Competition for Gear in a Multiple-Species Fishery

By

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The catch-per-unit-of-effort (CPUE) for one species may affect the CPUE of another species in a multiple-species fishery. This effect is studied by means of a simple stochastic model. The model provides a "competition free" CPUE which is never less than the CPUE obtained under competition, is strongly dependent on the empty space on or in the gear, and varies as a function of the proportion of each species. The multinomial structure of the model enabled the derivation of maximum-likelihood estimators of its parameters. Estimates of the large sample sampling variances and covariance of the parameters are also provided. Application of concepts of the model to other fishery problems is discussed and it is noted that the model also provides a maximum-likelihood estimate for the well-known yield equation.

Introduction

Many fisheries simultaneously harvest several species of fish. The unit of effort expended in a multiple-species fishery is usually applied to the catch of each species in that fishery. Thus, if 20 units of effort are expended to catch, simultaneously, 10 fish of species 1, and 30 fish of species 2, the catch-per-unit of-effort (CPUE) would be computed as 0·5 and 1·5, respectively. The possibility that the several species taken in a fishery compete for "space" on or in the gear has been infrequently acknowledged; see, however, KETCHEN (1964) and GULLAND (1964). Under the postulate of competition for space on or in the gear, the above values of 0·5 and 1·5 would be underestimates of the CPUE as an index of density for each species independent of the presence of other species. This paper considers the concept of competition for gear by more than one species. The consideration is based on a simple model contrived to demonstrate the concept.

Model

The model exploits the analytical simplicity of the known number of hooks on a longline-type of gear. GULLAND (1955) and MURPHY (1960) also took advantage of this feature to formulate models for gear competition. In our
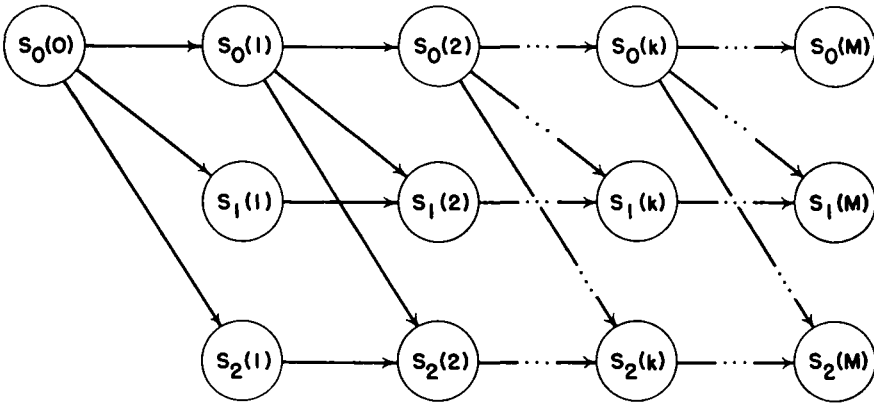
Figure 1. Diagram of possible routes of passage among the states for successive instants of time.

model a single, individual hook is placed in the water and examined for $k = 1, 2, \ldots, M$ successive instants of time (in reality, however, we can only examine the hook when we remove it from the water at the end of the M-th instant of time). During each instant the hook can remain empty, catch species 1, or catch species 2. Furthermore, we assume that the hook can remain empty for all instants of time, but if species 1 takes the hook it remains on that hook until the end of the M-th instant, and likewise for species 2. The events that the hook is empty in any instant, contains species 1, or contains species 2 are denoted as states S_0, S_1, S_2, respectively. This set of rules, which delimits the possible routes of passage among the states through the successive instants of time, is diagrammed in Figure 1. The probabilities of $S_i(k - 1)$ passing to $S_j(k)$ are q_{ij}, which for any i, j is constant for all k. In this particular model $q_{00} > 0$, $q_{01} > 0$, $q_{02} > 0$, $q_{11} = 1$, $q_{22} = 1$, all other $q_{ij} = 0$.

We define the probability that the system is in the i-th state at time M as Q_i,

$$P[S_i(M)] = Q_i, \qquad i = 0, 1, 2 \qquad (1)$$

The probabilities Q_i can be estimated from a set of independent hooks (a collection of hooks such that the probability of any hook containing a fish is unchanged by the event that any other hook in the collection contains a fish). The question of independence among hooks was considered by MAÉDA (1960) and MURPHY and ELLIOT (1954). As the hooks are removed from the water at the end of the M-th instant, the number that are empty (n_0), the number containing species 1 (n_1), and the number containing species 2 (n_2) provide an estimate of Q_i, viz: $\hat{Q}_i = n_i/N$, where $N = \Sigma n_i$. Note that 100 $\hat{Q}_1$ or 100 $\hat{Q}_2$ is the familiarly reported catch per hundred hooks of the Japanese longline fishery.

The Q_i ($i = 1, 2$), then, are the probabilities (and also CPUE for the longline fishery) of species i being on the hook in the presence of the possibility that species j could have occupied the hook instead. We now wish to derive from the Q_i's the probability of taking species i in an artificial situation where species j does not exist. This is analogous to the competing-risk-of-death

problem, an elementary exposition of which the reader will find in NEYMAN (1950).

To treat species i independent of species j we imagine, in considering i, that every time species j takes the hook the fish becomes invisible, thus allowing the hook to remain available to species i. All other things being equal, the probability of species i being caught under the fictional condition where species j is absent is,

$$P_{0i} = 1 - (1 - q_{0i})^M, \qquad i = 1, 2.$$

Before we can relate the P_{0i} to the Q_i, however, each must be defined in terms of the q_{ij}. Note that the probability of neither species being caught in any instant is $1 - (q_{01} + q_{02})$, and therefore the probability that the hook will contain neither species at the end of the M-th instant is,

$$P[S_0(M)] = Q_0 = (1 - q_{01} - q_{02})^M. \tag{2}$$

The event that species 1 is on the hook at the M-th instant is

$$
\begin{aligned}
S_1(M) \equiv\ & [S_1(1) \cap S_1(2) \cap \cdots \cap S_1(M)] \cup \\
& [S_0(1) \cap S_1(2) \cap \cdots \cap S_1(M)] \cup \cdots \\
& \cup [S_0(1) \cap S_0(2) \cap \cdots \cap S_0(M-1) \cap S_1(M)].
\end{aligned} \tag{3}
$$

Therefore,

$$
\begin{aligned}
P[S_1(M)] = Q_1 &= q_{01} + \sum_{k=2}^{M} (1 - q_{01} - q_{02})^{k-1} q_{01} \\
&= \frac{q_{01}}{q_{01} + q_{02}} [1 - (1 - q_{01} - q_{02})^M] \\
&= \frac{q_{01}}{q_{01} + q_{02}} (1 - Q_0).
\end{aligned} \tag{4}
$$

In the same manner we obtain,

$$Q_2 = \frac{q_{02}}{q_{01} + q_{02}} (1 - Q_0). \tag{5}$$

NEYMAN (1950) gave a method of evaluating these probabilities for a corresponding time-continuous process. A unit of time is divided into M intervals, each of duration $1/M$, and the transition probabilities are assumed proportional to the time lapse $1/M$; thus $q_{0i} = \lambda_i/M$. Time continuum is approached by letting $M \to \infty$, so that corresponding to P_{0i} we now have

$$P_{0i}{}^* = \lim_{M \to \infty} \left[1 - \left(1 - \frac{\lambda_i}{M} \right)^M \right] = 1 - e^{-\lambda_i} \tag{6}$$

Similarly, the Q_i are replaced by

$$
\begin{aligned}
Q_0{}^* &= e^{-(\lambda_1 + \lambda_2)} \\
Q_1{}^* &= \frac{\lambda_1}{\lambda_1 + \lambda_2} (1 - Q_0{}^*) \\
Q_2{}^* &= \frac{\lambda_2}{\lambda_1 + \lambda_2} (1 - Q_0{}^*).
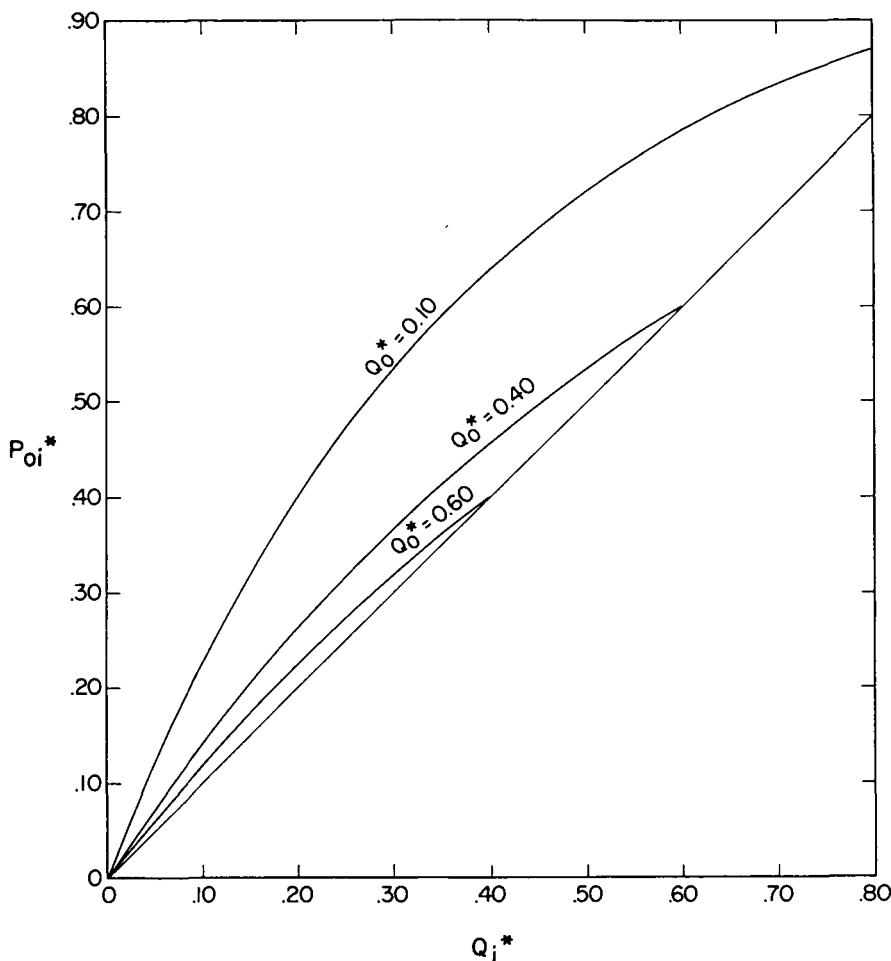\end{aligned} \tag{7}
$$

Figure 2. Relation between $P_{0i}*$ ($i = 1,2$) and Q_i* for various fixed values of Q_0*.

Noting that

$$e^{-\lambda_i} = [e^{-(\lambda_1+\lambda_2)}]^{\frac{\lambda_i}{\lambda_1 + \lambda_2}} = Q_0*^{\frac{Q_i*}{1 - Q_0*}}, \tag{8}$$

we obtain the relation

$$P_{0i}* = 1 - Q_0*^{\frac{Q_i*}{1 - Q_0*}}. \tag{9}$$

Given the Q_i* we can obtain the $P_{0i}*$. Figure 2 shows the relation between Q_i* ($i = 1, 2$) and $P_{0i}*$ for various fixed values of Q_0*. The figure shows that $P_{0i}*$ is never less than Q_i*, $P_{0i}*$ is strongly dependent on Q_0*, and for any fixed Q_0* the deviation of $P_{0i}*$ from Q_i* is not constant, but reaches a maximum at an intermediate mixture of species 1 and species 2. Thus – according to the model – in a multiple-species fishery the CPUE as a density index for species i will tend to underestimate the "true" CPUE because of the presence

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Table 1

Calculations showing adjustment of CPUE for multiple-species catch. Data are taken from Murphy and Shomura (1953). Each row represents the CPUE for a single day of fishing in the vicinity of Canton Island (lat. 3° S., long. 172° W.), July 1950

Catch/ 100 hooks yellowfin tuna	Total catch/ 100 hooks	$\hat{Q}_1$	$\hat{Q}_0$	$\hat{P}_{01}$	Catch/ 100 hooks yellowfin independent of other species	Increase in yellowfin CPUE
7·1	16·5	·071	·832	·076	7·6	0·5
3·2	7·8	·032	·922	·033	3·3	C·1
7·9	19·7	·079	·803	·084	8·4	0·5
1·3	2·7	·013	·973	·013	1·3	0·0
14·6	22·2	·146	·778	·159	15·9	1·3
0·0	3·9	·000	·961	–	–	0·0
13·2	17·8	·132	·822	·146	14·6	1·4

of species j, which competes with species i for space on the gear. The extent of underestimation is strongly related to the amount of empty space on the gear and to the apparent abundance of each species relative to the other. The tendency toward underestimation will be greatest in the years of highest apparent abundance.

In some fisheries – the Japanese longline fishery, for example – estimates of Q_0 may rarely be less than 0.90. Thus the underestimate of CPUE in these fisheries is likely to be small. The importance of a relatively small underestimate depends upon the manner in which the statistic is used. For example a seemingly small change in CPUE induced by correcting for a multiple-species catch might produce a large change in an estimate of total catch made from the relationship between total effort and CPUE.

The calculations for a specific example are given in Table 1. The data were obtained on cruise number 5 of the "Hugh M. Smith" in the vicinity of Canton Island (MURPHY and SHOMURA, 1953). These particular data were selected for illustrative purposes because they had a wider range of Q_0's than that usually found for tuna longline data. The species of interest is the yellowfin tuna (*Thunnus albacares*); all the other species caught were pooled as species 2. The data reiterate the conclusion that the most severe underestimations occur at maximum CPUE's.

Estimation of Parameters

The probabilities, Q_i, that the hook remains empty, takes species 1, or 2, at the terminal instant of time are, under the previously stated independence assumption, parameters of the multinomial probability law. MOOD and GRAYBILL (1963; p. 238) derived the maximum-likelihood estimator for the mean of this distribution and the large sample variance of the estimator.

It may be noted, however, that the λ_i's are a more fundamental statistic than the P_{0i}*'s since they can be computed for several components (year-classes, say) and then added to estimate a composite P_{0i}*, whereas the P_{0i}*'s themselves are not additive. When one works with components, the λ_i's can be reported as $-\log(1 - P_{0i}^*)$. It should be noted that the λ_i's have the properties of coeffi-

cients of instantaneous rates; for example, a λ_i of 1·0 represents an apparent abundance which is double that of a λ_i of 0·5. The maximum-likelihood estimator of the λ_i's is easily obtained from the n_i's by applying the invariant property of maximum-likelihood estimators to the Q_i*'s,

$$\hat{\lambda}_i = \frac{n_i}{n_0 - N} \log \frac{n_0.}{N} \tag{10}$$

If we make the reasonable assumption that large samples of the n_i's will have an approximately multivariate normal distribution, then the variances and covariance of the λ_i's are

$$\text{Var} (\hat{\lambda}_i) = \frac{1}{N} \left[\frac{\lambda_1 \lambda_2}{1 - Q_0} + \frac{\lambda_i^2 Q_i^2}{\lambda_i^2 Q_0 - Q_i^2} \right], \tag{11}$$

$$\text{Cov} (\hat{\lambda}_1, \hat{\lambda}_2) = \frac{1}{N} \left[\frac{\lambda_1 \lambda_2}{1 - Q_0} \right].$$

Estimates of $\text{Var} (\hat{\lambda}_i)$ and $\text{Cov} (\hat{\lambda}_1, \hat{\lambda}_2)$ are obtained by substituting the appropriate values of $\hat{\lambda}_i$ and $\hat{Q}_i$ into the above formulae.

Emphasis should be placed on the fact that the expressions in (11) are the *sampling* variances and the *sampling* covariance; they measure only the variation associated with repetitive sampling from a single population characterized by fixed λ_i's. These measures of variability must arise only from repetitive sampling since the model is constructed so that each λ_i is a constant and hence has zero variance. The constant λ_i's follow from the definition of the model where it is stated that the transition probabilities (the q_{ij}'s) are constant for all time intervals. In practice, samples may be drawn from several populations, each characterized by a different set of λ_i's. The resulting variance, in this situation, will be composed of at least two components: one owing to the variation among the several populations and the other owing to the chance variation which results from repetitive. sampling from the same population. The expressions in (11) provide an estimate of the latter.

Discussion

Unfortunately, the analytic simplicity of a known number of hooks is not always available. In these circumstances it is possible to conjecture that the empty "space" of a gear is the complement of the maximum CPUE. This maximum is not, in general, a well-defined quantity. The maximum catch of a gill-net, for example, is a fish in every "hole". Although the attainment of this event is improbable, it is not impossible. Should the "saturated" or "supersaturated" condition (cf. GULLAND, 1955) be used as a "maximum"? The concept of tolerance intervals (DIXON and MASSEY, 1957) may be helpful in establishing a lower bound on the maximum and, since the CPUE statistic is usually not easily treated parametrically, the table of distribution-free tolerance intervals given by MURPHY (1948) will be of use.

Some problems of interpretation may also arise in evaluating the number of hooks actually fishing in the longline situation. Aside from the obvious problem of either the bait or the hooked fish falling off of the hook, the relation of hook depth to fish depth must be considered. The longline is comprised of units

which hang in approximate catenaries from float lines. Each catenary contains several hooks which hang within a range of depths that may be as large as a few hundred metres. If the fish are excluded – by nature of their environmental tolerances – from some of the depths which the hooks occupy, then these hooks are in effect not fishing for the considered species. In the circumstance where hooks are recorded as actually fishing but are, in effect, not, Q_0 will be over-estimated and the effect of competition (*i.e.* a value of P_{0i} for a particular value of Q_i) will tend to be underestimated.

It is important to note that adjustment of CPUE to account for multiple-species catches increases the variability of the adjusted CPUE's over the un-adjusted ones. If environmental variables are important causes of fluctuation in abundance of any one species and these fluctuations tend to be minimized by multiple-species catches, then the environmental variables may be unrecognized or undetectable as causes of the fluctuations.

Finally, the competition model contains concepts that are applicable to other problems that arise in the study of harvested populations. These conceptual applications pertain to estimates of yield, estimates of survival rate, and optimum fishing strategies.

First, we note that natural and fishing mortality compete for the life of a fish just as species 1 and species 2 compete for a single hook. To express this competition we let S_0 denote the state where a fish escapes mortality; S_1, where the fish succumbs to fishing mortality; and S_2, where the fish succumbs to death from natural causes. Placing the problem in a more familiar notation, let $\lambda_1 = F$ (the coefficient of instantaneous fishing mortality) and $\lambda_2 = M$ (the coefficient of instantaneous natural mortality). Rewriting (7), gives

$$\frac{F}{F + M} (1 - e^{-(F + M)}), \qquad (12)$$

which is the well-known expression for the fraction of a year-class taken by a fishery in a unit-interval of time (annual expectation of death from fishing). Multiplying (12) by recruitment provides an expression for yield, in numbers. Thus we have derived, using a probabilistic argument, an expression for yield that is identical to that which is conventionally derived deterministically. Further-more, the assumption of an underlying multinomial-probability law enables estimates of the mean and variance of the λ_i's which are identical with F and M. Replacing F and M with their appropriate maximum-likelihood estimators from (10) gives a maximum-likelihood estimate of (12). The problem, here, of course, is to obtain data in terms of the number of fish that in some time interval are caught, succumb naturally, and remain alive. Perhaps tagging experiments are the most likely source of data of this type.

Second, survival rates are often estimated by means of a catch curve. Strong year-classes may arise and compete with other year-classes for space in the sampling gear. According to the model, the abundance of strong year-classes will be underestimated. Even under conditions of a stable age structure, several year-classes could compete with one another for space in the sampling gear. In a stable population the younger age-groups with the larger numbers of individuals would exert greater amounts of competition for becoming sampled than would older age-groups. It is conceivable that under some circumstances survival rate will tend to be underestimated as a result of competition among age-groups.

And, finally, the model suggests certain strategies for the fisherman. According to the model, the single-species CPUE resulting from fishing a fixed density of that species can be increased simply by setting the gear in a way or location such that the effect of competing species is reduced. The wisdom of this strategy is of course contingent on the monetary value of the competitor. Where the value of the competitor is often nil (*e.g.*, sharks in the Japanese high-seas longline fishery for tunas) it is to the fisherman's advantage to attempt to eliminate the competitor completely. On the other hand, the competing species may also be of value and the foregoing strategy would still increase the CPUE for the single species, but decrease the total CPUE. Thus, in a gradient of mixtures of two valuable species, the choice of fishing location would involve maximizing the monetary return from the combined catch.

Summary

The effect of multiple-species catches on the catch-per-unit-of-effort (CPUE) statistic for a single species is considered in terms of a simple stochastic model. The model enables computation for a single species of a CPUE which is independent of the effect of competition for space on the fishing gear by a competing species. According to the model, the adjusted single-species CPUE is never less than the multiple-species catch, the adjusted single-species CPUE is strongly dependent on the apparent abundances of the considered species, and the adjustment is not linear, but varies with the degree of species mixture. Various applications of the model are discussed and attention is called to the fact that the model provides a stochastic derivation of the well-known yield equation.

Acknowledgements

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