A stochastic integrated VPA for herring in the Baltic Sea using acoustic estimates as auxiliary information for estimating natural mortality

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Acoustic estimates on stock numbers at age for herring in the Central Baltic Sea are used in a stochastic integrated VPA to obtain estimates of mean natural mortality rates by ages for 1981–1987. The estimates of the natural mortality rates are inconsistent with the results of the ICES Multispecies VPA (MSVPA) carried out for the Central Baltic Sea. The high natural mortality rates of the young ages estimated by the MSVPA are not confirmed by the present analysis, where the natural mortality rates are estimated to be the same (0.18 year⁻¹) for ages 1 to 9. It is concluded that the acoustic survey underestimates the abundance of ages 1, 2, and 3. However, the actual natural mortality might lie somewhere between the estimates from the MSVPA and those presented here, because the MSVPA could be wrong by as much as a factor of 2 owing to uncertainties in the consumption data used for cod in the MSVPA.

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Introduction

The herring in the Central Baltic Sea (Subdivisions 25– 27, Fig. 1) are assumed to constitute one stock unit (Anon., 1988a). From 1981 to 1987 this stock was assessed by an acoustic survey conducted in October each year. Estimates of stock abundance from the survey are used to tune the VPA for this herring stock using the acoustic estimates as indices of stock abundance (Anon., 1988a). In this VPA the natural mortality rate, M, is assumed to be 0.2 year^{-1} for all ages and all years. However, since acoustic estimates can be regarded as absolute estimates of stock abundances, it should be possible to estimate both the natural and the fishing mortality rates from a combination of acoustic and catch-at-age data.

However, according to the Working Group on Multispecies Assessments of Baltic Fish (Anon., 1989) the natural mortality rate of the younger ages of this herring stock is much higher than 0.2 year^{-1} . As means over 1982–1984, the natural mortality rates were found by Anon. (1989) to be:

M year ⁻¹
1.45
0.62
0.27
0.27
0.24
0.20

The primary aim of this paper is to see whether these values of M could be verified using acoustic estimates of the stock abundance. As a side-product the presented procedure represents an alternative assessment method to the *ad hoc* tuning method used by the ICES Working Group on Pelagic Stocks in the Baltic (see Anon., 1988a).

In the present approach, which could be called a stochastic integrated model, all assumptions, e.g. probability distributions and modelling structure, are made

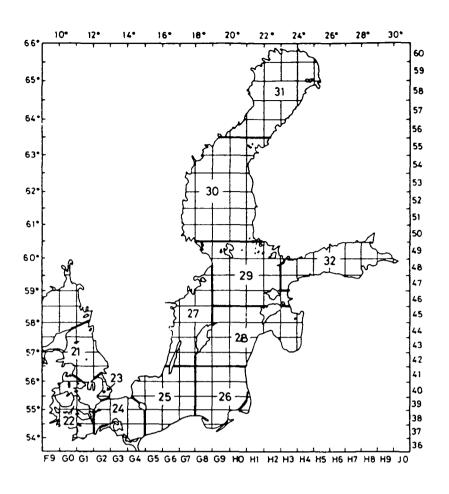


Figure 1. Chart of subdivisions in the Baltic Sea.

explicitly. Implausible assumptions such as no errors in the catch-at-age data are thus avoided.

Several methods which combine for example c.p.u.e. data with catch-at-age data have been published in recent years (Doubleday, 1981; Colie and Sissenwine, 1983; Fournier and Archibald, 1982; Pope and Shepherd, 1984; Shepherd and Nicholson, 1986; Deriso *et al.*, 1985; Lewy, 1988). All methods, except Lewy's use the lognormal distribution to specify the variance structure of observations such as catch-at-age and c.p.u.e. data. Lewy uses the normal distribution.

This paper presents a stochastic model which uses normal distributions and which simultaneously applies commercial catch-at-age data and acoustic estimates. Stock size, fishing mortality rate, and natural mortality rate are estimated. The model used is non-linear and estimates M as well as the usual VPA assessment parameters N and F (see, e.g., Gulland (1983)) by year and age or as mean values. This procedure differs from that used in many of the above-mentioned methods that apply catch-at-age data to multiplicative models. The latter are approximations to the VPA equations and the parameters estimated are not directly comparable to the VPA parameters. The main reason for using multiplicative models instead of the VPA equations is that computation time is considerably reduced; furthermore, log transformation of data converts the model into a linear form with normal variance structure if the original data are distributed lognormally. Another reason is that log transformation stabilizes the variance if the coefficient of variance (C.V.) is constant.

The present model differs from all the above-mentioned models in that M is estimated and not assumed known beforehand. Besides estimating the parameters M, N, and F the model gives approximate estimates of the variances of the observations, i.e. of catch-at-age number and acoustic estimates of number at age.

Material

The model is based on: (1) Catch-at-age in numbers of age groups 1 to 9 for Subdivisions 25–27 for 1981–1987; and (2) acoustic estimates of stock numbers at ages 1 to 9 for 1981–1985, and 1987. Data from 1986 are regarded as abnormal (see Anon., 1988a).

The acoustic estimates are based on the total integrated echoes combined with pelagic trawl catches to separate the echoes into species and age groups, taking into account the variation in target strength by species and age. Acoustic herring surveys are more easily carried out in the Baltic than in most other sea areas because of the small number of species found in the Baltic. In fact only herring and sprat can contribute significantly to the pelagic echoes because the only other common fish, cod, is distributed close to the bottom. Details of the method used and the results of the survey are given in Anon. (1988b).

Method

The stochastic model used is similar to that described by Sparholt (1989). Variability in both the acoustic estimates and in the catch-at-age data is assumed. It is also assumed that the residuals are normally distributed.

The parameters M, N, and F are estimated by the model relating these parameters to catch-at-age data and acoustic estimates by applying the usual VPA equations.

It is assumed that the natural mortality rate has the form:

 $M(a,y) = M1 + M2 \times exp[-M3 (a - 1)]$

where M1, M2, and M3 are estimated by the model, a is age and y year. The rationale of this equation is that the natural mortality consists of two additive components: one due to predation the other to causes such as disease, reproductive stress, damage by fishing gear, discarding, etc. The first component is an exponential decreasing function of age as shown by Anon. (1988c, 1989) for the most common prey-fish in the North Sea and in the Baltic. The other component is assumed to be age-independent.

Following Pope and Shepherd (1982), the fishing pattern is assumed to be separable, which means that the fishing mortalities at age can be separated into an age component common to all years and a year component common to fish of all ages in that year. The fishing mortality F(a,y) acting at year y on age a is thus given by:

$$F(a,y) = S(a) \times F(y)$$

where S(a) represents the age component and F(y) the year component.

The Laurec-Shepherd method (Laurec and Shepherd, 1983) was used with acoustic data as abundance indices to evaluate this assumption of separable fishing pattern. The F values obtained (Table 1) indicate a slight change in the exploitation pattern with decreasing F on the young ages and increasing F on the old ages from 1981 to 1987. The change was small, however, and the age effect, in a two-factor ANOVA of F with age and year class as variables, showed that the ANOVA explained 75% of the variation in the F values, i.e. only 25% of the variation remained for random "noise" and for possible trends. Thus, the results from the Laurec-

Table 1. Fishing mortality rates estimated by the Laurec-Shepherd method, where F on the oldest age was set to the average of the five younger ages and data were log transformed.

	Year												
Age	1981	1982	1983	1984	1985	1986	1987						
1	0.15	0.06	0.06	0.05	0.11	0.15	0.06						
2	0.33	0.29	0.14	0.12	0.14	0.17	0.24						
3	0.29	0.29	0.39	0.21	0.25	0.23	0.31						
4	0.31	0.19	0.26	0.40	0.32	0.33	0.32						
5	0.25	0.23	0.22	0.26	0.53	0.39	0.40						
6	0.23	0.20	0.33	0.27	0.40	0.40	0.42						
7	0.21	0.23	0.27	0.31	0.45	0.43	0.47						
8	0.17	0.21	0.29	0.28	0.55	0.60	0.52						
9	0.23	0.21	0.27	0.30	0.45	0.43	0.43						
Mean of													
ages 1–8	0.34	0.24	0.21	0.24	0.24	0.34	0.34						

Shepherd method did not seem to be in serious conflict with the assumption of a separable fishing pattern, especially when it is remembered that the main purpose of the present paper is to estimate a mean M over a period of years.

Model for the catch-at-age data

Catch-at-age in numbers C(a,y) is assumed to be normally distributed with mean

$$E[C(a,y)] = \tilde{N}(a,y) \times F(a,y)$$
(1)

where $\hat{N}(a,y)$ is the mean number of herring of age at present in year y.

Model for the acoustic estimates

The acoustic estimates, NA(a,y), are also assumed to be normally distributed with mean

$$E[NA(a,y)] = N(a,y) \times \exp\{-3/4 \times [F(a,y) + M(a,y)]\}$$
(2)

where N(a,y) is stock number at 1 January of age a in year y.

The factor 3/4 is used to correct for the difference in time between 1 January and October, when the acoustic survey is conducted.

Because 1-group herring are found mainly in shallow waters this age group is not covered properly in the acoustic survey and is probably underestimated. Formula (2) is therefore modified for age 1 to

$$E[NA(1,y)] = A \cdot N(1,y)$$

$$\times \exp\{-3/4[F(1,y) + M(1,y)]\}$$

where A is a constant which is estimated by the model.

In constrast to the deterministic approach of the VPA, all observations above are assumed to be subject to stochastic variations. The variance is assumed to be proportional to the observation with a proportionality coefficient of acoustic estimates 10 times as large as that of the catch-at-age data:

$$var[C(a,y)] = \mu \times C(a,y)$$
(3)

$$var[NA(a,y)] = 10 \times \mu \times NA(a,y)$$
(4)

where μ is a constant which is estimated by the model.

This means that the coefficient of variation of the acoustic estimates is assumed to be $\sqrt{10}$ times that of the catch-at-age data, given observations of equal size. However, as the acoustic estimates are usually higher than the catch-at-age figures and because the coefficient of variation decreases with an increase in the size of the observation, the acoustic C.V. would differ by less than $\sqrt{10}$.

Estimation procedure

The maximum likelihood method was used to estimate the parameters. This method can be used when a set of stochastic variables and their probability distributions – determined by a set of parameters – are known. The maximum likelihood estimators of the parameters are those values of parameters which maximize the joint probability distribution for given values of the observations.

In the present case the observations are catch-at-age in numbers and acoustic estimates, while the parameters are the size of the year classes, the fishing mortality rate and the natural mortality rate as described above.

Instead of maximizing the probability distribution the negative value of the logarithm of the probability can be minimized.

Under the assumption that all stochastic variables, catch-at-age in numbers and acoustic estimates are stochastically independent, the negative logarithm of the probability distribution, $L(\theta)$, can be written as:

$$L(\theta) = \ln p [C(a,y), NA(a,y)]$$

=
$$\sum_{a,y} [C(a,y) - E C(a,y)]^2/2 \times var[C(a,y)]$$

=
$$\sum_{a,y} [NA(a,y) - E NA(a,y)]^2/2 \times var[NA(a,y)]$$

=
$$\sum_{a,y} \ln \{\sqrt{var}[C(a,y)]\}$$

=
$$\sum_{a,y} \ln \{\sqrt{var}[NA(a,y)]\}$$

The parameters estimated by the maximum likelihood method are:

N (first age, y) = number of 1-groups at 1 January and y=1981, ..., 1987.

= the age component of the fishing mortality rate

M1, M2, M3, A = constants

S(a)

μ

= variance parameter

Altogether 36 parameters are estimated based on 117 observations. The model is implemented on a VAX 11/750 in FORTRAN 77. The minimization of the like-lihood function is done using the NAG routines EO4HBF and EO4JBF.

In the above procedure a basic assumption is that the errors in the observations are uncorrelated. As the acoustic estimates of number at age are probably multinomially distributed they are indeed correlated. However, as the number of age groups is large and as the age 10+ has been excluded from the analysis the correlation is probably of minor significance only. On the same basis the correlation in the catch data can also be neglected.

Results

The estimated parameters are given in Table 2. The observations which are used as input data in the model are given in Tables 3 and 4 together with the expected

Table 2. The estimated parameters, except for the fishing mortality rates, which are shown in Table 5.

Parameters	Estimated value
 N(1,1981)	6520 * 106
N(1,1982)	3854 * 106
N(1,1983)	3375 * 106
N(1,1984)	3387 * 106
N(1,1985)	3466 * 106
N(1,1986)	2163 * 106
N(1,1987)	1397 * 106
N(2,1981)	2123 * 106
N(3,1981)	1105 * 106
N(4,1981)	1238 * 106
N(5,1981)	877 * 106
N(6,1981)	649 * 10 ⁶
N(7,1981)	418 * 106
N(8,1981)	275 * 106
N(9,1981)	325 * 106
MI	0.18 per year
M2	0.00 per year
M3	0.01 per year
A	0.64 per year
μ	7.50 per year

	1981		19	82	19	83	19	84	19	85	19	86	19	87
Age	C _{OBS}	C _{EXP}	COBS	C _{EXP}	C _{OBS}	C _{EXP}								
1	815	665	235	328	215	275	221	257	418	346	324	230	178	183
2	564	420	1070	808	419	468	348	384	440	503	474	531	358	99
3	294	306	300	327	827	748	428	426	486	486	494	490	571	575
4	318	368	127	164	167	211	474	475	416	377	404	326	431	361
5	190	242	145	164	96	88	105	112	326	353	297	211	274	202
6	122	150	99	101	130	82	76	43	96	77	127	185	175	123
7	82	86	79	71	85	56	73	45	73	34	57	46	78	123
8	56	72	55	60	62	58	56	46	70	51	48	29	35	45
9	82	93	49	40	46	40	40	38	51	42	28	36	16	22

Table 3. Observed and expected values of catch-at-age in millions.

Table 4. Observed and expected values of acoustic estimates in millions.

	19	81	19	82	19	83	19	84	19	85	198	86	19	87
Age	N _{OBS}	N _{EXP}	N _{OBS} *	N _{EXP}	N _{OBS}	N _{EXP}								
1	4745	4205	3391	2486	1984	2176	1772	2184	1797	2335	2879	1395	2029	901
2	1695	2123	4931	4839	2089	2919	2316	2567	3028	2594	4730	2579	1358	1596
3	815	1105	1615	1390	3023	3305	2684	2012	2028	1794	4455	1709	2402	1671
4	1266	1238	931	645	1269	863	2322	2080	1766	1293	2500	1057	2221	982
5	909	877	924	700	847	390	650	530	521	1305	1671	738	965	587
6	937	649	708	512	793	435	433	246	210	341	566	769	564	424
7	451	417	387	405	656	336	358	289	163	166	263	214	222	474
8	501	275	222	270	464	274	256	230	111	200	153	108	81	137
9	514	326	168	164	304	171	127	176	54	150	71	120	30	64

* Not included in the estimation.

values estimated by the model. The natural mortality rates are estimated to be:

 $M(a) = 0.18 + 0.00 \times \exp(-0.01(a - 1))$

Thus, the natural mortality rates are the same (0.18 year^{-1}) for all ages.

Test of the model

The programming of the model is tested with a simulated data set of catch-at-age data and acoustic estimates of stock numbers at age without stochastic variation. The test data set is constructed on the basis of given values of the 35 parameters. The fitting procedure is then able to estimate the 35 parameters almost exactly.

If the model holds, the standardized residuals:

$$[C - E(C)]/std$$

or

[NA - E(NA)]/std

are approximately normal distributed with mean value 0 and standard deviation 1.

This was tested for the herring data and could not be rejected at the 5% significance level. Kolmogorov's D statistics were used to test the normality (Anon., 1985).

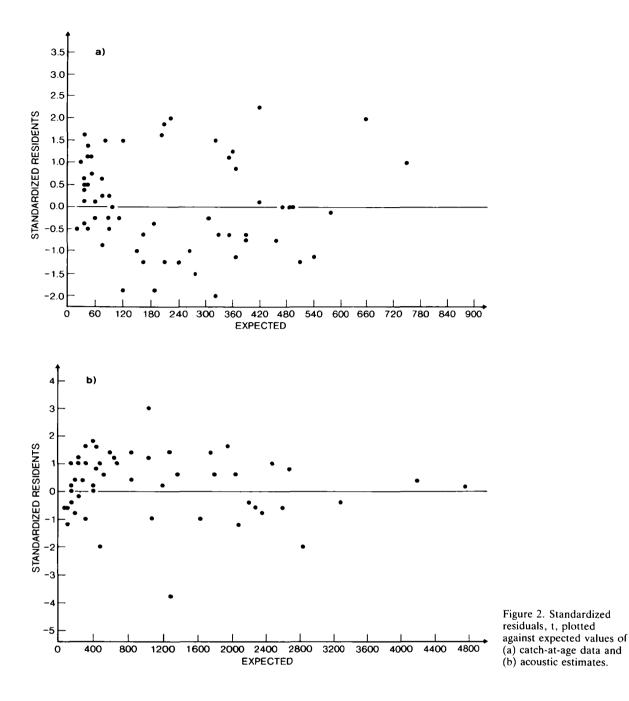
The proportion of the standardized residuals greater than 1.96 was $7/117 \sim 6.0\%$. A rough test of the total model is to compare this proportion to the mean value, which is 5%. Assuming a binomial distribution, the observed value of 6% does not differ significantly from 5% at the 5% level.

Another test of the total model is the quantity:

$$\chi^2 = \sum_i (C_i - E_{ci})^2 / V(C_i)$$

i.e. the sum of the squared standardized residuals. The test value is asymptotically χ^2 -distributed with degrees of freedom equal to the number of observations minus the number of parameters, i.e. 82. χ^2 is estimated to be 154.54, which is significant at the 0.1% level. Thus the model did not fit the data very well. This, however, was mainly caused by three outliers: the catch of 2-group fish in 1987, the acoustic estimate of 5-group fish in 1987.

Plots of the residuals against the expected observation are shown in Figure 2. Trends are not found in the





distribution of the residuals of either the catch-at-age data or the acoustic data.

It is usually very difficult to specify the variance structure of catch-at-age data and acoustic estimates. Sparholt (1988) iteratively estimated the variance structure of catch data, acoustic estimates, and young fish survey data in a stochastic integrated model very similar to the model presented in this paper for the herring stock in the Skagerrak-Kattegat and the Western Baltic. The criteria for stopping the iterations were: (1) lack of trends in plots of standardized residuals against expected values, and (2) as small as possible confidence intervals for the estimated parameters. The resultant variance structure for the young fish survey data was then compared with direct measurements made by Buijse and Daan (1986) and this 'test' confirmed the obtained variance structure.

In the present model only criterion (1) can be used because estimation of confidence intervals is not included in the model and because no direct measurements are available of the variance structure of the catch-at-age data or the acoustic estimates. The model was therefore run with different assumptions about the relative precision of the acoustic estimates compared with the catch-at-age data. In runs where the precision of the acoustic estimates was assumed to be better than specified in Equations (3) and (4), the mean standardized residuals of the acoustic estimates were larger than the mean standardized residuals of the catch-at-age data. This indicates that the assumed precision of the acoustic estimates was too optimistic. In runs where the relative precision of the acoustic estimates was assumed to be worse than specified in Equations (3) and (4) the opposite result was obtained. Thus the variance structure specified in Equations (3) and (4) seems to be appropriate.

Another approach to assessing the precision of the catch-at-age data and the acoustic estimates was to follow the idea of Pope (1977) and Pope and Shepherd (1982), namely to examine log catch ratios CD(a,y):

 $CD(a,y) = \ln [C(a + 1,y + 1)/C(a,y)],$

and log ratios of acoustic estimates:

ND(a,y) = ln [N(a + 1,y + 1)/N(a,y)].

These two matrices were then analysed by a conventional two-way analysis of variance (ANOVA). The mean square of the residuals was 0.14 for the catch data and 0.22 for the acoustic data. This suggests that the coefficient of variation was about 0.27 for the catch-atage data and about 0.34 for the acoustic estimates, indicating that the acoustic data are slightly "noisier" than the catch-at-age data. Based on Equations (3) and (4) the mean C.V. of the catch data is 0.18 and of the acoustic data 0.24; these are in reasonable agreement with the C.V. values from the above ANOVA. If the factor 10 in Equation (4) was replaced by 9.2 the relative precision of the catch data and the acoustic estimates would be identical in the ANOVA and in the model, but due to the very crude method used here for examining variance structure it seems reasonable to round the value 9.2 to 10.

The model is not very sensitive to the actual variance structure used as regards the estimated parameters. For instance the natural mortality rate varied between only 0.17 year^{-1} and 0.18 year^{-1} when the factor in Equation (4) was varied from 10 to between 4 and 16. However, as stated above, this procedure resulted in significant trends in the standardized residuals, suggesting that 10 was the most appropriate value.

Discussion

The model presented belongs to the family of analytical procedures which assume that all measurements are with errors (see, e.g., Lewy, 1988; Deriso *et al.*, 1985; Anon., 1988d), unlike the VPA and most *ad hoc* models

used to modify or "tune" the VPA to effort data, etc. The present model differs from other members of this family of methods in two aspects: (1) The natural mortality is estimated and not assumed known beforehand; and (2) the variance structure is justified (although very crudely) and not chosen arbitrarily. The model is of a general kind in its structure and it could easily be used on other stocks where catch-at-age data and acoustic estimates are available.

The estimation procedure takes about 10 c.p.u. minutes on a VAX 11/750. This computer time is probably acceptable for the relevant ICES Working Group wishing to use the model or a modification of it during a working group meeting. If the natural mortality is not estimated the computer time will probably be shorter.

The natural mortality rate is forced to fit into the model:

 $M(a) = M1 + M2 \times exp[-M3(a - 1)]$

where M1, M2, and M3 are assumed to be positive. This model is not the best one to fit the data. One where M2 was allowed to be negative fitted the data better. However, it was not considered realistic that the natural mortality could increase with increasing age of herring. The estimated natural mortality rate parameters resulted in an almost constant resultant rate of 0.18 year⁻¹ over the range of age in question. As M1, M2, and M3 were not allowed to be negative, bounds were put on these parameters in the fitting procedure and this unfortunately prevents conventional use of the Hessian matrix for estimating variance of the estimated parameters.

The present estimate of the natural mortality rate, M, is in two ways inconsistent with the estimates from the MSVPA model for the Baltic Sea; it is much smaller, and no age dependence is found. The present model shows that the 1-groups are underestimated in the acoustic survey. The low estimate of M on age groups 2 and 3 indicates that these two ages are also probably underestimated. A simple comparison between the numbers caught and the total decrease from age 2 to 3 and from age 3 to 4 according to the acoustic estimates showed that the catch was as high or even higher than the total decrease, with the unrealistic implication that M is zero or even negative for ages 2 and 3. However, the MSVPA estimates are still preliminary and according to Anon. (1989) can be wrong by as much as a factor of 2 simply due to uncertainties in the cod food consumption data. Furthermore, the assumption used in the MSVPA that the M1 component of natural mortality due to diseases, etc. (0.2), applies equally to both young and old fish may also be incorrect. Therefore, the actual M could very well be somewhere between the present estimates and the MSVPA estimates.

The C.V. estimated here is in general agreement with the mean C.V. of the catch-at-age data (C.V. = 0.19)

Age	19	81	19	82	19	983	19	984	19	985	19	86	19	87
	F _{VPA}	F _{EXP}												
1	0.16	0.12	0.07	0.10	0.07	0.09	0.06	0.09	0.12	0.12	0.20	0.12	0.06	0.15
2	0.33	0.24	0.31	0.20	0.15	0.19	0.14	0.18	0.15	0.24	0.18	0.25	0.33	0.32
3	0.30	0.36	0.30	0.30	0.41	0.28	0.22	0.26	0.29	0.35	0.23	0.37	0.32	0.47
4	0.32	0.39	0.20	0.32	0.29	0.31	0.47	0.29	0.35	0.38	0.39	0.34	0.32	0.51
5	0.28	0.36	0.23	0.30	0.25	0.28	0.30	0.26	0.54	0.35	0.43	0.37	0.50	0.47
6	0.24	0.29	0.24	0.24	0.35	0.23	0.32	0.21	0.41	0.28	0.40	0.30	0.47	0.38
7	0.23	0.25	0.22	0.21	0.36	0.20	0.34	0.19	0.53	0.25	0.43	0.27	0.42	0.33
8	0.22	0.33	0.21	0.28	0.28	0.26	0.43	0.25	0.48	0.33	0.77	0.35	0.45	0.44
9	0.19	0.37	0.26	0.31	0.28	0.29	0.24	0.27	0.59	0.36	0.34	0.39	0.53	0.48

Table 5. The estimated fishing mortality rates per year derived from the present study, F_{EXP} , compared with the VPA estimates, F_{VPA} , from Anon. (1988a).

and the acoustic estimates (C.V. = 0.29) which can be extracted from Sparholt (1989) for the herring stock in the Skagerrak-Kattegat and the Western Baltic. The F values estimated in the present model are generally similar to the F values from the VPA made by Anon. (1988a) (Table 5).

It is therefore suggested that the model developed here is a feasible alternative to the *ad hoc* tuning method currently used by the ICES Working Group on Pelagic Stocks in the Baltic. Its advantage, compared with *ad hoc* tuning methods, is that it is more statistically sound and more flexible as regards the variance structure used. Furthermore, it uses estimates of absolute stock size; terminal F's for the oldest age group do not have to be guessed. However, the extent to which it can improve the present overall assessment remains to be tested.

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