The Theoretical Effectiveness of Towed-Net Samplers as Related to Sampler Size and to Swimming Speed of Organisms

By

Richard A. Barkley

Bureau of Commercial Fisheries Biological Laboratory, Honolulu, Hawaii

Introduction

One of the vexing problems which confront the marine biologist is that of evaluating the effectiveness of towed nets as samplers of the marine biota; the obverse of this problem is that of designing towed nets to sample specific portions of the marine community. Studies of sampling effectiveness have thus far been based primarily upon statistical analysis of samples obtained in the field, often on an *a posteriori* basis; net design has been based upon past experience and trial and error. In the present analysis we shall depart from this pattern to consider, in a rather elementary way, the physical factors which ulimately limit a net's ability to catch a single organism. The results provide a basis for estimating the minimal effectiveness of existing towed nets and for improving net designs.

Method

Figure 1 shows the relationship which exists when a towed net approaches an individual organism. The organism is assumed to sense the oncoming net and to react to it by attempting to dodge. When it begins to react, the organism is located at point p, which is a distance x_0 ahead of the net and another distance r_0 away from the axis of motion of the net, which has a radius of R and is being towed at some speed U. The reaction of the organism carries it through the water at some speed u, in a direction which is at some angle θ to the net's axis of motion. As a result, the organism will either be outside or inside the rim of the net when the net catches up with it; if it is inside, it is assumed that the organism has been captured.

Since the objective of towed-net sampling is to obtain the maximum possible catch from the water column through which the net is towed, the net should be designed to minimize the effects of dodging; it will be seen that this can be

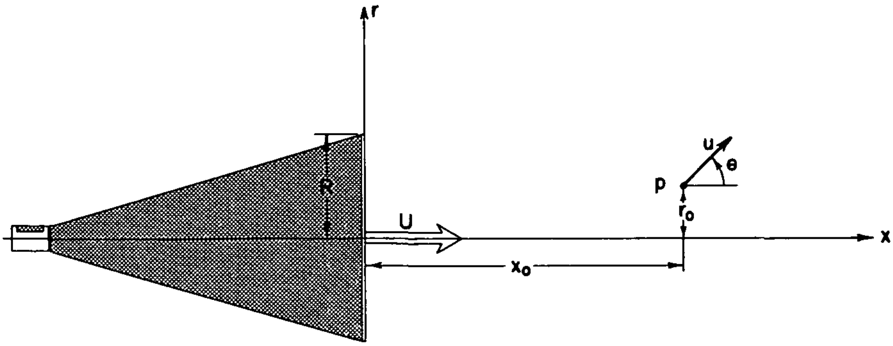


Figure 1. Coordinate system and notation used, showing conditions at the instant when the organism, at point p, begins to react to the presence of the net. For full description see text.

done by making the radius of the net's opening as large as possible and by towing at the highest possible speed. However, these two conditions conflict, because with a given amount of power a large net cannot be towed as fast as a smaller one, owing to drag effects. The problem can be stated thus: what is the optimum size of a towed-net sampler; what are the characteristics of this optimum-size net in terms of its efficiency in capturing organisms which can attempt to dodge it; and how can these characteristics be improved?

The problem stated above has been subjected to a mathematical analysis. The analysis is given in the Appendix, while the argument on which the analysis is based and its results, which are of more immediate interest, are discussed in the body of this paper. For the sake of simplicity it will be assumed that the net allows all the water in its path to pass through its opening. In practice a correction may have to be made for the fact that the net may push some of this water aside, so that R represents the effective filtering radius of the net. This effective filtering radius would depend, *inter alia*, upon the shape of the net, its mesh and cordage sizes, and the speed of the tow. It could be determined by experiments in towing tanks or flumes.

Results

In the Appendix an equation is derived which gives the minimal escape velocity (u_e) relationships for an organism in terms of its reaction distance (x_0) and initial offset (r_0), as well as the speed (u) and direction (θ) of the organism and the speed (U) and radius (R) of the net. This relationship can be shown in a vector diagram, in which distances are proportional to speeds; Figure 2 shows such a diagram for each of two similar cases, together with schematic drawings showing the relative positions of the nets in each example. In Figure 2, examples A and B differ only in that in the first case the initial offset of the organism is zero, while in the second the initial offset is 25 cm; the values of the other constants are: radius of net $R = 100$ cm, reaction distance $x_0 = 250$ cm, speed of tow $U = 150$ cm/sec (2·9 knots). The escape velocity is proportional to the distance between the starting point of the organism, p, and the line u_e, which is the locus of the escape velocity vectors. Two cases of particular

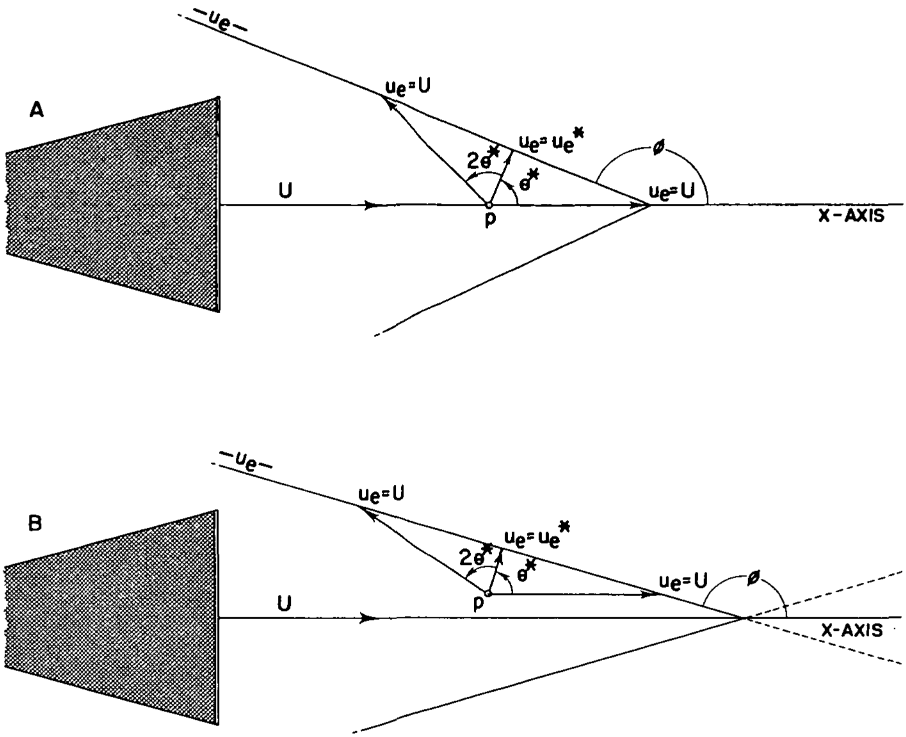


Figure 2. Escape velocity relationships for a net of 100 cm radius, moving at a speed of 150 cm/sec toward an organism which reacts at a distance of 250 cm. Initial offsets (r_0) of zero (example A) and 25 cm (example B). Escape velocity is proportional to the distance between point p and the line marked u_e.

interest are shown explicitly as vectors: the cases where the escape velocity equals the towing speed ($u_e = U$) and the case where the escape velocity is at a minimum, that is, where the vector is perpendicular to the curve u_e. This minimum escape velocity will be referred to by the symbol u_e^*. The angle corresponding to this minimum escape velocity is θ^*. It is worth noting that the escape velocity equals the velocity of the net when the organism swims parallel to the net's axis of motion ($\theta = 0$) or when it swims at an angle equal to $2\theta^*$; at intermediate angles the escape velocity is less than the speed of the net, and at larger angles it is greater than the towing speed.

For every set of values of R, U, x_0 and r_0 there is a minimum value of the escape velocity, u_e^*. It is possible to use this value to determine the minimum effectiveness of a net already in use, or as a criterion for improved net designs; that net having the highest value of u_e^* is desirable in that it has the highest threshold against escape by dodging. Equation 7 in the Appendix shows the way in which the minimum escape velocity varies with changes in the other variables in the problem; in general, the escape velocity increases proportionately with an increase in either the towing speed or the net radius, and decreases in proportion to increases in either the reaction distance or the initial offset of the organism.

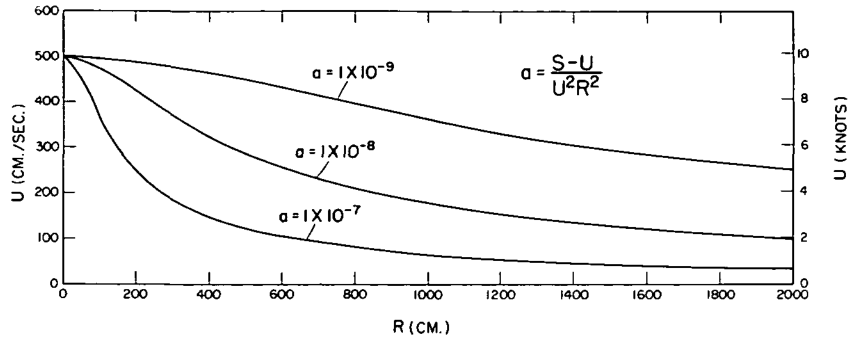
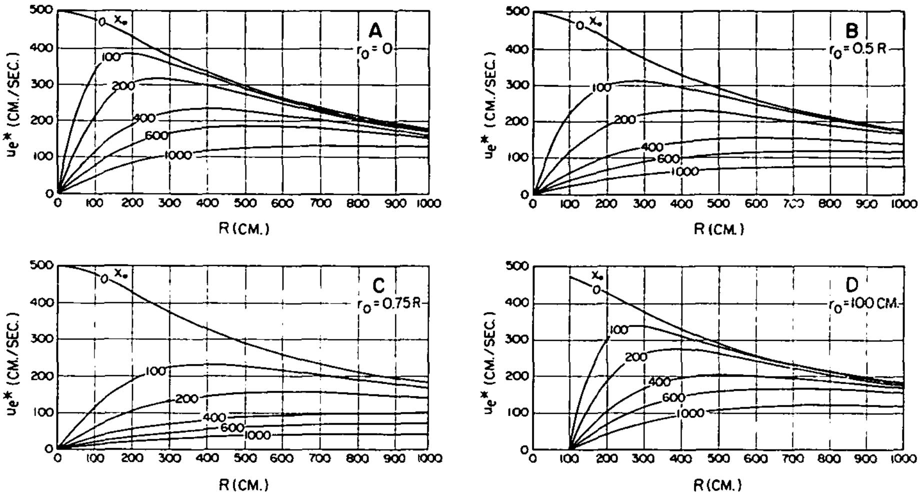


Figure 3. Assumed relationship between towing speed U and radius of net R, from equation (9) (Appendix), for three cases: Upper line, relatively powerful ship (or winch) or low-drag net design. Middle line, ship of medium power or medium-drag net design. Lower line, relatively low-powered ship or high-drag net design. See Appendix for a full discussion.

This suggests that both U, the towing speed, and R, the radius of the net, should be increased to increase the effectiveness of the net. But there is a limit to the advantage to be gained from such increases, since the towing speed cannot be greater than the maximum speed of the ship or the winch which does the towing, and this speed is in turn dependent upon the drag of the net, which increases with an increase in the net's radius. Figure 3 shows the maximum towing speeds which would result from changes in the radius of a single net design, for ships or winches of three ranges of power, or for nets of high, medium, and low inherent drag with one ship or winch. The towing speed is dependent upon the maximum speed of ship or winch without the net (S), the speed with the net in the water (U), and the size of the net's opening (R); the relationship which is assumed to exist between these factors is discussed in the Appendix.

It is obvious from the relationship shown in Figure 3, with towing speed dropping off with increased radius of the net towed, together with the relationships in equations (5) and (7), in the Appendix, where catch effectiveness (as measured by the escape velocity) increases with an increase in R, that there must be some combination of speed of tow and size of net which yields the highest, or optimum, value of u_e^*, the minimum escape velocity. Figure 4 shows examples of such relationships for a vessel of medium power (middle curve in Figure 3, where $a = 1 \times 10^{-8}$) and various combinations of x_0 and r_0. Each family of curves in Figure 4 shows the values of minimum escape velocity u_e^* as a function of net radius R for one value of r_0 and various values of x_0. In each case it is evident that there is an optimum net radius such that u_e^* is at a maximum.

Two important conclusions are evident from the curves presented in Figure 4. First, the minimum escape velocities decrease rapidly as R decreases below the optimum values, so that it is most inefficient to reduce the net opening to low values, because the gain in speed of tow is more than offset by the ease with which the smaller net may be avoided. Second, the importance of minimizing the reaction distance x_0 is obvious; a net which cannot be detected ($x_0 \leqq 0$) would have a minimum escape velocity equal to or greater than, the towing speed, while minimum escape velocities for nets detectable at 10 metres are



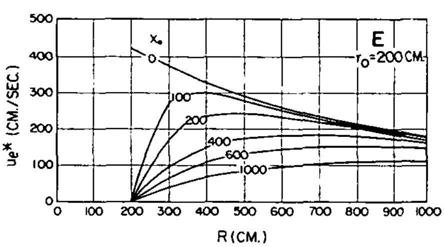


Figure 4. Minimum escape velocities (u_e*) as functions of net radius (R), from equations (5) and (11) (Appendix), for a vessel or winch of medium power or a medium-drag net design, with various values of initial offset r_0 and reaction distance x_0. Initial offsets are expressed as constant values (Panels D, E) or in terms of fractions of the net's radius (Panels B, C).

small fractions of the towing speed even at their optimum size, except under the most favourable conditions for extremely large nets.

Table 1 presents a comparison of the minimum escape velocities for nets of 50 cm radius (the well-known 1-metre net), 300 cm radius, and the optimum radius. It suggests that a net with 300 cm radius is several times more effective than the 50 cm net, and only slightly less effective than the optimum sizes, which tend toward completely unmanageable dimensions.

One final aspect of the present analysis deserves further consideration. In Figure 2 A, the locus of all values of the escape velocity u_e was presented as a line for one set of values for U, R, and x_0, when r_0 equalled zero. If x_0 and r_0 remained unchanged, and only R were allowed to vary, a series of lines, similar to those of Figure 2A, loci of u_e, would result, one for each value of R. If these were arranged in sequence by increasing values of R, the result would be a surface representing the field of values for u_e for a series of similar nets of varying size. Figure 5 is an example of such a surface, together with a schematic drawing of the sampling nets to the same scale in isometric projection, showing the assumed physical relationship between the organism (at the origin $u_e = 0$ when $t = 0$) and various possible sizes of net. The constants used in calculations for Figure 5 were: $r_0 = 0$, $x_0 = 400$ cm, $S = 500$ cm/sec, and $a = 1 \times 10^{-8}$ cm³/sec. Isotachs (solid heavy curved lines in Figure 5) are shown as intersections of cylinders of radius 100, 200, 300, 400 and 500 cm/sec with the surface of u_e. The point on the dashed curve marked with an encircled x represents the maximum value of u_e* for the surface depicted in the

Table 1

Minimum escape velocities for nets of various radii (50 cm, 300 cm, and optimum) as a function of reaction distance for various amounts of initial offset

Initial offset r_0	Reaction distance x_0	Minimum escape velocities u_e^* 50 cm	300 cm	Optimum	R_{opt}
0 cm	1,000 cm	25 cm/sec	110 cm/sec	135 cm/sec	(680) cm
	600	41	170	185	(525)
	400	62	225	235	(410)
	200	125	310	315	(275)
100 cm	1,000	0	75	120	(850)
	600	0	120	170	(660)
	400	0	170	200	(520)
	200	0	260	275	(390)
0·75 *R*	1,000	10	55	80	(> 1,000)
	600	20	90	120	(780)
	400	30	130	155	(600)
	200	60	220	230	(410)

figure. Figure 4A shows the same curve (in two dimensions) of u_e^* versus R, for $x_0 = 400$, as that shown in isometric projection on Figure 5. One most interesting relationship shown in Figure 5 is the range of values of u_e resulting from each of various values of the radius R. Considering only values of the angle θ between 0° and 90° we find, for example, that a net with radius 50 cm has a range of escape velocities from about 500 to 60 cm/sec, depending upon the value of θ, the direction in which the organism swims in attempting to escape. Similarly, at $R = 400$ cm, we observe that the escape velocity ranges from maximum values of 330 cm/sec at $\theta = 0°$ and 90°, to 230 cm/sec at $\theta = \theta^*$. Evidently the net with a radius of 400 cm is far less subject to the effects of changes in escape angle than is that with a radius of 50 cm, at least for the case where θ does not exceed 90°; that is, in those cases where the avoidance reaction does not involve swimming toward the oncoming net. Presumably, then, the effectiveness of a net of 400 cm radius would be only slightly affected by varying behaviour patterns of organisms with respect to their choice of direction for dodging the oncoming net; its selectiveness would then be largely a function of swimming speed. Comparison of catches with nets of different radii may make it possible to estimate both swimming speed and preferential directions of dodging for a number of species of organisms.

Discussion

The analysis thus far has assumed a circular net opening and a speed of tow limited only by the power available for towing, and it has treated only the capture of single organisms. Clearly such an analysis cannot begin to elucidate all of the complex problems involved in sampling the marine biota with towed nets. It can only indicate the magnitude of the limiting physical factors involved in the sampling process, suggest means for avoiding inherently poor physical designs, and point the way toward fruitful avenues for research on towed nets.

Some extensions of the simple theory are obvious: a net of noncircular opening can be treated by redefining R to mean the least distance between the centre of the opening and the rim of the opening, and by substituting A, the

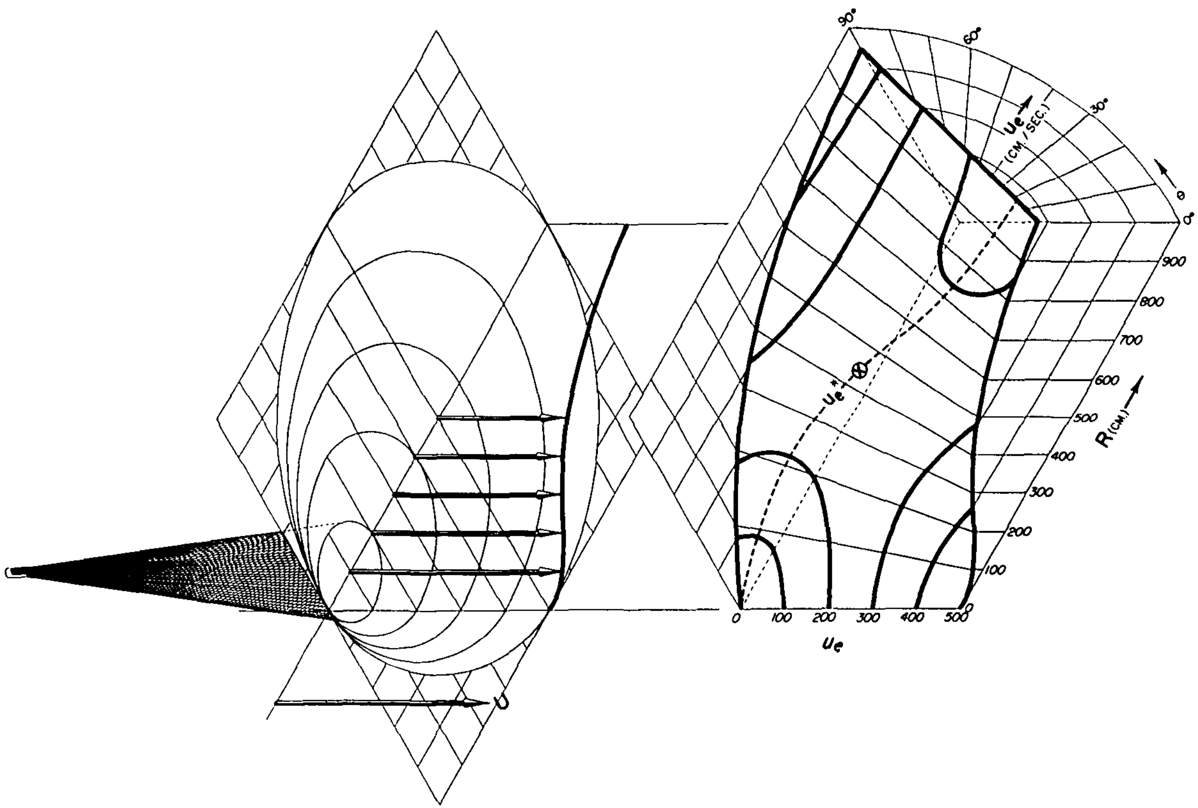


Figure 5. The field of escape velocities, as a function of escape angle θ, from equations (5) and (11) in the Appendix, for *various sizes of one net design, together with a schematic drawing of sampling nets illustrating the physical relationship* which is assumed to exist between the organism and various sizes of net. Escape velocity is proportional to the distance from the origin ($u_e = 0$) to the surface on which the isotachs (heavy curved lines) lie. For full description see text.

frontal area of the net, for R^2 in equation (9) in the Appendix. But this is a trivial substitution, because the frontal area of any net, regardless of shape, will vary as the square of the linear dimensions of the opening when the net design is scaled up or down in size.

The analysis might be extended to the case where the net is towed at less than the maximum possible speed. In this case the maxima in escape velocities as shown in Figure 4 would simply be truncated at the level where u_e^* is equal to the restricted towing speed.

A more fundamental extension of the theory would be that of considering a number of encounters instead of conditions for a single capture. This will require making assumptions about the distribution of organisms in the water. For example, it can be calculated that a randomly distributed population in the net's path would encounter the net in such a way that the initial offset r_0 would have an average value $\bar{r}_0$, equal to $\frac{R}{\sqrt{2}}$, because this is the radius of a circle whose area is one-half that of the net with an opening of radius R; one-half of the organisms encountered would be found inside of $\bar{r}_0$, and one-half between $\bar{r}_0$ and R. This elementary result assumes, of course, that there is no interaction between the organisms.

By these and other means it should be possible to determine the minimum escape velocities of a variety of net designs. More immediate tests of the conclusions reached would require systematic comparative trials of nets, such as that reported for smaller nets by KÜNNE (1929 and 1933), but over wider ranges of net size. An example of intercomparison of larger nets similar in design but differing in size is ARON's (1962) recent survey of equipment used for sampling the larger plankton. ARON (1962) concludes that the Isaacs-Kidd midwater trawl appears to be the best all-around device available today for sampling the larger plankton and small nekton. It is reasonable to assume that this conclusion is at least in part due to the near-optimum size of the Isaacs-Kidd trawl, as well as its ease of handling.

In addition to simple changes in size, the analysis suggests that there is much to be gained by greatly decreasing the distance ahead of the net at which organisms begin to react to its approach, as by reducing its visibility or making it move more silently. This idea is by no means novel, but its quantitative value is made evident here for the first time.

Finally, one of the most significant conclusions reached is that nets of non-optimum size are relatively sensitive to the direction in which an organism moves in attempting to avoid the net. In the absence of such effects, a towed net would select its catch primarily on the basis of swimming speed. Elimination of swimming direction as an important variable in sampling is itself a strong argument for increasing the size of sampling nets to more nearly optimum dimensions than the 0·5 m and 1·0 m nets now in common use.

Acknowledgement

The author is indebted to Dr. BRIAN J. ROTHSCHILD, of this Laboratory, for several helpful suggestions made during the course of preparations of this manuscript, and in particular for pointing out the fact that comparison of catches with nets of different physical properties may be useful in estimating behaviour-dependent variability in catches.

Summary

1. The "dodging" problem is analysed in geometic terms by determining the swimming velocity which an organism must attain in order to escape a net of given radius towed at a known speed.

2. For every set of conditions there exists a minimum escape velocity the value of which can be computed and used as a criterion for the design and evaluation of towed-net sampling gear.

3. Under the assumption that the net is towed at the highest possible speed, it is shown that for nets of too small a radius the ease with which the net may be avoided more than compensates for the fact that small nets can be towed at high speed.

4. Similarly, it is shown that too large a net results in a loss in towing speed which more than makes up for the difficulty an organism might have in avoiding a large net.

5. Accordingly, there exists an intermediate, optimum net size such that the minimum escape velocity is as large as possible.

6. For nets of optimum and near-optimum size, catch selectivity should be primarily a function af swimming speed; for non-optimum net sizes the direction in which an organism swims may be as important a factor as swimming speed.

7. The analysis suggests that the greatest penalty is incurred when a net is made much smaller than optimum size, rather than much larger.

8. Marked increases in effectiveness can be expected from new types of nets which are difficult to detect, so that the distance at which organisms begin to react to their approach is minimized.

APPENDIX

Mathematical Analysis

Figure 1 shows the geometry of the problem and the coordinate system and notation used. The coordinate system is a cylindrical one, fixed with respect to the water. The X-axis coincides with the axis of movement of the net. The origin is fixed at the location of the centre of the net opening at the instant when the organism begins to react to the net's approach, at time t_0. At time t_0 the organism is located at a distance x_0 ahead of the net and a distance r_0 from the X-axis. At a later time, t', the net overtakes the organism so that both the net opening and the organism are at a distance x' from the origin; at this same time, the organism is located at a distance r' from the X-axis. If capture is to occur, it is evident that r' must be less than R, the radius of the net's opening. In the time interval between t_0 and t', the net moves along the X-axis with an average velocity of U, while the organism moves with an average velocity of u at an angle θ measured counterclockwise from the direction of the X-axis. We are thus not concerned with the details of the movement of the organism, but only with the initial and final conditions.

In the time interval $t' - t_0$, the net moves a distance x_0 plus the distance travelled by the organism in the X-direction during this same period of time, given by $u \cos \theta (t' - t_0)$:

$$U(t' - t_0) = x_0 + u \cos \theta (t' - t_0) \qquad (1)$$

At the same time, the organism has moved through a distance $u \sin \theta(t' - t_0)$ perpendicular to the X-axis, so that its final distance r' is given by:

$$r' = r_0 + u \sin \theta(t' - t_0) \qquad (2)$$

which, for capture to occur, must be less than the radius of the net R. Rearranging equation (1) and dividing it into (2) yields:

$$\frac{R}{x_0} > \frac{r'}{x_0} = \frac{r_0 + u \sin \theta(t' - t_0)}{(U - u \cos \theta)(t' - t_0)} \qquad (3)$$

where the inequality sign indicates the condition for capture. If we replace R in (3) by R_{min}, and u by u_{max} to represent the maximum speed with which an organism can move and still be just within the rim of the net opening when overtaken, we obtain the following expression for the minimum radius of a net which will always enclose an organism moving with a speed u_{max} at an average angle θ, from a start at position (x_0, r_0), and having a time interval $(t' - t_0)$ in which to react to the net's approach at a speed U along the X-axis:

$$R_{min} = r_0 + x_0 \frac{(u_{max} \sin \theta)}{(U - u_{max} \cos \theta)} \qquad (4)$$

For a discussion of the effectiveness of a net, it is instructive to solve equation (4) for u_{max}, which we may re-name the escape velocity, u_e, if we assume that this escape velocity is infinitesimally greater than the maximum velocity for capture u_{max}, as defined above. Equation (4) then becomes:

$$u_e = \frac{U}{\left(\frac{x_0}{R - r_0}\right) \sin \theta + \cos \theta} \qquad (5)$$

Equation (5) gives the escape velocity for an organism as a function of the parameters of the problem. Equation (5) is the equation for a straight line which represents the locus of all values of u_e corresponding to one set of values for R, U, x_0, and r_0.

In each case there exists a minimum value of u_e, which will be referred to as u_e*, the minimum escape velocity. This value is illustrated in Figure 2; it corresponds to the perpendicular from point $p(x_0, r_0)$ to the line of equation (5).

It should be remembered that equation (5) only yields one branch of the locus u_e, namely that portion between the X-axis intercept and the value of θ where $\theta = \Phi$ (see Figure 2). The X-axis is an axis of symmetry, and those portions of the locus of u_e which lie on the side of the X-axis opposite of that where point p is found are solutions without physical meaning, although they are mathematically correct. In Figure 2B these invalid solutions for u_e are shown as broken lines. The branches of the curves shown in Figure 2 which lie below the X-axis were obtained from considerations of symmetry about the X-axis.

To determine u_e*, equation (5) is converted to the general form of a linear equation in Cartesian coordinates, $Ax + By + C = 0$, and thence by dividing by $\sqrt{A^2 + B^2}$, to the normal form, $x \cos \theta + y \sin \theta - p = 0$, whe·c p is the perpendicular distance from the origin to the line u_e and θ has the value of θ* (see, for example, ESHBACH (1952), p. 2–80, or any other handbook's section on analytic geometry). This procedure yields the equations

$$(R - r_0)x + x_0y - U(R - r_0) = 0$$

and

$$\frac{x + \dfrac{x_0}{R - r_0}y + U}{\sqrt{1 + \dfrac{x_0^2}{(R - r_0)^2}}} = 0 \qquad (6)$$

where the X- and Y-axes are centred on the position of the organism at time t_0, replacing the polar coordinate system used previously. The term with U in equation (6) is the desired value of the vector u_e^*:

$$u_e^* = \frac{U}{\sqrt{1 + \dfrac{x_0^2}{(R - r_0)^2}}} \qquad (7)$$

while the angle θ^* is given by

$$\cos \theta^* = \frac{1}{\sqrt{1 + \dfrac{x_0^2}{(R - r_0)^2}}} \qquad (8)$$

To design an effective net, it is necessary to determine the way in which u_e^* varies with changes in R, the radius of the net. We shall, therefore, consider the way in which U varies with changes in R, since U and R are the only variables in equation (7) which are assumed to vary with changes in R. Up to now the discussion has been completely general, applicable to all nets having a circular opening, or to nets which are not circular, in which case R is merely one-half the minimum diameter of the opening. It will be necessary, however, to introduce some assumptions to carry the analysis further. These assumptions are:

1. That the net is towed at the maximum possible speed, so that the major effect of increasing the net's radius is to increase the drag of the net, thus slowing the towing vessel down, and
2. That the drag of the net is proportional to the area presented by the net opening, and to the square of the velocity of tow.

The second assumption is based on the results of theoretical and experimental work on the drag of a body submerged in a moving fluid. Although it would be most desirable to study the details of fluid flow and drag for various towed-net designs, there is little reason to doubt the validity of the second assumption for present purposes. The assumptions lead to the following equation for U as a function of R:

$$U = S - aR^2U^2 \qquad (9)$$

where S is the speed of the ship with her engines set for towing, but without encumbrances, R is the radius of the net towed, U is the speed of tow with a given net, and a is an empirical constant combining the dimensions and numerical values of drag coefficients, the power exerted by the vessel, and other similar variables which are independent of R. It is possible to determine a for a given net by running the vessel over a measured distance to determine S, then setting the net and determining U, the speed of tow, over the measured course. The value of a is then given by:

$$a = \frac{S - U}{R^2 U^2} \qquad (10)$$

which is readily derived from equation (9). With units of centimetres and seconds, it will be found that for most vessels a is of the order of magnitude of 10^{-8} sec/cm^3, with larger values for less powerful vessels or nets with inherently higher drag and vice versa. Figure 3 shows curves of towing velocity as a function of net radius, for three values of a, computed from equation (9) in the following form:

$$U = \frac{-1 \pm \sqrt{1 + 4aSR^2}}{2aR^2} \qquad (11)$$

remembering that only positive values of U have physical meaning in the present context.

The above approach is of course valid not only for the case where the ship's main engines furnish the motive power, but also in cases where a winch, or a combination of winch and main engines furnish the power.

References

Aron, W., 1962. "Some aspects of sampling the macroplankton". Rapp. Cons. Explor. Mer, **153**: 29–38.

Eshbach, O.W., 1952. "Handbook of Engineering Fundamentals". (Second Edition) New York, Wiley & Sons, 80 pp.

Künne, C., 1929. "Vergleich der Fangfähigkeit verschiedener Modelle von Plankton-Netzen". Rapp. Cons. Explor. Mer, **59**: 1–32.

Künne, C., 1933. "Weitere Untersuchungen zum Vergleich der Fangfähigkeit verschiedener Modelle von vertikal fischenden Plankton-Netzen". Rapp. Cons. Explor. Mer, **83**: 1–35.