# Evaluating the parameters of a MSY control rule for the Bristol Bay, Alaska, stock of red king crabs 

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A maximum sustainable yield (MSY) control rule, which defines the level of overfishing, and determines the control rule parameters based on an age-, sex-, and size-structured assessment for Bristol Bay red king crabs (Paralithodes camtschaticus) is developed. $F_{\mathrm{x} \%}$ ( $F$ corresponding to $x \%$ spawning potential ratio) is used as a proxy for $F_{M S Y}$ and a minimum spawning-stock biomass (to open the fishery) for incorporation into the MSY control rule. The performance of the selected MSY control rule and the associated target control rule is evaluated using stochastic simulations. $F_{50 \%}$ is a reasonable proxy for $F_{\text {MSy }}$ when effective spawning biomass is used as the stock biomass in the stock-recruitment relationship. This method with appropriate modifications might be used for determining biological reference points and developing control rules for any crustacean stock with discrete growth, complex reproductive dynamics, and single sex exploitation.
Keywords: age-, sex-, and size-structured assessment, biological reference points, Bristol Bay, MSY control rule, red king crab.
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## Introduction

In the current Bering Sea and Aleutian Islands (BSAI) crab management plan, the maximum sustainable yield (MSY) control rule is based on setting the limit fishing mortality $\left(F_{\text {lim }}\right)$ equal to the value of natural mortality $(M)$, independent of biomass. If the current fishing mortality, $F$, exceeds $F_{\text {lim }}$, overfishing is said to be occurring. Moreover, the management plan defines the "overfished" status in relation to a minimum spawning stock threshold (MSST), which is equal to half the MSY mature stock biomass (NPFMC, 1999). For some stocks, if spawning-stock biomass declines below half MSST, the fishery will be closed. The MSY mature stock biomass is estimated as the average total mature biomass observed in annual resource surveys from 1983 to 1997, which is considered to have been an ecologically stable period (NPFMC, 1999).

The applicability of $M$ as an estimate of $F_{\text {MSY }}$, which in turn is then used as value for $F_{\text {lim }}$, and average total mature biomass as an estimate for MSY biomass has been criticized for lacking scientific basis. Here, we develop an analytical method to establish a MSY control rule for crabs similar to those for fish stocks (Caddy and Mahon, 1995; Restrepo and Powers, 1999) and apply it to the Bristol Bay, Alaska, stock of red king crabs. This stock is the most extensively studied crab stock in the BSAI; a length-based method has been used to estimate abundance and biological reference points, and to explore harvest strategies (Zheng et al., 1995a, 1997a; Siddeek, 2002). We used estimated values of assessment model parameters (Zheng, 2006) as inputs to an age-, sex-, and size-structured projection model to investigate the MSY control rule and to estimate a proxy for $F_{\text {lim }}$. We also considered a plausible range of stock-recruitment (S-R) steepness parameter ( $h$ )
values (Mace and Doonan, 1988) to develop different yield curves to determine $F_{\mathrm{x} \%}$ ( $F$ corresponding to $\mathrm{x} \%$ spawning potential ratio) as proxies for $F_{\text {lim }}$, following Clark's (1991) procedure. The selected control rule parameters are evaluated under stochastic simulation using a selected set of performance statistics with resource conservation and fishery productivity in mind. We also used simulations to select a minimum spawning-biomass reference point for opening the fishery.

## Material and methods

## Control rules

The notations used in the text, tables, and figures are defined in Appendix 1. Following Restrepo et al. (1998), the proposed MSY control rule is expressed in terms of $F$ on legal-sized males as a function of effective spawning biomass (ESB):

$$
\begin{gather*}
\text { If } \frac{\mathrm{ESB}}{\text { proxy } \mathrm{ESB}_{\mathrm{MSY}}}>1, \quad F_{\lim }=\operatorname{proxy} F_{\mathrm{MSY}} \\
\text { If } \beta<\frac{\mathrm{ESB}}{\text { proxy } \mathrm{ESB}_{\mathrm{MSY}}} \leq 1, \\
F_{\lim }=\operatorname{proxy} F_{\mathrm{MSY}} \frac{\left(\mathrm{ESB} / \text { proxy } \mathrm{ESB}_{\mathrm{MSY}}\right)-\alpha}{1-\alpha} .  \tag{1}\\
\quad \text { If } \frac{\mathrm{ESB}_{\text {proxy }} \mathrm{ESB}_{\mathrm{MSY}}}{} \leq \beta, \quad F_{\lim }=0
\end{gather*}
$$

The ESB is a function of the nominal mature female biomass, mature male abundance, and a male-to-female mating ratio (see

Appendix 2 for details of the calculation method). We used a mating ratio of 1:3 for the ESB calculation, following Paul and Paul (1997). In the current BSAI crab fisheries management plan, the MSY control rule is used to define whether or not overfishing is occurring, and a target control rule is used to set catch limits. The latter is based on principles of stock conservation and trade-offs between mean yield and variation of yield.

Following Restrepo et al. (1998), the target control rule used in this study is

$$
\begin{equation*}
F_{\text {target }}=0.75 \times \text { proxy } F_{\mathrm{MSY}} \tag{2}
\end{equation*}
$$

Performance statistics were calculated for both limit and target control rules.

The 1985-2005 recruitment estimates from the stock assessment of Bristol Bay red king crabs were used to estimate the maximum number of recruits ( $R_{\max }$ ); based on the top $50 \%$ of annual recruitments, the value was 29 million crabs. These were used as input to Clark's (1991) method to determine $F_{\mathrm{x} \%}$ and in stochastic projections. We assume that spawning biomass, either ESB or mature male biomass (MMB), can be measured precisely. The estimates of red king crab stock assessment parameters (Zheng, 2006) from the 1985-2005 surveys and catch data based on $M=0.18$ were used as input values in all simulations.

The $h$ specifies the underlying S-R curve. We fitted the Ricker S-R curve to 1977-2005 data, and determined $h$ using the following formulation (Booth, 2004):

$$
\begin{equation*}
h R_{0}=\gamma\left(0.2 S_{0}\right) \mathrm{e}^{-\theta\left(0.2 S_{0}\right)} \tag{3}
\end{equation*}
$$

where $R_{0}$ is estimated at $S_{0}\left(=\mathrm{ESB}_{0}\right.$ or $\left.\mathrm{MMB}_{0}\right)$ with $F=0$, and $\gamma$ and $\theta$ are parameters of the Ricker $\mathrm{S}-\mathrm{R}$ model.

Because annual values of recruitment of the Bristol Bay red king crab stock have been lower since the early 1980s than previously (Zheng and Kruse, 2006), perhaps because of a North Pacific regime shift in 1976/1977 (Hare and Mantua, 2000), we used the $h$ value applicable to the present low-productivity period as the base value in the control rule parameter estimation and evaluation.

The $h$ estimate for the Ricker S-R fit to the 1977-2005 data was 0.7 , with EBS as stock biomass $S$. A $h$ range of $0.53-1.57$ at steps of 0.26 for the Ricker S-R model, and a corresponding range of $0.46-0.77$ for the Beverton-Holt S-R model were used for $F_{\mathrm{x} \%}$ estimation, based on the ESB unit. For mature male stock biomass, the $h$ estimate from the Ricker S-R fit to the 1977-2005 data was 0.86 . The $h$ ranges for $F_{\mathrm{x} \%}$ estimation based on MMB spawning biomass unit were $0.79-2.0$ (at steps of 0.3 ) and $0.58-0.82$ under Ricker and Beverton-Holt S-R curves, respectively. The lower and upper limits of the Ricker $h$ range were obtained by fitting $\mathrm{S}-\mathrm{R}$ curves to the lowest $25 \%$ and the highest $25 \%$ recruitment points, respectively. The ESB-based $\mathrm{S}-\mathrm{R}$ fits for different combinations of data are shown in Figure 1. We calculated Clark's $(1991,2002)$ D parameter [which is a density-dependent multiple such that $(R / S)_{S=0}=$ $\left.D\left(R_{0} / S_{0}\right)\right]$ corresponding to Ricker $h$ values $\left[D=(5 h)^{5 / 4}\right]$, and converted it into Beverton-Holt $h$ values $\left[h=(1+4 / D)^{-1}\right]$ to generate relative yield curves under the latter $\mathrm{S}-\mathrm{R}$ model. Clark (1991, 2002) used a plausible range of $D$ to generate relative yield curves under the two $S-\mathrm{R}$ models to determine $F_{\mathrm{x} \%}$. $D$


Figure 1. Ricker S-R curve fitted to 1977-2005 red king crab S-R data (solid curve). The straight line is the replacement line passing through $(0,0)$ and ( $\mathrm{ESB}_{0}, R_{0}$ ) points. The dashed curves correspond to Ricker S-R curves for boundary values of the $h$ considered in the $F_{\mathrm{x} \%}$ estimation.
describes the level of spawner productivity at very low stock sizes in relation to spawner productivity at an unfished level.

## Evaluation of control rule parameters

The proposed control rules have five parameters: $\alpha, \beta$, proxy $F_{\mathrm{MSY}}$, and proxy $\mathrm{ESB}_{\mathrm{MSY}}$ from Equation (1), and $F_{\text {target }}$ from Equation (2). Although most analyses were done considering ESB in the S-R models, a few simulations with MMB in the S-R models were also carried out for sensitivity analysis. Although ESB may be a better estimate of crab spawning biomass, exact mating ratios can be uncertain in the wild. The MMB-based $F_{\mathrm{x} \%}$ estimate avoids the use of mating ratios in spawning biomass calculations.

Male red king crabs are assumed to be functionally mature by 120 mm cephalothorax length (CL) (Zheng et al., 1995a). Approximately $50 \%$ of females of 89 mm CL and $80 \%$ of females of 95 mm CL are mature (Otto et al., 1990). Size ranges of $65-200 \mathrm{~mm}$ CL for males and $65-165 \mathrm{~mm}$ CL for females were considered in the simulation models, to include immature sizes of crabs as initial recruits to the cohorts.

Simulations were initiated with a fixed number of immature new-shell recruits to the modelled population, divided equally between males and females and distributed between length bins by a probability function (Appendix 2). Full age structure was established by deterministically projecting the initial recruits through their entire lifespan up to a maximum age ( 26 y ) with a given set of values of mortality and growth parameters. For Clark's (1991) method, once the full equilibrium age structure was achieved, the projection process stopped, and the ESB $/ \mathrm{R}$ ratio (relative to ESB/R at $F=0$ ) and equilibrium yield were estimated using $S-R$ curves for a range of values of $h$. For the stochastic simulations, the recruits were generated by a stochastic $S-R$ model with lognormal random errors (variance $\sigma^{2}$ and temporal correlation $\rho$ ) for a 100-y fishing period, to estimate various reference points based on fishing mortality and biomass. The recent recruitment distributions indicated low temporal correlation, so a base value of 0.7 (based on a Ricker S-R fit to 1977-2005
data) for $\sigma$ and $\rho=0$ were used in the simulations with ESB, and $\sigma=0.54$ and $\rho=0$ were used for simulations with MMB.

Clark (1991) derived the $F_{\mathrm{x} \%}$ harvest rate for groundfish stocks for deterministic Beverton and Holt (1957) and Ricker (1954) S-R models. Our simulations followed the same approach, but with a number of modifications and improvements suitable for crab life history (see Appendix 2 for computation formulae). Clark's method was used only to locate an approximate $F_{\mathrm{x} \%}$, and detailed simulation analyses were carried out considering a number of $F_{\mathrm{x} \%}$ candidate values near this value to identify an appropriate $F_{\mathrm{x} \%}$ value as a proxy for $F_{\mathrm{MSY}}$.

The proxy $\mathrm{ESB}_{\mathrm{MSY}}$ parameter for the control rule was estimated from the stochastic simulation of a 100-year fishery, with a Ricker S-R curve for the base $h$ value and the selected fixed $F_{\mathrm{x} \%}$ value. The average $\mathrm{ESB} / \mathrm{ESB}_{0}$ ratio was used as proxy $\mathrm{ESB}_{\mathrm{MSY}} / \mathrm{ESB}_{0}$. $\mathrm{ESB}_{0}$ was estimated at $F=0$.

Three probable values for $\alpha$ and $\beta$ parameters were investigated: $\alpha(0.0,0.05$, and 0.1$)$ and $\beta(0.2,0.25$, and 0.3$)$. In the Alaska groundfish fisheries management Tier system, $\alpha=0.05$ (NPFMC, 1998), and in some crab fisheries management Tier systems $\beta=0.25$ are currently used. These parameters were evaluated by simulating the rebuilding of a hypothetical overfished stock and computing various performance statistics. The overfished stock began at a low level of 0.19 proxy ESB $_{\mathrm{MSY}}$. The stock was considered rebuilt upon reaching the proxy $\mathrm{ESB}_{\mathrm{MSY}}$ under a given $F_{\mathrm{x} \%}$ with stochastic recruitment. A number of performance statistics were estimated from a thousand simulations of a 100-year fishery with random recruitment, to explore the viability of selected values of control rule parameters: median and interquartile range (IQR) of rebuilding time (the time in years for ESB to first reach the proxy $\mathrm{ESB}_{\mathrm{MSY}}$ level), overfished proportion (when $\mathrm{ESB} \leq 50 \%$ proxy $\mathrm{ESB}_{\mathrm{MSY}}$ ), fishery closure proportion (when ESB $\leq 25 \%$ proxy $\mathrm{ESB}_{\mathrm{MSY}}$ ), mean yields over a short term (during the first 10 years of the rebuilding period) and a medium term (during the next 20 years of the rebuilding time period), and the 30th year $\mathrm{ESB} /$ proxy $\mathrm{ESB}_{\mathrm{MSY}}$ ratio. Additional performance statistics were computed to investigate the control rules: median values of mean yield and relative interannual yield difference $\left(\Sigma\left|y_{t}-y_{t-1}\right|\right) / \Sigma y_{t}$; Punt et al., 2002) in the short and the medium term (Butterworth and Punt, 1999), and the median and IQR of the final (100th) year ESB/proxy ESB MSY ratio. The 100th year ratio provides the long-term effect on the biomass of a given $F_{\mathrm{x} \%}$.

## Evaluation of alternative control rules

A range of sensitivity studies of limit and target control rules was carried out:
(i) Different $F_{\mathrm{x} \%}$, different stock productivity (low mating ratio and low steepness), and MMB-based $F_{\mathrm{x} \%}$ scenarios were considered to explore the sensitivity of the MSY control rule to rebuild the stock from a low biomass;
(ii) Different levels of recruitment error were used in the target control rule to explore that rule's sensitivity under different levels of recruitment error. The initial stock size was set at the 1997/1998 level for these simulations;
(iii) A zero fishing mortality was used in the control rule to compare the fishery performance statistics with those obtained under $F>0$. The initial stock size was set at the 1997/1998 level for this simulation;
(iv) Observation and implementation errors were introduced to evaluate the performance of the target control rule. Lognormal observation errors ( $\sigma_{1}=0.2$ ) were introduced to biomass estimates (i.e. ESB was set to ES $\hat{B} e^{z \sigma_{1}-\left(\sigma_{1}^{2} / 2\right)}$ ) and truncated normal errors ( $\sigma_{2}=0.1$ ) were added to catch estimates (i.e. $C$ was set to $\hat{C}\left(1+z \sigma_{2}\right)$, where $z \sim$ $N(0,1)$, and $z$ values were truncated at the $80 \%$ confidence limits). Although lognormal catch errors are used in some fishery simulations (Quinn and Deriso, 1999), we used truncated normal catch errors with a lower value of standard deviation because the implementation errors were generally low. Scenarios with MMB-based values for $F_{\mathrm{x} \%}$ were also considered in these simulations. The initial stock size was set at 1997/1998 and 2005 levels for these simulations. The 1997/1998 and 2005 biomass levels were equivalent to $\sim 68$ and $94 \%$ proxy $\mathrm{ESB}_{\mathrm{MSY}}$, respectively, providing true fishery biomass scenarios, with the former above half (the overfished level), and the latter closer to full $\mathrm{ESB}_{\mathrm{MSY}}$ levels.
(v) The $\beta$ parameter was set to 0 in the target control rule [Equation (2)], along with observation and implementation errors to investigate the necessity of this parameter in the control rule formula. The initial stock size was set at the 1997/1998 level for this simulation.

## Results

## Evaluation of control rule parameters

The $F_{\mathrm{x} \%}$ estimates following Clark's (1991) method resulted in $F_{51 \%}$ under ESB and $F_{37 \%}$ under MMB spawner units for Beverton-Holt and Ricker S-R curves with equilibrium yields, and with base parameter values (Figure 2 a and b). We chose $F_{50 \%}$ under ESB and $F_{35 \%}$ under MMB spawner units as proxy $F_{\mathrm{MSY}}$ candidates for further evaluation. Hereafter, we differentiate results based on the two spawning biomass units, ESB and MMB, by placing MMB in parenthesis for the MMB-based $F_{\mathrm{x} \%}$ estimate.

The average $\mathrm{ESB} / \mathrm{ESB}_{0}$ ratio from stochastic simulations of a 100 -year fishery with the Ricker S-R model using an estimated $h$ of 0.7 and $F_{50 \%}$ was 0.5661 . The $\mathrm{ESB}_{0}$ estimate at $F=0$ was 52274 t . Therefore, the proxy $\mathrm{ESB}_{\text {MSY }}$ estimate was 29593 t $\left(0.5661 \mathrm{ESB}_{0}\right)$. The proxy $\mathrm{MMB}_{\text {MSY }}$ was estimated to be 43842 t $\left(0.4617 \mathrm{MMB}_{0}\right)$ by the same procedure. These proxy values were used in the control rule formula to evaluate control rule parameters. The rebuilding of the stock from a hypothetical severely overfished level of 0.19 proxy $\mathrm{ESB}_{\text {MSY }}$ was done under the proposed limit control rule with $F_{50 \%}$, a stochastic Ricker S-R model with an estimated $h$ value of 0.7 , the ESB spawner unit, and a mating ratio of $1: 3 . F_{\mathrm{x} \%}$ values of $F_{45 \%}, F_{50 \%}$, and $F_{55 \%}$, and different combinations of the three different values of $\alpha$ and $\beta$, were used. The results were similar for different values of $F_{\mathrm{x} \%}$, so only the results for $F_{50 \%}$ are given in Table 1. There was no significant change in the rebuilding time, overfished proportion, or fishery closure proportion for different values of $\alpha$ and $\beta$. The median 30th year ESB/proxy $\mathrm{ESB}_{\mathrm{MSY}}$ ratio and the median medium-term mean yield increased as $\alpha$ and $\beta$ values increased. The opposite was true for the median short-term yield. The stock rebuilt above proxy $\mathrm{ESB}_{\text {MSY }}$ with $p>0.5$ in the 30th year for all combinations of $\alpha$ and $\beta$. This suggests that $F_{50 \%}$ is a reasonable proxy for $F_{\text {MSY }}$ to rebuild the stock from low levels for all $\alpha$ and $\beta$ combinations chosen in this study. For $\alpha$ fixed at 0.1 , the median short-term mean yield under $\beta=0.25$ was


Figure 2. Estimate of $F_{\mathrm{x} \%}$ under Beverton - Holt (broken curves) and Ricker (solid curves) S-R models. (a) ESB as stock size; (b) MMB as stock size. Red king crab base parameter values were used as input to the simulations.
higher, but the median medium-term mean yield was lower than under $\beta=0.3$. Although higher $\beta$ values tend to produce higher medium-term yields, it may increase the fishery closure proportions at low stock levels (results not shown). Therefore, we selected $\alpha=0.1$ and $\beta=0.25$ as defaults because the stock was rebuilt within a reasonable time period (median rebuilding time of 19 years compared with mean generation time of 14.2 years, estimated following Restrepo et al., 1998), and because the median 30th year ESB/proxy $\mathrm{ESB}_{\mathrm{MSY}}$ ratio and the medium-term mean yield were larger than those at $\beta=0.2$ (Table 1).

## Evaluation of alternative control rules

The median rebuilding time, overfished proportion, and fishery closure proportion increased and the 100th year ESB/ESB MSY ratio decreased for lower steepness $(h=0.61)$, lower mating ratio (1:2), and constant $F_{50 \%}$ (i.e. $F$ independent of biomass) compared with those for $F_{50 \%}, F=2 M$, and $F_{35 \%}$ (MMB) with the control rule. The performance statistics among the latter three were similar except for $F_{35 \%}$ (MMB), which produced slightly higher yields because of slightly greater $\mathrm{S}-\mathrm{R}$ productivity (higher value of the $h$ ). The median short-term yields were high for $F_{35 \%}$ (MMB) and constant $F_{50 \%}$, and median medium-term yields were high for $F_{35 \%}$ (MMB), $F_{50 \%}$, and $F=2 M$. The constant $F_{50 \%}$ produced a high proportion of fishery closures, and a low 100th ESB/ESB MSY $^{\text {ratio and medium-term mean yield }}$ (Table 2). The control rules behaved as expected as a function of initial depletion.

Table 1. Performance statistics (median and IQR) for different $\alpha$ and $\beta$ parameters of the MSY control rule of a hypothetical severely overfished red king crab stock for the base case scenario. RBT, rebuilding time; short term, first 10 years of the rebuilding period; medium term, next 20 y of the rebuilding period.

| Control rule parameter $\boldsymbol{\alpha}$ | 0.0 | 0.05 | 0.1 | 0.0 | 0.05 | 0.1 | 0.0 | 0.05 | 0.1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Control rule parameter $\boldsymbol{\beta}$ | 0.2 | 0.2 | 0.2 | 0.25 | 0.25 | 0.25 | 0.3 | 0.3 | 0.3 |
| RBT (y) | 19 (18-21) | 19 (18-21) | 19 (18-21) | 19 (18-21) | 19 (18-21) | 19 (18-21) | 19 (18-21) | 19 (18-21) | 19 (18-21) |
| Overfished proportion | 0.17 (0.16-0.17) | 0.17 (0.16 0.17) | 0.17 (0.16-0.17) | 0.16 (0.16-0.17) | 0.16 (0.16-0.17) | 0.16 (0.16-0.17) | 0.16 (0.16-0.17) | 0.16 (0.16-0.17) | 0.16 (0.16-0.17) |
| Fishery closure proportion | 0.09 (0.06-0.11) | 0.09 (0.06-0.11) | 0.09 (0.06-0.11) | 0.09 (0.06-0.11) | 0.09 (0.06-0.11) | 0.09 (0.06-0.11) | 0.09 (0.06-0.11) | 0.09 (0.06-0.11) | 0.09 (0.06-0.11) |
| 30th year ESB/ proxy ESB MSY | 1.05 (0.9-1.21) | 1.06 (0.92-1.23) | 1.09 (0.94-1.25) | 1.09 (0.95-1.27) | 1.11 (0.96-1.28) | 1.13 (0.98-1.30) | 1.15 (1.0-1.33) | 1.16 (1.01-1.34) | 1.17 (1.02-1.35) |
| Short-term mean yield ( t ) | 541 (307-898) | 485 (266-828) | 426 (220-751) | 492 (261-846) | 446 (230-782) | 392 (195-711) | 422 (138-765) | 387 (123-708) | 346 (105-653) |
| Medium-term mean yield ( t ) | 8172 (7 199-9 407) | 8314 (7323-9589) | 8484 (7480-9835) | 8555 (7584-9922) | 8645 (7651-10026) | 8770 (7728-10 146) | 9014 (7901-10351) | 9068 (7955-10 399) | 9128 (7999-10 502) |

Table 2. Performance statistics (median and IQR) for MSY control rule under different proxy $\mathrm{F}_{\text {MSY }}$ and stock productivity parameters.

| $\mathrm{F}_{\text {x\% }}$ | $\mathrm{F}_{50 \%}$ | Constant $\mathrm{F}_{50 \%}$ | $\mathrm{F}_{50 \%}$ | $\mathrm{F}_{50 \%}$ | $F=2 M$ | $\mathrm{F}_{35 \%}$ (MMB) | $F_{35 \%}$ (MMB) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| S-R variability parameter $\boldsymbol{\sigma}$ | 0.7 | 0.7 | 0.7 | 0.7 | 0.7 | 0.7 | 0.54 |
| Steepness | 0.7 | 0.7 | 0.61 | 0.7 | 0.7 | 0.7 | 0.86 |
| Mating ratio | 1:3 | 1:3 | 1:3 | 1:2 | 1:3 | 1:3 | Not applicable |
| RBT (y) | 19 (18-21) | 50.5 (0-78) | 32 (31-35) | 38 (34-53) | 19 (18-21) | 19 (18-21) | 16 (15-17) |
| Overfished proportion | 0.16 (0.16-0.17) | 0.71 (0.55-0.9) | 0.24 (0.20-0.28) | 0.26 (0.23-0.31) | 0.16 (0.15-0.17) | 0.17 (0.16-0.17) | 0.11 (0.09-0.12) |
| Fishery closure proportion | 0.09 (0.06-0.11) | 0.38 (0.28-0.6) | 0.16 (0.13-0.18) | 0.18 (0.15-0.2) | 0.09 (0.06-0.11) | 0.09 (0.06-0.11) | 0.06 (0.05-0.07) |
| 100th Year ESB/proxy ESB MSY $_{\text {or }}$ $M M B /$ proxy $\mathrm{MMB}_{\text {MSY }}$ (last column's result) | 0.99 (0.85-1.18) | 0.69 (0.21-0.93) | 0.93 (0.8-1.11) | 0.76 (0.66-0.9) | 1.14 (0.98-1.36) | 0.92 (0.78-1.09) | 0.99 (0.86-1.15) |
| Short-term mean yield (t) | 392 (195-711) | 1405 (1141-1727) | 0 | 0 | 292 (145-532) | 458 (228-828) | 476 (270-700) |
| Medium-term mean yield ( t ) | 8770 (7728-10 146) | 2127 (1667-2595) | 3991 (2739-5650) | 2673 (1642-4076) | 7353 (6432-8489) | 9527 (8423-11 069) | 9758 (8934-10632) |



Figure 3. Proportion of years in a 100-year fishery that the ESB was below $25 \%$ proxy $\mathrm{ESB}_{\mathrm{MSY}}$ vs. relative $\mathrm{ESB} / R$ for the red king crab stock. The estimates were made under stochastic Beverton-Holt (BH) (broken curves) and Ricker (RC) (solid curves) stock-recruitment models, with ESB as an index of spawning biomass, constant $F$, and using base input parameter values. The arrows point to $F_{45 \%}, F_{50 \%}$, and $F_{55 \%}$ levels of fishing mortality.

We also considered a cross-over scenario of applying a policy developed for MMB when the S-R relationship was a function of ESB, but it affected the results only slightly (Table 2).

Among different $F_{\mathrm{x} \%}$ estimates, from $F_{30 \%}$ to $F_{60 \%}$, the proportion of years ESB was depleted below $25 \%$ ESB $_{\text {MSY }}$ (leading to fishery closure) was 0 for $h \geq 0.52$ under $F_{50 \%}$ (Figure 3). Therefore, considering a number of performance statistics, $F_{50 \%}$ appears to be a robust proxy for $F_{\text {MSY }}$ for the red king crab (limit) MSY control rule.

Following Restrepo and Powers (1999), a default target harvest (optimum yield) control rule was used with $75 \% F_{\mathrm{x} \%}$ (i.e. $F_{\text {target }}$ ) and the $\alpha=0.1$ and $\beta=0.25$ combination to investigate the performance of the target control rule. This was done in two steps. First, the performance of target $F_{50 \%}$ was investigated for two combinations with higher temporal correlation and recruitment variability ( $\sigma=0.7$ and $\rho=0.5 ; \sigma=0.9$ and $\rho=0.9$ ). The results were compared among themselves and with the performance of $F=0$, assuming no observation and implementation errors. The initial abundance was kept at the estimated total mature biomass obtained from length-based stock assessment for 1997/1998 ( $\sim 58400 \mathrm{t}$ ) in these simulations. Second, similar simulation analyses were carried out with default target harvest control rules of $F_{50 \%}$ and $F_{35 \%}$ (MMB), with observation and implementation errors. The initial abundances were kept at total mature biomass estimates obtained for 1997/1998 (low, $\sim 58400 \mathrm{t}$ ) and 2005 (high, $\sim 87700 \mathrm{t}$ ) in these simulations.

At the base $\sigma$ and $\rho$ values, the median rebuilding time was lowest for $F=0$. The median overfished proportion was highest for $F_{50 \%}$ with higher values of $\sigma$ and $\rho$. The median 100th year ESB/proxy ESB $_{\text {MSY }}$ ratio was highest for $F=0$ and lowest for $F_{50 \%}$ with highest values of $\sigma$ and $\rho(0.9)$. The median shortand medium-term mean yields were the highest, and relative yield differences were the lowest for $F_{50 \%}$ with base values of $\sigma$ and $\rho$ (Table 3).

Table 3. Performance statistics (median and IQR) for target (optimum yield) control rule with $75 \%$ of ESB-based $F_{50 \%}$ and zero fishing mortality.

| $\begin{aligned} & \hline F_{\mathrm{x} \%} \\ & \mathrm{~S}-\mathrm{R} \text { variability parameters } \sigma, \rho \end{aligned}$ | $\begin{aligned} & F_{50 \%} \\ & 0.7,0.0 \end{aligned}$ | $\begin{aligned} & \hline F_{50 \%} \\ & 0.7,0.5 \end{aligned}$ | $\begin{aligned} & \hline F_{50 \%} \\ & 0.9,0.9 \end{aligned}$ | $\begin{aligned} & F=0.0 \\ & 0.7,0.0 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: |
| RBT (y) | $10(8-13)$ | $10(8-13)$ | 20 (10-32) | $8(7-10)$ |
| Overfished proportion | 0.06 (0.05-0.07) | 0.06 (0.05-0.07) | $0.35(0.28-0.43)$ | 0.05 (0.05-0.06) |
| Fishery closure proportion | 0 | 0 | 0.06 (0.03-0.12) | 0 |
| 100th year ESB/proxy ESB MSY | 1.13 (0.97-1.34) | 1.12 (0.94-1.38) | 0.72 (0.43-1.31) | 1.69 (1.48-1.95) |
| Short-term mean yield ( t ) | $2402(1794-3227)$ | 2324 (1649-3278) | 865 (519-2 035) | 0 |
| Short-term relative yield difference | 0.23 (0.19-0.26) | 0.23 (0.19-0.27) | 0.26 (0.23-0.32) | 0 |
| Medium-term mean yield ( t ) | 8959 (8006-10011) | 8854 (7772-10 176) | 5762 (3158-10 676) | 0 |
| Medium-term relative yield difference | 0.09 (0.07-0.11) | 0.10 (0.09-0.12) | 0.23 (0.20-0.27) | 0 |

Under observation and implementation errors, as expected, the median rebuilding times were lower at higher initial biomass levels, and the median overfished proportion was slightly higher for higher observation and implementation errors ( $\sigma_{1}=\sigma_{2}=$ 0.3 ). The two levels of initial biomasses rebuilt or maintained the stock above MSY level by the 100th year with $p>0.5$ for both forms of target control rule (ESB and MMB). For the data sets considered in this analysis, the mean yields under $F_{50 \%}$ were slightly lower than those under $F_{35 \%}(\mathrm{MMB})$, but the other statistics were similar. The higher initial biomass (2005) produced higher median short- and medium-term mean yields than the lower initial biomass (1997/1998). Both median mean yields were lowest for higher observation and implementation errors for $F_{50 \%}$ (Table 4).

## Discussion

We used the stochastic Ricker S-R model in most cases to investigate the performance of the control rules because the red king crab S-R scatter points suggested that the Ricker model was more appropriate for the stock. The $\mathrm{S}-\mathrm{R} h$ indicates that the productivity of a stock and the optimal harvest are highly sensitive to the value of the parameter. A higher $h$ means greater productivity. The $h$ ranges used here are slightly more conservative than the ranges used by Clark (1991) for groundfish stocks. Crab recruitment in the eastern Bering Sea has been weaker during the past two decades than historically (Zheng and Kruse, 2006). For example, the values were 0.7 for ESB and 0.86 for MMB for Bristol Bay red king crabs when the 1977-2005 S-R data were fitted to the Ricker model. On the other hand, when the whole data series (1968-2005) was considered, they were 0.97 and 1.16 , respectively

Table 4. Performance statistics (median and IQR) for target (optimum yield) control rule with $75 \%$ of ESB-based $F_{50 \%}$ and MMB-based $F_{35 \%}$ under observation and implementation errors.

| $F_{\text {x\% }}$ | $F_{50 \%}$ | $F_{50 \%}$ | $F_{50 \%}$ | $F_{50 \%}$ | $F_{35 \%}$ (MMB) | $F_{35 \%}$ (MMB) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| S-R variability parameters $\boldsymbol{\sigma}, \boldsymbol{\rho}$ | 0.7, 0.0 | 0.7, 0.0 | 0.7, 0.0 | 0.7, 0.0 | 0.54, 0.0 | 0.54, 0.0 |
| Control rule parameters $\alpha, \beta$ | 0.1, 0.0 | 0.1, 0.25 | 0.1, 0.25 | 0.1, 0.25 | 0.1, 0.25 | 0.1, 0.25 |
| Observation error $\sigma_{1}$, implementation error $\sigma_{2}$ | 0.2, 0.1 | 0.2, 0.1 | 0.3, 0.3 | 0.2, 0.1 | 0.2, 0.1 | 0.2, 0.1 |
| Initial total mature biomass ( $\mathbf{t}$ ) | 58400 | 58400 | 58400 | 87700 | 58400 | 87700 |
| RBT (y) | $10(8-12)$ | $10(8-12)$ | $10(8-12)$ | $8(5-10)$ | $9(8-12)$ | $6(4-8)$ |
| Overfished proportion | 0.05 (0.04-0.07) | 0.05 (0.04-0.07) | 0.06 (0.05-0.08) | 0 (0-0.01) | 0.03 (0.02-0.04) | 0 (0-0.01) |
| Fishery closure proportion | 0 | 0 | 0 (0-0.01) | 0 | 0 | 0 |
| 100th year ESB/proxy $\mathrm{ESB}_{\mathrm{MSY}}$ or MMB/ proxy $\mathrm{MMB}_{\text {MSY }}$ (last two columns' results) | 1.11 (0.92-1.4) | 1.11 (0.92-1.4) | 1.07 (0.84-1.41) | 1.11 (0.92-1.4) | 1.10 (0.91-1.38) | 1.10 (0.91-1.38) |
| Short-term mean yield ( t ) | 2352 (1735-3 175) | $2352(1734-3175)$ | 2263 (1622-3 051) | 5143 (4184-6182) | 3127 (2479-4000) | 6025 (5312-6933) |
| Short-term relative yield difference | 0.25 (0.22-0.29) | 0.26 (0.22-0.3) | 0.34 (0.28-0.41) | 0.15 (0.12-0.18) | 0.27 (0.23-0.31) | 0.18 (0.15-0.21) |
| Medium-term mean yield ( t ) | 8643 (7680-9745) | 8666 (7701-9748) | 8233 (7212-9415) | 8723 (7708-9804) | 9411 (8579-10280) | 9282 (8457-10 187) |
| Medium-term relative yield difference | 0.13 (0.12-0.15) | 0.13 (0.12-0.15) | 0.27 (0.24-0.31) | 0.13 (0.12-0.15) | 0.15 (0.13-0.17) | 0.15 (0.13-0.17) |

(S-R curves are not shown). We chose the conservative $h$ values of 0.7 for ESB and 0.86 for MMB in control rule evaluation simulations to reflect the low productivity in recent years. The underlying principle behind this choice was that if an overfished stock could be rebuilt or a healthy stock could be maintained with a selected control rule under this low productive regime, then the same control rule could very well rebuild an overfished stock or maintain a healthy stock under greater productivity (i.e. higher $h$ ) regimes. The trade-off is that a strategy that enhances recovery time will generally lead to lower catches in the short term.

Unlike in finfish, the mating ratio plays an important role in crabs when computing ESB for fitting S-R models. Mating of crabs is complex and its success depends on the spatial distribution, sex ratio, and size difference between mature females and males. In confined environments, large male red king crabs ( $>140 \mathrm{~mm}$ CL) in Kodiak are capable of mating with 7-9 female crabs successfully (Powell et al., 1974). Another laboratory study shows that most male red king crabs $\geq 140 \mathrm{~mm}$ CL can fertilize at least three females during the brief period when most multiparous females breed (Paul and Paul, 1997). Small male red king crabs ( $120-139 \mathrm{~mm}$ CL) are generally successful at fully fertilizing egg clutches of at least 2-4 females, and male red king crabs $>90 \mathrm{~mm}$ CL can generally fertilize at least one female in a laboratory (Paul and Paul, 1990). Mating ratios in the wild may be lower than those in the laboratory because of spatial limitation. Zheng et al. (1995a) assumed mating ratios from 1:1 to 1:3 based on the carapace length of mature males, with both new-shell and oldshell males participating in mating. In this analysis, a 1:3 mating ratio was used with mature males that are at least 10.5 months beyond their last moult, as a conservative measure to compute reference points. Under the current harvest strategy, old-shell mature male abundance is generally less than new-shell mature male abundance for Bristol Bay red king crabs.

Use of a constant mating ratio to compute ESB is debatable because the mean mating ratio in the wild can change with sex ratio, shell condition, size, and spatial distributions. Therefore, we also considered a few scenarios of performance statistics under MMB-based $F_{\mathrm{x} \%}$, which disregards mating ratio in the calculation of spawning biomass. Overall performance statistics by the two spawning-biomass units (ESB and MMB) were similar, even though the $F_{\mathrm{x} \%}$ values were different.

There are other potential options for measuring spawning biomass when calculating crab $\mathrm{S}-\mathrm{R}$ relationships. Available options are (i) total mature female biomass, (ii) total male and female mature biomass, (iii) fully mated (effective) mature female biomass, (iv) fully mated mature female and effective MMB, and (v) total MMB (mentioned above). As larvae come from fully mated mature females, effective mature female biomass is a better measure of larval abundance than other biomass indices. Therefore, we used option (iii) as the base biomass unit in our study. Zheng et al. (1995a, b) derived a Ricker S-R model with varying spawning-biomass units for Bristol Bay red king crabs. The one with effective mature female biomass, option (iii), provided a good fit, which supported our choice of ESB unit in the S-R models for a major part of this analysis. Reference point mortality estimates under option (iv) were similar to those under option (iii) for Bristol Bay red king crabs.

We investigated the performance of the proposed constant fishing mortality MSY control rule with a proxy $F_{\text {MSY }}$ and a default target control rule ( $=75 \%$ proxy $F_{\mathrm{MSY}}$ ) under stochastic simulations.

A constant fishing mortality was used in the control rule formula following the current practice of defining overfishing for BSAI crab stocks (NPFMC, 1999). Although performance statistics on finfish management are usually evaluated under target control rules, we provided the performance statistics under both limit and target control rules. Therefore, once a reasonable limit control rule has been established, the limit catch estimated from this control rule can be used as an upper limit for the target catch that may be taken considering various biological and non-biological factors.

Siddeek (2003) investigated the relationship between $F_{\text {MSY }}$ and $M$ and determined the MSY level harvest rate of legal male red king crabs using deterministic Beverton-Holt and Ricker S-R models under a more conservative $h$ range of $0.35-0.52$ and a higher $M$ range of $0.2-0.4$. Although the results of the previous and the current studies are not comparable because of different approaches and input parameter values used in the two studies, the previous study indicated that $F_{\text {MSY }}$ was higher than $M$ for a wide range of $h$ for the mating ratio 1:3. The fishing mortality values corresponding to proposed $F_{50 \%}$ or $F_{35 \%}$ (MMB) were higher than $2 M$. These results reflect the fact that the crab fishery is male-only, with a size limit above the size at maturity. Hence, an overfishing control rule with $M$ as a proxy for $F_{\mathrm{MSY}}$ on commercial size crab is conservative.

The performance statistics of the $F_{50 \%}$ control rule under observation and implementation errors did not show any unusual results, so $F_{50 \%}$ appears to be a reasonable proxy for $F_{\text {MSY }}$ to rebuild or to maintain stock levels with plausible observation and implementation errors. However, our analysis did test other plausible $F_{\mathrm{x} \%}$ candidates for defining an MSY control rule with ESB as a spawning-biomass unit [e.g. any $F_{\mathrm{x} \%}$ value between $F_{45 \%}$ and $F_{50 \%}$ (Figure 3), or $F=2 M$ (Table 2)].

When the $\beta$ parameter was set to 0 in the target control rule with observation and implementation error (column 1 of Table 4), all performance statistics were similar to those obtained under an identical simulation set up with $\beta=0.25$ (column 2, Table 4), except for a very slight increase in median medium-term mean yield for the latter. However, higher initial abundances produced higher increases (results not shown). Therefore, inclusion of a non-zero $\beta$ parameter to the control rule has some beneficial effects, such as increasing medium-term yield.

Based on the current population parameters, $F_{50 \%}$ results in a mature male harvest rate of $17.7 \%$, which is slightly higher than the current maximum mature male harvest rate of $15 \%$. The current harvest rates are based on computer simulation studies conducted by Zheng et al. (1997a, b). The study by Zheng et al. (1997a) focused on the robustness of harvest strategies under changes in $M$ and different handling mortality rates, and the study by Zheng et al. (1997b) evaluated the performance of alternative harvest strategies for population rebuilding. Under the current harvest strategy, mature red king crab abundance in Bristol Bay has increased greatly during the past 10 years, especially mature female abundance (Zheng, 2006). Therefore, the current harvest strategy is considered to be a conservative one. The harvest rates corresponding to $F_{50 \%}$ serve as a limit, and the target harvest rates are lower than that limit. Because the current harvest rates are a step function of ESB, to be consistent with the MSY control rule proposed in this study, the current harvest rates may need to be slightly modified to serve as the target harvest rates.

The rebuilding analysis with $F_{50 \%}$ in the control rule provided a low probability of the stock being overfished and of incidences of fishery closure. The $F$ for this $F_{\mathrm{x} \%}$ was closer, but slightly lower
than the $F_{\text {MSY }}$ level. The $F_{50 \%}$ fared well with a number of diagnostic tests. Therefore, we suggest $F_{50 \%}$ as a precautionary proxy for $F_{\text {MSY }}$ in the proposed MSY control rule for BSAI king crab stocks.

The method developed in this paper can be used (with modification) for determining biological reference points and developing MSY control rules for managing any crustacean stock with discrete growth and complex reproductive dynamics, where a fishery targets a single sex (e.g. male-only BSAI crab fishery).

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## Appendix 1

## Glossary

$a, b, c$, and $d=$ parameters in the auxiliary models,
$c_{t}=$ retained catch of legal-sized male in year $t$,
$C=$ a general term used for catch in numbers,
$\hat{\mathrm{C}}=$ predicted catch in numbers,
$D=$ a density-dependent multiple for stock productivity,
$\mathrm{EFSSN}_{t}=$ a multiplying factor to calculate effective spawning biomass in year $t$,
$\mathrm{ESB}_{t}$ and $\mathrm{ESB}_{0}=$ effective spawning biomasses (adjusted for male to female mating ratio) corresponding to a fishing mortality $F$ in year $t$ and $F=0$, respectively,
$\mathrm{ESB}=$ a general term used for effective spawning biomass,
$\mathrm{ESB}_{\mathrm{MSY}}=$ effective spawning biomass at the MSY producing level,
$\mathrm{ESB}_{t} / R$ and $\mathrm{ESB}_{0} / R_{0}=$ effective spawning biomass-per-recruit corresponding to a fishing mortality $F$ in year $t$ and $F=0$, respectively,
$\mathrm{ESB} / R=\mathrm{a}$ general term used for effective spawning biomass-per-recruit,
$e t=$ average time elapsed between the mid-moulting date (i.e. start of a biological year) and start date of a fishing period as a fraction of a year,
$F_{\text {lim }}=$ limit instantaneous fishing mortality $\left(=F_{\mathrm{MSY}}\right)$,
$F_{\text {MSY }}=$ instantaneous fishing mortality that will produce MSY at the MSY-producing biomass,
$\mathrm{FSSN}_{1, t}=$ total primiparous female (first time spawners) abundance in number in year $t$,
$\mathrm{FSSN}_{2, t}=$ total multiparous female (previously spawned spawners) abundance in number in year $t$,
$F_{\text {target }}=$ target instantaneous fishing mortality,
$F_{\mathrm{T}}=$ instantaneous bycatch fishing mortality by the trawl fishery, a fixed value of 0.01 was used,
$F_{t}=$ instantaneous fishing mortality in year $t$,
$F_{\mathrm{x} \%}=$ instantaneous fishing mortality that results in $\mathrm{x} \%$ equilibrium spawning potential ratio,
$g=$ lapsed time between moulting and mating times as a fraction of a year,
$h=$ steepness parameter of a $S-R$ curve,
$h m=$ proportion of discarded males and females that died due to capture and release to sea
$\mathrm{HM}_{i, t}^{s}=$ instantaneous handling mortality of sex $s$, length class $i$, and year $t$,
immat $\mathrm{N}_{i, k, t}^{\mathrm{s}}=$ new-shell immature abundance of sex $s$, length class $i$, age $k$, and year $t$,
immat $\mathrm{O}_{i, k, t}^{s}=$ old-shell immature abundance of sex $s$, length class $i$, age $k$, and year $t$,
immolt ${ }_{i}^{s}=$ immature crab moult probability of sex $s$ and length class $i$,
$k=$ age in years,
$L_{c}=$ minimum legal size,
$M=$ instantaneous natural mortality,
mat $_{i}^{s}=$ maturity probability of sex $s$ and length class $i$,
mat $\mathrm{N}_{i, k, t}^{s}=$ new-shell mature abundance of sex $s$, length class $i$, age $k$, and year $t$,
mat $\mathrm{O}_{i, k, i}^{s}=$ old-shell mature abundance of sex $s$, length class $i$, age $k$, and year $t$,
$\mathrm{MMB}_{t}=\mathrm{MMB}$ corresponding to a fishing mortality $F$ in year $t$, $\mathrm{MMB}_{0}=\mathrm{MMB}$ corresponding to a fishing mortality $F=0$,
$M M B=$ a general term used for $M M B$,
$\mathrm{MMB}_{\mathrm{MSY}}=\mathrm{MMB}$ at the MSY producing level,
mmolt $_{i}^{s}=$ mature crab moult probability of sex $s$ and length class $i$, $\operatorname{MSSN}_{t}=$ total mature male abundance in number in year $t$,
MSY $=$ maximum sustainable yield,
$n=$ total number of length intervals available in a cohort for $P_{i, j}$ estimation,
$\mathrm{N}_{i, k, t}^{s}=$ new-shell stock abundance in number of sex $s$, length class
$i$, age $k$, and year $t$,
$\mathrm{O}_{i, k, t}^{s}=$ old-shell stock abundance of sex $s$, length class $i$, age $k$, and year $t$,
$P_{i}^{s^{*}}=$ probability of recruits of sex $s$ falling into length class $i$,
$P_{i, j}^{s}=$ probability of crabs of sex $s$ in length class $i$ growing into length class $j$,
$R=$ a general term used for total number of recruits,
$R_{0}=$ number of recruits at $F=0$,
$R_{0, t}=$ number of recruits at age 0 , and year $t$,
$R_{\max }=$ maximum number of recruits,
$s_{i}^{\prime}=$ trawl bycatch selectivity for length class $i$,
$s_{i}=$ pot fishery retained/discard selectivity for length class $i$,
$S=$ a general term used for spawning biomass,
$S_{0}=$ spawning biomass at $F=0$,
$W_{i}^{\varsigma}=$ mean weight of crabs of sex $s$ in length class $i$,
$x=$ a random variable representing the annual growth increment,
$y_{t}=$ retained yield of legal-sized males in year $t$,
$Z_{i, t}^{s}=$ instantaneous total mortality of sex $s$, length class $i$, and year $t$,
$\tau_{i}=$ midlength of length class $i$ for $P_{i, j}^{s}$ estimation,
$\alpha, \beta=$ control rule parameters,
$\phi, \omega=$ growth increment model parameters,
$\sigma_{\mathrm{e}}=$ standard deviation of the interannual variability of recruitment error,
$\rho=$ temporal correlation parameter, and
$\delta=$ duration of average fishing period as a fraction of a year
(handling and fishing mortalities occur during this period).

## Appendix 2

## Simulation model equations

An age-, sex-, and length-based model was used in all simulations.
The following assumptions were made to simplify the derivation in the analyses:
(i) M is constant;
(ii) Timing of events: moulting and mating of primiparous females (first-time spawners) on 15 February, moulting of males on 1 April; and moulting and mating of multiparous females (previously spawned spawners) on 1 May;
(iii) Initial recruits to simulation models have a $1: 1$ sex ratio; and
(iv) All female red king crabs were assumed to moult annually.

## The population dynamics model

The abundance of different stages and shell conditions of crabs of sex $s$ (in number) and age $k$ (last age is plus group) growing from smaller-size classes $i$ into a larger-size class $j$ at the start of year $t+1$ is:

$$
\begin{equation*}
\text { when } k=0 \text {, immat } \mathrm{N}_{j, 0, t+1}^{s}=\left(R_{0, t} / 2\right) \times P_{j}^{s *} \tag{2.1}
\end{equation*}
$$

( $R_{0, t}$ is first set to $R_{\max }$ to build the age structure; thereafter, it is set to an $R_{0, t}$ value generated by the $S-R$ model) and
when $1 \leq k<$ maximum age,

$$
\begin{align*}
\operatorname{mat} \mathrm{N}_{j, k, t+1}^{s}= & \sum_{i=1}^{j}\left[\left(\operatorname{mat~}_{\mathrm{N}}^{\mathrm{i}, k-1, t}\right.\right. \\
& +\left(\operatorname{mat~O}_{i, k-1, t}^{s}\right) \operatorname{mmolt}_{i}^{s} P_{i, j}^{s} \\
& \left.\times \operatorname{immolt}_{i, k-1, t}^{s}+\operatorname{mat}{ }_{j}^{s} P_{i, j}^{s}\right] \mathrm{e}^{-\boldsymbol{z}_{i, t}^{s}} \tag{2.2}
\end{align*}
$$

where $Z_{i, t}^{s}=M+F_{\mathrm{T}} s_{i}^{\prime}+\left(F_{t} s_{i}+\mathrm{HM}_{i, t}^{s}\right) \delta$ for males and $Z_{i, t}^{s}=$ $M+F_{\mathrm{T}} s_{i}^{\prime}+\mathrm{HM}_{i, t}^{s} \delta$ for females. $F_{t}$ is determined by the MSY or target control rule.

$$
\begin{align*}
\text { mat } \mathrm{O}_{j, k, t+1}^{s}= & \sum_{i=1}^{j}\left[\left(\operatorname{mat~}_{\mathrm{N}, k-1, t}^{s}+\text { mat }_{\mathrm{O}}^{i, k-1, t} s\right.\right. \\
& \left.\left.\times \mathrm{e}^{-Z_{i, t}^{s}}\right)\left(1-\operatorname{mmolt}_{i}^{s}\right)\right] \tag{2.3}
\end{align*}
$$

$$
\begin{align*}
\text { immat } \mathrm{N}_{j, k, t+1}^{s}= & \sum_{i=1}^{j}\left[\left(\operatorname{immat} \mathrm{~N}_{i, k-1, t}^{s}+\operatorname{immat} \mathrm{O}_{i, k-1, t}^{s}\right)\right. \\
& \left.\times \operatorname{immolt}_{i}^{s}\left(1-\text { mat }_{j}^{s}\right) \mathrm{P}_{i, j}^{s}\right] \mathrm{e}^{-Z_{i, t}^{s}} \tag{2.4}
\end{align*}
$$

$$
\begin{align*}
\text { immat } \mathrm{O}_{j, k, t+1}^{s}= & \sum_{i=1}^{j}\left[\left(\operatorname{immat} \mathrm{~N}_{i, k-1, t}^{s}\right.\right. \\
& \left.\left.+ \text { immat } \mathrm{O}_{i, k-1, t}^{s}\right)\left(1-\text { immolt }_{i}^{s}\right)\right] \mathrm{e}^{-Z_{i, t}^{s}} \tag{2.5}
\end{align*}
$$

Effective spawning biomass $\left(\mathrm{ESB}_{t}\right)$

$$
\begin{equation*}
\mathrm{ESB}_{t}=\sum_{j, k} \operatorname{mat~}_{j, k, t}^{s} \times \mathrm{W}_{j}^{S} \times \mathrm{EFSSN}_{t} \times \mathrm{e}^{-g \mathrm{Z}_{j, t}^{s}} \tag{2.6}
\end{equation*}
$$

where

$$
\operatorname{EFSSN}_{t}=\min \left(1, \frac{\operatorname{MSSN}_{t} \times \text { Mating ratio }}{\mathrm{FSSN}_{1, t}+\mathrm{FSSN}_{2, t}}\right)
$$

and

$$
\begin{gathered}
\operatorname{MSSN}_{t}=\sum_{j, k}\left(\operatorname{mat~}_{j, k, t}^{S}+\text { mat } \mathrm{O}_{j, k, t}^{S}\right) \times \mathrm{e}^{-g Z_{j, t}^{S}} \\
\operatorname{FSSN}_{1, t} \text { or } \mathrm{FSSN}_{2, t}=\sum_{j, k} \text { mat } \mathrm{N}_{j, k, t}^{S} \times \mathrm{e}^{-g Z_{j, t}^{S}}
\end{gathered}
$$

## Stochastic spawner-recruit models

For stochastic simulations, the number of recruits was predicted by the Beverton-Holt and Ricker S-R models. The S-R model parameters were reparameterized in terms of steepness parameter, $h$, virgin recruitment $\left(R_{0}\right)$, and virgin effective spawning biomass-per-recruit $\left(\mathrm{ESB}_{0} / R_{0}\right)$ as:

$$
\begin{equation*}
R_{0, t}=\frac{4 h \mathrm{ESB}_{t}}{(1-h)\left(E S B_{0} / R_{0}\right)+\left((5 h-1) / R_{0}\right) \mathrm{ESB}_{t}} \mathrm{e}^{\epsilon_{t}^{-} \sigma_{\epsilon}^{2} / 2} \tag{2.7}
\end{equation*}
$$

(Beverton and Holt, 1957)

$$
\begin{equation*}
R_{0, t}=\frac{(5 h)^{5 / 4}}{\left(E S B_{0} / R_{0}\right)} \mathrm{ESB}_{t} \mathrm{e}^{\left(-1.2 \frac{\ln (5 h)}{R_{0}\left(\mathrm{ESB}_{0} / R_{0}\right)} \mathrm{ESB}_{t}\right)} \mathrm{e}^{\epsilon_{t}-\sigma_{\epsilon}^{2} / 2} \tag{2.8}
\end{equation*}
$$

(Ricker, 1954)

$$
\text { where } \epsilon_{t}=\rho^{\star} \varepsilon_{t-1}+\mathrm{e}_{t} \text { and } \mathrm{e}_{t} \sim N\left(0, \sigma_{\mathrm{e}}^{2}\right) \text {. }
$$

Note, for $F_{\mathrm{x} \%}$ estimation by the equilibrium method, the recruitment random errors were set to zero.

The $R_{0}$ is related to $R_{\text {max }}$ for the two S-R models as follows:

$$
\begin{aligned}
R_{\max } & =\frac{4 R_{0} \mathrm{e}^{-1}(5 h)^{5 / 4}}{5 \ln (5 h)}(\text { Ricker } \mathrm{S}-\mathrm{R} \text { model }) \\
R_{\max } & =\frac{4 h R_{0}}{(5 h-1)}(\text { Beverton }- \text { Holt } \mathrm{S}-\mathrm{R} \text { model })
\end{aligned}
$$

## Catch

Estimation of legal-sized male retained catch and abundance at the fishing time in year $t$ :

$$
\begin{align*}
c_{t}=\sum_{j=L_{c}, k} & \left(\mathrm{~N}_{j, k, t}^{s}+\mathrm{O}_{j, k, t}^{s}\right)\left(\frac{F_{t} s_{j}}{Z_{j, t}^{s}}\right) \mathrm{e}^{-\left(M+F_{\mathrm{T}}^{s} s_{j}^{\prime}\right) e t}\left(1-\mathrm{e}^{-Z_{j, t}^{s} \delta}\right)  \tag{2.9}\\
y_{t}= & \sum_{j=L_{c}, k}\left(\mathrm{~N}_{j, k, t}^{s}+\mathrm{O}_{j, k, t}^{s}\right) \\
& \times\left(\frac{F_{t} s_{j}}{\left(Z_{j, t}^{s}\right)}\right) \mathrm{e}^{-\left(M+F_{\mathrm{T}} s_{j}^{\prime}\right) e t}\left(1-\mathrm{e}^{-Z_{j, t}^{s} \delta}\right) W_{j}^{s} \tag{2.10}
\end{align*}
$$

Discarded catch was computed using the same equations ((2.9) and (2.10)) replacing $F_{t} s_{j}$ in the numerator by $\mathrm{HM}_{j, t}^{s}$ (i.e. sizespecific handling mortality).

Note, in the stochastic simulations, the annual total catch and abundance were averaged for a number of years of the fishery to estimate relevant statistics for each simulation.

## Auxiliary models

The instantaneous handling mortality for sex $s$ and size $j, \mathrm{HM}_{j, t}^{s}$, is defined as a function of $F_{t}$ with discard selectivity $s_{j}$, ignoring $M$ and trawl and other bycatch mortality as follows:

$$
\begin{gather*}
1-\mathrm{e}^{-\mathrm{HM}_{\mathrm{j}, t^{s}} \delta}=h m\left(1-\mathrm{e}^{-F_{t} j^{\delta}}\right) \\
\mathrm{HM}_{\mathrm{j}, t}^{\mathrm{s}}=\frac{-1}{\delta} \ln \left(1-h m\left(1-\mathrm{e}^{-F_{t} j^{\mathrm{\delta}}}\right)\right) . \tag{2.11}
\end{gather*}
$$

The moult probability for sex $s$ and a given size class $j$ is described by the function:

$$
\begin{gather*}
\operatorname{mmolt}_{j}^{s}=1-\frac{1}{1+\mathrm{e}^{-a(j-b)}} \text { (if males) } \\
\text { mmolt }_{j}^{s}=1(\text { if females }) . \tag{2.12}
\end{gather*}
$$

The maturity probability, retained selectivity, female discard selectivity, and trawl bycatch selectivity for a given size are described by the logistic function.

The male discard selectivity for a given size $j$ is described by the double logistic function:

$$
\begin{equation*}
S_{j}=\frac{1}{1+\mathrm{e}^{-a(j-b)}} \frac{1}{1+\mathrm{e}^{c(j-d)}} . \tag{2.13}
\end{equation*}
$$

Weight of crab of sex $s$ at size $j$, following Beyer (1987), is defined by the function:

$$
\begin{equation*}
W_{j}^{s}=\left(\frac{1}{5}\right)\left(\frac{a}{b+1}\right)\left((j+2.5)^{b+1}-(j-2.5)^{b+1}\right) \tag{2.14}
\end{equation*}
$$

The expected proportion of moulting crabs of sex $s\left(P_{i, j}^{s}\right)$ growing from size class $i$ to size class $j$ during a year is described by the gamma distribution as follows:

$$
\begin{equation*}
P_{i, j}^{s}=\frac{\int_{j_{1}-\tau_{i}}^{j_{2}-\tau_{i}} \operatorname{gamma}\left(x / \phi_{i}, \omega\right) \mathrm{d} x}{\sum_{j=1}^{n} \int_{j_{1}-\tau_{i}}^{j_{i}-\tau_{i}} \operatorname{gamma}\left(x / \phi_{i}, \omega\right) \mathrm{d} x}, \tag{2.15}
\end{equation*}
$$

$$
\text { where } \operatorname{gamma}\left(x / \phi_{i}, \omega\right)=\frac{x^{\phi_{i}-1} \mathrm{e}^{-x / \omega}}{\omega^{\phi_{i}} \Gamma\left(\phi_{i}\right)} .
$$

and where $x$ is the growth increment per moult, $\phi_{i}$ is the expected growth increment of size interval $i$ divided by the shape parameter $\omega, j_{1}$ and $j_{2}$ are lower and upper limits of the receiving length inter$\mathrm{val} \mathrm{j}, \tau_{\mathrm{i}}$ is the midpoint of the contributing size interval $i$, and $n$ is the total number of receiving size intervals. The summation in the denominator is a normalizing factor for the discrete gamma function.

