TH. HESSELBERG. Ein neuer Ausdruck für den Austauschkoeffizienten. Annalen der Hydrographie usw. LVII Jahrg. (1929). Heft X. Berlin 1929.

The word "Austauschkoeffizient" which can probably best be given in English by "coefficient of exchange" was introduced into meteorological and hydrographical literature by W. SCHMIDT (1917). It can be defined as the amount of some attribute of the fluid (e.g. heat, salt, plankton, momentum etc.) which is carried in unit time through a unit surface in the fluid by certain turbulent movements, when the decrease of the characteristic quality (e.g. temperature, salinity, amount of plankton per kilo, velocity etc.) given to the fluid by the attribute is one unit per unit length in the direction vertical to the surface in question.

If, for example, we take the salinity of the seawater as the quality to be considered in this connection and let Q be the amount of salt carried through a unit area, S the salinity of the water, t the time and z the distance in the direction perpendicular to the area in question, we have the coefficient of exchange η determined by the equation:—

$$\frac{dQ}{dt} = -\eta \cdot \frac{dS}{dz}$$

which is analogous to the equations used in dealing with conduction of heat and with diffusion.

TH. HESSELBERG has formerly published various investigations of the coefficient of exchange for the atmosphere and in the present article now *gives a new expression for the coefficient of exchange applicable to the conditions both in the atmosphere and in the sea.*

The following summary of the article should, however, be taken only with reference to conditions in the sea.

The actual movements of the water particles may be resolved into 1) the resultant motion, which is common to all the particles considered, and 2) the turbulent motion.

The turbulent motions of the water particles which account for the exchange of water from one side of a surface in the sea to the other, and *vice versa*, may be resolved into components parallel to an area A of the surface considered and components perpendicular to it. The exchange of water through the area is due only to the last components.

These components perpendicular to the area A are supposed to be of a certain nature, and, if the coefficient of exchange in the space considered is to be regarded as a constant, HESSELBERG's assumptions may, I think, be formulated as follows:—

1) The conditions are stationary in the space considered.
2) The sum of the components, perpendicular to the area A, of the velocities, of the particles in a volume, two of whose dimensions are given by the area A, is equal to zero for every interval of time.
3) A water particle will retain its velocity of turbulent motion unaltered through a fixed interval of time $\frac{P}{2}$. After the lapse of that time the

velocity is suddenly altered and the particle will keep its new velocity through the time $\frac{P}{2}$, when it will again be altered, and so on.

4) When the turbulent velocity of a water particle is altered, the particle will acquire the qualities of the water at that point at which the alteration occurred, and it will then keep those qualities (e.g. the velocity parallel to the area A, the temperature, the salinity, etc.) through the time $\frac{P}{2}$.

5) The points at which the velocities are altered for the different particles are equally distributed in the space considered.

6) The frequency of the above-mentioned velocity components perpendicular to the area A is given by Maxwell's Law at any time over the space considered.

TH. HESSELBERG then expresses the coefficient of exchange with regard to temperature, salinity and the momentum parallel to the area A by

$$\eta = \varrho \cdot \frac{P}{2} \cdot \int_0^\infty F \cdot u^2 \cdot du \qquad (1)$$

ϱ being the density of the sea-water, $\frac{P}{2}$ the interval of time during which a particle keeps its turbulent velocity unaltered, F the frequency according to Maxwell's Law and u the velocity component normal to the area A.

The expression (1) is also given by HESSELBERG in the following form:—

$$\eta = \frac{1}{4} \cdot \varrho \cdot P \cdot \bar{u}^2 \qquad (2)$$

where $\bar{u}^2$ indicates the mean of the squares of the velocity components.

Taking an expression from the theory of errors, $\bar{u}^2$ can be denominated as the square of the standard deviation of the velocity components.

The expression found by HESSELBERG is of special interest in that, under certain definite assumptions, it gives an insight into the mechanics of the exchange of water.

It seems, however, that the assumptions adopted are more restrictive than necessary and it may be doubted if they are in accordance with fact.

It follows directly from the assumptions, as HESSELBERG shows, that the coefficient of exchange should be the same both for momentum parallel to the area A and, for example, for salinity.

Some time ago the reviewer found that the "mixing coefficient", which differs from the coefficient of exchange only by the factor ϱ in the vertical direction, is different for the horizontal momentum and for salinity, and has since confirmed this.

Instead of the assumptions 3), 4) and 6) it would perhaps be more in accordance with the facts to make the supposition that:—

7) The frequency of the paths z of the elements of volume of water, which are carried away with unaltered "conservative" qualities (temperature,

salinity etc.) by the turbulent movements from a definite unit volume in a definite direction and in an arbitrary unit of time, is given by the law of normal distribution (MAXWELL's expression).

The coefficient of exchange through an area A, when z is vertical to A, can then be written:—

$$\eta = \frac{1}{2}\varrho \cdot \bar{z}^2 \tag{3}$$

which signifies in words half the product of the specific gravity and the "square of the standard deviation of the paths" referred to the unit of time used.

According to supposition 7), it should be possible for a water particle in motion to alter its velocity parallel to the area A without altering, for example, its salinity. Then the paths of a particle with unaltered qualities would be different for momentum parallel to the area A and for salinity. The momentum can be said to have a looser connection with the water than, for instance, the salt and can be partly transferred from sheet to sheet independently of the transfer of the water particles themselves.

Then, for momentum, a supposition analogous to 7) should be made, that:—

The frequency of the paths z for different uniform fractions of the momentum parallel to the area A, which is found in a certain volume and carried away by the turbulent movements in a direction perpendicular to A in an arbitrary unit of time, is given by the law of normal distribution.

Then the coefficient of exchange for momentum is also expressed by formula (3), with the special value of $\bar{z}^2$ referring to the momentum.

Besides the above-mentioned principles, the article discusses the exchange of energy and various other questions. The study of the coefficient of exchange is undoubtedly more advanced for the atmosphere than for the sea, but, even if it should not be possible to admit all the results obtained, it will undoubtedly be of great interest to study them by planning investigations at sea, which is the only way in which we are able to decide upon the reality of the theoretical deliberations leading to formula (1) or to formula (3).   J. P. JACOBSEN.

J. P. JACOBSEN. Eine graphische Methode zur Bestimmung des Vermischungskoeffizienten im Meere. Gerlands Beiträge zur Geophysik. Bd. XVI, Heft 4. Leipzig 1927.

In this paper Dr. JACOBSEN attacks a problem of great importance in hydrographical research, namely, how to obtain a measure of the extent to which, under oceanic conditions, intermixture takes place between superimposed, homogeneous water-masses possessing different physical characteristics. Incidentally it is claimed that the method expounded by the author has a wider application than the determination merely of a quantity termed "the mixing coefficient".