \qquad \delta_w = \left(\frac{2n^2}{\omega \mu_0 \sigma_0} \right)^{1/2n}$$

$$(13)$$

(13) is readily solved in terms of Bessel functions as

$$A_x = \zeta^{\frac{1}{2}} C_1 \left[J_{1/2n}(\sqrt{(2i)}\,\zeta^n) + C_2\, J_{-(1/2n)}(\sqrt{(2i)}\,\zeta^n) \right] \quad (14)$$

C_1 and C_2 are arbitrary constants—subject to satisfying boundary conditions and $\sqrt{(2i)} = (1+i)$. As $z \to \infty$ (N.B. z of this section has the opposite sign to z in the previous section), $A_x \to 0$ since we assume no excitation from within the conductor; noting the appropriate assymtotic expansions for Bessel functions evaluated in the first quadrant this condition is satisfied only if

$$C_2 = e^{+\pi i/2n}. \quad (15)$$

We then obtain

$$A_x = \zeta^{\frac{1}{2}} C_1 \left[J_{1/2n}(\sqrt{(2i)}\,\zeta^n) + e^{\pi i/2n} J_{-(1/2n)}(\sqrt{(2i)}\,\zeta^n) \right] \quad (16)$$

We assume that our measurement is made at $\zeta = 0$, i.e. at $z = 0$; we obtain, noting the appropriate power series developments for Bessel functions,

$$\frac{1}{\delta_w A_x} \frac{\partial A_x}{\partial \zeta} = \frac{(-1)\,\Gamma(1 - 1/(2n))}{\delta_w\,\Gamma(1 + 1/(2n))\,2^{\frac{1}{n}}\left(\cos\dfrac{\pi}{4n} + i\sin\dfrac{\pi}{4n}\right)}$$

$$= \frac{1}{A_x}\frac{\partial A_x}{\partial z} \quad (17)$$

Comparing this to (5) we obtain

$$\mathrm{Re}\left(\frac{B_z(0)}{\dfrac{\partial B_x(0)}{\partial x} + \dfrac{\partial B_y(0)}{\partial y}}\right) = p = +\frac{\delta_w\,2^{\frac{1}{n}}\left(\Gamma 1 + 1/(2n)\right)}{\Gamma(1 - 1/(2n))}\cos\left(\frac{\pi}{4n}\right) \quad (18a)$$

$$\mathrm{Im}\left(\frac{B_z(0)}{\dfrac{\partial B_x(0)}{\partial x} + \dfrac{\partial B_y(0)}{\partial y}}\right) = a = \frac{\delta_w\,2^{\frac{1}{n}}\,\Gamma(1 + 1/(2n))}{\left(\Gamma 1 - 1/(2n)\right)}\sin\left(\frac{\pi}{4n}\right) \quad (18b)$$

Γ is the usual Gamma function. The distance δ_w is the penetration depth to which the W.K.B. solution to (12) would lead us, i.e.

$$1 = \int_0^{\delta_w} \sqrt{\left(\frac{\omega\mu\sigma(z)}{2}\right)}\, dz. \quad (19)$$

The important conclusion of this computation is that the electrical conductivity at the depth p is related to the attenuation length a in a way which is essentially independent of the shape of the conductivity profile. We define a conductivity σ' at the depth p according to:

$$\sigma'(p) = \frac{1}{2\omega\mu_0\,a^2}. \quad (20)$$

If we compare σ' to the actual value of σ at the depth p as a function of n we obtain the results shown in Fig. 2. We note that even with $n = 4$, which corresponds to the
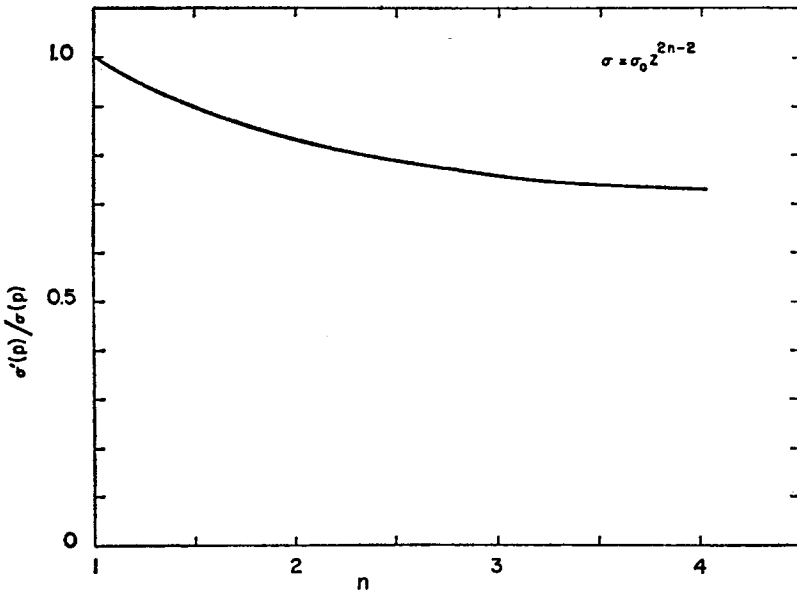
FIG. 2. Ratio of the apparent conductivity derived from the approximate analysis and the true conductivity at the penetration depth p as a function of the parameter n which indicates the non-uniformity of the conductivity profile analysed.

conductivity increasing with the sixth power of depth, the value of σ' obtained from (20) is only 30 per cent lower than the true value. With such a strong dependence upon depth this corresponds to a 5 per cent error in the depth p at which the conductivity has been evaluated.

Conductivity profile mapping

It has been shown that the conductivity profile of a body can be made from localized field measurements without detailed knowledge of the entire modal decomposition of the fields. To make a conductivity profile map, i.e. statements about how the conductivity profile varies between adjacent regions, it is important

FIG. 3. Idealization considered in the analysis to evaluate the lateral resolution intrinsic to the theory.

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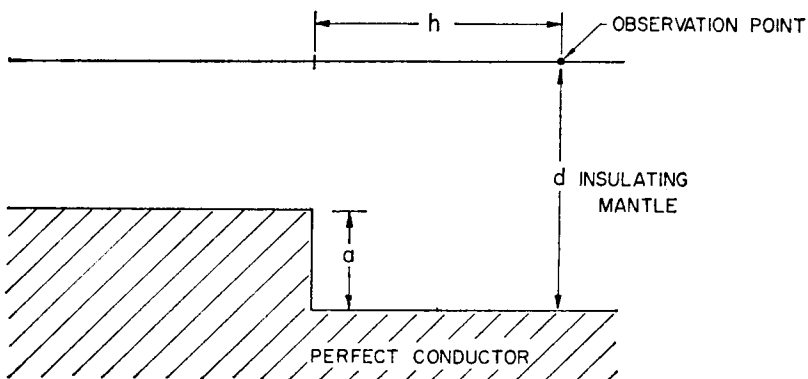to estimate the intrinsic lateral resolution the method offers. To investigate this question, we consider the response of a two-dimensional step in a perfectly conducting medium at depth as shown in Fig. 3. This configuration is amenable to exact treatment using a conformal mapping method and we will compare the penetration depth given by our approximate treatment with the actual depth.

The conformal mapping method transforms a given set of boundaries to a geometry more amenable to simple analysis. We solve the problem shown in Fig. 3 by transforming the step boundary into a plane—an appropriate transformation function for accomplishing this is:

$$z = \sqrt{(w+1)} \sqrt{(w-1)} + \ln\left(w + \sqrt{(w+1)} \sqrt{(w-1)}\right) \tag{21}$$

where both z and w are complex variables (the x, y, z variables are unrelated to x, y, z of earlier sections!):

$$\left.\begin{array}{l} z = x + iy \\[2mm] w = \rho\, e^{i\psi}. \end{array}\right\} \tag{22}$$

In evaluating (21) we always use the evaluations of square root and logarithm functions which lie in the first two quadrants; we consider x, y, ρ, ψ to be real variables $y > 0$ and $0 < \psi < \pi$. The mapping (21) is indicated in Fig. 4. It is well known that any analytic function of the complex variable z or in the transformed space w satisfies Laplace's equation. We can introduce a complex potential satisfying Laplace's equation $W(z)$ with the magnetic fields given by

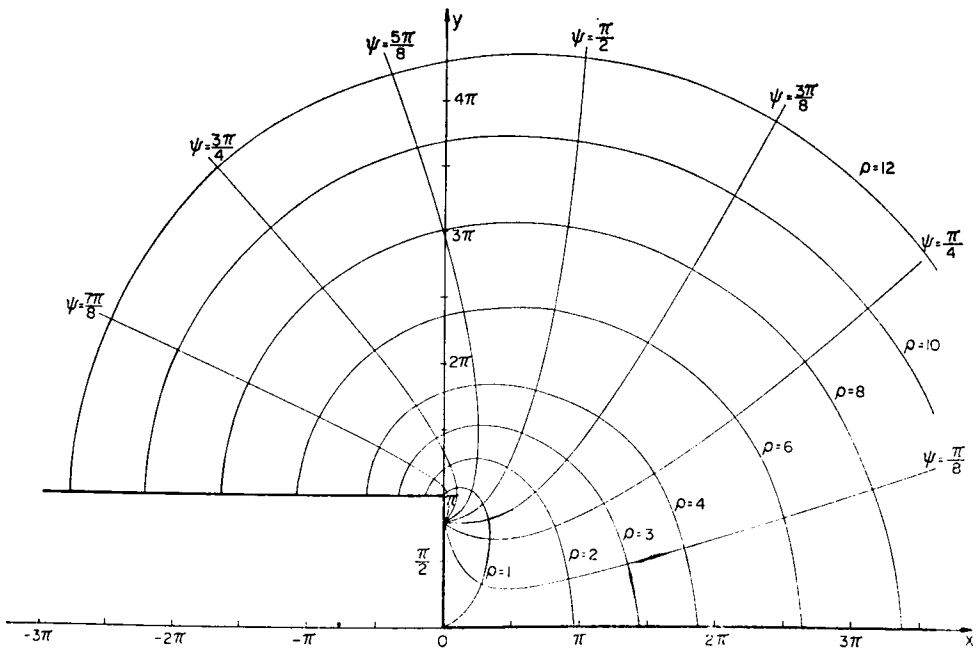$$B_x = \mathrm{Re}\left(\frac{dW}{dz}\right) \qquad B_y = \mathrm{Im}\left(\frac{dW}{dz}\right). \tag{23}$$

FIG. 4. Schematic map indicating the relation between the parameters x and y which identify a point in z space and the parameters ρ and ψ which identify a point in the transformed space w. The continuing step indicated in z space is transformed to the plane defined by Re $w = 0$.

W must satisfy the boundary conditions associated with a perfect conductor, i.e. $B_y = 0$ or Im $(\partial W/\partial z) = 0$ on the boundary. To approximate a non-uniform field above the step with $(1/B_x) dB_x/dx \simeq 1/L$ we consider the potential in w space

$$W(w) = \left(w + \frac{w^2}{2L} \right). \tag{24}$$

To evaluate the precision of the prescription (5) for estimating the depth to the conducting layer, we will consider the difference between the actual height between a point of observation and the conducting plane and that given by (5). To do so we will consider ρ to be large and ψ to be somewhat less than $\pi/2$. Similar conclusions are obtained considering $\pi > \psi > \pi/2$.

We expand (21) in inverse powers of w to obtain

$$\left. \begin{aligned} x &= \rho \cos \psi + \ln 2\rho - \frac{\cos \psi}{2\rho} - \frac{\cos 2\psi}{2\rho^2} - \frac{\cos 3\psi}{8\rho^3} \cdots \\[2mm] y &= \psi + \rho \sin \psi + \frac{\sin \psi}{2\rho} + \frac{\sin 2\psi}{2\rho^2} + \frac{\sin 3\psi}{8\rho^3} \cdots. \end{aligned} \right\} \tag{25}$$

Similarly expanding B_x and B_y we obtain

$$\left. \begin{aligned} B_x &= \frac{\rho \cos \psi}{L} + \left(1 - \frac{1}{L} \right) - \left(1 - \frac{1}{2L} \right) \frac{\cos \psi}{\rho} \\[2mm] &\qquad + \left(1 - \frac{1}{L} \right) \frac{\cos 2\psi}{2\rho^2} - \left(1 - \frac{3}{4L} \right) \frac{\cos 3\psi}{2\rho^3} \cdots \\[2mm] B_y &= \frac{\rho \sin \psi}{L} + \left(1 - \frac{1}{2L} \right) \frac{\sin \psi}{\rho} - \left(1 - \frac{1}{L} \right) \frac{\sin 2\psi}{2\rho^2} \\[2mm] &\qquad\qquad + \left(1 - \frac{3}{4L} \right) \frac{\sin 3\psi}{3\rho^3} \cdots \end{aligned} \right\} \tag{26}$$

We note that

$$\left(\frac{\partial B_x}{\partial x} \right)_y = \left[\frac{\partial}{\partial x} \left(\text{Re} \, \frac{dW}{dz} \right) \right]_y$$

$$= \left(\frac{\partial}{\partial \psi} \, \text{Re} \, \frac{dW}{dz} \right)_\rho \left(\frac{\partial \psi}{\partial x} \right)_y + \left(\frac{\partial}{\partial \rho} \, \text{Re} \, \frac{dW}{dz} \right)_\psi \left(\frac{\partial \rho}{\partial x} \right)_y. \tag{27}$$

Evaluating the appropriate Taylor series in inverse powers of w we obtain

$$\left(\frac{\partial B_x}{\partial x} \right)_y = \frac{1}{L} - \frac{\cos \psi}{\rho L} + \frac{\cos 2\psi}{\rho^2} + \frac{\cos 3\psi}{\rho^3} \left(-2 + \frac{1}{L} \right) \cdots. \tag{28}$$

The fractional error E made in evaluating the depth to the conducting layer is ($\psi < \pi/2$)

$$E = \frac{1}{y} \left[B_y \Big/ \left(\frac{\partial B_x}{\partial x} \right) - y \right]. \tag{29}$$

Using the first two terms in the series expansion of $\sin \psi$ and $\cos \psi$ in powers of ψ

to approximate these functions we obtain

$$E = -\psi^2 \left[\frac{1}{2\rho} + \frac{1}{\rho^2} \frac{7L}{3} + \tfrac{1}{3} \right] \cdots \approx -\psi^2 \frac{7L}{3\rho^2}. \tag{30}$$

If we rewrite this expression in a form applicable to measurements made a horizontal distance h away from a step of height a which is at a depth d we obtain for the fraction error E (see Fig. 3):

$$E = -\frac{7d^2 La}{3\pi h^4}. \tag{31}$$

The greatest part of the error in (30) and (31) is associated with the vertical field generated by the interaction of the uniform part of B_x with the step. It is easy to see that, in general, the interaction of a uniform field with a steadily changing penetration depth d in the horizontal direction x will yield a vertical field component. A heuristic analysis similar to that presented earlier yields a vertical field B_y from this effect as

$$B_y = B_x \frac{\partial d}{\partial x}. \tag{32}$$

For many data it appears that this effect is a correction factor to the analysis. In any case, it can be identified and eliminated (or studied) by analysing the vertical field, at a given frequency, as a linear combination of the excitation B_x and $\partial B_x / \partial x$. By choosing several data with different signs and magnitudes for $1/L$ the quantity $\partial d / \partial x$ should be easy to identify and estimate. For three dimensions similar conclusions hold with the vertical field expanded as a linear combination of both horizontal field components and longitudinal derivatives.

Conclusion

It has been shown that we can make considerably better and simpler use of magnetic fluctuation data for deducing geologic mantle conductivity profiles than has been the case to date. When the fluctuating magnetic field penetration depth into a body is much less than the lateral scale length of the exciting field, a very considerable simplification in deducing the conductivity profile is possible. Since the method averages over a lateral radius which is comparable to the penetration length, systematic conductivity profile 'mapping' becomes an interesting new prospect.

In the past, the out-of-phase part of the vertical field has usually been considered less reliable than the in-phase part of the field. We have shown that this part of the field is intrinsically the more sensitive indicator of the electrical conductivity at depth. Some of the non-uniqueness difficulty of ignoring this part of the field may be related to the fact that this component is indicating an interesting structure which we have been unable to study with the methods of analyses which have been attempted. The in-phase part effectively indicates only how far down the attenuating region is. If the conductivity profile is characterized by relatively insulating mantle followed by a rapid rise in conductivity this work indicates that the out-of-phase part may contain the best experimental data for deducing the actual conductivity at depth.

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H mag.

15

10

5

●	WAR	MOM	ROC	GRB
				WAS
✱	SWS	OTT	DVE	CHP
		OSC	PAR	STD
○	LAW	COR	FTD	WIL
	MAN	UNI		MIC
△	LIN	LEX	DRA	AKA
	WAK			

H phase

90

60

30

0

●	WAR	MOM	ROC	GRB
				WAS
✱	SWS	OTT	DVE	CHP
		OSC	PAR	STD
○	LAW	COR	FTD	WIL
	MAN	UNI		MIL
△	LIN	LEX	DRA	AKA
	WAK			

D mag

25

20

15

10

●	AKA	MIL	WIL	STD		WAS	GRB
✱	DRA			CHP			
				FTD	PAR	DYE	ROC
○	LEX			UNI	COR	OSC	OTT	MOM
△	WAK	LIN	MAN	LAW	SWS	WAR

D phase

-25

-30

-35

-40

●	AKA	MIL	WIL	STD		WAS	GRB
✱	DRA			FTD	PAR	DYE	ROC
○	LEX			UNI	COR	OSC	OTT	MOM
△	WAK	LIN	MAN	LAW	SWS	WAR

Z mag

15

10

5

●	WAR	MOM	ROC	GRB
				WAS
✱	SWS	OTT	DVE	CHP
		OSC	PAR	STD
○	LAW	COR	FTD	WIL
	MAN	UNI		MIL
△	LIN	LEX	DRA	AKA
	WAK			

Z phase

-30

-40

-50

-60

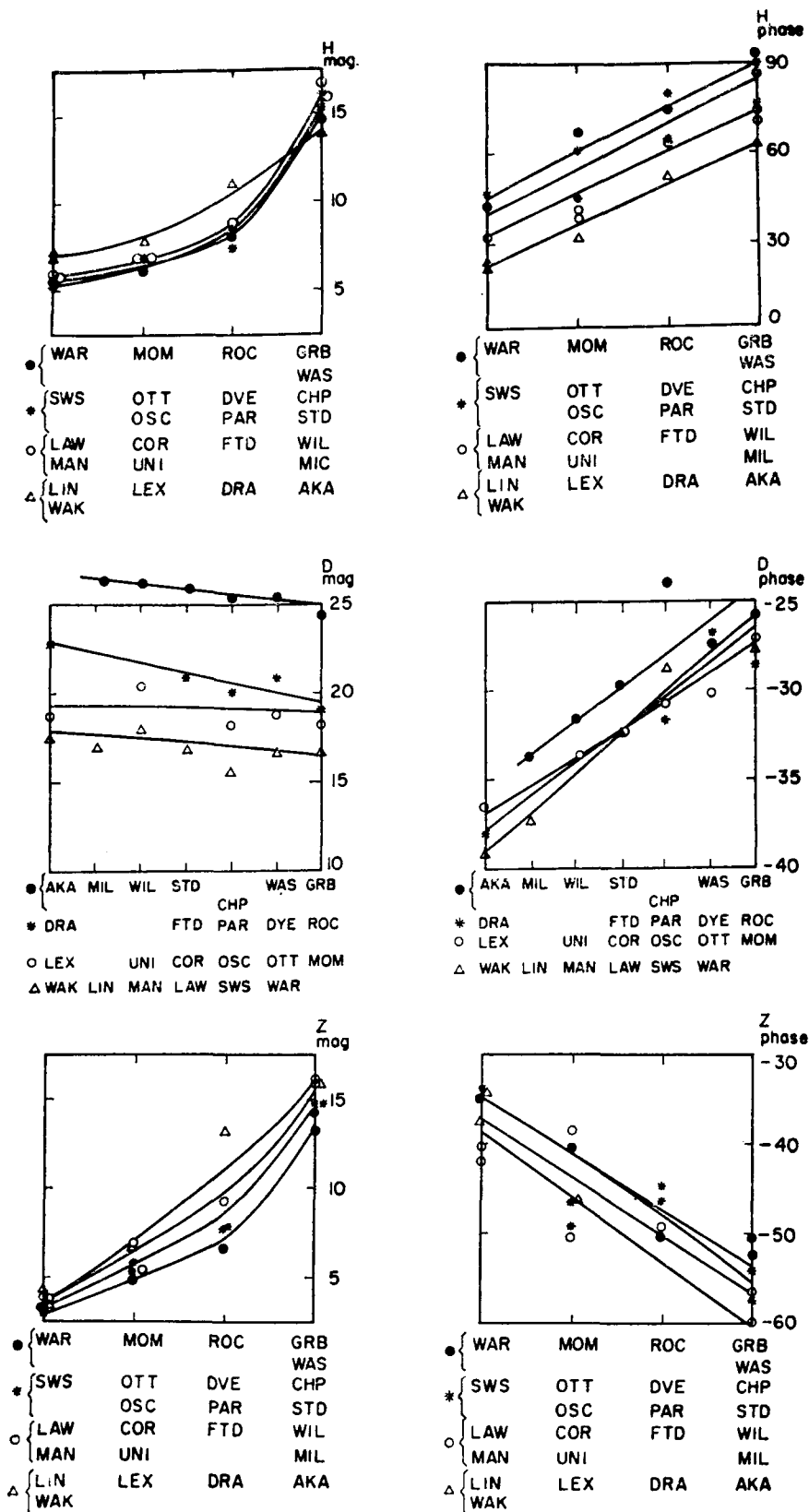|				
●	WAR	MOM	ROC	GRB
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	MAN	UNI		MIL
△	LIN	LEX	DRA	AKA
	WAK			

FIG. 5. A sample of Fourier analysed showing the relative amplitude and phase of 2-hr fluctuations indicated by the Great Plains data of Porath & Dziewonski between 8 a.m. and 12 a.m. on 1969 November 27. The abscissa abbreviations are for field station sites as defined by Porath & Dziewonski. The *H* data are plotted in north–south progression and the *D* station in east–west progression to facilitate estimating the field gradients required by the analysis.

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Addendum

It has been pointed out to the author that Schmucker (1970) has considered a method of analysing magnetic field fluctuation phenomena which is closely related to the ideas put forth above. While Schmucker also demonstrated the utility of his method, it has not received widespread attention.

The utility of the field penetration method of analysis was examined by analysing the published magnetometer array data of Porath & Dziewonski (1970) and of Chapman & Bartels (1940).   Time intervals were selected from these data and Fourier analysed to determine the parameters p and a defined by equation (18). In Fig. 5 the results of Fourier analysing the amplitude and phase of the three components of magnetic field are shown.   The quantities were plotted in the manner indicated to facilitate evaluation of the quantities p and a.   The abscissa of this figure is proportional to the approximate latitude of the sites indicated in the plot for H and the approximate longitude for D.   The real and imaginary quantities defined by equation (18) were evaluated at the centre of the array to determine the magnitude of the penetration depth $(p^2 + a^2)^{\frac{1}{2}}$ at the period of 2 h.   In Fig. 6 many such results as a function of period are plotted.   The points from 300-s to 7200-s period were derived from the Porath–Dziewonski data.   The points on Fig. 6 for periods $2 \cdot 16 \times 10^4$ s to $8 \cdot 64 \times 10^4$ s were taken from averaged world-wide data given by Chapman & Bartels (1940). These data were digitized using a micrometer slide under a microscope followed by similar digital analysis.