8. Appendix

Analysis of Target Discrimination with Sector Scanning Equipment

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Close to the ship, sector scanning equipment can discriminate between single fish, even when they are close together. Further away, discrimination is less fine, and discrete echoes may be returned from several individual fish. In two dimensions the discrimination may be described by the difference which can be detected in range (dr) or in bearing ($d\theta$). That is, the target area can be divided into small compartments, area $r\,d\theta\,dr$, such that individual targets in different compartments will be recorded separately, but individuals in the same compartment, however numerous, will only produce a single record. In three dimensions the angular discrimination $d\theta$ can be replaced·by a solid angle $d\varphi$, not necessarily circular, and the volume of the discrimination compartments will be $r^2\,d\varphi\,dr$.

Thus in unit area there will be $\dfrac{1}{r\,d\theta\,dr} = \dfrac{1}{ar}$ compartments; suppose also that there are N targets (fish) per unit area. Then the probability of there being a target in a particular compartment is Nar. Therefore if the targets are randomly distributed the number of compartments per unit area containing 0, 1, 2 ... targets will be given by the terms in the Poisson distribution

$$\frac{1}{ar}e^{-Nar}\left(1,\ Nar,\ \frac{(Nar)^2}{2!}\ \dots\right).$$

The number of empty compartments $= \dfrac{1}{ar}e^{-Nar}$ and the number of records (compartments with one or more targets) $= \dfrac{1}{ar}(1 - e^{-Nar})$. Note that as $r \rightarrow 0$, this expression $\rightarrow \dfrac{1}{ar}[1 - (1 - Nar)] = N$. For the three-dimensional case the corresponding number of records is

$$\frac{1}{ar^2}(1 - e^{-Nar^2t}),$$

where $a = d\varphi dr$.

In Figure 13, the numbers of discrete echoes, expressed as a percentage of the number of targets, estimated from these expressions have been plotted against range, for the two-dimensional case (above), and the three-dimensional (below,
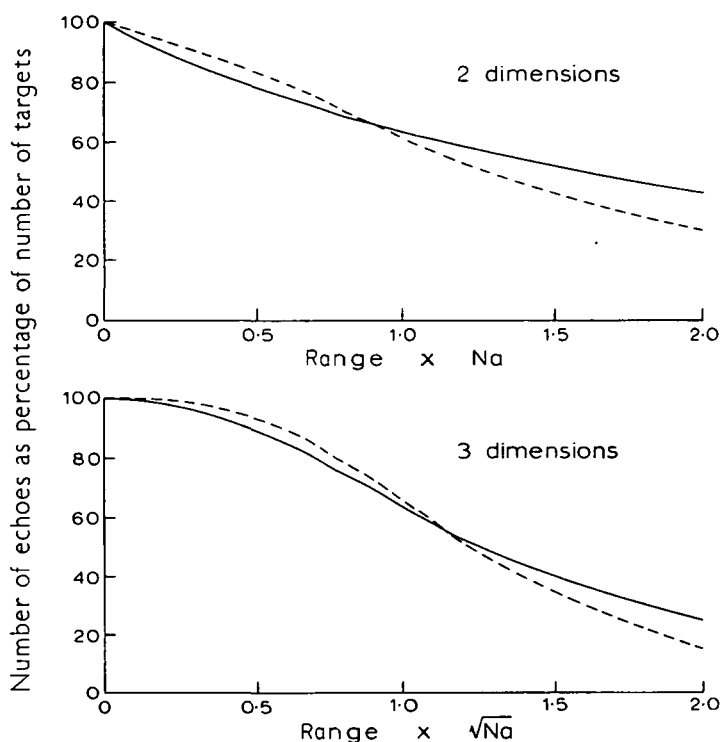
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Figure 13. The number of discrete echoes, exprsesed as a percentage of targets, plotted against range.

full lines). These curves resemble quite closely the actual observations plotted in Figure 6, suggesting that this model does give a reasonable picture of the events. Assuming that the three-dimensional model is applicable then the position of the observations in Figure 6 can be estimated. At relatively short ranges ($r\sqrt{Na} < 0.5$) a doubling in range causes little change in the number of echoes; at long ranges ($r\sqrt{Na} > 3$), doubling the range reduces the number of echoes by nearly four times; the observations cover a two-fold difference in range over which the number of echoes halves, suggesting that the observations correspond to a moderate range $r\sqrt{Na} \simeq 1$, i.e. $r^2a \simeq 1/N$. Thus the size of the discrimination compartments is about equal to the average volume occupied by a single target. The discrimination of the equipment used is 8 cm in range, $\times 0.33° \times 5$–$10°$, which at 100 m range is 8 cm $\times$ 60 cm $\times$ 9–17 m. The vertical extent is very large, but the horizontal dimensions seem reasonable for the average distance between individual fish.

Clearly the model cannot be applied too closely. Fish are not distributed randomly, but normally occur in shoals. At long ranges, when a fish shoal occupies only a small fraction of the volume of sea examined, the fish are clumped together, and the real chance of several targets occurring in the same compartment is greater than suggested by the random model – i.e. there are

fewer echoes. At close range where the shoal occupies much of the target area, the fish are over-dispersed – they keep their distance from each other – and the chances of multiple echoes are less than in the random model (for instance, the compartment may be so small that two targets in the same compartment would imply that the fish were touching). Allowing for the non-random distribution of fish, the actual curves of number of echoes against range may therefore be like the broken lines in Figure 13.