Note on the Theoretical Courses of Ocean-Currents.

By

V. Walfrid Ekman, Lund.

In a paper[1]) of 1923 I deduced dynamically some mathematical equations showing a close dependence of a current on the topography of the sea-bottom and the direction of the North. Particularly the investigation had for its object the laws governing the currents produced by gravity in homogeneous water when the sea-surface is inclined to the horizon. Such currents, when running from a deeper towards a shallower part of the sea or *vice versa,* were proved to be subject to a "topographic rotational effect" causing them to deviate cum sole or contra solem respectively (i. e. to the right or to the left in northern latitudes). Likewise a current not moving due east or west would be subject to a "planetary rotational effect" causing it in most cases to deviate eastwards. The two effects combined would govern the actual course of the current.

Now this simple theory appears to need improvement on an important point; and since its results have actually been referred to for the interpretation of oceanographical observations, I think it would be proper at an early convenience to point out its principal defect and the directions in which its results are likely to be modified. I am indebted to the Editor, who has at rather short notice kindly found room for this note. It is hoped that a fuller account of the subject will appear within not too long a time in another place.

The original simple theory was founded on the assumption that the real forces acting horizontally on a mass of water are sufficiently accurately balanced by the Coriolean force due to the earth's rotation, so that the acceleration of the water relative to the earth can be left

[1]) "Über Horizontalzirkulation bei winderzeugten Meeresströmungen", Arkiv f. Mat. Astr. o. Fys. 17 Nr. 26, Stockholm 1923.

See also the article "Meeresströmungen" in "Handbuch der physikalischen und technischen Mechanik" herausg. von F. Auerbach und W. Hort, B. 5, Leipzig 1927.

out of account in the dynamical equations. In a previous paper[1]) I have given reasons for this assumption, and no doubt it is for most purposes legitimate, when actual stationary ocean-currents are being considered. However, in the theory in question even a very minute acceleration will be of importance, if the depth of the sea is many times greater than the "depth of frictional influence". For this particular purpose therefore, the assumption is not legitimate.

The principal alterations to be made are best explained by the help of the following two simplified equations representing the original and the revised theory respectively. In both equations terms which, though not insignificant, are of an inferior order of importance have been omitted. The equations are in the case of the original theory:

$$\frac{D''}{2\pi d} \cdot \frac{1}{G} \operatorname{curl} G = \frac{1}{d} \frac{\delta d}{\delta s} - \frac{\cos \alpha}{R \tan \varphi} \tag{1}$$

and in the case of the revised theory:

$$\frac{D''}{2\pi d} \cdot \frac{1}{G} \operatorname{curl} G + \frac{1}{2\omega \sin \varphi} \frac{\delta \operatorname{curl} G}{\delta s} = \frac{1}{d} \frac{\delta d}{\delta s} - \frac{\cos \alpha}{R \tan \varphi}. \tag{2}$$

Here d is the depth of the sea and D'' is the "bottom" value of the "depth of frictional influence", so that at all levels at a greater distance than D'' above the sea-bottom, the current has — in any case within the homogeneous deep water — one direction and velocity; this direction and velocity being denoted by the letter G. R is the earth's radius, ω its angular velocity and φ the latitude; and α is the angle between the meridian (drawn towards increasing latitudes) and the direction of the current G. Finally curl G is the well-known vector operation of G equal to twice the average velocity of rotation of the water, this rotation being reckoned positive when running contra solem; and $\delta/\delta s$ means a derivative taken along the course of the current itself, so that for instance $\delta d/\delta s$ is the slope of the sea-bottom in the direction of the current.

The interest attached to these equations is chiefly due to the circumstance that they might give values of curl G greater than would be *consistent with any current of a not altogether local character. Currents* carrying water from one part of the ocean to another must follow such courses that this does not happen.

It is difficult at present to say anything for certain about the relative

[1]) "Beiträge zur Theorie der Meeresströmungen", Ann. d. Hydrographie u. marit. Meteorologie, 34, 1906.

magnitude and importance of the two terms on the left hand side of equation (2), since it partly depends on the velocity of the current and other circumstances which are still unknown or varying from one part of the ocean to another. The new, second term may under certain circumstances be of inferior importance, but in other cases it might be the dominating one.

The first term on the right hand side represents the topographic rotational effect. According to both theories this effect leaves the current unaffected only in the case when the latter is following the level lines of the sea-bottom; and it very effectively prevents the current (by deviating it to the right or to the left) from traversing a bottom ridge or groove or from entering into a deeper or a shallower part of the sea, if the differences of depth are large enough.

The important alterations are the following:

Firstly. According to the original theory (equation 1) the curvature of the stream-lines is directly connected with the slope $\delta d/\delta s$ of the sea-bottom, the current turning contra solem when running in the direction of deeper water and cum sole when running in the direction of shallower water. According to the revised theory the connection is a more complex one, so that there is a tendency to contra solem rotation not only where the water is deepening in the direction of the current but also where the depth is a maximum, and vice versa.

Secondly. According to the original theory the power of a submarine ridge (or groove) to prevent a current from traversing it would depend simply on its absolute height as compared to D''. According to the revised theory this power will depend on various circumstances. A certain height as compared to D'' will always be necessary for the purpose, but the depth of the sea and the velocity of the current are of consequence as well. For even if the height of the ridge (or the depth of the groove) exceeds D'', it will not effectively check the current, if the area of its cross-section is sufficiently small as compared to the area swept out by a drifting plumb-line reaching from the surface down to the bottom and drifting with the current for one pendulum day (24 sidereal hours divided by sin φ).

As far as the planetary effect alone is concerned (represented by the second term on the right hand side) the results of the theory seem to be in the main unaltered, the directions of currents being still in low latitudes restricted to east or west approximately.

Finally I wish to emphasize the observation in my 1923-paper that the above-mentioned laws refer only to the gradient currents in homogeneous water and to some extent to convection currents carried along on top of such gradient currents. They do not refer to convection currents in the upper layers in cases where the deeper water-layers have no appreciable motion. As to the apparent connection between the bottom-topography and the various bends in the paths of ocean-currents which may be observed on oceanographical maps, its dynamical explanation does not seem to be quite clear yet, and I would not like to enter upon this matter for the present.