

Some Characteristic Features in the Variation of Surface Temperature in the North Atlantic.

By

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Preface.

AT the time of his death in June 1946 Dr. J a c o b s e n, Hydrographer to the Council, had in hand work on the variation of surface temperature in the North Atlantic. Though the work was not quite completed, it was considered that even in this incomplete state the paper contained so much of interest that it ought to be published. At its Meeting in 1947 the Hydrographical Committee recommended that the paper should be printed in the publications of the Council (see Rapp. & Proc.-Verb., Vol. CXXI, 1ère partie, p. 39).

Elaboration of methods takes up a considerable part of the paper, and its main purpose was evidently to give methods that might be of value for the working-up of comparable material in the future. As to the hydrographical results of the work attention should be drawn particularly to Table 3. This table gives for each of the localities considered correlation-coefficients between the surface temperature for one month and the surface temperature one, two, etc., to six or nine months later. From this table it appears that the decrease of the correlation-coefficient r_{pq} with increasing values of the difference $q-p$ months varies from one locality to another. This decrease may be approximately expressed in various ways, and Dr. J a c o b s e n has chosen the expression

$$r_{pq} = r[(q-p)^E]$$

As seen from Figure 6 it is possible with this formula — by choosing suitable values of E — to obtain a fairly good approximation to the actual variation of r_{pq} with $q-p$ and thus to characterize this variation by the quantity E . The value of E is a measure of the rate at which a temperature anomaly vanishes. As appears from Figure 5 E varies fairly regularly with locality. Thus, E has high values for the areas influenced by the polar front (10 and 11). For the areas to the south and southwest of Iceland (12, 13, 14, and 15) E has considerably lower values. As emphasized by Dr. J a c o b s e n, this may be explained by the fact that in the latter areas the temperature anomalies of the Atlantic Current play a prominent part: as these anomalies are of long duration they give low values of E . For the Faroe Islands (Station 18) also we find a

rather low value of E . However, high values of E are found for an area north of Scotland (16), for the stations off the Norwegian coast (19, 20, 21), and for a station in the easternmost part of the North Sea (22).

At the request of Prof. Martin Knudsen I have gone over the manuscript and prepared it for publication. Only minor alterations and additions have been undertaken.

Jens Smed.

1. Introduction.

On the suggestion of Professor Otto Pettersson the Bureau in the years 1915—1918 put in hand the working up of data on surface temperature in the North Atlantic for the years 1900—1913. The results are published in the Bulletin Hydrographique 1919. These data relate to selected areas or stations. For each area or station the mean value t of the surface temperature is computed for each month in the years considered, namely 1900—1913. The general monthly means ϑ and the anomalies $t - \vartheta$ are also computed. The values of t , ϑ , and $t - \vartheta$ are given in tables, and $t - \vartheta$ in graphs also.

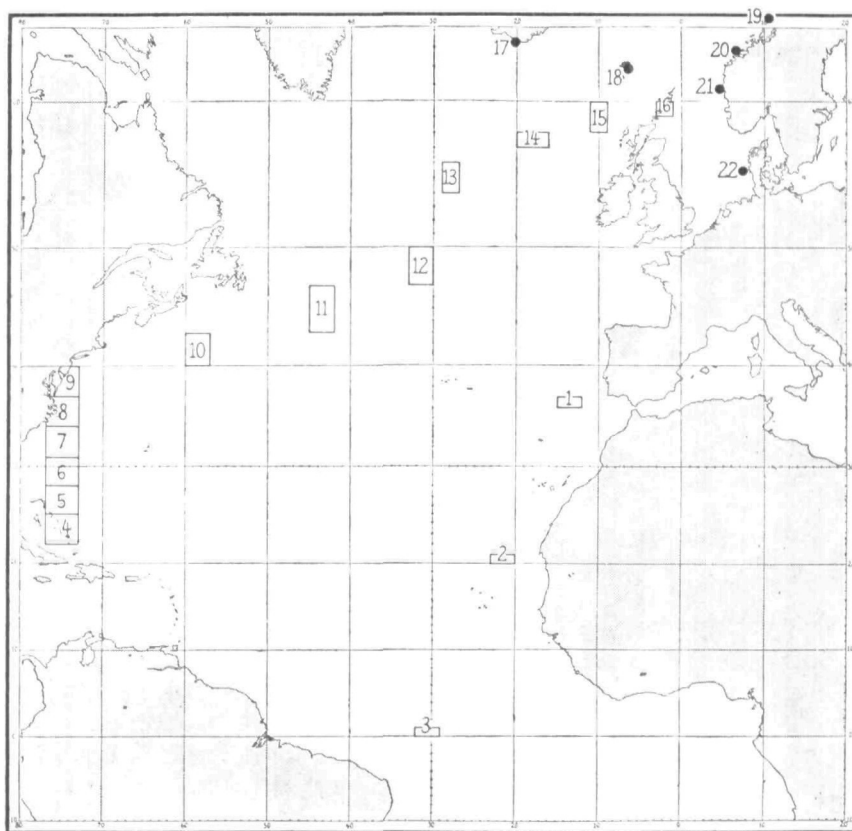


Fig. 1. Location of areas and stations.

In accordance with the plan for this work the actual results from the material considered were given in a comprehensive and easily accessible form. A discussion of the variations of the temperature with time and space has, however, never been taken up and at the request of Professor Martin Knudsen I have tried to investigate this problem more closely.

This investigation in the first place aims at an examination of the regularities in the variation of the temperature for each of the areas or stations. The main problem is, however, a discussion of the anomalies occurring in the temperature. At some places an anomaly may be maintained for several months, whereas at other places it may vanish in a short time. In order to characterize the different behaviours of the anomalies I have introduced a quantity *E* that may be termed the "effacement" constant. For areas in which the meteorological and hydrographical conditions are more or less alike the "effacement" constants may be expected to be similar also. Thus it should be possible to localize such areas geographically according to the effacement constant. This is in fact found to be the case to some extent. The present paper, however, may be considered only as an attempt at the solution of the problems connected with the temperature anomalies occurring in the North Atlantic. Should it be found worth while, the far larger collection of temperature data from the North Atlantic could perhaps be treated on similar lines as the material here dealt with. Furthermore, a comparison of the "effacement" of anomalies of both temperature and salinity might perhaps make it possible to distinguish between the various influences on the water temperature at a given locality of the currents, the atmosphere, and the solar radiation.

In the present investigation it was necessary to have at disposal all — or nearly all — the monthly temperature means for the period 1900—1913 for each area. For this reason, only the areas and stations indicated in Figure 1 and numbered 1—22 are taken into consideration. The positions of these areas and stations are given in Table 1.

Table 1.

No.	Lat. N.	Long. W.	Nationality	No.	Lat. N.	Long.	Nationality
1	36°—37°	12°—15°	German	15	58°—60°	9°—11°W.	Danish
2	20°—21°	20°—23°	"	16	59°—60°	1°—3°W.	"
3	0°—1°	29°—32°	"	17	63°26'	20°15'W.	Icelandic (Vestmanø)
4	22°—25°	73°—77°	Dutch	18	62°02'	6°45'W.	Danish (Thorshavn)
5	25°—28°	73°—77°	"	19	64°48'	10°33'E.	Norwegian (Nordøerne)
6	28°—31°	73°—77°	"	20	62°52'	6°33'E.	Norwegian (Ona)
7	31°—34°	73°—77°	"	21	60°45'	4°43'E.	Norwegian (Hellisø)
8	34°—37°	73°—77°	"	22	55°34-1'	7°19-5'E.	Danish (Horns Rev)
9	37°—40°	73°—77°	"				
10	40°—43°	57°—60°	"				
11	43°—47°	42°—45°	"				
12	47°—50°	30°—33°	"				
13	54°—56°	27°—29°	Danish				
14	57°—58°	16°—20°	"				

Table 2.
Constants relating to Temperature Variation at the

Loca- lity	Annual period			Semi-annual period			A	B
	Annual mean <i>m</i> °C	$R_1 \cos \frac{2\pi}{12} (q - q_1)$		$R_2 \cos \frac{2\pi}{6} (q - q_2)$				
		Ampl. <i>R</i> ₁ °C	Date of occurrence of maximum	Ampl. <i>R</i> ₂ °C	Dates of occurrence of maxima			
1	2	3	4	5	6	7	8	9
1	17.6	3.2	IX 3	0.4	II 27	VIII 28	0.3	0.1
2	21.7	2.4	IX 19	0.5	IV 6	X 6	0.4	0.1
3	26.2	0.8	III 24	0.3	V 9	XI 7	0.2	0.0
4	25.6	2.2	VIII 22	0.1	II 4	VIII 6	0.1	0.1
5	25.1	2.8	VIII 21	0.2	II 8	VIII 10	0.2	0.1
6	23.9	3.5	VIII 18	0.5	II 12	VIII 14	0.4	0.1
7	23.1	3.8	VIII 15	0.4	II 3	VIII 5	0.3	0.1
8	21.4	4.6	VIII 12	0.3	II 2	VIII 4	0.4	0.3
9	14.6	8.2	VIII 16	0.9	II 5	VIII 7	0.7	0.3
10	16.2	5.1	VIII 4	0.6	II 14	VIII 16	0.7	0.5
11	13.2	4.1	VII 3	1.3	II 18	VIII 20	1.2	0.7
12	13.4	2.3	VIII 9	0.6	II 12	VIII 14	0.4	0.2
13	9.7	2.5	VIII 16	0.4	II 13	VIII 15	0.3	0.1
14	10.2	2.2	VIII 15	0.5	II 10	VIII 12	0.4	0.1
15	9.9	2.2	VIII 17	0.5	II 8	VIII 10	0.4	0.1
16	8.9	2.7	IX 1	0.3	II 5	VIII 7	0.2	0.1
17	7.2	3.2	VII 29	0.6	I 25	VII 27	0.4	0.1
18	7.7	2.5	VIII 18	0.2	II 8	VIII 10	0.1	0.1
19	7.2	4.3	VIII 20	0.8	I 28	VII 30	0.6	0.2
20	8.2	4.0	VIII 24	0.4	I 30	VIII 1	0.3	0.1
21	8.4	4.3	VIII 28	0.5	I 28	VII 30	0.4	0.2
22	9.1	6.6	VIII 22	0.4	I 9	VII 10	0.3	0.1

2. Regular Variations of the Surface Temperature.

The values given in Table 2 have been found from the grand monthly means, ϑ , and the monthly means, t . Each line in the table contains the values referring to the locality indicated by its number in the first column of the table. The annual mean, m , is recorded in Column 2. By harmonic analysis the constants R_1 , q_1 , R_2 and q_2 were obtained in the expression $\vartheta = m + R_1 \cos \frac{2\pi}{12} (q - q_1) + R_2 \cos \frac{2\pi}{6} (q - q_2)$ for the temperature ϑ at the time q months after the beginning of the year. The amplitude, R_1 , of the annual period is recorded in Column 3, and the amplitude, R_2 , of the semi-annual period in Column 5. Instead of the displacement constants q_1 and q_2 , in Table 2, Columns 4, 6, and 7, the times have been recorded for the maxima of the terms with annual and semi-annual periods respectively.

It is seen from the table that the values of R_1 and R_2 are extraordinarily large for areas 9, 10, and 11. These areas are situated on the

Localities 1—22 (see Figure 1) in the North Atlantic.

$$\left[\frac{1}{14} \sum (\tau - \vartheta)^2 \right]^{1/2}$$

Locality	J 10	F 11	M 12	A 13	M 14	J 15	J 16	A 17	S 18	O 19	N 20	D 21
1	0.5	0.4	0.4	0.5	0.5	0.5	0.6	0.4	0.4	0.5	0.6	0.4
2	0.6	0.7	0.8	0.9	0.6	0.6	0.6	0.5	0.5	0.5	0.5	0.5
3	0.2	0.3	0.3	0.3	0.3	0.4	0.4	0.4	0.3	0.3	0.3	0.3
4	0.7	1.0	0.7	0.5	0.7	0.5	0.4	0.5	0.5	0.7	0.7	0.7
5	0.7	0.8	0.8	0.5	0.5	0.4	0.4	0.5	0.5	0.7	0.7	0.6
6	0.6	0.8	0.9	0.6	0.7	0.6	0.6	0.6	0.5	0.5	1.0	0.7
7	1.2	0.9	0.6	0.7	0.6	0.5	0.6	0.6	0.6	0.5	1.0	1.0
8	1.7	1.5	1.3	1.0	1.2	0.8	0.8	0.6	1.2	0.8	1.0	1.3
9	1.3	1.6	1.8	1.0	1.1	1.3	1.0	1.1	1.1	1.1	1.4	1.4
10	2.1	1.5	1.4	1.5	1.1	1.5	1.4	1.6	1.6	1.8	2.1	1.9
11	1.3	1.3	0.9	1.2	1.1	1.2	1.8	2.9	1.9	2.1	2.0	2.6
12	0.5	1.0	0.9	1.0	0.8	1.0	1.3	1.4	1.0	0.9	0.6	0.7
13	0.8	0.7	1.0	0.9	1.3	0.8	0.8	0.8	0.6	0.8	0.7	0.6
14	0.7	0.5	0.5	0.6	0.7	0.6	0.6	0.6	0.6	0.7	0.5	0.5
15	0.7	0.6	0.4	0.5	0.6	0.7	0.7	0.8	0.6	0.4	0.5	0.3
16	0.6	0.5	0.4	0.3	0.4	0.5	0.7	0.7	0.6	0.6	0.5	0.9
17	0.7	0.6	0.5	0.6	0.6	0.6	0.5	0.6	0.5	0.8	0.6	0.7
18	0.3	0.4	0.4	0.4	0.6	0.4	0.5	0.5	0.5	0.4	0.4	0.4
19	0.5	0.7	0.4	0.4	0.6	0.9	1.1	1.1	0.9	0.9	0.8	0.9
20	0.4	0.5	0.6	0.5	0.5	0.5	1.2	1.5	0.9	0.9	0.9	0.6
21	0.4	0.9	0.5	0.4	0.5	0.6	1.4	1.9	0.8	0.7	0.5	0.4
22	0.5	0.7	1.0	0.7	0.7	0.8	0.9	1.0	0.7	0.8	0.8	1.1

polar front or near it. For Station 22 (the lightship "Horns Rev") in the south-eastern part of the North Sea the value of R_1 is 6.6. For the annual period the temperature maximum in most cases occurs between the 4th and 24th of August. Exceptions are found in the localities 1, 2, 3, 11, 16, 17, and 21.

For the semi-annual period the first temperature maximum occurs between 25th January and 14th February. Exceptions are found in the localities 1, 2, 3, 11, and 22. Notwithstanding the fact that the amplitude R_2 for the semi-annual period is of the order of magnitude 0.5 only, the time of occurrence of the temperature maximum is found to be the same for most of the localities. Thus the reality of the semi-annual period cannot be doubted. The principal cause for the semi-annual period may reside in the different stability states of the water in summer and winter. The heating of the water in summer reaches down to a relatively small depth only and accounts for a relatively high increase in the surface temperature. In winter the cooling of the water

reaches down to relatively great depths on account of mixing, and the cooling of the very surface will be less than it would be without the mixing. Possibly also other circumstances, especially the air conditions, are connected with the semi-annual period.

Columns 8 and 9 of Table 2 contain the values of the expressions:—

$$A = \left[\frac{1}{12} \sum (\vartheta - m - R_1 \cos \frac{2\pi}{12} (q - q_1))^2 \right]^{1/2} \text{ and}$$

$$B = \left[\frac{1}{12} \sum (\vartheta - m - R_1 \cos \frac{2\pi}{12} (q - q_1) - R_2 \cos \frac{2\pi}{6} (q - q_2))^2 \right]^{1/2}$$

These expressions may be interpreted as the standard deviations of the differences between the grand monthly means ϑ and the monthly means estimated when the annual period only is considered and when both annual and semi-annual periods are considered. Apart from the areas 9, 10, and 11, the means of the values in Columns 8 and 9 of Table 2 are 0.3° and 0.1° respectively.

Columns 10–21 in Table 2 contain the values of the expression $\left[\frac{1}{14} \sum (t - \vartheta)^2 \right]^{1/2}$. These values represent the standard deviations of the monthly means t . For most of the localities they are of the order of magnitude 0.5 – 1.0 . For Area 3, situated on the Equator, all values are less than 0.5 . For areas influenced by the polar front — especially Areas 9, 10, and 11 — many of the values are greater than 1.0 and in some few cases greater than 2.0 . The standard deviation of a grand mean ϑ may be estimated from a standard deviation for a monthly mean t by dividing by $\sqrt{14}$. If the above-mentioned areas 9, 10, and 11, are disregarded the standard deviation for a grand monthly mean, ϑ , may thus be estimated at 0.2 . Comparing this value with the mean values 0.3 and 0.1 of the differences in Columns 8 and 9 — Areas 9, 10, and 11 excepted — we see that the regular variations of the temperature may be given by the expressions found for the annual and semi-annual period. If the semi-annual period is not considered, the differences between the actual and the calculated values of t are of the order of magnitude 0.3 , i. e., greater than the estimated value 0.2 for the standard deviation of a value of ϑ .

3. Stability of the Anomalies of Temperature.

The investigation is based on calculated values for the correlation between the monthly anomalies $t - \vartheta$ mentioned in the introduction. For each locality the anomalies for months with the same name are correlated with the anomalies for the following months. For instance, the anomalies for April are correlated with the anomalies for May, June, and so on. The values found for the correlation coefficients are then averaged.

For each locality, $12 \times 14 = 168$ anomalies $t - \vartheta$ are at disposal. For convenience an anomaly ($t - \vartheta$) is designated by x . The month to

which it refers is given by an index in accordance with the form A below. In this form each of the anomalies $x_{13} - x_{168}$ are recorded in two different places.

Form A.

Year	Month												Year
	J	F	M	A	M	J	J	A	S	O	N	D	
1900	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	1901
	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}	x_{21}	x_{22}	x_{23}	x_{24}	
1901	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}	x_{21}	x_{22}	x_{23}	x_{24}	1902
	x_{25}	x_{26}	x_{27}	x_{28}	x_{29}	x_{30}	x_{31}	x_{32}	x_{33}	x_{34}	x_{35}	x_{36}	
1912	x_{145}	x_{146}	x_{147}	x_{148}	x_{149}	x_{150}	x_{151}	x_{152}	x_{153}	x_{154}	x_{155}	x_{156}	1913
	x_{157}	x_{158}	x_{159}	x_{160}	x_{161}	x_{162}	x_{163}	x_{164}	x_{165}	x_{166}	x_{167}	x_{168}	
1913	x_{157}	x_{158}	x_{159}	x_{160}	x_{161}	x_{162}	x_{163}	x_{164}	x_{165}	x_{166}	x_{167}	x_{168}	

Correlation coefficients are calculated for anomalies recorded in the columns in Form A. Designating by p and q two different indices for x , where $p \leq 12$ and where $p < q \leq 24$ we obtain a correlation coefficient r_{pq} from:—

$$r_{pq} = \frac{x_p \cdot x_q + x_{p+12} \cdot x_{q+12} + x_{p+24} \cdot x_{q+24} + \dots}{(x_p^2 + x_{p+12}^2 + x_{p+24}^2 + \dots)^{1/2} \cdot (x_q^2 + x_{q+12}^2 + x_{q+24}^2 + \dots)^{1/2}}$$

For $q \leq 12$, 14 pairs of values are at disposal for the calculation of r_{pq} , the last pair being x_{p+156} and x_{q+156} . For $q > 12$, 13 pairs only are at disposal, the last pair being x_{p+144} and x_{q+144} .¹⁾

A correlation coefficient r_{pq} is said to refer to the interval of $q - p$ months. For each locality the values of r_{pq} may be recorded as shown in Form B.

At the top of the form, the months in two successive years are indicated by their names and by the numbers 1, 2, 3, ... 24. A correlation coefficient r_{pq} is placed midway between the columns for the months p and q . Coefficients on the same horizontal line refer to the same interval of time, viz., $q - p$ months, recorded on the left-hand side of the form.

Instead of Form B a slightly different Form C has been used for recording the calculated correlation coefficients r_{pq} . Form C differs from Form B in that the coefficients r_{1214} , r_{1215} , r_{1115} , r_{1216} , etc. are placed on the left-hand side of the form, whereas they are placed on the right-

¹⁾ Strictly speaking instead of the values $t - \vartheta$ we should in this case ($q > 12$) have used for x $t - \vartheta'$ where ϑ' is the grand monthly mean for the 13 years in question. As this would certainly not influence the results to any essential degree we have for the sake of simplicity used the values of $t - \vartheta$ throughout the investigation.

Form B.

q—p	J	Month												M	D
		M	A	M	J	J	A	S	O	N	D	J	F		
1	r ₁	3	4	r ₃₄	r ₄₅	r ₅₆	r ₆₇	r ₇₈	r ₈₉	r ₉₁₀	r ₁₀₁₁	r ₁₁₁₂	r ₁₂₁₃	15	24
2		r ₂₄	r ₃₅	r ₄₆	r ₅₇	r ₆₈	r ₇₉	r ₈₁₀	r ₉₁₁	r ₁₀₁₂	r ₁₁₁₃	r ₁₂₁₄			
3		r ₂₅	r ₃₆	r ₄₇	r ₅₈	r ₆₉	r ₇₁₀	r ₈₁₁	r ₉₁₂	r ₁₀₁₃	r ₁₁₁₄	r ₁₂₁₅			
4		r ₁₅	r ₂₆	r ₃₇	r ₄₈	r ₅₉	r ₆₁₀	r ₇₁₁	r ₈₁₂	r ₉₁₃	r ₁₀₁₄	r ₁₁₁₅	r ₁₂₁₆		
5		r ₁₆	r ₂₇	r ₃₈	r ₄₉	r ₅₁₀	r ₆₁₁	r ₇₁₂	r ₈₁₃	r ₉₁₄	r ₁₀₁₅	r ₁₁₁₆	r ₁₂₁₇		
6		r ₁₇	r ₂₈	r ₃₉	r ₄₁₀	r ₅₁₁	r ₆₁₂	r ₇₁₃	r ₈₁₄	r ₉₁₅	r ₁₀₁₆	r ₁₁₁₇	r ₁₂₁₈		
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Form C.

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.	Mean for the year
1	r ₁₂	r ₂₃	r ₃₄	r ₄₅	r ₅₆	r ₆₇	r ₇₈	r ₈₉	r ₉₁₀	r ₁₀₁₁	r ₁₁₁₂	r ₁₂₁₃
2	r ₁₃	r ₂₄	r ₃₅	r ₄₆	r ₅₇	r ₆₈	r ₇₉	r ₈₁₀	r ₉₁₁	r ₁₀₁₂	r ₁₁₁₃	r ₁₂₁₄
3	r ₁₄	r ₂₅	r ₃₆	r ₄₇	r ₅₈	r ₆₉	r ₇₁₀	r ₈₁₁	r ₉₁₂	r ₁₀₁₃	r ₁₁₁₄	r ₁₂₁₅
4	r ₁₅	r ₂₆	r ₃₇	r ₄₈	r ₅₉	r ₆₁₀	r ₇₁₁	r ₈₁₂	r ₉₁₃	r ₁₀₁₄	r ₁₁₁₅	r ₁₂₁₆
5	r ₁₆	r ₂₇	r ₃₈	r ₄₉	r ₅₁₀	r ₆₁₁	r ₇₁₂	r ₈₁₃	r ₉₁₄	r ₁₀₁₅	r ₁₁₁₆	r ₁₂₁₇
6	r ₁₇	r ₂₈	r ₃₉	r ₄₁₀	r ₅₁₁	r ₆₁₂	r ₇₁₃	r ₈₁₄	r ₉₁₅	r ₁₀₁₆	r ₁₁₁₇	r ₁₂₁₈
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hand side in Form B. Furthermore, a column headed "Mean for the year" on the right-hand side of Form C contains the means r_1, r_2, r_3, \dots of the coefficients for the values 1, 2, 3, \dots of $q - p$.

In accordance with Form C the calculated values of 100 r_{pq} are given in Table 3 for the localities stated in Table 1.

An inspection of the values for r_{pq} shows that their variation must be very largely subject to accidental causes. This is easily understood, as each value for r_{pq} is based on 13 or 14 pairs of anomalies only. A few anomalies deviating to a particularly high extent from the normal may make their influence felt in a series of values for r_{pq} in Table 3. This is, for instance, the case with the anomalies in the month of April for Area 12. For this area the mean temperatures for the months March, April, and May in 1903 are 11.2° , 14.6° and 11.9° with anomalies -0.5° , 2.3° , and -1.0° . For the same months in 1909 the mean temperatures are 12.5° , 10.9° , and 13.4° , with the anomalies 0.8° , -1.4° , and 0.5° . Thus the mean temperature for April in 1903 was exceptionally high and in 1909 exceptionally low. This is the cause of the peculiar distribution of the positive and negative values of r_{pq} in Table 3 for Area 12. The values $r_{45} = -0.03$, $r_{46} = 0.21$, $r_{47} = -0.39$, $r_{48} = -0.41$, $r_{49} = -0.44$, $r_{410} = -0.48$, $r_{411} = -0.12$, $r_{412} = -0.55$, $r_{413} = -0.07$ lie on a sloping line. With one exception only these values are all of negative sign. The values of r_{pq} in the neighbourhood of this line are mainly of positive sign. Similar characteristic distributions of particularly high or low (negative) values for r_{pq} along sloping lines are found at many places in Table 3. This is in accordance with the fact that the anomaly for a month has been used for calculating the values of r_{pq} along such a line as seen in Form C.

Such exceptionally high or low anomalies as those mentioned above are ordinarily to be considered as accidental. Their exceptional value may be due to the temperature conditions themselves but may also arise from the distribution in space and time of the temperature observations used for the computation of the monthly means. For some localities — Danish, Icelandic, and Norwegian coastal stations — the monthly means are computed from daily observations. For the rest of the localities — the areas — the monthly means are based on fewer observations distributed over the area. The number, n , of observations on which the monthly means are based is stated in the Bulletin. Observations from areas marked by D are collected by Deutsche Seewarte, and with regard to these observations it is stated in the Bulletin that the average date for each monthly mean, t , was calculated and that of these average dates by far the greater number were found to lie close to the middle of the month. A similar statement is not found for the other areas. The number, n , of the observations used for the calculation of the monthly temperature means is, however, ordinarily so great that the value found may be considered as applicable to the middle of the month.

Disregarding the accidental variations of r_{pq} , one has to look for

Table 3. Values of $100 \cdot r_{pq}$ as defined in the Explanation of Forms B and C, page 24.

Locality	q-p	Month												Mean for the year			
		J	F	M	A	M	J	J	A	S	O	N	D				
36°-37°N. 12°-15°W. D	1	70	65	77	08	12	46	49	38	53	31	44	33	44			
	2	10	54	70	08	-12	01	23	25	55	13	32	-19	22			
	3	13	55	01	-08	-18	-56	-05	11	16	-04	-32	06	-02			
	4	17	36	29	-27	-28	-42	-63	12	09	14	-02	-24	-06			
	5	30	54	35	08	-42	-13	-27	-40	38	-09	-27	19	02			
	6	43	26	12	33	-14	-17	-46	03	-15	59	-18	21	07			
20°-21°N. 20°-23°W. D	1	73	90	67	88	65	56	39	69	71	40	08	47	59			
	2	-03	57	85	71	62	43	49	-04	19	35	-04	-08	34			
	3	-04	75	88	31	53	30	34	-26	07	-05	19	-36	22			
	4	-23	03	74	65	42	39	03	-16	-09	10	26	06	18			
	5	-16	15	50	57	08	01	-24	-18	17	51	-08	07	12			
	6	13	-23	-18	36	28	-31	-37	-48	-12	-12	17	-25	-09			
	7	16	-41	-29	44	-10	-41	-23	-22	-25	-03	-15	-04	-13			
	8	14	08	-65	10	35	-39	-44	-38	-03	-11	-10	26	-10			
	9	08	16	01	28	-01	-45	-09	-12	25	-13	00	30	02			
0°-1°N. 29°-32°W. D	1	71	68	88	53	74	57	48	45	66	85	87	81	69			
	2	66	57	70	44	62	53	73	64	63	48	88	64	63			
	3	78	48	40	51	63	33	66	58	79	64	55	45	57			
	4	47	60	32	59	61	61	34	34	54	68	50	50	51			
	5	31	49	47	68	61	40	-02	45	58	44	13	58	43			
	6	44	19	58	57	69	17	51	15	47	65	18	43	42			
	7	52	45	64	51	59	33	65	11	45	38	20	31	43			
	8	34	66	29	52	45	68	57	50	39	-03	41	04	40			
	9	36	58	47	45	34	85	32	40	-03	17	40	06	36			
22°-25°N. 73°-77°W. H	1	45	45	-14	25	53	17	44	49	29	03	44	44	32			
	2	36	-18	-26	-11	42	15	29	57	26	21	51	03	19			
	3	28	33	-12	-02	28	05	40	07	44	06	18	08	17			
	4	00	52	-33	-37	58	46	13	09	48	19	-03	52	19			
	5	23	-05	-26	35	-17	26	15	42	24	08	67	55	21			
	6	31	15	03	08	-19	23	-04	39	46	-13	32	72	19			

5	1	57	31	15	41	37	01	27	16	50	11	15	71	31
25°-28°N.	2	67	61	-18	-03	48	19	04	73	-11	28	17	43	27
73°-77°W.	3	34	12	-05	-44	67	-09	51	46	54	00	43	11	22
H	4	48	00	-09	-62	33	23	46	27	36	-11	38	32	17
	5	56	28	-43	-03	08	71	13	37	-17	13	33	52	21
	6	36	07	-16	36	32	02	47	-01	48	24	-10	53	22
6	1	59	46	14	38	39	-10	43	33	24	30	03	30	29
28°-31°N.	2	55	04	-32	-28	34	-10	-11	34	07	-17	29	05	06
73°-77°W.	3	45	08	-21	-29	19	-45	05	43	28	-25	32	09	06
H	4	21	27	-04	-18	22	-10	32	46	18	-15	34	17	14
	5	-08	07	-27	14	-08	10	20	52	-04	-22	-20	14	02
	6	16	-56	12	07	-11	-18	07	05	08	07	-18	-09	-04
7	1	60	19	61	30	26	04	72	45	27	34	49	46	39
31°-34°N.	2	57	28	06	08	60	-05	10	36	57	11	18	26	26
73°-77°W.	3	41	47	07	16	-09	-29	04	40	39	-26	28	33	16
H	4	18	36	42	-09	12	-09	16	27	33	-10	-23	05	12
	5	17	06	52	-30	-01	-04	-14	72	02	07	-29	16	08
	6	07	-18	-15	-39	-01	-08	-29	24	52	19	-23	-18	-04
8	1	09	04	29	18	51	53	40	02	26	33	03	11	23
34°-37°N.	2	00	30	11	32	27	53	08	04	00	46	00	03	18
73°-77°W.	3	13	46	-11	22	03	20	17	-30	12	03	-27	-11	05
H	4	48	31	65	-22	09	-03	37	-13	33	35	02	-12	18
	5	13	38	49	-36	-06	-11	-37	22	33	-43	01	-18	00
	6	01	-03	28	35	-16	29	21	08	-22	24	-14	69	13
9	1	64	05	48	18	59	39	42	53	59	53	46	44	44
37°-40°N.	2	30	57	-12	24	18	03	35	-14	55	25	43	16	23
73°-77°W.	3	-34	45	57	-11	-03	-05	17	-01	29	25	22	-05	11
H	4	12	24	44	-03	-22	28	14	72	12	15	-06	29	18
	5	52	30	-16	-23	-49	32	24	42	-14	07	13	07	09
	6	11	04	-13	-21	-19	-06	44	41	24	04	-17	-39	01

Table 3 continued.

Locality	q-p	Month												Mean for the year			
		J	F	M	A	M	J	J	A	S	O	N	D	28			
10	1	-17	07	64	00	25	52	41	57	70	71	60	25	38			
40°-43°N.	2	-23	04	38	-01	-03	32	22	11	38	52	58	08	20			
57°-60°W.	3	11	07	-48	-06	-24	57	-19	-21	45	69	62	-36	08			
H	4	52	-52	33	10	-52	-13	47	-05	-19	20	69	-37	04			
	5	-01	14	-12	-16	-24	-26	27	-05	-29	47	-45	29	-03			
	6	11	15	13	-03	-33	-17	-06	42	-15	30	-03	19	04			
11	1	30	42	58	65	05	30	74	44	90	83	71	18	51			
43°-47°N.	2	-03	14	45	41	01	-21	41	30	33	68	45	43	28			
42°-45°W.	3	20	37	19	21	-33	-22	07	28	16	44	41	07	15			
H	4	28	30	49	21	07	-52	14	04	14	13	27	09	14			
	5	65	61	23	-24	-11	-04	24	-05	15	-03	-03	-05	11			
	6	34	82	05	03	-23	-18	05	03	-40	28	-15	-11	04			
12	1	26	71	28	-03	73	56	77	80	68	55	47	60	53			
47°-50°N.	2	56	40	21	57	21	55	33	88	45	35	37	22	42			
30°-33°W.	3	48	38	53	59	-39	38	51	60	19	36	26	35	35			
H	4	09	19	21	51	27	-41	55	36	15	09	34	21	21			
	5	-16	65	44	43	27	-44	46	21	36	23	26	03	23			
	6	43	52	57	14	34	27	-48	-03	32	52	-05	27	24			
	7	44	27	34	32	34	25	-12	15	60	26	12	36	28			
	8	44	48	49	26	37	52	06	-55	35	27	24	09	25			
	9	51	33	70	39	32	06	-04	-07	39	37	40	36	31			
13	1	71	39	79	44	73	42	82	76	68	13	67	63	60			
54°-56°N.	2	55	52	52	29	60	23	52	85	75	06	25	50	47			
27°-29°W.	3	33	74	39	37	17	39	58	61	-01	25	12	18	34			
Da	4	05	16	54	72	13	23	39	55	-05	11	26	33	28			
	5	09	19	72	59	23	33	55	28	07	18	50	-15	30			
	6	06	13	45	43	52	20	17	24	36	-03	42	-24	23			
	7	-35	27	35	39	53	02	-09	61	25	33	-19	17	19			
	8	-26	07	42	40	59	47	-38	18	19	49	-22	08	17			
	9	24	-17	46	39	46	-18	00	04	40	-08	-04	-24	11			

14	1	35	20	75	74	48	66	59	34	20	14	25	32	42
57°—58°N.	2	63	55	42	64	65	41	18	37	02	—13	11	09	33
16°—20°W.	3	42	48	47	56	56	26	60	07	—10	21	02	38	33
Da	4	18	41	30	40	59	36	32	11	42	62	—30	07	29
	5	—01	30	45	03	49	45	06	24	77	18	—16	16	25
	6	01	20	34	37	—24	53	16	11	44	44	38	24	25
	7	—42	39	—09	36	31	—20	09	61	07	60	15	—08	15
	8	18	43	25	—01	11	33	—02	45	18	20	62	23	25
	9	16	15	27	24	01	41	51	11	67	61	41	32	32
15	1	—08	00	26	45	42	88	73	80	23	13	19	—01	33
58°—60°N.	2	00	—03	32	45	16	36	81	78	21	02	31	—12	27
9°—11°W.	3	06	26	22	60	26	13	76	—07	—27	34	—05	24	21
Da	4	—06	02	49	—27	44	—06	33	—18	01	45	30	—07	12
	5	27	66	33	—15	26	24	—46	—16	52	08	30	12	17
	6	16	—01	40	55	—36	33	—26	16	43	20	07	35	17
	7	35	14	10	31	—16	—42	19	—34	34	22	48	23	12
	8	72	18	—13	30	47	12	—22	03	32	23	26	07	20
	9	—02	—19	13	24	—01	26	19	23	63	36	16	63	22
16	1	—08	46	67	71	74	52	62	78	39	54	45	18	50
59°—60°N.	2	37	—51	18	48	32	74	71	59	34	19	44	23	34
1°—3°W.	3	—05	—03	—17	13	32	53	43	—05	—03	15	07	57	16
Da	4	42	—07	—25	—42	29	07	35	31	—03	03	33	22	10
	5	26	—30	—46	—20	—05	—09	—14	02	—23	12	—41	11	—11
	6	28	—19	—34	—13	—39	—19	—34	—17	—26	32	—39	—04	—15
	7	15	—51	—28	—38	—57	—43	—39	—51	—04	—34	13	33	—24
	8	05	—21	—03	—45	—06	—23	—39	—52	—11	—31	—15	67	—14
	9	—26	09	—60	—58	03	—47	—28	—31	—26	31	18	45	—14
17	1	20	52	22	48	54	56	24	65	—02	62	26	51	40
63°26'N.	2	22	28	33	36	50	46	32	34	—32	—27	45	—03	22
20°15'W.	3	41	52	45	13	44	—24	15	—21	—50	—08	24	50	15
Vestmæ	4	—16	26	29	10	33	—17	—36	—50	—20	26	—18	38	00
	5	24	03	19	18	—14	—41	—24	—42	—31	15	—09	—06	—07
	6	52	—12	14	29	—36	22	—05	—24	—42	13	—03	02	01
	7	08	—23	32	—35	—22	33	05	—31	27	42	46	04	07
	8	34	19	13	—35	—16	25	—07	—03	06	—03	—01	—17	01
	9	27	10	—24	—02	15	—02	09	45	05	—02	—02	39	10

Table 3 continued.

Locality	q-p	Month												Mean for the year			
		J	F	M	A	M	J	J	A	S	O	N	D				
62°02'N. 6°45'W. Thorshavn	1	46	27	40	62	35	80	85	66	48	56	68	53	56			
	2	57	17	45	75	12	47	77	69	62	52	61	10	49			
	3	29	51	22	25	16	50	59	56	22	38	11	23	34			
	4	33	39	43	43	42	18	38	34	21	32	14	45	31			
	5	12	18	56	50	27	25	45	10	11	20	15	45	25			
	6	57	12	30	62	46	20	33	09	12	05	05	40	24			
	7	38	15	22	50	14	39	41	33	18	05	48	04	20			
	8	16	10	08	17	50	28	07	34	36	36	28	47	18			
	9	14	18	21	18	47	18	13	21	24	21	18	50	20			
64°48'N. 10°33'E. Nordørne	1	34	56	49	31	26	61	70	72	78	41	82	77	56			
	2	14	02	14	62	30	34	23	53	36	25	18	44	27			
	3	33	08	49	21	19	47	14	50	05	26	21	30	12			
	4	44	21	12	18	28	10	58	19	12	18	01	40	01			
	5	02	43	39	30	31	05	52	10	01	03	27	70	06			
	6	30	64	38	06	30	39	39	50	09	17	03	51	05			
60°52'N. 6°33'E. Ona	1	14	77	34	83	48	33	77	58	00	78	50	36	49			
	2	27	13	34	43	15	10	07	38	10	28	45	49	14			
	3	40	43	47	30	14	21	44	15	16	31	47	23	09			
	4	38	14	46	34	39	40	18	01	17	40	33	23	06			
	5	39	15	10	45	30	50	02	25	33	34	36	28	03			
	6	39	21	32	12	51	34	04	23	23	26	26	21	12			
60°45'N. 4°43'E. Helliø	1	61	47	27	48	22	02	65	62	24	72	57	33	43			
	2	17	15	27	07	03	00	27	23	12	01	45	54	14			
	3	26	19	63	23	15	27	09	05	48	32	53	18	13			
	4	12	39	06	26	20	20	47	32	24	02	19	33	05			
	5	06	51	18	11	00	29	26	27	14	13	09	08	01			
	6	51	36	04	11	01	29	57	06	24	00	09	11	07			
55°34'N. 7°19'5"E. Horns Rev	1	05	47	80	57	57	41	65	67	30	69	56	52	51			
	2	44	01	22	68	37	36	49	23	15	22	51	19	25			
	3	30	05	51	55	35	62	68	24	10	43	47	33	22			
	4	11	31	07	68	27	01	59	10	51	30	32	11	09			
	5	00	16	11	22	10	09	12	30	16	60	34	22	00			
	6	25	27	06	28	25	30	16	22	06	54	42	04	02			

some regularity in its variation. For a constant value of $q-p$ an annual variation of r_{pq} may be assumed. In the regions influenced by the polar front — the areas 9, 10, and 11 — the temperature conditions are less stable in the early part of the year than later on. The case is otherwise in the regions between the British Islands and Iceland, where the weather conditions in the autumn are unstable. A low degree of stability of the temperature conditions may be assumed to exhibit itself by low values of r_{pq} for the same value of $q-p$ and thus an annual variation of r_{pq} may be accounted for. This question will be discussed in a future section (Section 5, p. 40).

The most striking regularity in the variation of r_{pq} is its decrease with the increase of $q-p$. That means that the correlation of the monthly means is the less the farther off they are from one another. This could of course be imagined beforehand, but it is an interesting fact that the decrease of the correlation is to a considerable extent different for the different localities.

An investigation of the decrease of r_{pq} with the increase of $q-p$ may be undertaken by means of the mean values for the year given in Table 3. This procedure requires that the actual circumstances are compared with some stationary conditions which in reality are the object of the investigation.

In Form A three successive columns are considered. They may be the columns with the anomalies x_{11} , x_{12} , and x_{13} at the top. Instead of the symbols used in Form A, the symbols stated in Form D are used in the following consideration. The number of groups of connected anomalies is designated by N . The number N is either 13 or 14, and in Form D is equal to 13. For the investigation of correlations between connected anomalies symbols and definitions from the textbook by Yule¹⁾ are used.

Form D.

x'_1	x'_2	x'_3
x''_1	x''_2	x''_3
\vdots	\vdots	\vdots
x^{XIII}_1	x^{XIII}_2	x^{XIII}_3

Referring to Form D we have:—

$$\Sigma x_1 = \Sigma x_2 = \Sigma x_3 = 0^2)$$

¹⁾ G. U. Yule: An Introduction to the Theory of Statistics. (Second edition. London, 1912).

²⁾ In fact, this and the following is exactly fulfilled for $N=14$ only (cf. footnote, p. 23).

The mean deviations of the zero order σ_1 , σ_2 , and σ_3 are determined by:—

$$\sigma_1^2 = \frac{1}{N} \Sigma x_1^2, \sigma_2^2 = \frac{1}{N} \Sigma x_2^2, \text{ and } \sigma_3^2 = \frac{1}{N} \Sigma x_3^2$$

The total correlations of zero order r_{12} , r_{23} , and r_{13} are determined by:—

$$r_{12} = \frac{\Sigma x_1 x_2}{(\Sigma x_1^2 \cdot \Sigma x_2^2)^{1/2}} = \frac{\frac{1}{N} \Sigma x_1 x_2}{\sigma_1 \cdot \sigma_2}, \quad r_{23} = \frac{\Sigma x_2 x_3}{(\Sigma x_2^2 \cdot \Sigma x_3^2)^{1/2}} = \frac{\frac{1}{N} \Sigma x_2 x_3}{\sigma_2 \cdot \sigma_3},$$

$$r_{13} = \frac{\Sigma x_1 x_3}{(\Sigma x_1^2 \cdot \Sigma x_3^2)^{1/2}} = \frac{\frac{1}{N} \Sigma x_1 x_3}{\sigma_1 \cdot \sigma_3}$$

The partial correlation $r_{13.2}$ of the first order may be taken as the expression for the correlation between the x_1 - and the x_3 -values if the x_2 -values were all equal to zero, i. e., if the monthly means used for the calculation of the anomalies x_2 were all equal. The partial correlation $r_{13.2}$ may be found in the following way.

By means of the regression-equations

$$x_1 \sim b_{12}x_2 \text{ and } x_3 \sim b_{32}x_2$$

the regression-coefficients b_{12} and b_{32} are found from

$$b_{12} = \frac{\Sigma x_1 x_2}{\Sigma x_2^2} = r_{12} \cdot \frac{\sigma_1}{\sigma_2} \text{ and } b_{32} = \frac{\Sigma x_2 x_3}{\Sigma x_2^2} = r_{23} \cdot \frac{\sigma_3}{\sigma_2}$$

The mean deviations $\sigma_{1.2}$ and $\sigma_{3.2}$ of the first order are defined by

$$\sigma_{1.2}^2 = \frac{1}{N} \Sigma (x_1 - b_{12}x_2)^2 \text{ and } \sigma_{3.2}^2 = \frac{1}{N} \Sigma (x_3 - b_{32}x_2)^2$$

By means of the above expressions for b_{12} and b_{32} we obtain

$$\sigma_{1.2}^2 = \sigma_1^2(1 - r_{12}^2) \text{ and } \sigma_{3.2}^2 = \sigma_3^2(1 - r_{23}^2)$$

The deviations $x_1 - b_{12}x_2$ and $x_3 - b_{32}x_2$ may be taken as the values that should have been found for x_1 and x_3 if the monthly means used for the calculation of the anomalies x_2 were all equal. Thus the partial correlation $r_{13.2}$ may be written as

$$r_{13.2} = \frac{\Sigma (x_1 - b_{12}x_2)(x_3 - b_{32}x_2)}{[\Sigma (x_1 - b_{12}x_2)^2 \cdot \Sigma (x_3 - b_{32}x_2)^2]^{1/2}}$$

By means of the expressions developed above we obtain

$$r_{13.2} = \frac{r_{13} - r_{12} \cdot r_{23}}{\sqrt{1 - r_{12}^2} \cdot \sqrt{1 - r_{23}^2}}$$

Under stationary conditions the values of r_{12} and r_{23} may be replaced by a common value r represented by the mean value of the correlations for successive months. If, further, the anomalies accounting for the correlations r_{12} , r_{23} , and r_{13} are effaced in a short time, less than a month, the partial correlation $r_{13.2}$ may be put equal to zero. In this case we get $r_{13} = r^2$. Under these conditions a general expression for an arbitrary correlation r_{pq} valid for the interval $q-p$ months is $r_{pq} = r^{q-p}$.

This case relates to an extreme condition. Another extreme may occur when the effacement of an anomaly is very — infinitely — slow. Then an arbitrary correlation r_{pq} valid for the interval $q-p$ months may be equal to the normal value r of the correlation for successive months.

A general formula for r_{pq} embracing the extreme cases considered may be given as:

$$r_{pq} = r^{[(q-p)^E]}$$

where E is a characteristic constant that may be taken as a measure of the effacement of the anomalies. For $E = 1$ we have $r_{pq} = r^{q-p}$ and for $E = 0$ we get $r_{pq} = r$. In the following E is termed the "effacement constant" or simply the "effacement". The expression for r_{pq} is not based on any other considerations than those stated above and for values of E between 0 and 1 empirical justification for the expression must be sought.

According to what has been explained it seems reasonable to use $r^{[(q-p)^E]}$ as an approximate expression for the values of r_{pq} , but it must be emphasized that the expression is only approximate. In Table 3 some negative values for r_{pq} are found also in the means for the year. These may be accidental but the possibility exists that they are real and that the effacement of an anomaly may take place in such a way that a positive anomaly is normally followed by a negative anomaly and *vice-versa*. Thus it is to be remarked that the above expression for r_{pq} can be used for positive values only.

To what degree the expression for r_{pq} for a suitable value of E is in accordance with the real variation of r_{pq} may be illustrated by two examples. In Table 4 are recorded the values for r_{pq} obtained as means for the year from Table 3 for Areas 10 — 15. Means are computed for groups of the areas, viz., for the areas 10 and 11 and for the areas 12, 13, 14, and 15. For comparison with these means the values of $r_{pq} = r^{[(q-p)^E]}$ are calculated for $r = 0.44$ and $E = 0.80$, and for $r = 0.47$ and $E = 0.41$ respectively. The calculated values are given in the table. The comparison is shown graphically in Figures 2 and 3, where the actual means are given by the dots and calculated values of r_{pq} by the curves. It is seen that the calculated values for r_{pq} are in quite good accord with the actual means and that the difference in the effacement of the anomalies for the two groups of areas are characterized by the values of the effacement constant E .

Table 4.

Area	q—p								
	1	2	3	4	5	6	7	8	9
10	0.38	0.20	0.08	0.04	—0.03	0.04			
11	0.51	0.28	0.15	0.14	0.11	0.04			
Means for Areas 10 and 11	0.44	0.24	0.12	0.09	0.04	0.04			
$r_{pq} = 0.44^{[(q-p)^{0.80}]}$	0.44	0.24	0.13	0.08	0.05	0.03			
12	0.53	0.42	0.35	0.21	0.23	0.24	0.28	0.25	0.31
13	0.60	0.47	0.34	0.28	0.30	0.23	0.19	0.17	0.11
14	0.42	0.33	0.33	0.29	0.25	0.25	0.15	0.25	0.32
15	0.33	0.27	0.21	0.12	0.17	0.17	0.12	0.20	0.22
Means for Areas 12, 13, 14, and 15	0.47	0.37	0.31	0.22	0.24	0.22	0.18	0.22	0.24
$r_{pq} = 0.47^{[(q-p)^{0.41}]}$	0.47	0.37	0.31	0.27	0.23	0.22	0.19	0.17	0.16

Incidentally the actual mean values 0.44 and 0.47 for r in the two cases are about the same, and it is accidental that these mean values are found to be the most suitable for insertion in the expression $r_{pq} = r^{(q-p)^E}$. In order to obtain the best possible agreement between the actual and the calculated values of r_{pq} the most suitable values should be chosen for both r and E . For this purpose a graphical process has been used, which is illustrated by Figure 4.

The coordinate system in the figure has its zero-point at O , the axis of abscissae OP , and the axis of ordinates OQ . The inserted scales — the $(q-p)$ -scale and the r_{pq} -scale — hold for the axis of abscissae and the axis of ordinates respectively. With the unit of length OP the $(q-p)$ -scale has divisions for $\log (q-p)$ equal to 0.000, 0.301, 0.477, 0.602, ..., i. e., for $(q-p)$ equal to 1, 2, 3, 4, ... The divisions are marked with the last figures. With the same unit of length the r_{pq} -scale has its principal divisions for $-\log (-\log r_{pq})$ equal to 0.000, 0.156, 0.282, 0.400, ..., i. e., for r_{pq} equal to 0.1, 0.2, 0.3, 0.4, ...

The actual means for r_{pq} for Areas 10 and 11 and for Areas 12, 13, 14, and 15 as found in Table 4 are plotted in the figure according to the scales for $q-p$ and r_{pq} . If the expression $r_{pq} = r^{[(q-p)^E]}$ with suitably chosen values of r and E should be in complete agreement with the actual means for r_{pq} the dots representing these values in the figure should lie on straight lines. This is approximately the case, and the lines L_1 and L_2 are drawn freehand to follow the plotted values in the best way. When the lines L_1 and L_2 are drawn, the values of r and E may be determined. Their points of intersection with the axis of ordinates give the values of r . These values are 0.44 and 0.47. The values of E may be read off by the arrangement shown in the upper part of the figure. Through the point A on the ordinate axis the line AB is drawn parallel to the axis of abscissae, intersecting the line PB

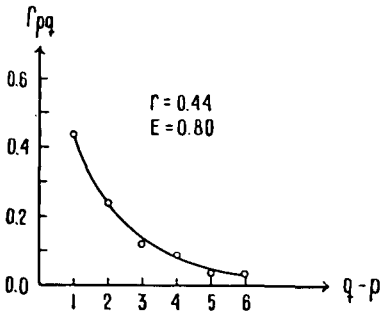


Fig. 2. Curve showing the variation of r_{pq} with $q-p$ for Areas 10 and 11. The dots indicate actual mean values for the two areas (see Table 4).

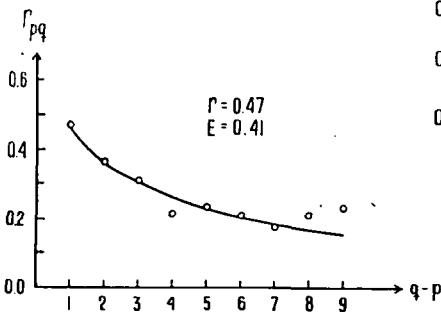


Fig. 3. Curve showing the variation of r_{pq} with $q-p$ for Areas 12, 13, 14, and 15. The dots indicate actual mean values for these areas (see Table 4).

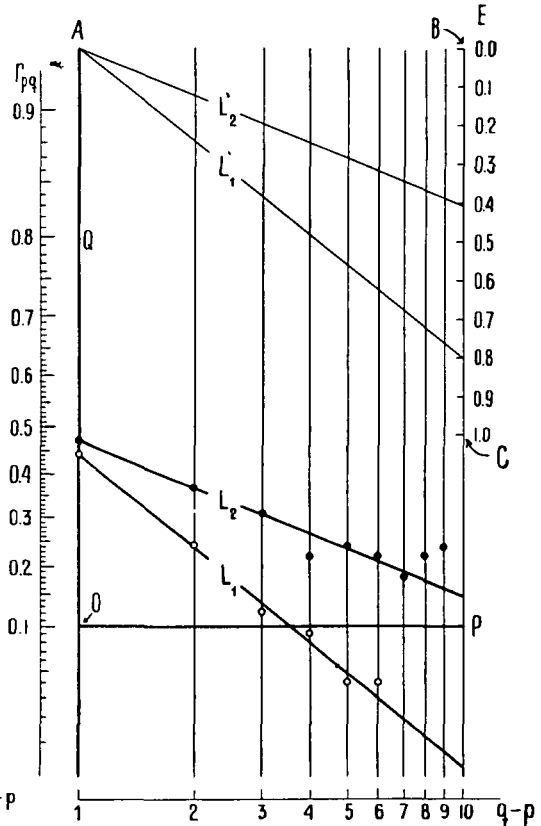


Fig. 4. Diagram for determining the "effacement" constant E . The outline circles relate to Areas 10 and 11, the full circles to Areas 12, 13, 14, and 15 (see Table 4).

parallel to the axis of ordinates in B. The distance from B to the point C on the line PB is equal to the unit of length OP. The distance BC is divided into ten equal parts and forms a scale for E with the divisions 0.0, 0.1, ... When the lines L'_1 and L'_2 are drawn through the point A parallel to the lines L_1 and L_2 their intersections with the E -scale give the values of E . These values are 0.80 and 0.41.

The curves in Figures 2 and 3 are based on diagram Figure 4. The ordinates of the curves in Figures 2 and 3 for $q-p$ equal to 1, 2, 3, ... are found by reading off on the r_{pq} -scale in Figure 4 the lengths of the ordinates of the lines L_1 and L_2 for the same values 1, 2, 3, ... of $q-p$.

4. Dependence of the Effacement Constant on Locality.

As in the preceding section, we shall disregard the seasonal variation of r_{pq} and take the yearly mean values recorded in Table 3 as valid for certain stationary conditions. By the process given in detail above the characteristic constants r and E for each of the areas considered can be found from the yearly mean values r_1, r_2, \dots . The values found are recorded in Table 5 and are also indicated in Figure 5. The column headed f in Table 5 is explained below (p. 43).

Table 5.

Locality	r	E	f	Locality	r	E	f
1	0.44	0.90	1.5	12	0.54	0.53	1.0
2	.58	.89	1.1	13	.61	.64	1.0
3	.71	.49	1.0	14	.42	.28	1.0
4	.31	.32	1.0	15	.34	.35	1.0
5	.31	.22	1.0	16	.52	.87	1.2
6	.26	.67	1.5	17	.40	.67	1.2
7	.40	.61	1.1	18	.58	.56	1.0
8	.25	.40	1.2	19	.56	(1.25)	—
9	.43	.74	1.2	20	.46	0.93	1.5
10	.40	.90	1.6	21	.43	.93	1.6
11	.50	.82	1.2	22	.51	.88	1.3

Figure 6 gives for each of the areas considered a graphical representation of the yearly means r_1, r_2, r_3, \dots and of the curve $r_{pq} = r^{[(q-p)E]}$. Each part-figure is analogous to Figures 2 and 3.

An inspection of Figure 5 shows that the effacement constant E varies more or less regularly with place. As was seen already in Figures 2 and 3, E is large for Areas 10 and 11 which are influenced by the polar front, whereas it is essentially smaller for Areas 12, 13, 14, and 15 which lie to the south and southwest of Iceland. In the latter areas the temperature anomalies of the Atlantic Current appear fairly clearly, and as these anomalies are of long duration one gets low values of E . The same applies to Station 18 (Thorshavn, in the Faroe Islands). On the other hand, the effacement constant attains high values for Area 16 north of Scotland, for Stations 19, 20, and 21 off the Norwegian coast, and for Station 22 in the eastern part of the North Sea. For coastal waters the meteorological conditions are probably the decisive factors in the temperature anomalies.

In this connection we may examine the variation of surface salinity which may be supposed to be influenced mainly by the currents, whereas surface temperature is probably highly dependent on the air temperature. Suitable data are at hand for Station 22, "Horns Rev" light-vessel. From monthly means of surface salinity for the years 1900—1913, published in the Danish Nautical-Meteorological Annals, we have computed correlation coefficients for salinity in the same way as was done for temperature. The results are given in Table 6.

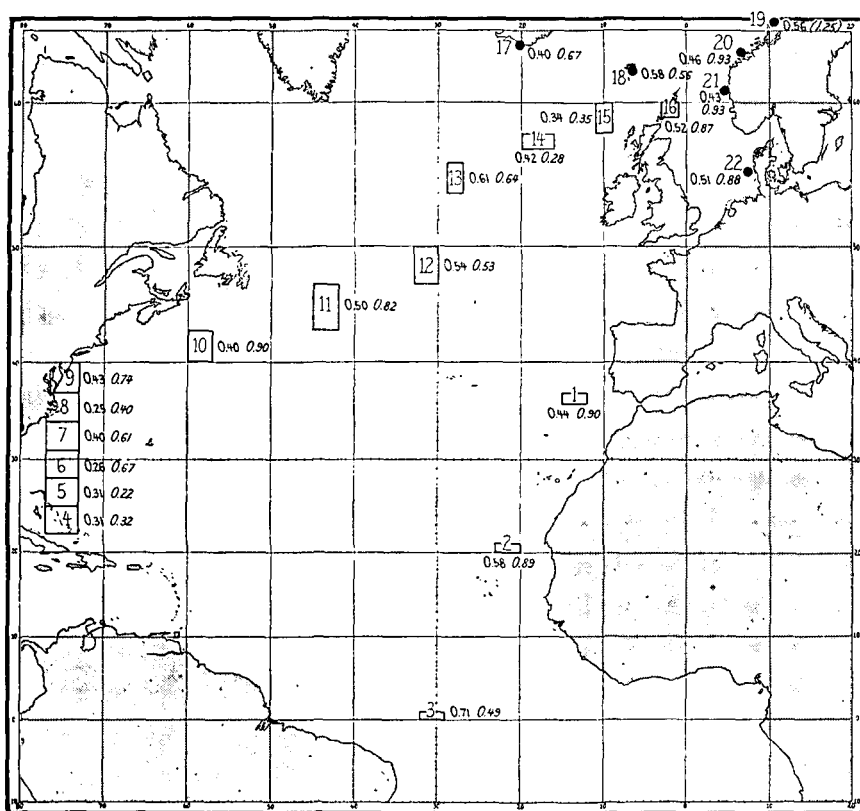


Fig. 5.

Chart showing values of the constants r (figures sloping to the left) and E (figures sloping to the right) for the various areas and stations.

As was done for surface temperature, for the surface salinity we use $r^{[(q-p)E]}$ as an approximate expression for the variation of r_{pq} with $q-p$. Proceeding as explained for the temperature, we obtain the following values: $r = 0.54$, $E = 0.66$. With these constants the variation of r_{pq} will be as shown by the curve in Figure 7 in which the dots indicate the actual yearly means.

A comparison with the constants found for surface temperature at the same station ($r = 0.51$, $E = 0.88$) shows that while the values of r are practically identical, the value of E for salinity is essentially smaller than that for temperature. Thus — as was to be expected — the effacement of a salinity anomaly is slower than the effacement of a temperature anomaly.

For the same locality (22, "Horns Rev" lightvessel) we have also computed correlation coefficients for air temperature as was done for surface temperature and salinity. The material used is the monthly

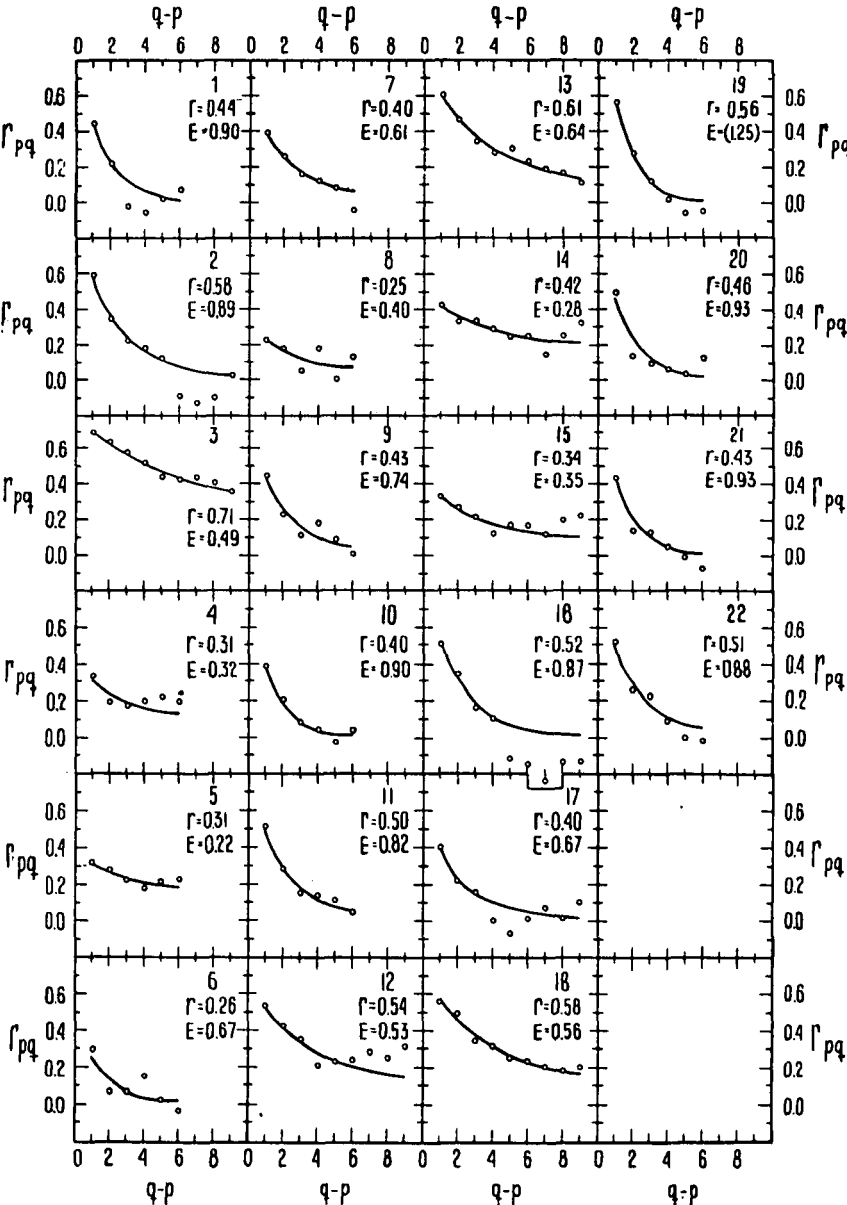


Fig. 6. Curves showing the variation of r_{pq} with $q-p$ for surface temperature at localities 1—22. The dots indicate actual values.

means of air temperature at 8 a. m. during the years 1900—1913, published in the Danish Nautical-Meteorological Annals. The correlations found are given in Table 7.

From an approximation to the curve $r_{pq} = r^{[(q-p)^E]}$ we find the

Table 6. Values of $100 \cdot r_{pq}$ for Surface Salinity.

Locality	q-p	Month												Mean for the year
		J	F	M	A	M	J	J	A	S	O	N	D	
55°34.1'N.	1	77	65	26	37	85	57	55	59	20	34	40	85	53
	2	48	54	17	69	28	52	40	48	14	24	33	32	38
7°19.5'E.	3	29	41	51	55	40	48	10	-07	18	09	20	24	28
	4	-25	31	52	41	37	17	27	52	13	22	-02	00	22
"Horns Rev"	5	-01	27	61	23	21	-31	56	24	-16	29	-24	02	14
	6	-30	-31	39	29	34	32	-05	44	03	-05	-04	-35	06

Table 7. Values of $100 \cdot r_{pq}$ for Air Temperature.

Locality	q-p	Month												Mean for the year
		J	F	M	A	M	J	J	A	S	O	N	D	
55°34.1'N.	1	16	42	08	48	39	56	55	70	69	24	45	20	38
	2	-12	-19	08	41	13	45	51	12	-13	71	-10	-12	15
7°19.5'E.	3	15	-39	65	41	33	76	34	-42	17	18	25	-55	16
	4	-07	20	-06	46	20	16	67	-63	-18	41	08	-13	09
"Horns Rev"	5	-05	23	18	-05	-05	06	-07	-33	43	-02	-65	-26	-05
	6	-19	-50	31	-58	20	-11	10	25	09	12	-64	-12	-09

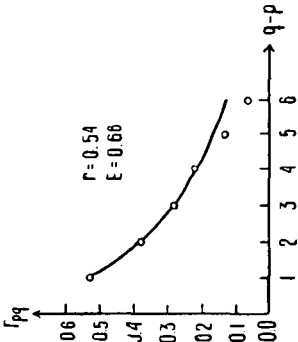


Fig. 7. Curve showing the variation of r_{pq} with $q-p$ for surface salinity at locality 22. The dots indicate actual values.



Fig. 8. Curve showing the variation of r_{pq} with $q-p$ for air temperature at locality 22. The dots indicate actual values.

Form E.

q—p	Jan.	Feb.	Month Mar.	Apr.	May	June
1	$\frac{r_{12\ 13} + r_{12}}{2}$	$\frac{r_{12} + r_{23}}{2}$	$\frac{r_{23} + r_{34}}{2}$	$\frac{r_{34} + r_{45}}{2}$	$\frac{r_{45} + r_{56}}{2}$	$\frac{r_{56} + r_{67}}{2}$
2	$\frac{r_{11\ 13} + r_{13}}{2}$	$\frac{r_{12\ 14} + r_{24}}{2}$	$\frac{r_{13} + r_{35}}{2}$	$\frac{r_{24} + r_{46}}{2}$	$\frac{r_{35} + r_{57}}{2}$	$\frac{r_{46} + r_{68}}{2}$
3	$\frac{r_{10\ 13} + r_{14}}{2}$	$\frac{r_{11\ 14} + r_{25}}{2}$	$\frac{r_{12\ 15} + r_{36}}{2}$	$\frac{r_{14} + r_{47}}{2}$	$\frac{r_{25} + r_{58}}{2}$	$\frac{r_{36} + r_{69}}{2}$
4	$\frac{r_{9\ 13} + r_{15}}{2}$	$\frac{r_{10\ 14} + r_{26}}{2}$	$\frac{r_{11\ 15} + r_{37}}{2}$	$\frac{r_{12\ 16} + r_{48}}{2}$	$\frac{r_{15} + r_{59}}{2}$	$\frac{r_{26} + r_{6\ 10}}{2}$
5	$\frac{r_{8\ 13} + r_{16}}{2}$	$\frac{r_{9\ 14} + r_{27}}{2}$	$\frac{r_{10\ 15} + r_{38}}{2}$	$\frac{r_{11\ 16} + r_{49}}{2}$	$\frac{r_{12\ 17} + r_{5\ 10}}{2}$	$\frac{r_{16} + r_{6\ 11}}{2}$
6	$\frac{r_{7\ 13} + r_{17}}{2}$	$\frac{r_{8\ 14} + r_{28}}{2}$	$\frac{r_{9\ 15} + r_{39}}{2}$	$\frac{r_{10\ 16} + r_{4\ 10}}{2}$	$\frac{r_{11\ 17} + r_{5\ 11}}{2}$	$\frac{r_{12\ 18} + r_{6\ 12}}{2}$
Mean	m_1	m_2	m_3	m_4	m_5	m_6

constants $r = 0.38$, $E = 0.70$. Figure 8 gives the curve $r_{pq} = r^{[(q-p)E]}$ for these values of r and E while the dots indicate the actual values of r_{pq} . The value of E suggests that the anomalies of air temperature are of longer duration than the anomalies of surface water temperature. As regards the air temperature, it should be emphasized, however, that the irregular variation with $q-p$ of the yearly means of r_{pq} involves much uncertainty in the estimate of E .

5. Seasonal Variation of the Correlation Coefficients.

It was mentioned above that the great dispersion of the r_{pq} -values springs naturally from the fact that the number of pairs of values on which they are based is rather small (13 or 14). Moreover, it was explained that if anomalies for months of the same name differ exceptionally from anomalies in the preceding and the subsequent months, a series of correlations situated along sloping lines in a table of r_{pq} will differ in an exceptional way from the neighbouring correlations in the table. Such exceptional correlation values will influence the yearly mean values r_1, r_2, r_3, \dots , but do not prevent these values from varying evenly with $q-p$. On the other hand, the exceptional values may cause the variation of r_{pq} for a fixed value of $q-p$ to become irregular to such a degree that it is impossible to judge whether a regular seasonal variation is present or not. In an examination of the seasonal variation of the correlation coefficients one must therefore set off possible variation by taking averages.

Month						Mean for the year
July	Aug.	Sept.	Oct.	Nov.	Dec.	
$\frac{r_{67} + r_{78}}{2}$	$\frac{r_{78} + r_{89}}{2}$	$\frac{r_{89} + r_{910}}{2}$	$\frac{r_{910} + r_{1011}}{2}$	$\frac{r_{1011} + r_{1112}}{2}$	$\frac{r_{1112} + r_{1213}}{2}$	r_1
$\frac{r_{57} + r_{79}}{2}$	$\frac{r_{68} + r_{810}}{2}$	$\frac{r_{79} + r_{911}}{2}$	$\frac{r_{810} + r_{1012}}{2}$	$\frac{r_{911} + r_{1113}}{2}$	$\frac{r_{1012} + r_{1214}}{2}$	r_2
$\frac{r_{47} + r_{710}}{2}$	$\frac{r_{58} + r_{811}}{2}$	$\frac{r_{69} + r_{912}}{2}$	$\frac{r_{710} + r_{1013}}{2}$	$\frac{r_{811} + r_{1114}}{2}$	$\frac{r_{912} + r_{1215}}{2}$	r_3
$\frac{r_{37} + r_{711}}{2}$	$\frac{r_{48} + r_{812}}{2}$	$\frac{r_{59} + r_{913}}{2}$	$\frac{r_{610} + r_{1014}}{2}$	$\frac{r_{711} + r_{1115}}{2}$	$\frac{r_{812} + r_{1216}}{2}$	r_4
$\frac{r_{27} + r_{712}}{2}$	$\frac{r_{38} + r_{813}}{2}$	$\frac{r_{49} + r_{914}}{2}$	$\frac{r_{510} + r_{1015}}{2}$	$\frac{r_{611} + r_{1116}}{2}$	$\frac{r_{712} + r_{1217}}{2}$	r_5
$\frac{r_{17} + r_{713}}{2}$	$\frac{r_{28} + r_{814}}{2}$	$\frac{r_{39} + r_{915}}{2}$	$\frac{r_{410} + r_{1016}}{2}$	$\frac{r_{511} + r_{1117}}{2}$	$\frac{r_{612} + r_{1218}}{2}$	r_6
m_7	m_8	m_9	m_{10}	m_{11}	m_{12}	

Form E is derived in an easily comprehensible way from Form C by averaging. On the right-hand side of both forms are recorded the mean values r_1, r_2, \dots, r_6 which are identical in the two forms.

In the case of some of the correlations r_{pq} used in obtaining averages, $q \leq 12$, while for the others $q > 12$. As explained above the former were computed from 14 pairs of values, the latter from 13 pairs only. Of the mean values recorded under January in Form E, $r_{1213}, r_{1113}, \dots, r_{713}$ have thus been computed on the basis of the January anomalies for the years 1901—1913 while $r_{12}, r_{13}, \dots, r_{17}$ were computed from the January anomalies for the years 1900—1913. Disregarding this difference we may say that the mean values in the January column are computed from correlations between the January anomalies and the anomalies for all the other months of the year. Among these, however, the anomalies for July are used twice, in both r_{713} and r_{17} , either of which occurs for $q-p=6$. Analogous considerations hold for the other months in Form E.

The mean values m_1, m_2, \dots, m_{12} for each month are recorded at the foot of form E. These mean values are computed from the mean values given in the various columns. In taking these averages the last mean value — for $q-p=6$ — is given the weight 0.5 while the other mean values are given the weight 1. For instance one has thus

$$m_1 = \frac{r_{12} + r_{13} + r_{14} + r_{15} + r_{16} + \frac{r_{17} + r_{713}}{2} + r_{813} + r_{913} + r_{1013} + r_{1113} + r_{1213}}{11}$$

As explained above, r_{17} may be taken as equal to r_{713} and consequently $\frac{r_{17} + r_{713}}{2}$ might be replaced by r_{17} . The January anomalies therefore occur 11 times and the anomalies for each of the other months once in the expression for m_1 . Analogous conditions apply to the other months.

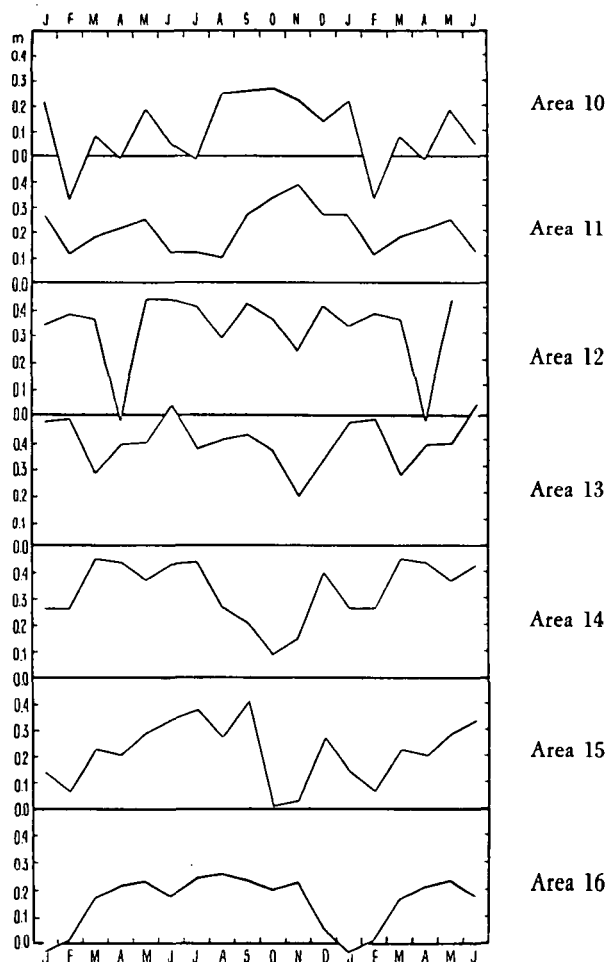


Fig. 9. Seasonal variation of the correlation coefficients for surface temperature at Localities 10–16. For method of computation of the plotted means m , see text.

The seasonal variation of the mean values m found from Form E is given in Figure 9 for some of the areas considered (10–16).

If it is supposed that at a certain time of the year a disturbance of otherwise stationary conditions in the anomalies normally occurs, this

must be expected to become apparent in a typical variation of the mean values m_1, m_2, \dots . That the variation is typical may appear either from it being homogeneous for several neighbouring areas or from it being in accordance with the hydrographical and meteorological conditions in the area considered. A false form of the variations in the m -values may arise if the observations in several areas have been taken by the same ships at short intervals of time. A variation that cannot be brought into accord with known hydrographical or meteorological conditions should therefore be submitted to a closer examination before it can be regarded as real.

The significance of the mean values m_1, m_2, \dots under stationary conditions may to some degree be elucidated by the following consideration. In Section 3 it has been explained that under stationary conditions it may be reckoned that $r_{pq} = r^{[(q-p)E]}$ where r indicates a smoothed value for the correlation between the anomalies of two neighbouring months and where E is the characteristic termed the effacement constant. From the expression $r_{pq} = r^{[(q-p)E]}$ values can be found that are analogous to the correlations in the expression for m_1 . A mean value M analogous to m_1, m_2, \dots can be computed from the formula:—

$$M = \frac{2r + 2r^{(2E)} + 2r^{(3E)} + 2r^{(4E)} + 2r^{(5E)} + r^{(6E)}}{11}$$

In many cases the value of M does not differ much from the value of $r^{(3E)}$. This is to say that when r_{pq} can be given by $r_{pq} = r^{[(q-p)E]}$ a mean value m can in many cases be reckoned as equal to the correlation for an interval of 3 months.

Table 8 gives the ratio $f = \frac{M}{r^{(3E)}}$ for various values of r and E .

From this table are taken the values of f recorded in Table 5.

Table 8.

TABLE 5.											
Values of $f = \frac{M}{r^{(3E)}}$ for various Values of r and E .											
r	E	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.1	1.0	1.1	1.2	1.4	1.7	2.3	3.5	5.7	10.2	20.2	
0.2	1.0	1.0	1.1	1.2	1.3	1.5	1.9	2.6	3.7	5.7	
0.3	1.0	1.0	1.0	1.1	1.2	1.3	1.4	1.7	2.2	2.9	
0.4	1.0	1.0	1.0	1.1	1.1	1.1	1.2	1.4	1.6	1.9	
0.5	1.0	1.0	1.0	1.0	1.0	1.0	1.1	1.1	1.2	1.3	1.4
0.6	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.1	1.1	1.1	1.2
0.7	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.1
0.8	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
0.9	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0