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Dome C coherence time statistics from DIMM data

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ABSTRACT
We present a reanalysis of several years of DIMM data at the site of Dome C, Antarctica, to provide measurements of the coherence time τ_0. Statistics and seasonal behaviour of τ_0 are given at two heights above the ground, 3 and 8 m, for the wavelength $\lambda = 500$ nm. We found an annual median value of 2.9 ms at the height of 8 m. A few measurements could also be obtained at the height of 20 m and give a median value of 6 ms during the period June–September. For the first time, we provide measurements of τ_0 in daytime during the summer, which appears to show the same time dependence as the seeing with a sharp maximum at 5 PM local time. Exceptional values of τ_0 above 10 ms are met at this particular moment. The continuous slow variations of turbulence conditions during the day offers a natural test bed for a solar adaptive optics system.

Key words: atmospheric effects – methods: data analysis – site testing.

1 INTRODUCTION

In the years 2000–2012, a long-term program of site testing was conducted at the Concordia station on the site of Dome C (Antarctica) by the University Côte d'Azur (France) under the names of Concordiastro and Astroconcordia. A large variety of instruments were deployed, mainly to measure properties related to optical turbulence. Long-term statistics of turbulence parameters as well as vertical profiles of turbulence were obtained. In particular, we pointed out the bimodal distribution of the seeing caused by a strong turbulent surface layer whose thickness was estimated to about 30 m (Trinquet et al. 2008; Aristidi et al. 2009; Aristidi 2012; Aristidi et al. 2015).

The first instrument that we exploited at Concordia from 2002 was the differential image motion monitor (DIMM). This monitor, introduced in the early 90s (Sarazin & Roddier 1990) has become a standard in seeing measurements. Using several DIMMs during 10 yr, we obtained seeing statistics at three different elevations [3, 8, and 20m above the ground level (AGL)]. Isoplanatic angle measurements could also be done by modifying the pupil mask of the DIMM (Aristidi et al. 2005b), and the outer scale was measured by combining two DIMMs into a generalized seeing monitor (GSM) configuration (Ziad et al. 2008). However, no measurements of the coherence time τ_0 were made with the DIMMs, the only published values so far come from the Australian automated observatory AASTINO (Lawrence et al. 2004), balloon-borne radiosoundings (Trinquet et al. 2008), and the single star SCIDAR (SSS) (Giordano et al. 2012). All these measurements were made at night-time during the period 2004–2006. The coherence time in the surface layer (up to an altitude of 45 m) was also measured during several years by a set of sonic anemometers placed on the 45 m high US tower (Aristidi et al. 2015).

Recent theoretical advances (Ziad et al. 2012) allowed us to derive the coherence time from DIMM data. The method gives satisfactory results and is used routinely to process measurements from our Generalized DIMM (Aristidi et al. 2019) operated at the Calern observatory (south of France) as a part of the Calern Atmospheric Turbulence Station (Ziad et al. 2018). We then decided to re-process Dome C DIMM data using this method, to extract the coherence time τ_0. We present statistics for this parameter and comparisons with previous determinations. We also present the first daytime measurements of the coherence time during the summer.

The paper is organized as follows. Section 2 recalls theoretical concepts about the coherence time and presents the method to derive it from DIMM data. The data processing and error analysis are presented in Section 3. Statistics for τ_0 at two elevations, and daytime values and behaviour are shown in Section 4.

2 THEORETICAL BACKGROUND

Atmospheric optical turbulence is described by a set of quantities such as the seeing ϵ, the isoplanatic angle θ_0, the coherence time τ_0, or the spatial coherence outer scale $\mathcal{L}_0$. These parameters are sometimes denoted as 'integrated parameters' as they result from an integration along the line of sight of local quantities (wind speed, refractive index structure constant...). A review of optical turbulence in astronomy is found in Roddier (1981), and we will focus here on the coherence time τ_0.

The coherence time relevant for AO is defined by (Roddier 1981)

$$\tau_0 = 0.31\,\frac{r_0}{v_e} \qquad (1)$$

with r_0 the Fried parameter and $v_{\rm e}$ the effective wind speed defined as a weighted average of the wind speed $V(h)$ over the whole

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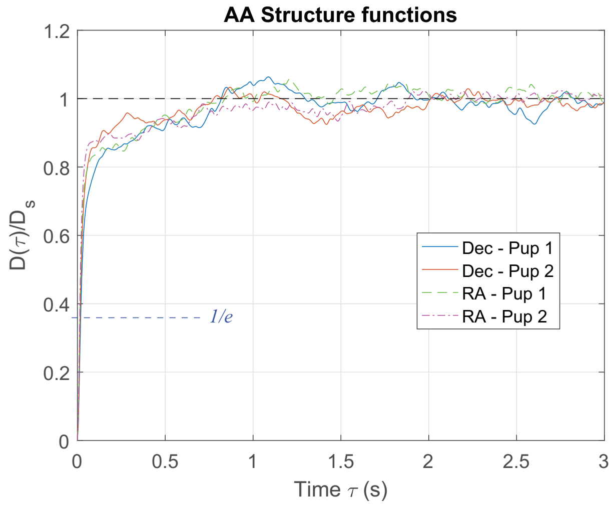

Figure 1. Example of AA structure functions $D_A(\tau)$ calculated on data from the ground DIMM, on 2011 January 18 at 0:33 local time. The four curves displayed correspond to y (RA) and x (dec.) motions for the two sub-images. One can see that all curves saturate at large temporal lag τ, structure functions were divided by their saturation value D_s. The intersection with the line $D_A(\tau)/D_s = 1/e$ (the dashed blue line) gives the AA correlation time τ_A. For this example, we found $\tau_{A,\,x1} = 19$ ms, $\tau_{A,\,x2} = 16$ms, $\tau_{A,\,y1} = 15$ ms and $\tau_{A,\,y2} = 14$ ms.

atmosphere:

$$v_e = \left[\frac{\int_0^\infty |V(h)|^{5/3}\, C_n^2(h)\, dh}{\int_0^\infty C_n^2(h)\, \mathrm{d}h}\right]^{3/5}, \tag{2}$$

where $C_n^2(h)$ is the refractive index structure constant at an altitude h. It has been shown (Conan et al. 2000; Ziad et al. 2012; Aristidi et al. 2019) that $v_{\rm e}$ can be estimated from DIMM data. It is deduced from the temporal structure functions $D_{x|y}(\tau)$ of the Angle of Arrivals (AA) in the x- and y-directions (x being parallel to the right ascension and y to the declination axis):

$$D_{x|y}(\tau) = \langle (x|y(t) - x|y(t+\tau))^2 \rangle_t, \tag{3}$$

where $\langle\rangle_t$ stands for the temporal average. This function is zero for $\tau = 0$ and saturates to a value D_s for large τ. We define the AA correlation time $\tau_{A,\,x|y}$ as the time τ for which $D_{x|y}(\tau) = \frac{D_s}{k'}$, i.e. when the curve of is $D_{x|y}(\tau)/D_s$ is $1/k'$ above zero, as shown in Fig. 1. The constant k' is here taken as $k' = e$ to be consistent with the GSM (Ziad et al. 2012). Typical values for τ_A are 10–30 ms. The effective wind speed $v_{\rm e}$ is given by equation (5.16) of Conan et al. (2000):

$$v_e = 10^3 D\, G^{-3} \left[\tau_{A,x}^{\frac{1}{3}} + \tau_{A,y}^{\frac{1}{3}}\right]^{-3}, \tag{4}$$

where D is the sub-pupil diameter and G a constant given by equation (13) of Ziad et al. (2012):

$$G = 7.30\, k'^{-1}\, K^{\frac{1}{3}} + 7.01\,(1 - k'^{-1}) \tag{5}$$

with $K = \frac{\pi D}{\mathcal{L}_0}$ and $\mathcal{L}_0$ the outer scale, taken equal to 7 m, the median value at the Dome C site (Ziad et al. 2008).

3 OBSERVATIONS AND DATA PROCESSING

The data analysed in this paper come from three DIMMs. Two of them were located at ground level (with a pupil height of about 3 m AGL), as a part of the GSM experiment. When the two telescopes observed simultaneously, we took the mean value of τ_0 given by the two instruments. The third DIMM was on top of a platform, at an elevation of 8 m. The data set covers the period 2006 December–2011October for the ground DIMMs, and 2005 December–2012 September for the DIMM at 8 m.

In 2005 and 2012, one of the ground DIMMs was put on the roof of the calm building of the Concordia station, at an elevation of 20 m AGL. Data were collected for several months during the second half of the two winterovers, and allowed to measure the seeing at 20 m. However, because of strong vibrations of the building and several technical failures, the number of data exploitable for coherence time measurement is small, and limited to the period June–September of the 2 yr.

The instrumentation and the data processing for seeing estimation are described in details in Aristidi et al. (2005b). The raw data consist of a series of photocentre positions for the twin images produced by the telescope. Coordinates lists are divided into short sequences of 2 mn. They are pre-processed to remove bad points, drift, and vibrations. For every sequence, the AA structure function is computed, the AA correlation time τ_A is estimated, and we derive the effective wind speed $v_{\rm e}$. The Fried parameter is also calculated by the classical DIMM method. This lead, through equation (1), to an estimation of the coherence time τ_0 every 2 mn. The high framerate of about 100 frames s^{-1} allows to properly sample AA structure functions and to reduce the statistical error, as several thousands of photocentre coordinates are measured in a 2 mn time interval.

3.1 Error analysis

The calculation of the error budget on the coherence time is described in section 5.3 of Aristidi et al. (2019). The uncertainty $\delta\tau$ on the AA correlation time is given by

$$\delta\tau = \frac{\delta D}{D'(\tau_A)} \tag{6}$$

with δD the uncertainty on the structure function, estimated in the saturation zone as the standard deviation of values and $D'(\tau_A)$ its slope for $\tau = \tau_A$, which can be calculated as a finite difference. Errors on τ_A were computed for all three DIMMs for subsets of 3 months of data. We found a relative uncertainty $\frac{\delta\tau_A}{\tau_A} \simeq 7$ per cent for the dec. axis and 10 per cent for the RA axis. These values propagate into the effective wind speed, whose uncertainty, obtained by differentiation of equation (4), is $\frac{\delta v_e}{v_e} \simeq 8$ per cent.

Finally, the relative error on the coherence time is obtained from equation (1), assuming that errors on r_0 and $v_{\rm e}$ are uncorrelated:

$$\frac{\delta\tau_0}{\tau_0} = \left[\left(\frac{\delta v_e}{v_e}\right)^2 + \left(\frac{\delta r_0}{r_0}\right)^2\right]^{\frac{1}{2}} \simeq 8 \text{ per cent}, \tag{7}$$

where we used a relative error on the Fried parameter r_0 of $\simeq 1$ per cent (Aristidi et al. 2005b).

4 RESULTS

4.1 Statistical analysis

A total amount of about 700 000 values of the coherence time were obtained from the DIMMs at heights 3 and 8 m during the considered period. This large number of measurements allows to undertake significant statistical studies. For the 20 m DIMM, the number of

Table 1. Statistics of coherence time τ_0 calculated at the wavelength $\lambda = 500$ nm. Values are in milliseconds.

Elevation		3 m			8 m		20 m
	Summer	Winter	Total	Summer	Winter	Total	June–Sep
Nb of points	73 042	148 570	437 986	62 126	65 378	262 951	15 213
Median	5.1	1.3	1.9	8.2	1.7	2.9	6
Mean	6.3	1.8	3.1	9.9	2.4	5.2	7
1st quartile	3.2	1.0	1.3	5.0	1.2	1.6	3
3rd quartile	7.8	1.8	3.2	13	2.5	6.0	8
1st centile	1.4	0.50	0.55	1.5	0.57	0.63	1
Last centile	25	6.8	15	32	11	24	35

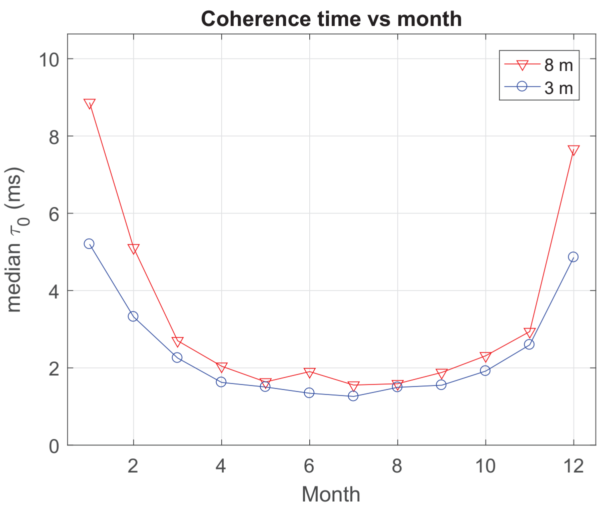

Figure 2. Monthly median values of τ_0 at the two altitudes 3 and 8 m.

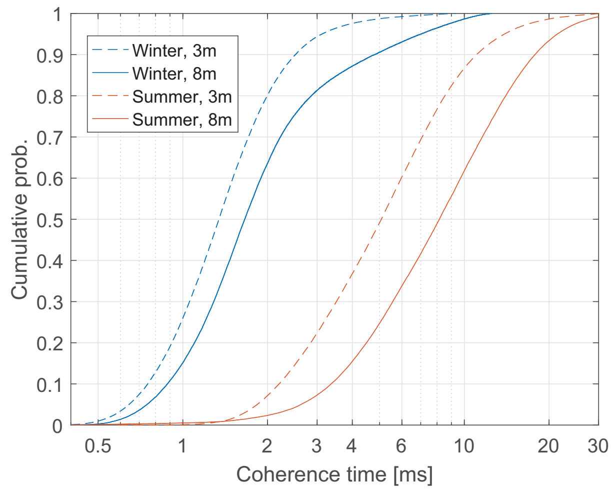

Figure 3. Cumulative distributions of the coherence time at altitudes 3 and 8 m, during the summer (December–January) and the winter (June–August).

values is much smaller, about 15 000, and limited to the period June–September.

Table 1 presents statistics of τ_0 obtained at the three heights 3, 8, and 20 m. All values are calculated for the wavelength $\lambda = 500$ nm. For the 3 and 8 m DIMMs, global and seasonal values are given for the summer (December–January) and the winter (June–August). Very different conditions are observed between these two seasons: best conditions are met during the summer with a median value of 8.2 ms measured by the 8 m DIMM, while it is 1.7 ms in the middle of the winter. The coherence time follows the same seasonal dependence as the seeing (Aristidi et al. 2009) and this was expected since it is proportional to the Fried parameter (equation 1).

Fig. 2 shows the monthly median values of τ_0 at the two heights (3 and 8 m). The best period is December–January, the closest to the summer solstice: this is why we chose to denote this period as ‘summer’ in this paper. In winter, bad conditions are explained by the presence of the turbulent surface layer. This surface layer was observed at many locations of the Antarctic plateau (Marks et al. 1999; Bonner et al. 2010; Okita et al. 2013) and is predicted by climatic models (Swain & Gallée 2006). Its consequence is a poor winter seeing around 1.7 arcsec at the height of 8 m (Aristidi 2012) and a poor coherence time (1.7 ms at 8 m).

Cumulative histograms of τ_0 in winter and summer are presented in Fig. 3 for both altitudes 3 and 8 m. In winter, the two distributions are close to each other with small difference between the two altitudes (the median values differ by 0.3 ms). This difference becomes larger for large values of τ_0. Hence, the nineth decile, in winter, is 5 ms at 8 m and 2.5 ms at 3 m. This may be due to situations where the thickness of the surface layer is lower than the altitude of the 8 m DIMM, which is then in the free atmosphere (FA). Such situations, mentioned in Aristidi et al. (2009) and Fossat et al. (2010) are likely to occur 12 per cent of the time in winter (Aristidi 2012). The situation is more favourable at 20 m height, the DIMM being above the surface layer 45 per cent of the time. We found a median value of τ_0 of 6 ms at this altitude. Summer curves show larger values of τ_0 and will be discussed in Section 4.2.

Table 2. Coherence time median values from DIMMs at altitudes 3, 8, and 20 m, compared to other Dome C mesurements and to other sites. The column 2 reports the effective height above which measurements were done. For the 20 m DIMM, the statistics is poor and measurements are available only during the period June–September.

	τ_0 (ms)	Height	Reference
DIMM 3 m	1.9	3 m	
DIMM 8 m	2.9	8 m	
DIMM 20 m	6	20 m	
SSS	3.4	8 m	Giordano et al. (2012)
Balloons	5.7	8 m	Trinquet et al. (2008)
AASTINO	7.9	30 m	Lawrence et al. (2004)
South Pole	1.6	2 m	Marks et al. (1999)
Paranal	3.2	2 m	Ziad et al. (2012)
Mauna Kea	2.4	2 m	Ziad et al. (2012)

Table 2 compares median annual values of τ_0 obtained by the DIMMs with other available Dome C measurements. The balloon value is based on 34 radiosoundings in 2005. But part of these flights suffers from a bias due to a lack of measurement of the wind speed near the ground and/or a poor vertical sampling of the surface layer. The AASTINO value was measured by a MASS and gives the coherence time in the FA (indeed, it compares well with the FA value of 6.8 ms measured by the ballons above 33 m (Trinquet et al. 2008), and it is consistent with our value of 6 ms at the height of 20 m). The SSS value is based on a large set of vertical profiles obtained in 2006 and the instrument was at the same height as the 8 m DIMM. The SSS estimation of τ_0 is a little more optimistic than the DIMM one, but the small diameter of the telescope (40 cm) prevents the measure of the turbulence in layers where the wind speed is higher than ∼40 m s^{-1}. This happens in particular at high altitudes during the winter where strong stratospheric winds are triggered by the polar vortex (Giordano et al. 2012; Aristidi et al. 2005a). The coherence time is dependent on both $C_n^2(h)$ and $V(h)$ profiles (equations 1and 2) and some values of τ_0 may be overestimated by the SSS.

Table 2 compares also the Dome C coherence time with other sites in the world. The effective height of these measurements is about 2 m AGL. The South Pole value is derived from 15 radiosoundings in the winter 1995. It appears to be similar to the winter τ_0 measured by our DIMMs, but the statistics is too weak to derive reliable conclusions. The two other values at Paranal and Mauna Kea were obtained by the GSM (Ziad et al. 2000) using the same method as this work. A more complete list of locations measured by the GSM (including La Silla, Palomar, La Palma...) can be found in Ziad et al. (2012), they show values of τ_0 in the range 1.2–6.6 ms. They suggest that the Dome C coherence time at the height of 8 m AGL is comparable to the one observed above temperate astronomical sites. At the height 20 m, however, Dome C becomes better that the majority of sites.

4.2 The summer situation

The site of Dome C offers very particular turbulence conditions during the summer. We have discovered (Aristidi et al. 2005b, 2009) that every day, the planetary boundary layer (responsible for more than 90 per cent of the optical turbulence) almost completely vanishes around 5 PM local time, due to the cancellation of the temperature gradient between the ground and the upper atmosphere. Excellent seeing values around 0.4 arcsec are observed during this transition from the day to the night (though there is no night is summer, but a period where the Sun in low above the horizon). Similar conditions were reported at other locations of the Antarctic plateau, such as Dome F (Okita et al. 2013) and Taishan station (Tian et al. 2020). The coherence time being proportional to the Fried parameter (equation 1), it was natural to suspect that τ_0 might be very high at this particular moment, and this was comforted by visual observations at the telescope showing very slow image motions. However, we did not have quantitative measurements at the time of the observations.

The present data set contains six summer campaigns, at heights 3 and 8 m. Statistics for the summer are presented in Table 1. From the cumulative distributions displayed in Fig. 3, it can be seen that the summer coherence time at 8 m is very high, with values greater than 10 ms 38 per cent of the time (13 per cent at 3 m).

Fig. 4 shows the hourly median value of τ_0 as a function of the local time. For the 8 m DIMM, values around 5 ms are observed at night hours, and the coherence time progressively increases at the end of the morning, to attain a maximum near 12 ms at 17 h local time. This peak is coincident with the seeing minimum observed every day in mid-afternoon in summer (fig. 6 of Aristidi et al. 2005b).

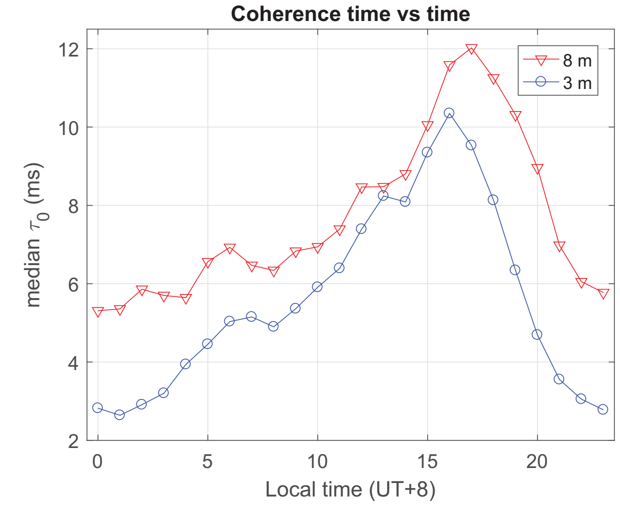

Figure 4. Hourly median values of τ_0 in the Summer (December–January) the two altitudes 3 and 8 m.

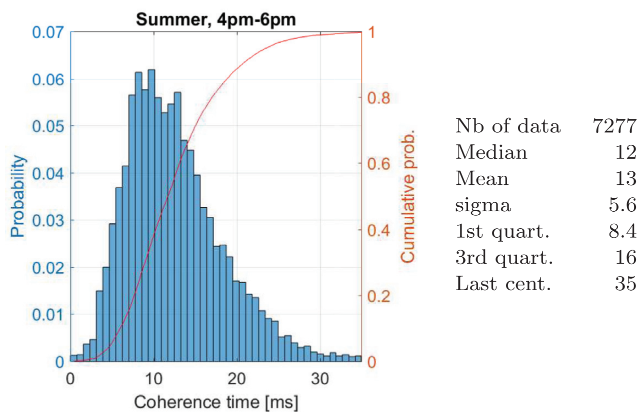

Figure 5. Histogram of τ_0 between 4 and 6 PM during the Summer (December–January), for the DIMM at altitude 8 m.

This particular period around 5 PM gives an opportunity to infer statistical properties of the coherence time in the FA. This statistics, made possible by the large amount of data, is presented in Fig. 5. More than 7000 values of τ_0 were gathered by the 8 m DIMM between 4 and 6 PM during the months of December and January. Their histograms show a lognormal distribution of median 12 ms. Note that this summer FA coherence time is greater than FA measurements given by the balloons (6.8 ms; Trinquet et al. 2008) and the MASS (7.9 ms; Lawrence et al. 2004). The difference is explained by stronger high-altitude wind speed in the winter.

Table 1 shows that the coherence time, for the 8 m DIMM, can be greater than 32 ms in 1 per cent of cases. Indeed, superb conditions were sometimes met as in the case of Fig. 6, obtained in the afternoon of 2009 December 25. The coherence time remained continuously above 20 ms between 4 and 7 PM, and attained values close to 50 ms at 5 PM.

5 CONCLUSIONS

We have presented statistics for the coherence time at Dome C, during the summer and the winter, at three elevations above the ground. They were obtained by reanalysis of a large data set of

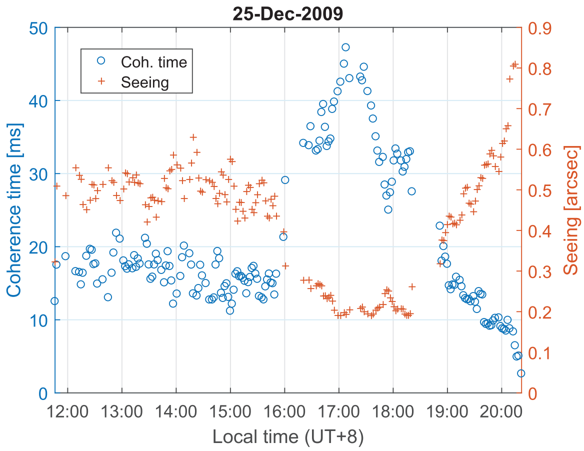

Figure 6. Exceptional conditions observed on 2009 December 25 during the afternoon: the coherence time, measured by the 8 m DIMM, attained values close to 50 ms at 5 PM local time.

DIMM data covering several years, using recent algorithms. This analysis was made possible because of the fast temporal sampling of DIMM images. The presence of data obtained during the daytime allowed, for the first time, to measure the coherence time at Dome C in the summer.

As expected the coherence time near the ground is rather poor in winter because of the strong surface layer that contains about 90 per cent of the turbulence. This surface layer is much less active in summer and the coherence time is multiplied by 5 at the altitude of 8 m. Indeed, in the past, we measured the coherence time $\tau_{0,s}$ inside the surface layer (between 8 and 45 m AGL) with a set of sonic anemometers placed on a tower. Fig. 15 of Aristidi et al. (2015) shows that values of $\tau_{0,s}$ vary from 5 to 6 ms in winter to 30 ms in summer: this is also a factor 5. Also, the surface layer is rather thin and we have reported that a telescope placed at a height of 20 m would be in the FA almost 50 per cent of the time (Aristidi 2012). Measurements made by the 20 m DIMM showed very good coherence time values with a median of 6 ms.

The behaviour of τ_0 with time during the summer is another interesting result of this work. As for the seeing, very favourable conditions are met every day around 5 PM local time. Superb values greater than 12 ms can be measured 50 per cent of the time during this period: this fraction is less than 10 per cent for Mauna Kea and Paranal at night (Ziad et al. 2012). Combined to low seeing values around 0.4 arcsec, this offers interesting perspective for high angular resolution solar imaging and solar adaptive optics (AO). Indeed a large coherence time reduces the delay error of an AO system, while a small seeing value allows to benefit from a good correction (Cheng 2009; Carbillet et al. 2017).

Solar AO is a well developed tool for investigating solar surface structures such as faculas, sunspots, and solar granulation (Rimmele & Marino 2011). Solar astronomers have already expressed their interest for a solar AO instrument at Dome C (Damé et al. 2010), recognizing that Dome C is a privileged site for solar observations. Indeed, Dome C summer conditions offer a unique test bench for AO. The continuous slow improvement, then degradation, of atmospheric turbulence conditions during the day offers a natural testing laboratory for an AO system.

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DATA AVAILABILITY STATEMENT

Data availability statement: There are no new data associated with this article.

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