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Multiple sound scattering by densely packed shoals of marine animals

I. B. Andreeva and A. V. Belousov

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Brief information on packing density and size of fish and krill oceanic shoals is given. The sound field scattered by a mass of discrete inhomogeneities is calculated using the modified Börn approximation. For shoals of the above-mentioned animals, a validity range for the single scattering approximation is established. It is shown that the effect of multiple sound scattering is considerable only for densely packed shoals of small fishes and krill.

Key words: marine animals, multiple scattering, shoal, sound scattering.

I. B. Andreeva, and A. V. Belousov: Andreev Acoustical, Institute, Russian Academy of Sciences, ul Shvernika 4, Moscow, 117036 Russia. Correspondence to Andreeva [tel. +7 095 126 90 52, fax: +7 095 126 84 11].

Introduction

Pelagic animals of different types form shoals in the ocean. Hundreds, and sometimes thousands, of shoals of various kinds may exist simultaneously in biologically rich aquatic regions extending for hundreds of kilometres. The density n (no. of animals m^{-3}) can be very high. Until now, the acoustic scattering properties of shoals have been studied under the assumptions of single-scattering theory. This article considers the practical limits of applying the single-scattering theory of concentrations of discrete inhomogeneities. The first correction offered by the theory of multiple scattering is also obtained. These results are applied to dense shoals of fish and crustacea in the ocean.

The following brief information on the structure of pelagic oceanic shoals is based on the work of Yudanov (1992), which summarizes numerous experimental data from field observations of several hundred shoals. This information, illustrated in Figure 1, is in approximate agreement with other publications (e.g. see Misund, 1993). Region 1 in Figure 1 shows the characteristic dependence of the density, N, on the animal length, l. Curve 2 corresponds to the maximum observable density, when the average distance, D, between adjacent animals is equal to 2l $\{N=(2l)^{-3}\}$. Region 3 defines the values of N which correspond to a relative volume of animals approximately equal to 0.1% of the shoal values. Shoals with the highest concentration are formed by relatively small animals, such as small fishes (sprat, khamsa, etc.) and krill. The numerical density of the larger fishes is always lower. Horizontal dimensions of the shoals vary from a few tens of metres to several kilometres, while the vertical dimensions are much smaller and, as a rule, do not exceed 50–100 m.

Theory

Let us evaluate the acoustical properties of dense shoals of animals. The scattered field is calculated using the modified Börn approximation. A shoal is considered as a finite volume of randomly distributed discrete inhomogeneities with extinction cross-sections similar to those of marine animals.

In order to calculate the acoustical field scattered by shoals in any direction, we assume that a monochromatic wave from a transmitter with a gain function $\mathbf{A}_1(\mathbf{e}_0)$ is incident upon this volume, where $\mathbf{e}_0$ is a unit vector in the direction of radiation. The scattered power is detected by a distant remote receiver with a gain function $\mathbf{A}_2(\mathbf{e}_0)$. The scattered specific intensity $I(\mathbf{r},e)$ satisfies a well-known equation of radiation transfer (Ishimaru, 1978).

$$\mathbf{I}(\mathbf{r}+D\mathbf{e},\mathbf{e})=\mathbf{I}(\mathbf{r},\mathbf{e})\eta+\int\mathbf{I}(\mathbf{r},\mathbf{e}_0)\frac{\sigma(\mathbf{e},\mathbf{e}_0)}{4\pi D^2}d\Omega_0+\in(\mathbf{r},\mathbf{e}) \quad (1)$$

where $\varepsilon(\mathbf{r},\mathbf{e})$ is the equivalent source function describing the transfer of energy from a coherent to an incoherent form, $\sigma(\mathbf{e},\mathbf{e}_0)$ is the scattering cross-section of one scatterer in a given direction, D is the mean distance between scatterers, $\eta=1-\sigma_e/D^2$, σ_e is the extinction

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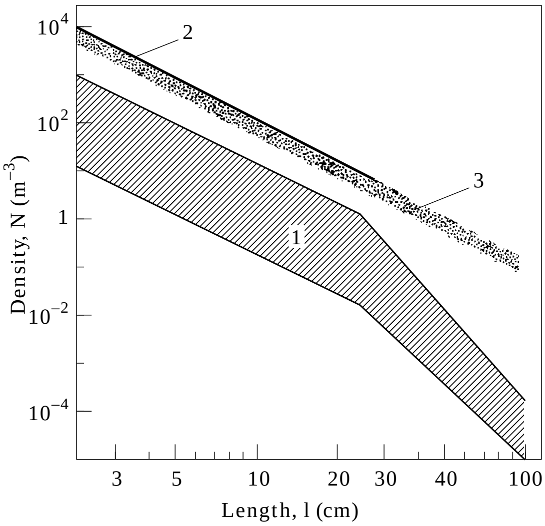

Figure 1. Packing density N of oceanic fishes and krill shoals versus their length l. (1) Typical values; (2) maximum value $N_{max} = (2l)^{-3}$; (3) region with a relative volume of scatterers about 0.1%.

cross-section of one scatterer, which is equal to a sum of the total scattering and absorption cross-sections, and Ω_0 is a solid angle. The value $\sigma(\mathbf{e},\mathbf{e}_0)$ defines the fraction of the scattered power propagating from each ensonified scatterer in the direction of the receiver within a unit solid angle.

Generally, the solution of equation (1) can be written as a series in orders of scattering:

$$\mathbf{I}(\mathbf{r},\mathbf{e}) = \sum_{h=1}^{\infty} I^{(h)}(\mathbf{r},\mathbf{e}), \tag{2}$$

$$\mathbf{I}^{(l)}(\mathbf{r},\mathbf{e}) = \sum_{j=0}^{q} \eta^j \epsilon(\mathbf{r}_j,e), \tag{3}$$

$$\mathbf{I}^{(h)}(\mathbf{r},\mathbf{e}) = \sum_{j=0}^{q} \eta^j \int \mathbf{I}^{(h-1)}(\mathbf{r}_j,\mathbf{e}_0)d\Omega_0, \tag{4}$$

$$\mathbf{r}_j = \mathbf{r} - \mathbf{e}D(j+1)$$

The limit of summation q is determined by the assumption that the scattering volume is finite.

Let us consider formula (3), which defines specific intensity in the modified Börn approximation in more detail. The intensities of individual scattered waves from all targets located within the common volume of gain function A_1 and A_2 are summarized, taking into account their attention due to energy losses caused by scattering and absorption along the paths of incident and scattered sound. We assume the modified Börn approximation to be applicable when the relative volume of scatterers does not exceed 10^{-3} of the total volume (Ishimaru, 1978). This corresponds to the shoal densities shown in Figure 1 where the narrow bar 3 is adjacent to the maximum observed values of N at small l. Thus, our approach can be successfully applied up to the highest densities found in natural conditions. Andreeva *et al.* (1994) gave an explicit expression for $\varepsilon(\mathbf{r}_j,\mathbf{e})$. Substituting this into Equation (3), we have:

$$\mathbf{I}^{(l)}(\mathbf{r},\mathbf{e}) = \sum_{j=0}^{q} \eta^j \frac{A_l[\mathbf{e}_0(\mathbf{r}_j)]P_t e^{-(\beta_1-\beta_2)r_Q(e_0)-\beta_2 r_j}\sigma(\mathbf{e},\mathbf{e}_0)}{(4\pi)^2 r_j^2 D^2} \tag{5}$$

where P_t is the power of the transmitter, β_1 is the sound absorption coefficient in water, $\beta_2 = \ln \eta/D$ is the additional coefficient of extinction due to scatterers, $\mathbf{e}_0$ is the unit vector in direction $\mathbf{r}_j$, and $\mathbf{r}_Q(\mathbf{e}_0)$ is the radius vector of the first point where the scattering volume boundary is crossed by a ray emerging from a radiation point in the direction $\mathbf{e}_0$. One must distinguish between "narrow" and "wide" transmitting and receiving patterns when using the modified Börn approximation. If the volume ν formed by the overlapping gain functions does not exceed that occupied by the scatterers, then we call the patterns narrow; otherwise we consider them to be wide. In the case of wide patterns,

$$\mathbf{r}_j = \mathbf{r}_Q(\mathbf{e}) + Dj\mathbf{e}, \tag{6}$$

where $\mathbf{r}_Q(\mathbf{e})$ is the radius vector of the first point at which the boundary of the cloud of scatterers is crossed by the ray emerging from a reception point in the direction $-\mathbf{e}$. In the case of a narrow pattern, r_Q means the boundary of the volume ν.

Expression (5) determines the power per unit solid angle received by the antenna from the direction **e**. In order to find the total received power J(**r**), it is necessary to integrate expression (5) over all rays arriving at the observation point and to take into account the directional pattern of the receiver. Furthermore, the gain functions of the transmitter and the receiver are assumed to be Gaussian (Ishimaru, 1978)

$$A_{1,2}(\mathbf{e}) = A_{1.2}(\mathbf{e}') \exp\{-(\ln 2)(2\theta/\theta_{1,2})^2 + (2\varphi/\varphi_{1,2})^2\} \tag{7}$$

where the angles θ and φ are measured from the vector **e′** in the vertical and horizontal directions, respectively, and the angles $\theta_{1,2}$ and $\varphi_{1,2}$ are the half-power beam widths. Let us assume that σ $(\mathbf{e},\mathbf{e}_0)$ is constant (the isotropic case). Omitting intermediate calculations, we obtain the final expression for the received power in the form

$$J(\mathbf{r}) = \frac{P_t \sigma A_1(\mathbf{e}_0')A_2(\mathbf{e}')}{(4\pi)^2 R_1^2 R_2^2} e^{-\beta_2(\Delta r_1 + \Delta r_2)} \nu N \tag{8}$$

where $R_{1,2}$ are the average distances from the transmitter and the receiver to the volume ν, and $\Delta r_{1,2}$ are the lengths of the ray paths from the bundary of ν to the point of intersection with the pattern maxima (vectors $\mathbf{e}'_0$ and $\mathbf{e}'$) for the transmitter and the receiver, respectively. If J_N is the ratio of $J(\mathbf{r})$ to the received power $J^{(1)}(\mathbf{r})$ scattered by a single scatterer located at the point of intersection with the pattern maxima, then

$$J_N = \frac{J(\mathbf{r})}{J^{(1)}(\mathbf{r})} = N\nu e^{-\beta_2(\Delta r_1 + \Delta r_2)} \quad (9)$$

Expression (9) enables one to estimate the effect of the density of the scatterers on the received power. According to Ishimaru (1978), the volume ν can be expressed as follows for Gaussian patterns. For the bistatic case, we have

$$\nu = 1.2 \frac{R^3 \theta_1 \theta_2 \varphi_1 \varphi_2 \{\sin^2[(\mathbf{e}_0 \mathbf{e}_s)]\}\{\sin^2[(\mathbf{e}'\mathbf{e}_s)]\}}{\{\sin^4[\theta_s]\}[\![\{\sin^2[(\mathbf{e}_0\mathbf{e}_s)]\}\varphi_1^2 + \{\sin^2[(\mathbf{e}'\mathbf{e}_s)]\}\varphi_2^2]\!](1/2)} \quad (10)$$

where R is the distance between the transmitter and the receiver, θ_s is the scattering angle and $\mathbf{e}_s$ is the unit vector in the direction from the transmitter to the receiver. For monostatic sounding, ν depends upon the duration t_0 of the transmitted pulse. If R_0 is now the distance from the transmitter to the boundary of shoal, and c is the sound velocity, then

$$\nu = 0.57\ \theta_1\ \varphi_1\ R^2 c t_0, \quad (11)$$

In Figure 2, J_N is plotted as a function of the density of the scatterers. Curve 1 corresponds to bistatic sounding and curve 2 corresponds to monostatic sounding. It can be seen that the linear dependence of the received power upon the density of the scatterers occurs only at small densities. With an increase in the number of scatterers, the number of scattered waves arriving at the observation point increases, resulting in growth of the scattered field. With a further increase in density, there is more attenuation within the shoal and the growth of the scattered field decelerates. In the case of narrow directional patterns, a further increase in N produces a sharp drop beyond some critical value N_{cr}. In the case of wide directional patterns, the saturation begins at N_{cr} because the scattering is primarily at the boundary of the cloud. The critical value of the density corresponds to the maximum of the curve $J_N(N)$ and is given by the expression

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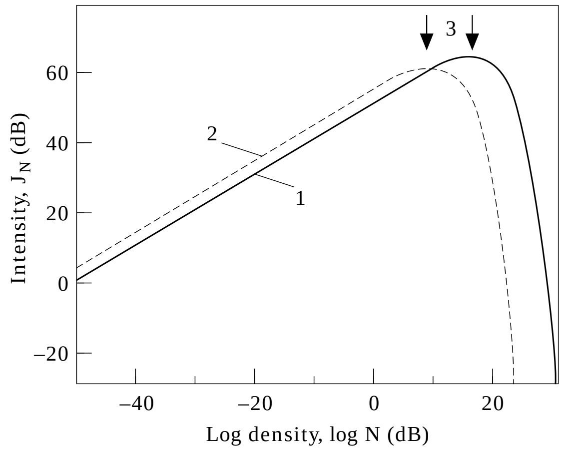

Figure 2. Dependence of received power upon density of scatterers for narrow gain functions. (1) Bistatic sounding, (2) monostatic sounding, (3) values of the critical density N_{cr}.

If we write N(l) and $\sigma_e(l,f)$ as length-dependent functions, and r as the length of sound path inside the scattering volume (equal to $\Delta r_1 + \Delta r_2$), then the critical path length, r_{cr}, follows immediately from (12):

$$r_{cr}(l,f) = [\sigma_e(l,f)\ N(l)]^{-1} \quad (13)$$

Equation (13) is the criterion for the validity limit of the single-scattering approximation. The scattered field can be estimated with the single-scattering approximation if $r \le r_{cr}$. If r is more than this, the scattered field must be evaluated using the multiple-scattering approximation.

We apply this criterion to estimate the effect of multiple sound scattering in shoals of oceanic fishes and krill, based on typical sizes of shoals and the known acoustic properties of marine animals.

Results

Multiple scattering in shoals is important only if the shoal size is more than or comparable with r_{cr}. As mentioned above, the size is seldom more than a few 10s or 100s of metres.

The acoustic cross-section of fish and krill has been studied by a number of authors (e.g. Andreeva, 1964; Love, 1977; Macaulay, 1994). For present purposes, we have used the results of Andreeva *et al.* (1994) who describe the scattering cross-sections σ for different oceanic animals at $l/\lambda \le 100$. For $l/\lambda \ge 1$, the actual value of σ depends strongly on both the directions $(\mathbf{e},\mathbf{e}_0)$ and the orientation of the animal. But according to the

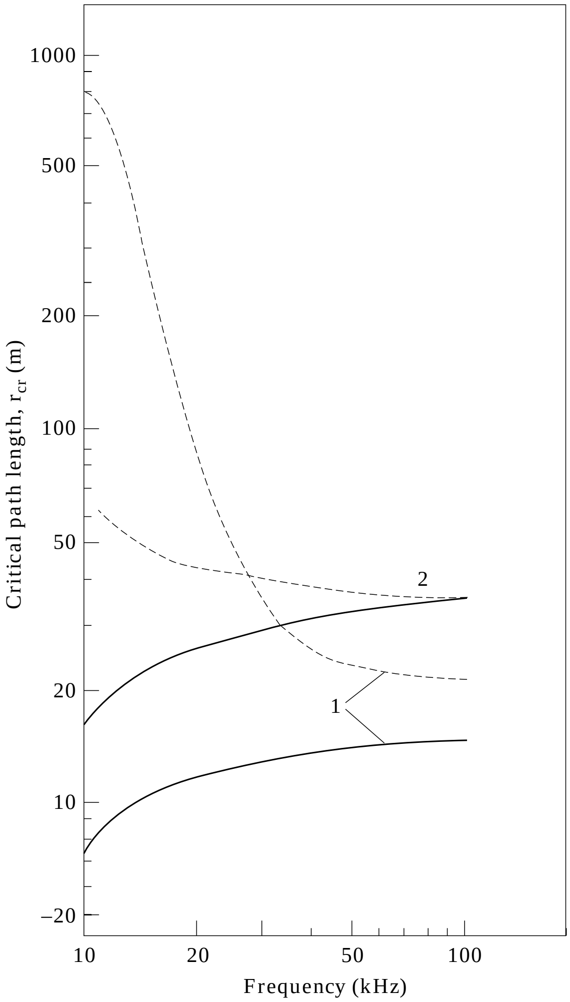

Figure 3. Frequency dependence of the critical path length r_{cr} in shoals of small fish at $N=(2l)^{-3}$. (1) l=5 cm, (2) l=10 cm. Solid lines=fishes with a swimbladder, dashed line=without.

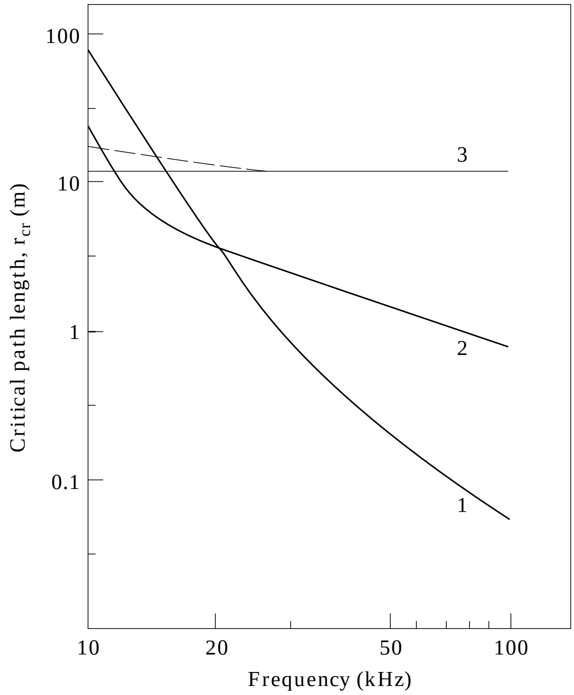

Figure 4. The frequency dependence of critical path length r_{cr} inside shoals of some ocean animals. (1) Krill, $N=(2l)^{-3}$, l=3 cm; (2) krill, $N=(2l)^{-3}$, l=10 cm; (3) fishes, l=30 cm, $N=0.1\ m^{-3}$. Solid lines=fishes with a swimbladder, dashed line=without.

developed scattering model, we ignore this dependence and, in numerical calculations we assume that σ_0 is simply the average value $<\sigma(\mathbf{e}_0,\mathbf{e})>$, at any frequency. Besides, owing to the lack of reliable data on sound absorption in marine animal tissues for different acoustic frequencies, we neglected this effect. Such an approach implies that the criterion (13) will overestimate the critical path length r_{cr}. Accordingly, we can write that $\sigma_e=\sigma_0$.

The critical range r_{cr} for shoals of different animals is shown in Figures 3 and 4 for frequencies 10–100 kHz. The single-scattering approximation is applicable within the areas under each curve, whereas one must take multiple scattering into account in regions above the curves. The results show that multiple scattering may be noticeable only in dense shoals of small animals (3–10 cm), because the critical path length is small enough ($r_{cr}\leq 1$ km) only in these cases. In other cases, the length r_{cr} is too large, that is, larger than any likely size of shoal. The greatest effect corresponds to small fishes with a swimbladder, which have high values of the scattering cross-section.

Our results are not highly accurate because of the simplifying assumptions and inaccurate data on shoal structure. In spite of the fact that it is clear that the single scattering approximation may be inadequate for solving direct and inverse problems of sound scattering by some shoals of sea animals, at frequencies of tens of kiloHertz and more. Thus, we have proposed a method to estimate the validity range of the single scattering approximation for actual shoals of marine animals, and a theory to evaluate the scattered field in terms of the modified Börn approximation.

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