Mon. Not. R. Astron. Soc. **370,** 1513–1518 (2006) doi:10.1111/j.1365-2966.2006.10574.x

On the dynamical evolution of hotspots in powerful radio-loud active galactic nuclei

N. Kawakatu[1,2★] and M. Kino[1,3★]
[1]*SISSA, via Beirut 2-4, 34014 Trieste, Italy*
[2]*National Observatory of Japan, 181-8588 Mitaka, Japan*
[3]*Department of Earth and Space Science, Osaka University, 560-0043, Toyonaka, Japan*

Accepted 2006 May 15. Received 2006 February 24; in original form 2005 December 12

ABSTRACT

We describe the dynamical evolution of hotspots velocity, pressure and mass density in radio-loud active galactic nuclei (AGNs), taking proper account of (1) the conservations of the mass, momentum and kinetic energy flux of the unshocked jet, (2) the deceleration process of the jet by shocks and (3) the cocoon expansion without assuming the constant aspect ratio of the cocoon. By the detailed comparison with two-dimensional relativistic hydrodynamic simulations, we show that our model well reproduces the whole evolution of relativistic jets. Our model can explain also the observational trends of the velocity, the pressure, the size and mass density of hotspots in compact symmetric objects (CSOs) and Fanaroff–Riley type II (FR II) radio galaxies.

Key words: hydrodynamics – shock waves – galaxies: active – galaxies: jets.

1 INTRODUCTION

Which evolutionary tracks are radio-loud active galactic nuclei (AGNs) (radio galaxies) passing through? This is one of the primal issues in the study of AGNs (Ryle & Longair 1967; Carvalho 1985; Fanti et al. 1995; De Young 1997). Stimulated by the observational progress (e.g. Turland 1975; Readhead, Cohen & Blandford 1978; Bridle & Perley 1984), a number of hydrodynamic simulations of jet propagations have been performed to examine their physical state of the jet (e.g. Norman et al. 1982; Wilson & Scheuer 1983; Smith et al. 1985; Clarke, Norman & Burns 1986; Lind et al. 1989; Clarke, Harris & Carilli 1997; Martí et al. 1997). These numerical studies have confirmed that the jet is composed of 'light' (i.e. lower mass density) materials compared with that of the surrounding an ambient medium to reproduce the observed morphology of the expanding *cocoon* (e.g. Norman et al. 1982). However, it is hard to examine the whole duration of powerful radio-loud AGNs with sufficiently large dynamical range because of the limitation of computational powers.

A new population of radio sources so-called 'compact symmetric objects (CSOs)' has been recently noticed. The CSOs were first identified by Phillips & Mutel (1980, 1982) and more complete samples were presented by Wilkinson et al. (1994) and Readhead et al. (1996a,b). Concerning the origin of CSOs, two scenarios were initially proposed. One is so-called 'frustrated jet scenario' in which the ambient medium is so dense that jet cannot break its way through, so sources are old and confined (van Breugel, Miley & Heckman 1984). The other is 'youth radio source scenario' in which CSOs are the young progenitor of Fanaroff–Riley type II (FR II) radio galaxies (e.g. Shklovsky 1965; Phillips & Mutel 1982; Carvalho 1985; Fanti et al. 1995; Begelman 1996; Readhead et al. 1996a; O'Dea & Baum 1997). Recent observations reveal that their speeds are better understood within youth radio source scenario because of their age with 10^{3-5} yr, which is much shorter than the age of FR II sources with 10^{6-7} yr (e.g. Murgia et al. 1999; Owsianik, Conway & Polatidis 1999; Taylor et al. 2000). This indicates the possibility of CSOs as the progenitor of FR II sources although their evolutionary tracks are poorly understood.

The hotspot, which is identified as the reverse-shocked (RS) region of the decelerating jet, is one of the most important ingredients in the whole jet system. The evolution of the hotspot is tightly linked to that of cocoon because the cocoon consists of the shocked plasma escaped from the hotspot (see Fig. 1). Observationally, the correlations between the hotspot properties (the velocity, the pressure, the size and the mass density) and projected linear size have been reported for CSOs and FR II sources (Readhead et al. 1996a; Jeyakumar & Saikia 2000; Perucho & Martí 2002). These observational trends would also reflect the evolutionary tracks of radio-loud AGNs. Thus, in order to clarify the physical relation between CSOs and FR II sources, it is inevitable to model the dynamical evolution of hotspots in radio-loud AGNs. However, little attention has been paid to this point in spite of the fact that lots of theoretical evolutionary models have been proposed based on cocoon dynamics (e.g. Falle 1991; Begelman 1996; Kaiser & Alexander 1997). Thus, the goal of this paper is to construct an appropriate dynamical model of hotspots in the radio-loud AGNs.

The plan of this paper is organized as follows. In Section 2, we outline and model a dynamical evolution of hotspots connected with the cocoon dynamics. In Section 3, we compare with

★E-mail: kawakatu@th.nao.ac.jp (NK); kino@vega.ess.sci.osaka-u.ac.jp (MK)

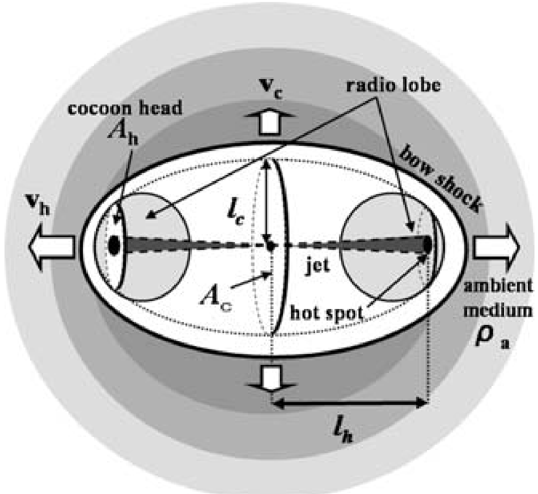

Figure 1. A schematic picture of the co-evolution of hotspots and cocoons. Most of the kinetic energy jet is injected via the termination shock of the jet which is identified as the hotspot. The sideways expansion speed of cocoon is v_c. The area of the radio lobe at the position of hot spots A_h is larger than that of hotspots. The head part of the cocoon advances with speed v_h.

previous theoretical and observational works. Conclusions are given in Section 4.

2 DYNAMICAL EVOLUTION OF HOTSPOTS CONNECTED WITH THE COCOON EXPANSION

2.1 Outline

In this paper, we model a dynamical evolution of hotspots with the aid of cocoon dynamics (Begelman & Cioffi 1989: hereafter BC89; Kino & Kawakatu 2005: hereafter KK05). Specifically, the evolution of the hotspot velocity (v_{HS}), the hotspot pressure (P_{HS}) and the hotspot density (ρ_{HS}) is discussed. These quantities are described in terms of the length from the centre of the galaxy to the hotspot (l_h).

Concerning v_{HS}, radio observations of powerful FR II radio galaxies show us that hotspots are always reside at the tip of the radio lobe (e.g. Myers & Spangler 1985; Readhead et al. 1996b). Thus, it is natural to impose the relation of

$$v_{HS} = v_h, \tag{1}$$

where v_h is the advance speed of the cocoon head. The velocity v_h is significantly affected by the two-dimensional (2D) effect. However, it can be reasonably handled by the phenomenological description as follows. Consider a pair of jets propagating in an ambient medium (see Fig. 1). At the hotspot, the flow of the shocked matter is spread out by the oblique shocks that then deflects (Lind et al. 1989), the vortex occurs via shocks (e.g. Smith et al. 1985) and/or the effect of jittering of the jet (e.g. Williams & Gull 1985; Cox, Gull & Scheuer 1991) which behaves like the 'dentist drill' (Scheuer 1982). Thus, the effective 'working surface' for the advancing jet is larger than the cross-section area of the hotspot A_j, which was pointed out by BC89. BC89 introduced the effective cross-section area of the cocoon head A_h as that of the effective 'working surface'.[1] Thus, we can determine the reasonable value of v_h by the expanding cocoon process.

[1]Before BC89, the head advance velocity is estimated by purely the 1D momentum balance (e.g. Begelman, Blandford & Rees 1984).

As for P_{HS} and ρ_{HS}, we deal with them through one-dimensional (1D) shock junctions. Since the hotspot is identified with the RS region of the jet, P_{HS} and ρ_{HS} can be obtained as a function of v_h by combining with equation (1).

2.2 Unshocked jet

Here we set up the mass, momentum and energy flux of the jet with three assumptions. Our main assumptions are as follows:

(i) We assume that the speed of the jet is relativistic on the large scale (~100 kpc) and the jet is consist of the cold medium. Although the jet speed on large scales is still open issue, several jets are suggested to be relativistic ones (e.g. Tavecchio et al. 2000; Celotti, Ghisellini & Chiaberge 2001; Uchiyama et al. 2005).

(ii) The mass, energy and momentum of jets are conserved in time. Namely, we do not include the entrainment effect of the ambient medium. This is justified by the numerical simulations for highly relativistic jet flows (e.g. Scheck et al. 2002; Mizuta, Yamada & Takabe 2004).

(iii) We ignore the dynamical effect of magnetic fields because of the kinetic flux dominance in FR II radio galaxies (e.g. Hardcastle & Worrall 2000; Leahy & Gizani 2001; Isobe et al. 2002).

Then, the mass (J_{1D}), energy (L_{1D}) and momentum (Q_{1D}) flux are given (Blandford & Rees 1974);

$$J_{1D} = \Gamma_j A_j \rho_j c, \tag{2}$$

$$L_{1D} = \Gamma_j^2 A_j \rho_j c^3, \tag{3}$$

$$Q_{1D} = \Gamma_j^2 A_j \rho_j c^2, \tag{4}$$

where Γ_j and ρ_j are the Lorentz factor and the mass density of the jet, respectively. Note that the kinetic energy flux L_{1D} denoted here satisfies the relation of $L_j = (A_h/A_j)L_{1D}$, where L_j is the total kinetic power shown in KK05.

$$\Gamma_j = \text{constant}, \tag{5}$$

$$\rho_j A_j\, c = \text{constant}. \tag{6}$$

Interestingly, the Lorentz factor Γ_j does not depend on l_h. In other words, the speed of jet is relativistic even on the large scale.

2.3 Shock junctions between the jet and ambient medium

We briefly review the 1D shock jump conditions which governs the deceleration of the relativistic jet by the surrounding ambient medium (Kino & Takahara 2004, for details). We can determine P_a, ρ_a from X-ray observations where P_a and ρ_a are the pressure and the mass density of the ambient medium, respectively. The assumption of a cold jet is written as $P_j = 0$. We regard Γ_j as a parameter.

Since v_{HS} is estimated to be in the range of $0.01c$ to $0.1c$ for both FR II sources (Liu, Pooley & Riley 1992; Scheuer 1995) and CSOs (Conway 2002 and references therein), the forward-shocked (FS) region quantities are determined by the shock jump conditions in non-relativistic limit (Landau & Lifshitz 1959). By using the pressure balance along the contact discontinuity between the hotspot and ambient medium, we can obtain the expression of P_{HS} as functions of two observable quantities $v_{\rm HS}$ and $\rho_{\rm a}$ such as

$$P_{\rm HS} = \frac{4}{15}\frac{[5-(1/\mathcal{M}^2)]}{[1-(1/\mathcal{M}^2)]^2}\rho_{\rm a} v_{\rm HS}^2, \tag{7}$$

where $\mathcal{M} = v_{\rm FS}/\sqrt{(5P_{\rm a}/3\rho_{\rm a})}$ and $v_{\rm FS}$ are the Mach number and the velocity of the upstream of FS, respectively. We adopted the adiabatic index of the downstream of FS as 5/3. In the RS region, we employ the relativistic shock jump conditions in the strong shock limit (Blandford & McKee 1976). Then, the equation of state and the mass continuity in the RS region can be written as

$$\rho_{\rm HS} = \frac{3P_{\rm HS}}{(\Gamma_{\rm j}-1)c^2}, \tag{8}$$

$$\rho_{\rm j} = \frac{3P_{\rm HS}}{(4\Gamma_{\rm j}+3)(\Gamma_{\rm j}-1)c^2}, \tag{9}$$

where we set the adiabatic index in the RS region as $4/3$. Thus, $\rho_{\rm HS}$ and $\rho_{\rm j}$ also can be given by $\rho_{\rm a}$ and $v_{\rm HS}$.

2.4 Dynamical evolution of the cocoon

To determine the velocity of the cocoon head $v_{\rm h}$ with considering 2D sideways expansion, we prepare the solutions of cocoon dynamics based on KK05. In KK05, by solving the equation of motion along the jet axis, perpendicular to the axis (i.e. sideways expansion), and energy injection into the cocoon, we obtained the $v_{\rm c}$, $v_{\rm h}$, $P_{\rm c}$ and $A_{\rm h}$ in terms of $l_{\rm h}$, where $v_{\rm c}$, and $P_{\rm c}$ are the velocity of cocoon sideways expansion and the pressure of cocoon, respectively. The declining mass density of the ambient medium is assumed to be $\rho_{\rm a}(d) = \rho_{\rm a0}(d/d_0)^{-\alpha}$, where d, d_0 and $\rho_{\rm a0}$ are the radial distance from the centre of the galaxy, the reference position and the mass density of the ambient medium at d_0, respectively. In order to convert t-dependence of the results of KK05 to $l_{\rm h}$-dependence, we use the equation $l_{\rm h} = \int_0^t v_{\rm h}(t')\,{\rm d}t'$ and $l_{\rm c} = \int_0^t v_{\rm c}(t')\,{\rm d}t'$ where $l_{\rm c}$ is the radius of the cocoon body. The obtained cocoon quantities in KK05 are as follows:

$$v_{\rm c} = v_{\rm c0}\left(\frac{l_{\rm h}}{l_{\rm h0}}\right)^{(0.5X-1)/[X(-2+0.5\alpha)+3]}, \tag{10}$$

$$P_{\rm c} = P_{\rm c0}\left(\frac{l_{\rm h}}{l_{\rm h0}}\right)^{[X(1-\alpha/2)-2]/[X(-2+0.5\alpha)+3]}, \tag{11}$$

$$v_{\rm h} = v_{\rm h0}\left(\frac{l_{\rm h}}{l_{\rm h0}}\right)^{[2-X(2-0.5\alpha)]/[X(-2+0.5\alpha)+3]}, \tag{12}$$

$$A_{\rm h} = A_{\rm h0}\left(\frac{l_{\rm h}}{l_{\rm h0}}\right)^{[X(\alpha-2)(-2+0.5\alpha)+3\alpha-4]/[X(-2+0.5\alpha)+3]}, \tag{13}$$

where X is the power-law index of the effective cross-section area of the cocoon body $A_{\rm c}(t) \propto t^X$ (see Fig. 1). Throughout this paper, we use the normalization of $l_{\rm h0} = v_{\rm h0}t_7$, where $t_7 = 10^7$ yr and $v_{\rm h0}$ is the velocity of the cocoon head at $l_{\rm h0}$ and the coefficients of each physical quantities are denoted with the subscript 0. In KK05, we selected $A_{\rm c}$ as an unknown parameter because we could not obtain the solution for $\alpha = 2$. However, by comparing with previous works, it is worth to show the power-law index β of the effective cross-section area of the cocoon head $A_{\rm h} = A_{\rm h0}(l_{\rm h}/l_{\rm h0})^\beta$ as a function of α and X. From equation (13), we obtain the following relation as

$$\beta = \frac{X(\alpha-2)(-2+0.5\alpha)+3\alpha-4}{X(-2+0.5\alpha)+3}. \tag{14}$$

From that, it is clear that the head growth can be expressed accurately in KK05 even for $\alpha = 2$.

Self-similar models are also useful tools to explore the evolution of the cocoon expanding (e.g. Falle 1991; Begelman 1996; Kaiser & Alexander 1997). However, the problem has been pointed out on the assumption of the constant aspect ratio of cocoon employed in self-similar models. By the comparison of the young and grown-up sources, they claim that the condition of the constant aspect ratio is not fulfilled incidentally (e.g. Readhead et al. 1996a; De Young 1997; Komissarov & Falle 1998; O'Dea 1998; Scheck et al. 2002; Carvalho & O'Dea 2002a,b; Tinti & de Zotti 2006).

2.5 Dynamical evolution of hotspots

Combining with (i) the mass, momentum and kinetic energy of the jet (Section 2.2), (ii) the deceleration process of the jet by shock (Section 2.3) and (iii) the cocoon expansion (Section 2.4), we can finally obtain the dynamical evolution of hotspots. From equations (1), (5), (7)–(9) and (12), the quantities $v_{\rm HS}$, $P_{\rm HS}$, $\rho_{\rm HS}$ and $\rho_{\rm j}$ as follows:

$$v_{\rm HS} = v_{\rm HS0}\left(\frac{l_{\rm h}}{l_{\rm h0}}\right)^{[2-X(2-0.5\alpha)]/[X(-2+0.5\alpha)+3]}, \tag{15}$$

$$P_{\rm HS} = P_{\rm HS0}\left(\frac{l_{\rm h}}{l_{\rm h0}}\right)^{[X(2-0.5\alpha)(\alpha-2)+4-3\alpha]/[X(-2+0.5\alpha)+3]}, \tag{16}$$

$$\rho_{\rm HS} = \rho_{\rm HS0}\left(\frac{l_{\rm h}}{l_{\rm h0}}\right)^{[X(2-0.5\alpha)(\alpha-2)+4-3\alpha]/[X(-2+0.5\alpha)+3]}, \tag{17}$$

$$\rho_{\rm j} = \rho_{\rm j0}\left(\frac{l_{\rm h}}{l_{\rm h0}}\right)^{[X(2-0.5\alpha)(\alpha-2)+4-3\alpha]/[X(-2+0.5\alpha)+3]}. \tag{18}$$

Note that $P_{\rm HS0}$, $\rho_{\rm HS0}$ and $\rho_{\rm j0}$ can be expressed by only observable quantities $\rho_{\rm a0}$ and $v_{\rm HS0}$ if we assume $\Gamma_{\rm j}$ (see equations 7–9). Thus, it is possible to know not only $l_{\rm h}$-dependence but also the absolute quantities of hotspots and jets. The aspect ratio of the cocoon $\mathcal{R} \equiv l_{\rm c}/l_{\rm h}$ is the intriguing quantity for studying the dynamical evolution of hotspots. The $l_{\rm h}$-dependence of the aspect ratio of cocoon is then given by

$$\mathcal{R} = \mathcal{R}_0\left(\frac{l_{\rm h}}{l_{\rm h0}}\right)^{[X(2.5-0.5\alpha)-3]/[X(-2+0.5\alpha)+3]}, \tag{19}$$

where $\mathcal{R}_0 = (v_{\rm c0}/v_{\rm HS0})[(X(-2+0.5\alpha)+3)/(0.5X)]$. As a consistency check of our assumption of constant $A_{\rm h}/A_{\rm j}$, we can easily find that $r_{\rm HS}$ ($\propto A_{\rm j}^{1/2}$) shows the same $l_{\rm h}$-dependence as $A_{\rm h}^{1/2}$ from equations (6) and (18). From above results, we obtain the slope of all physical quantities as functions of two key physical quantities, namely α (the slope index of the ambient matter density) and X (the growth rate of cross-section of cocoon body). In the case of constant $\mathcal{R}$, our results agree with self-similar models of cocoon expansions (e.g. Falle 1991; Begelman 1996; Kaiser & Alexander 1997). However, we stress that these self-similar models assume that $P_{\rm HS}/P_{\rm c}$ and $\mathcal{R}$ are both constant in $l_{\rm h}$, whilst we do not impose these assumptions and also predict the dynamical evolution of $\rho_{\rm HS}$ and $\rho_{\rm j}$.

The relation between $P_{\rm HS}$ and $P_{\rm c}$ is also the interesting topic. From equation (7), the hotspot pressure is written by $P_{\rm HS} = 4\rho_{\rm a}(l_{\rm h})v_{\rm HS}^2/3$ for $\mathcal{M} \gg 1$, while the overpressured cocoon requires $P_{\rm c} = \rho_{\rm a}(l_{\rm c})v_{\rm c}^2$.

Thus, the ratio of $P_{\rm HS}$ to $P_{\rm c}$ is

$$\frac{P_{\rm c}}{P_{\rm HS}} = \frac{3}{4}\left[\frac{0.5X}{X(-2+0.5\alpha)+3}\right]^2 \mathcal{R}_0^{\alpha}\mathcal{R}^{2-\alpha}. \quad (20)$$

This implies that $P_{\rm c}/P_{\rm HS}$ is controlled by $\mathcal{R}$ and α. Since $\mathcal{R} < 1$ and $\mathcal{R}_0 < 1$ are satisfied by definitions, $P_{\rm c}/P_{\rm HS}$ should be less than unity for $0 < \alpha < 2$. In the case of $\mathcal{R} = \mathcal{R}_0 =$ constant or $\alpha = 2$, it reduces to the interesting relation of

$$\frac{P_{\rm c}}{P_{\rm HS}} = \frac{3}{4}\mathcal{R}_0^2.$$

This shows that $P_{\rm c}/P_{\rm HS}$ is determined only by $\mathcal{R}_0^2$. We stress that our model predicts that $P_{\rm c}$ is smaller than $P_{\rm HS}$ as long as $\mathcal{R}_0 < 1$. Additionally, rewriting of the explicit form of the $P_{\rm c}$ in terms of the quantities of the jet may be also stimulating, which is given by $P_{\rm c} = \mathcal{R}_0^2\Gamma_{\rm j}^2\rho_{\rm j}c^2$ for $\mathcal{R}_0 =$ constant. From this, one can find that the larger $\Gamma_{\rm j}$ leads to the larger $P_{\rm c}$ which is actually seen in relativistic hydrodynamic simulations (see fig. 5 in Martí et al. 1997). To comprehend the energy injection process into the cocoon via the hotspot with the duration of $t_{\rm inj}$, we rewrite the equation (20) as

$$P_{\rm c0}v_{\rm c0}S_{\rm c0} \approx P_{\rm HS0}v_{\rm esc0}S_{\rm HS0} \equiv L_{\rm j}t_{\rm inj}, \quad (21)$$

where $S_{\rm HS0} \equiv 4\pi r_{\rm HS0}^2$, and $S_{\rm c0} \equiv 2\pi l_{\rm c0}\, l_{\rm h0}$, $v_{\rm esc0} \equiv c/[2(0.5X)^2] \sim (0.5-0.7)c$. This describes the *continuous* energy injection of AGN jets (i.e. $t_{\rm inj} = t_{\rm age}$). On the contrary, Blandford & Rees (1974) used the relation of $P_{\rm c0}(v_{\rm c0}S_{\rm c0})^{\hat{\gamma}_{\rm c}} \approx P_{\rm HS0}(v_{\rm esc0}S_{\rm HS0})^{\hat{\gamma}_{\rm HS}}$ where $\hat{\gamma}$ is the adiabatic index in each region. We claim that this relation is appropriate for the instantaneous (i.e. $t_{\rm inj} \ll t_{\rm age}$) injection seen in supernovae (SNe) or gamma-ray bursts (GRBs).

3 COMPARISON WITH PREVIOUS WORKS

3.1 Comparison with numerical simulations

Scheck et al. (2002; hereafter S02) examined the long-term evolution of the powerful jet propagating into a uniform ambient medium ($\alpha = 0$) with '2D' relativistic hydrodynamic simulations. S02 showed that the evolution of the jet proceeds in two different phases appear (they are shown in table 4 and fig. 6 in S02). *'1D' phase*. In the initial phase ($t < 1.2 \times 10^5$ yr), the jet shows ballistic propagation with $A_{\rm h} =$ constant and $v_{\rm HS} =$ const. *'2D' phase*. This phase starts when the first large vortices are produced near the tip of the jet. Then, the cross-section area of the cocoon head begins to increase. The hotspot then starts decelerating, but the jet speed remains the same relativistic one during whole simulations. Below we compare the present work with the hydrodynamic simulation of S02 in Table 1.

In the '1D' phase, the results of S02 can be well described by our model with $\beta = 0$ and $\alpha = 0$. Note that this '1D' phase corresponds to the evolutionary model with constant $A_{\rm h}$ (BC89). For $v_{\rm HS}$, the power-law index is slightly (∼10 per cent,) different from our model (also BC89) and the results of S02. In this case, $P_{\rm c} \propto l_{\rm h}^{-1}$ and $P_{\rm HS} =$ const are predicted by this work and BC89, which coincides with the numerical results of S02 (see fig. 6c for $P_{\rm c}$ and $P_{\rm HS}$ in S02). In addition, our model can reproduce the constant $\rho_{\rm j}$ (see fig. 5a in S02). For comparison, let us briefly comment on the self-similar model of expanding cocoons in which the growth of the cocoon head is included (e.g. Begelman 1996: hereafter B96). As already pointed out (e.g. Carvalho & O'Dea 2002a), the self similar mofel of B96 cannot represent the behaviour of the '1D' phase. The behaviour of $P_{\rm c}/P_{\rm HS}$ is also the intriguing issue. The decrease of $P_{\rm c}/P_{\rm HS}$ with time is reported in fig. 6 of S02. Using our model, this behaviour

Table 1. Comparison with 2D hydrodynamic simulations and self-similar models.

	$v_{\rm HS}$	$A_{\rm h}$	$P_{\rm c}$	$P_{\rm HS}$	$\rho_{\rm j}$	$\mathcal{R}$
			'1D' phase[a]			
S02	$l_{\rm h}^{-0.11}$	Constant	$l_{\rm h}^{-0.95}$	Constant	Constant	$l_{\rm h}^{-0.45}$
BC89	Constant	Constant	$l_{\rm h}^{-1}$	Constant	–	$l_{\rm h}^{-0.5}$
B96	$l_{\rm h}^{-2/3}$	$l_{\rm h}^{4/3}$	$l_{\rm h}^{-4/3}$	$l_{\rm h}^{-4/3}$	–	Constant
This work	Constant	Constant	$l_{\rm h}^{-1}$	Constant	$l_{\rm h}^{0}$	$l_{\rm h}^{-0.5}$
			'2D' phase[b]			
S02	$l_{\rm h}^{-0.55}$	$l_{\rm h}^{0.90}$	$l_{\rm h}^{-1.30}$	$l_{\rm h}^{-1.1}$	$l_{\rm h}^{-1.0}$	$l_{\rm h}^{-0.09}$
B96	$l_{\rm h}^{-2/3}$	$l_{\rm h}^{4/3}$	$l_{\rm h}^{-4/3}$	$l_{\rm h}^{-4/3}$	–	Constant
This work	$l_{\rm h}^{-0.56}$	$l_{\rm h}^{1.1}$	$l_{\rm h}^{-1.30}$	$l_{\rm h}^{-1.1}$	$l_{\rm h}^{-1.1}$	$l_{\rm h}^{-0.08}$

[a]The 1D phase corresponds to our model with $\beta = 0$ and $\alpha = 0$.
[b]The 2D phase corresponds to our model with $\beta = 1.1$ and $\alpha = 0$.

is clearly explained by the decrease of the cocoon aspect ratio (see equation 20).

The '2D' phase of S02 is well described by our model with $\beta = 1.1$ and $\alpha = 0$. We adopt $\beta = 1.1$ to reproduce the $P_{\rm c}$ evolution in fig. 6(c) of S02 because the other quantities show much larger fluctuations in fig. 6 of S02. The present model predicts the evolution of the hotspot pressure and mass density of the jet as $P_{\rm HS} \propto l_{\rm h}^{-1.1}$, $v_{\rm HS} \propto l_{\rm h}^{-0.56}$ and $\rho_{\rm j} \propto l_{\rm h}^{-1.1}$. These coincide with the average values of $P_{\rm HS}$, $v_{\rm HS}$ and $\rho_{\rm j}$ (see figs 5a and 6 in S02). In the '2D' phase, the cross-section of cocoon head grows as $A_{\rm h} \propto l_{\rm h}^{1.1}$ unlike the '1D' phase ($A_{\rm h} =$ constant). Thus, the velocity of hotspot decreases with $l_{\rm h}$. Actually, the growth of the cross-section area of the cocoon head can be seen in their simulations (fig. 3 in S02). In this phase, B96 also explained these results of S02. Moreover, the cocoon pressure is proportional to $P_{\rm HS}$ in this phase of S02. From equation (20), it can be understood with a constant $\mathcal{R}$. From above detailed comparison with '2D' relativistic hydrodynamic simulations, we found that the model represented in this paper can describe the flow and cocoon behaviours seen in the '1D' and '2D' phases very well. It should be stressed that our analytic model is more useful than numerical simulations when investigating a longer term evolution of jets. The length of jets performed by numerical simulations of jets achieves at most the length of the order of 100 times of a jet beam size, while the spacial sizes of actual jets in AGNs are spread in six order of magnitude (i.e. from parsec to megaparsec scale).

3.2 Comparison with observations

Based on a number of recent reports of indicating that the constant speed of hotspot advance ($0.01 < v_{\rm HS}/c < 0.1$) (e.g. Carilli et al. 1991; Readhead et al. 1996b; Conway 2002), we here examine the case of $v_{\rm HS} =$ constant. Observationally, $P_{\rm HS}$ and $\rho_{\rm HS}$ are inferred by using the minimum energy assumption and the neglecting the effect of thermal components (Readhead et al. 1996a; Jeyakumar & Saikia 2000; Perucho & Martí 2002). From equation (15), the relation of $2 - X(2 - 0.5\alpha) = 0$ is required for the constant hotspot velocity. By eliminating the parameter X, our model reduces to the following forms; $v_{\rm c} \propto l_{\rm h}^{-(\alpha-2)/(\alpha-4)}$, $P_{\rm c} \propto l_{\rm h}^{4/(\alpha-4)}$, $P_{\rm HS} \propto l_{\rm h}^{-\alpha}$, $\rho_{\rm HS} \propto l_{\rm h}^{-\alpha}$, $r_{\rm HS} \propto l_{\rm h}^{\alpha/2}$, $\rho_{\rm j} \propto l_{\rm h}^{-\alpha}$ and $\mathcal{R} \propto l_{\rm h}^{-(\alpha-2)/(\alpha-4)}$. Here we used of mean density

Table 2. Comparison with observations.

	$v_{\rm HS}$	$P_{\rm HS}$	$r_{\rm HS}$	$\rho_{\rm HS}$
Observations[a]	Constant	$l_{\rm h}^{-(1.3\text{–}1.7)}$	$l_{\rm h}^{0.7\text{–}1.3}$	$l_{\rm h}^{-(1.9\text{–}2.9)}$
This work[b]	Constant	$l_{\rm h}^{-(1.5\text{–}2.0)}$	$l_{\rm h}^{0.75\text{–}1}$	$l_{\rm h}^{-(1.5\text{–}2.0)}$

[a]The results are adopted from Readhead et al. (1996a), Jeyakumar & Saikia (2000) and Perucho & Martí (2002); [b]we set the slope index of the ambient density $\alpha = 1.5$–2.

profiles of obtained by a large number of sample clusters of galaxies, which is $\rho_{\rm a}(d) \propto d^{-(1.5\text{–}2)}$ (e.g. Mulchaey & Zabludoff 1998).

We show the comparison with observational data for CSOs and FR II sources in Table 2. This indicates that our model well reproduce observational trends within the error bars. These agreements are likely to support 'youth radio source scenario' basically. At the same time, the large dispersion of the observational data could tell us other possibilities of evolutionary tracks of radio-loud AGNs usually discussed. To explore it, it must be valuable to inquire into the origin of their large dispersion. Furthermore, we emphasize that the deviation from the self-similar evolution are frequently indicated by several authors (e.g. De Young 1997; Gilbert et al. 2004).

It is worth to show the reliability of the relation of the opening angle of hotspots derived by equation (21), namely $\theta_{\rm HS} = (v_{\rm HS}/c)^{1/2}\mathcal{R}_0^2$. For this aim, we adopt this equation to an archetypal radio galaxy Cygnus A. Using the values of $v_{\rm HS} \approx 0.01c$ (Carilli et al. 1991) and $\mathcal{R} = 0.6$ (Wilson, Young & Shopbell 2000), our model predicts $\theta_{\rm HS} \simeq 0.036$, while the direct estimate of $\theta_{\rm j}$ with $r_{\rm HS} = 2$ kpc and $z = 60$ kpc indicates $\theta_{\rm HS} \simeq 0.033$. Thus, we can verify the reliability of the relation for the opening angle of hotspots and then we propose a new way of the estimation of $v_{\rm HS}$ from two observable quantities $\mathcal{R}$ and $\theta_{\rm HS}$. It would be worth to compare with evaluations from the kinematic studies.

4 CONCLUSIONS

In this work, we model a dynamical evolution of hotspots in radio-loud AGNs. In this model, the unshocked flow satisfies the conservations of the mass, momentum and kinetic energy. We take account of the deceleration process of the jet by shocks, and the cocoon expansion which is identified as the by-product of the exhausted flow. The model describes the evolution of physical quantities ($v_{\rm HS}$, $P_{\rm HS}$ and $\rho_{\rm HS}$) in the hotspot in terms of $l_{\rm h}$. Below we summarize the main results based on this model.

(i) We find that the ratio of $P_{\rm c}/P_{\rm HS}$ is controlled by the aspect ratio of the cocoon $\mathcal{R}$ and slope index of the ambient medium α. If $\mathcal{R}$ remains to be constant during the jet propagation, the value $P_{\rm c}/P_{\rm HS}$ is proportional to $\mathcal{R}^2$ with the coefficient of the order of unity. This naturally explains the basic concept of the elongated cocoon with larger $P_{\rm HS}$ than $P_{\rm c}$. Concerning $P_{\rm c}$, it is proportional to the bulk kinetic power of the jet in given $\rho_{\rm a}$. This is originated from our treatment of adiabatic injection of the dissipation energy of the jet into the cocoon. In addition, we suggest a new method to evaluate the velocity of hotspots from the aspect ratio of cocoon and the opening angle of hotspots.

(ii) Our analytic model can well explain the results of 2D co-evolution of jets and cocoons obtained by relativistic hydrodynamic simulations. This clearly guarantees the reliability of our model during the overpressure cocoon phase. Since the dynamical length of jets obtained by numerical simulations is about a few 100 times of the jet beam size, our analytic model must be an useful tool to explore a longer term dynamical evolution of jets than this scale.

(iii) Our model prediction reasonably coincides with the recent observational trends of hotspots seen in CSO and FR II sources. Furthermore, we predict $\mathcal{R} \propto l_{\rm h}^{-(0.2\text{–}0)}$ and $A_{\rm h} \propto l_{\rm h}^{1.5\text{–}2}$ although little is done about systematic studies on these quantities.

Lastly, we should keep in mind that the present model does not take account of the details of (i) the absolute value of the mass density of the ambient medium, and (ii) radiative cooling effect which may be important for younger radio galaxies. In order to investigate whole story of evolutionary track of the radio-loud AGNs, the study of above two points will be inevitably required. We plan to investigate both of them in the near future.

ACKNOWLEDGMENTS

We thank A. Celotti, H. Ito and F. Takahara for useful discussions and comments. We acknowledge the Italian MIUR and INAF financial support. We also thank the anonymous referee for her/his helpful comments to improve this paper.

REFERENCES

Begelman M. C., 1996, in Carilli C. L., Harris D. E., eds, Cygnus A – Study of a Radio Galaxy. Cambridge Univ. Press, Cambridge, p. 209 (B96)
Begelman M. C., Cioffi D. F., 1989, ApJ, 345, L21 (BC89)
Begelman M. C., Blandford R. D., Rees M. J., 1984, RvMP, 56, 255
Blandford R. D., McKee C. F., 1976, Phys. Fluids, 19, 1130
Blandford R. D., Rees M. J., 1974, MNRAS, 169, 395
Bridle A. H., Perley R. A., 1984, ARA&A, 22, 319
Carilli C. L., Perley R. A., Dreher J. W., Leahy J. P., 1991, ApJ, 383, 554
Carvalho J. C., 1985, MNRAS, 215, 463
Carvalho J. C., O'Dea C. P., 2002a, ApJS, 141, 337
Carvalho J. C., O'Dea C. P., 2002b, ApJS, 141, 371
Celotti A., Ghisellini G., Chiaberge M., 2001, MNRAS, 321, L1
Clarke D. A., Norman M. L., Burns J. O., 1986, ApJ, 311, L63
Clarke D. A., Harris D. E., Carilli C. L., 1997, MNRAS, 284, 981
Conway J. E., 2002, New AR, 46, 263
Cox C. I., Gull S. F., Scheuer P. A. G., 1991, MNRAS, 252, 558
De Young D. S., 1997, ApJ, 490, L55
Falle S. A. E. G., 1991, MNRAS, 250, 581
Fanti C., Fanti R., Dallacasa D., Schilizzi R. T., Spencer R. E., Stanghellini C., 1995, A&A, 302, 317
Gilbert G. M., Riley J. M., Hardcastle M. J., Croston J. H., Pooley G. G., Alexander P., 2004, MNRAS, 351, 845
Hardcastle M. J., Worrall D. M., 2000, MNRAS, 319, 562
Isobe N., Tashiro M., Makishima K., Iyomoto N., Suzuki M., Murakami M. M., Mori M., Abe K., 2002, ApJ, 580, L111
Jeyakumar S., Saikia D. J., 2000, MNRAS, 311, 397
Kaiser C. R., Alexander P., 1997, MNRAS, 286, 215
Kino M., Kawakatu N., 2005, MNRAS, 364, 659 (KK05)
Kino M., Takahara F., 2004, MNRAS, 349, 336
Komissarov S. S., Falle S. A. E. G., 1998, MNRAS, 297, 1087
Landau L. D., Lifshitz E. M., 1959, Fluid Mechanics. Pergamon Press, Oxford
Leahy J. P., Gizani N. A. B., 2001, ApJ, 555, 709
Lind K. R., Payne D. G., Meier D. L., Blandford R. D., 1989, ApJ, 344, 89
Liu R., Pooley G., Riley J. M., 1992, MNRAS, 257, 545
Martí J. M. A., Müller E., Font J. A., Ibanez J. M. A., Marquina A., 1997, ApJ, 479, 151
Mizuta A., Yamada S., Takabe H., 2004, ApJ, 606, 804
Mulchaey J. S., Zabludoff A. I., 1998, ApJ, 496, 73
Murgia M., Fanti C., Fanti R., Gregorini L., Klein U., Mack K.-H., Vigotti M., 1999, A&A, 345, 769

Myers S. T., Spangler S. R., 1985, ApJ, 291, 52
Norman M. L., Smarr L., Winkler K. H. A., Smith M. D., 1982, A&A, 113, 285
O'Dea C. P., 1998, PASP, 110, 493
O'Dea C. P., Baum S. A., 1997, AJ, 113, 148
Owsianik I., Conway J. E., Polatidis A. G., 1999, New AR, 43, 669
Perucho M., Martí J. M., 2002, ApJ, 568, 639
Phillips R. B., Mutel R. L., 1980, ApJ, 236, 89
Phillips R. B., Mutel R. L., 1982, A&A, 106, 21
Readhead A. C. S., Cohen M. H., Blandford R. D., 1978, Nat, 272, 131
Readhead A. C. S., Taylor G. B., Pearson T. J., Wilkinson P. N., 1996a, ApJ, 460, 634
Readhead A. C. S., Taylor G. B., Xu W., Pearson T. J., Wilkinson P. N., Polatidis A. G., 1996b, ApJ, 460, 612
Ryle M. S., Longair M. S., 1967, MNRAS, 136, 123
Scheck L., Aloy M. A., Martí J. M., Gómez J. L., Müller E., 2002, MNRAS, 331, 615, (S02)
Scheuer P. A. G., 1982, in Heeschen D. S., Wade C. M., eds, IAU Symp. 97, Extragalactic Radio Sources. Reidel Publishing Co., Dordrecht, p. 163
Scheuer P. A. G., 1995, MNRAS, 277, 331
Shklovsky I. S., 1965, Nat, 206, 176
Smith M. D., Norman M. L., Winkler K.-H. A., Smarr L., 1985, MNRAS, 214, 67
Tavecchio F., Maraschi L., Sambruna R. M., Urry C. M., 2000, ApJ, 544, L23
Taylor G. B., Marr J. M., Pearson T. J., Readhead A. C. S., 2000, ApJ, 541, 112
Tinti S., de Zotti G., 2006, A&A, 445, 889
Turland B. D., 1975, MNRAS, 172, 181
Uchiyama Y., Urry C. M., Van Duyne J., Cheung C. C., Sambruna R. M., Takahashi T., Tavecchio F., Maraschi L., 2005, ApJ, 631, L113
van Breugel W. J. M., Miley G. K., Heckman T. A., 1984, AJ, 89, 5
Wilkinson P. N., Polatidis A. G., Readhead A. C. S., Xu W., Pearson T. J., 1994, ApJ, 432, L87
Williams A. G., Gull F. G., 1985, Nat, 313, 34
Wilson M. J., Scheuer P. A. G., 1983, MNRAS, 205, 449
Wilson A. S., Young A. J., Shopbell P. L., 2000, ApJ, 544, L27

This paper has been typeset from a TEX/LATEX file prepared by the author.