On the Relationship between Dynamic Topography and Direction of Current under the Influence of External (Climatic) Factors.

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IN spite of all the pertinent reservations and restrictions made in the abstract deduction of the modern hydrodynamic equations, it is commonly stated that the dynamic contour lines can to all practical purposes be regarded as representative of true lines of flow, and this has become an almost universally adopted rule in the practical application of hydrodynamic methods (even, one may find, to waters in contact with sloping bottoms and other topographical complications which shall not concern us here). In the course of certain marine investigations of which he has recently been in charge, the writer has been considerably troubled with the difficulty of reconciling, except in the grossest general features, the picture of ocean currents obtained from this "practical" interpretation of dynamic topography with other hydrographic evidence bearing upon the transportation of water masses. Outstanding examples of such discrepancies are those cases in which dynamic contour lines take their departure from solid land in regions in which no upwelling can occur. A practical method of evaluating dynamic topography in terms of actual directions of flow is plainly needed, and under a certain very common and important set of conditions applying to the open ocean as well as to inshore waters it may not be impossible to devise such a method.

Let us assume that a body of water actually flows from a lower to a higher latitude on the northern hemisphere, as illustrated at the upper left in our Figure 1. In the course of this flow it becomes chilled, and other things remaining equal its surface therefore loses dynamic height. Dynamic contour lines would consequently have to be drawn at right angles to the actual lines of flow which had been followed. According to the current "practical" interpretation of dynamic topography this

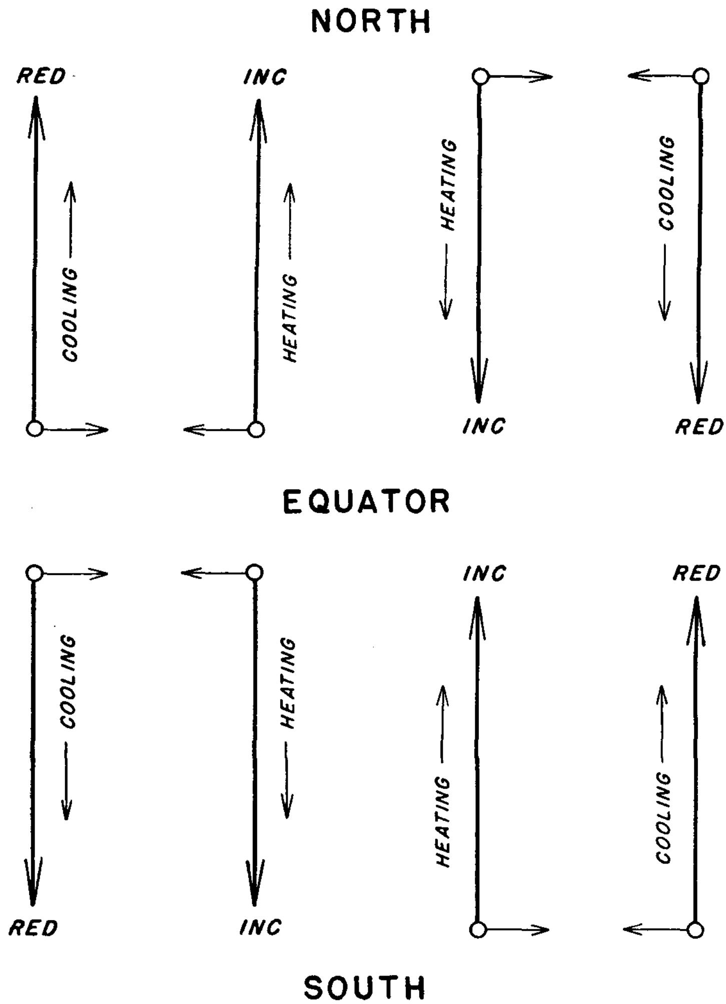

Figure 1.
Eight theoretical situations in which the apparent direction of flow (thin horizontal arrows) indicated by the customary interpretation of dynamic contour lines would be at right angles to actual direction of flow (thick vertical arrows) on both hemispheres. Dynamic height decreasing (r e d.) in the direction of cooling, increasing (i n c.) in the direction of heating.

would then be taken to mean that the water could only have flowed at an angle of 90° to the actual course of the current, in the direction indicated by the finer horizontal arrow.

This introduces the principle that whenever dynamic heights in the direction of a current's flow are changed by the influence of external non-kinematic forces (heating, cooling, condensation, dilution etc.) the current must have a component at 90° to the direction of the dynamic contour lines. It is also immediately apparent from an inspection of the other cases illustrated in our Figure 1 that when dynamic height is thus reduced in the direction of flow the actual current will deviate to the left from the dynamic contour lines and when dynamic height is increased the current will deviate to the right on the northern hemisphere; with the inverse relationships obtaining south of the equator.

Regions of different external influence will, in other words, create a field of energy in the intervening area tending to cause the dynamic contour lines to pass at right angles to a line between such opposite regions. If this field of external energy is not opposed by internal energies in the ocean both the dynamic contour lines and the actual lines of flow must ultimately take this direction at right angles to the gradient of external influence. This principle has the important implication that when dynamic contour lines do *not* run at right angles between the centres of different external influences, but have a component in the direction from one to the other, this in itself is evidence that the actual lines of flow are neither determined by external nor by internal forces alone, but by an interaction between the two, and must therefore deviate from the dynamic contour lines in the manner stated in our first thesis. To establish the actual lines of flow it is therefore necessary to take both internal and external fields of energy into consideration.

In our Figure 2 the external field of energy is represented by the parallels EXT. 1 — EXT. 3, the internal field by parallels INT. 1 — INT. 4, both expressed in the same units in terms of the topographic gradient of the isobaric surface under consideration which either field of energy would produce independently in the absence of the other, but with the abstract assumption of the continued flow of the same current; the contour lines being numbered in sequence from lower to higher dynamic levels. Under the conditions illustrated, the internal field of energy would obviously contribute the current component A→B to the flow of water from point A, the external field contributing the component A→E. None of the restrictions attached to the use of the hydrodynamic equations would apply to a determination of the separate components by this method. The resultant actual flow from point A is consequently indicated by the arrow from A to C. But in flowing from A to C the waters of the current suffer a total cumulative loss within themselves of 1 unit dynamic height under the influence of the external field of energy (e.g., by chilling). At point C the surface of the current consequently has an actual dynamic height one unit lower than the topographic height of the intercept between EXT. 2 and INT. 2 at point A, or equal to the height of the intercept between EXT. 2 and INT. 1. The actual, that is, the observed dynamic contours consequently run from the latter point to C and correspondingly from A to D, and so on, as shown by the lines: OBS. 1 — OBS. 7.

For accurate determination of the actual lines of flow a knowledge

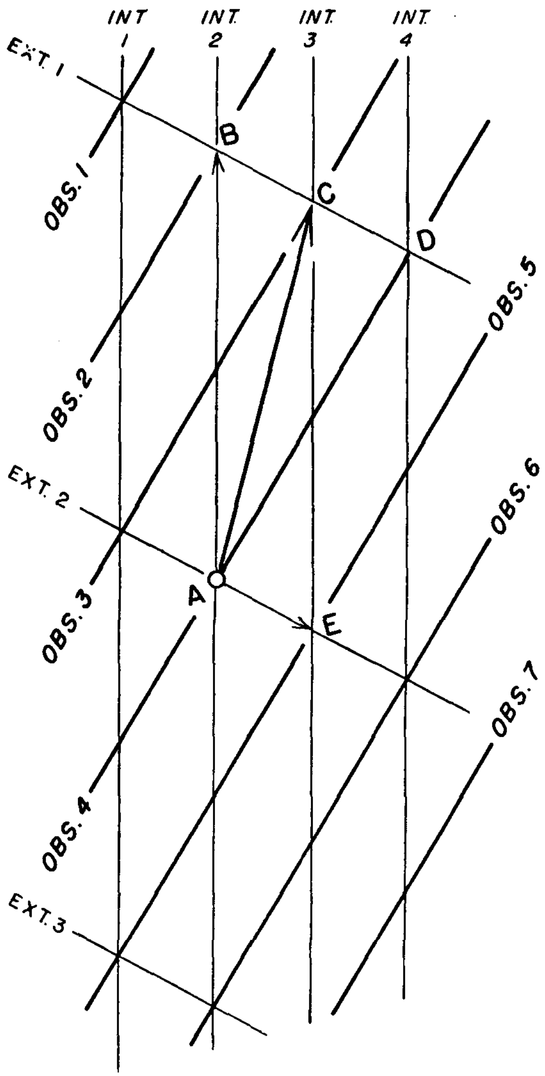

Figure 2.
Relationship between internal and external fields of energy, actual direction of current and observed dynamic topography.

of at least two of the following three sets of data is required: 1) The actual topography, and 2) the topography of the internal field of energy alone, or 3) the nature of the external field of energy in terms of its direct influence upon the dynamic topography of the region considered. The actual topography is derived directly from observations, and is usually presented as a matter of oceanographic routine. To plot approximately the external field of energy in terms of its influence upon the actual topography does not seem beyond possibility[1]).

[1]) In a plot of this sort one would have to take into account the actual character of the waters in the region investigated with reference to the rate at which their densities would be changed under the influence of the external forces.

An interesting aspect of the external field would be the seasonal change of its gradient of influence. The available data for plotting in two dimensions the (influence of) the external field are probably still inadequate for most oceanic regions, but a practical method for determining approximately the true lines of flow from data given in one dimension only may be derived from the following considerations.

Given an actual topography determined directly from observations, such as that indicated by the contour lines OBS. 1 — OBS. 7 in our Figure 2 we may attempt to determine (e.g., from observed temperature differences for equal or corresponding salinities in waters of traceable identity) the total loss of dynamic height which would occur due to the influence of external forces (mainly chilling, but also condensation or dilution) alone if the waters had really moved along an actual contour line from A to D. Finding that approximately one unit dynamic height would have been lost in this manner over the distance considered, we know that a body of water flowing from an actual topographic level of OBS. 4 at point A must have reached topographic contour line OBS. 3 at a point C corresponding to point D on OBS. 4 or conversely that at body of water arriving at D must have departed from actual topographic contour line OBS. 5, at a point E corresponding to a point A on OBS. 4. In practice one should be mainly guided by the version of the above method which gives the least distance between established points on the actual contour line along which the calculations are made and the "corresponding" points on other contour lines, which must be fixed by personal judgement only, in the absence of precise knowledge of the gradient of external influence. With actual contour lines converging in the direction of flow, the first version, extrapolating from D to C should therefore be given main attention, the second version when contour lines diverge, but the best result would probably as a rule be obtained by a comparison of opposite versions for next adjacent actual contour line. With these precautions the personal judgement will probably as a rule not be greatly in error, and will certainly give us a much more nearly true picture of actual direction of flow than that obtained from the actual topographic contour lines.

In regard to a method for ultimate determination of the entire field of influence of external factors, we may suggest that the loss in dynamic height which we estimate would have occurred along an actual dynamic contour line (OBS. 1 — OBS. 7, in Figure 2) according to observed differences of external origin be designated as the dynamic discrepancy. If, starting from one extreme area of external influence, we estimate the dynamic discrepancies along the different contour lines of the actual topography at a sufficient number of points, we can draw the curves of equal dynamic discrepancies, and these would directly represent the contour lines of the field of influence of the external forces (N.B. *not* the field of external energy itself) determined from hydrographic observations alone. The actual lines of flow would then run diagonally between the intercepts of these contour lines (EXT. 1 — EXT. 3) and the contour lines of actual topography (OBS. 1 —

OBS. 7, see Figure 2). To apply this method with great precision would be very difficult, but to do it in sufficient approximation for general guidance in the interpretation of dynamic topography in terms of actual lines of flow seems quite feasible. The writer shall not go to the extent of attempting an actual application of this entire procedure in this short article, but shall merely endeavour to show in the following the importance of giving consideration to the principles here developed by taking a well-known example from the literature as a rough illustration.

If we take the dynamic topography of the North Atlantic as presented by Jacobsen[1]) according to standard oceanographic methods, and begin with the assumption that the dynamic contour lines also represent actual lines of flow, we may follow in our imagination the movement of a cube of surface water a hundred metres deep, wide, and long from the Florida Current south of Cape Hatteras along the line of 0 relative dynamic height to its extreme north-eastern point, north-west of the Iberian peninsula. If we now compare its characteristics at the time it left the Florida waters with its features when it arrives at the extreme north-eastern point[2]) we find that it will, on the annual average, have suffered a reduction in surface temperature from about 25° C. to 15° C. or less, with the surface salinity remaining almost unchanged (about 35·5 ‰), which gives a reduction in (anomaly of) specific volume at the surface of about 250×10^{-5}. Since the changes at 100 metres depth, while not identical with those at the surface, have probably been of about the same magnitude with reference to density, we may, for the sake of this rough illustration, assume the loss of specific volume at the surface to be representative of the entire 100 metre thickness of our cube, which has thus within itself alone suffered a loss of dynamic height in passage of no less than 25 dynamic cm. or one-third of the total dynamic slope throughout the entire North Atlantic current-system. Other things being equal, the actual path of our cube of surface water should therefore have been crossed by no less than 5 such dynamic contour lines as those shown in Jacobsen's chart. In other words, to arrive at the extreme north-eastern end of the 0 dynamic contour line our cube of water must have started from about 25 cm. relative dynamic height in the region south of Cape Hatteras; or, if it started from 0 dynamic height, it must arrive proportionately farther to the north-east than indicated by the 0 dynamic contour. That is, the actual line of flow from 25 cm. relative dynamic height south of Cape Hatteras must extend approximately to the extreme point reached by the 0 contour line (or beyond), and the line of flow from 0 height in the south-west

[1]) J. P. Jacobsen: Contribution to the Hydrography of the North Atlantic. Danish "Dana" Exp. 1920—22. Oceanogr. Rep. No. 3. Copenhagen, 1929, Fig. 53.

[2]) Since this is merely meant as a very rough illustration, the writer has considered it adequate for our present purposes to take his information from the charts of surface salinities and average annual surface temperatures published by G. Schott in his *Geographie des Atlantischen Ozeans.* Hamburg, 1926 (Plates VIII and X).

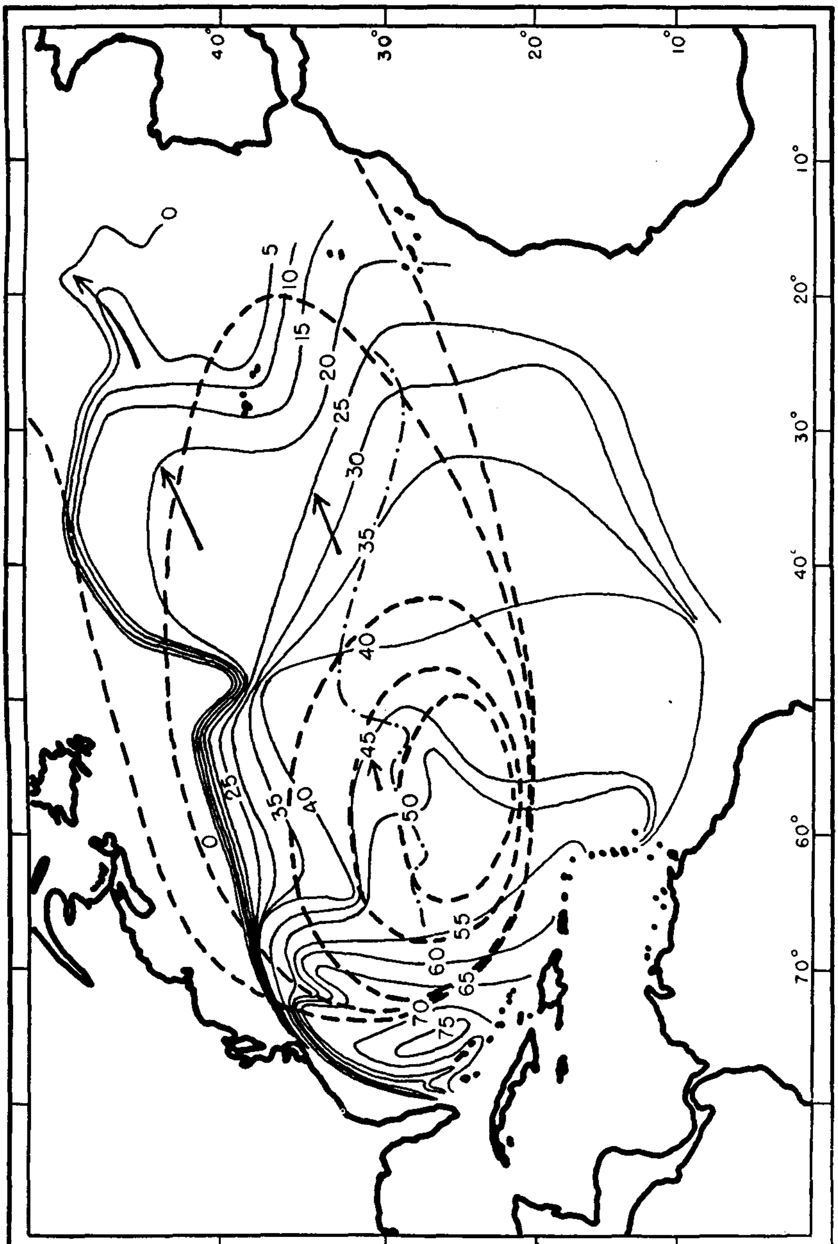

Figure 3.
Dynamic topography of the North Atlantic according to Jacobsen (solid contour lines) with distribution of successive stages of eel larvae according to Schmidt (broken lines), probable actual directions of flow (arrows) suggested according to methods herein developed, and northern subtropical convergence according to Meyer (dot and dash).

must extend proportionately farther to the north-east. Similar evaluations of dynamic discrepancies at the locations on the contour lines for 20, 25, and 45 cm. relative actual height towards which our arrows in Figure 3 are pointing give values of about 20, 15 and 13 cm. In accordance with the principles here developed, applied to these approximate evaluations, arrows have been entered in our figure to indicate general actual direction of flow. It is evidently utterly impossible that a cube of water could have followed any single dynamic contour line at any level to which the effect of external factors may penetrate, since the same reasoning would apply to any particular case we might choose to consider.

For corroborative evidence we may now again turn to the literature. In Figure 3 we first compare the distribution of successive stages of larval European eels, according to Schmidt[1]), with the dynamic topography of the North Atlantic according to Jacobsen. This comparison has already previously been used in confirmation of the standard interpretation of dynamic contour lines as approximate actual lines of flow. But a closer scrutiny will immediately reveal that the comparison on this basis really offers great obstacles in its details, and it will also be seen that these obstacles would be entirely removed and a very much better fit would be obtained if the lines of actual flow were dispersed much farther in a north-easterly direction, as suggested by the arrows entered according to the principles here introduced.

For final and conclusive proof of the justification of our methods, we will now compare the general directions and location of our arrows with the current chart prepared by Meyer[2]) from actual observations of the drift of ships etc. To facilitate the comparison the northern subtropical convergence has been entered (dot and dash) in our figure as a line of reference; and a comparison discloses a perfect agreement in general terms between our interpretation of dynamic topography and actually observed directions of flow, where an enormous discrepancy would exist if the "practical" rule of regarding the topographic contour lines as approximate actual lines of flow were applied in this case.

Fully aware that this is not a finished mathematical treatment of the problem, and it is not intended as such, the writer is hopeful that these simple considerations and principles may serve to extract in general terms from the dynamic topography of the upper isobaric levels some really helpful information to correlate with the problems of biological distribution and the horizontal distribution of water masses, where topographic information under current practical rules seems to offer mainly obstacles.

It is evident, of course, that the influence of the external field (mainly represented by heating and cooling) on the dynamic contour

[1]) Johs. Schmidt: Danish Eel Investigations during 25 years. Copenhagen, 1933. Publ. by the Carlsberg Foundation. And earlier papers by the same author.

[2]) H. F. Meyer: Wasserbewegung a. d. Oberfläche im Februar. Fig. 51 opposite p. 528 in Zeitschr. Gesell. f. Erdkunde, Berlin. Sonderband 1928.

lines will rapidly diminish from the surface downwards. In our example from the North Atlantic current-system we have simply assumed that the drop in temperature throughout the upper 100 metres along the 0 contour line has been solely due to external chilling alone. This will probably be very nearly, but not entirely, true. On the other hand, the error of the assumption will perhaps be more than compensated for by the fact that the influence of external factors will undoubtedly in some degree penetrate deeper than the upper 100 metres. On the whole it will therefore be fair to assume that the discrepancy equivalent of 25 dynamic cm. may approximately represent the total cumulative discrepancy equivalent from surface to bottom along the area of that path of the North Atlantic current-system which we have here considered.

It is obvious from the apparent magnitude of this discrepancy that a formula for evaluating dynamic topography in terms of actual lines of flow is absolutely essential.