On the first generation of stars

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ABSTRACT

We argue that the first stars may have spanned the conventional mass range rather than be identified with the very massive objects ($\sim$100–10^3 M$_\odot$) favoured by numerical simulations. Specifically, we find that magnetic field generation processes acting in the first protostellar systems suffice to produce fields that exceed the threshold for magneto-rotational instability (MRI) to operate, and thereby allow the MRI dynamo to generate equipartition-amplitude magnetic fields on protostellar mass scales below $\sim$50 M$_\odot$. Such fields allow primordial star formation to occur at essentially any metallicity by regulating angular momentum transfer, fragmentation, accretion and feedback in much the same way as occurs in conventional molecular clouds.

Key words: stars: formation – stars: general – stars: magnetic fields – galaxies: formation – cosmology: theory.

1 INTRODUCTION

The formation of the first stars should, in principle, be simpler to understand than present-day star formation. The usual prediction that the mass of the first stars $\mathcal{O}(100$–$1000)$ M$_\odot$ is attributable to dominance of H$_2$ cooling and consequent high temperature and accretion rate (Abel, Bryan & Norman 2002; Bromm, Coppi & Larson 2002). It is appropriate to question this result that only very massive stars formed, for at least four phenomenological reasons.

First, one finds solar mass stars at [Fe] $\lesssim -4$. These may be contaminated by companion stars that exploded as hypernovae, whose ejecta are depleted in Fe relative to N and C and this interpretation is consistent with the measured abundance ratios for a handful of extreme metal-poor stars. However not all metal-poor halo stars display such anomalies, and a substantial fraction at the 10^{-4} solar level have abundance ratios that are consistent with conventional Type II supernovae (SNII) precursors. There are at least two halo stars known at [Fe] $\lesssim -5$. HE 0107$-$5240 is a giant with [Fe/H] $= -5.3$, but enhanced nitrogen, carbon and oxygen: [N/H] $= -3.0$, [C/H] $= -1.3$ and [O/H] $= -2.9$ (Christlieb et al. 2004). HE 1327$-$2326 is a main sequence star (or subgiant) with an iron abundance about a factor of 2 lower than HE 0107$-$5240. In this latter case, both nitrogen and carbon are enhanced relative to iron by about 4 dex, while there is only a comparable upper limit on oxygen (Frebel et al. 2005).

Appeal to pre-enrichment by a core collapse $\sim$25 M$_\odot$ supernova of Population III (Pop III) abundance with fallback fits the abundance patterns well (Umeda & Nomoto 2003), although oxygen may possibly be overproduced in the case of HE 0107$-$5240 (Bessell, Christlieb & Gustafsson 2004). The example of HE 1327$-$2326 eliminates any internal enrichment source (e.g. convective dredge-

up), or is there evidence for a binary companion in the case of HE 0107$-$524, thereby also suggesting that mass transfer from an asymptotic giant branch star is an unlikely explanation. The high CNO abundances argue for a common explanation involving enrichment of the primordial cloud by SNII of primordial abundance. Indeed, the abundance patterns in extremely metal-poor halo stars suggest that enrichment was produced by Pop III stars in the mass range 20–130 M$_\odot$ (Umeda & Nomoto 2005).

Secondly, the broad emission line regions of very high redshift quasars reveal high elemental abundances that appear to have also been generated by conventional SNII precursors. In particular, the nearly constant Fe II/Mg II emission line ratios over $0 < z < 5$ require intense SNII activity at a redshift as high as 9 (Dietrich et al. 2003).

Thirdly, the chemical yield predictions from primordial very massive stars, when normalized to the inferred ionizing photon output required to reionize the Universe at high redshift, do not correspond to observed abundances in any primitive environments. In particular, the pair-instability supernova nucleosynthetic signatures generated by stars with initial masses in the range 130–260 M$_\odot$ are not seen either in the intergalactic medium (IGM), including both Lyman α forest and damped Lyman α absorption systems, or in the extremely metal-poor halo stars (Daigne et al. 2006). The primordial star mass range 10 to 100 M$_\odot$ may provide an adequate ionization source (Tumlinson, Venkatesan & Shull 2004), especially in view of the lowered requirements on reionization from WMAP-3.

Finally, from the theoretical perspective with regard to cooling, the primordial metal abundance pattern has profound consequences for the thermal balance and chemical composition of the gas, and hence for the equation of state of the parental cloud. Spaans & Silk (2005) find that the polytropic index is soft for low oxygen abundance enhancements, [O/H] < -3, as appropriate for Pop III, but stiffens to a polytropic index γ larger than unity for [O/H] $> 10^{-2}$ due to the large opacity in the CO and H$_2$O cooling lines. Hence Pop III star formation is efficient, especially before hypernova

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enrichment and associated oxygen enhancement has occurred. There should be no obstacle to forming stars over a wide range of masses even in the absence of significant fine-structure cooling. On the contrary, once the polytropic index stiffens, at [O/H] $\gtrsim$ −2, the initial mass function (IMF) should change and, specifically, flatten, as argued by Spaans & Silk (2000). Star formation is subsequently less efficient, in part because of massive star feedback in addition to the stiffening of the equation of state, as is required in most discussions of galaxy formation in order to avoid premature consumption of the gas reservoir.

Of course, to do justice to the very massive primordial star hypothesis, the typical expectation of one very massive star per primordial cloud is unlikely to produce enough metals to raise the mean IGM metallicity to a value of ∼10^{-4} to $10^{-2.5}$ solar, as observed in the Lyman α forest (Schaye et al. 2003) and as also required to significantly modify the cooling and thereby enhance fragmentation. This is especially plausible, as the ejecta are mostly not retained in the shallow potential wells of the pre-galactic clouds (Norman, O'Shea & Paschos 2004). Rather, one has the impression that the first stars that produced metals in significant amounts, in particular as monitored in the abundance patterns of the Lyman α forest at modest overdensity and in halo stars at [Fe/H] $\lesssim$ −4, spanned the conventional stellar mass range.

A more fundamental issue may be that the elegant state-of-the-art simulations with cosmological initial conditions have nothing directly to say about the primordial stellar initial mass function, or for that matter about the physics underpinning the inferred transfer of angular momentum, but only about the mass function of gas clumps. The ultimate fate of these gas clumps has not yet been convincingly demonstrated.

Note that recent simulations by Saigo, Matsumoto & Umemura (2004) show that Pop III stars are prone to form in binary systems through the formation of a rotationally supported disc. For that to happen, the initial angular momentum of the parent cloud needs to be small, the centrifugal force being at most 10–30 per cent of the pressure force. However, the accretion efficiency seems not affected by the binary formation, and the binary Pop III stars are again predicted to be very massive.

Of course, all of this begs the question as to whether a softened equation of state (EOS) can lead to low or intermediate mass star formation in situations of very low metallicity, or at any rate, reduce the characteristic stellar mass to below the lower limit for pair instability SN. Certainly, the clouds from which stars with [Fe/H] $\lesssim$ −5 formed seem to have had a soft equation of state. The minimum fragment mass is lowered at [O/H] $\gtrsim$ −3 due to enhanced fine-structure cooling (Bromm & Loeb 2003), but it is likely that turbulent fragmentation (Klessen et al. 2004) enhanced by the softening of the equation of state is the key to low-mass fragmentation, for example, via disc formation. According to this viewpoint, it is the equation of state in the initial collapsing cloud that controls the IMF.

An alternative view is that the final fragment mass remains high, as suggested by the numerical simulations, but that a second mode of fragmentation associated with the onset of dust cooling at a very high density may lead to low-mass star formation (Omukai et al. 2005). The latter study finds that a metallicity of $Z \gtrsim 10^{-5}\,Z_{\odot}$ at a density in excess of $10^{10}\,\mathrm{cm}^{-3}$ marks this transition. However, Spaans & Silk (2005) argue that the gas properties, and in particular the equation of state, at much lower densities, typically the range associated with saturation of H_2 cooling at ∼$10^4\,\mathrm{cm}^{-3}$, most likely determine the IMF.

The purpose of this paper is to demonstrate that a plausible alternative to dust cooling in primordial clouds involves recourse to a mechanism that is universally accepted as being an essential key to controlling the IMF in conventional molecular clouds at the onset of collapse, namely magnetically regulated fragmentation and accretion. The essence of our argument will be to derive the magnetic field strength in primordial star formation. An important conclusion is that one should continue to find low mass, and especially abundance signatures of intermediate mass, stars down to essentially zero metallicity. Note that the possible effects of magnetic fields on primordial star formation have also been investigated in a pioneering paper by Tan & Blackman (2004), and more recently by Machida et al. (2006).

2 THE TRANSITION FROM PRIMORDIAL STAR FORMATION

For the moment, pending higher resolution simulations with adequate exploration of the EOS, it may be grasping at straws to argue that Pop III could contain a substantial or even a dominant low-mass component. Nevertheless, an analytic exploration is warranted. In this note, therefore, an alternative approach is addressed. A reasonable hypothesis, prevalent in the star formation community, is that magnetic fields are essential to the generation of the initial stellar mass function in conventional star-forming clouds (e.g. Mestel 1965; Lizano & Shu 1989; Shu, Li & Allen 2004), even if precise details of the actual regulation mechanism still remain to be worked out. If this is indeed the case, one may then ask what happened in the first clouds, when presumably magnetic fields were absent at any significant dynamical level? We refer to this as primordial star formation. And was there a seamless transition to current epoch star formation, involving no drastic change in the underlying physics?

One common, and generally accepted, viewpoint among cosmologists is that metallicity was a primary driver. When the metallicity was less than about 0.001 of the solar value, the dominant coolants were H atoms and H_2 molecules. These sufficed only to cool to about 200 K. The inefficient cooling is the primary reason that both analytic and numerical calculations of the minimum fragmentation mass in primordial clouds yield high accretion rates on to protostellar cores ($\dot{M}_{\star} \approx c_s^3/G$) that result in characteristic primordial stellar masses of ∼100–1000 $M_{\odot}$.

However, the protostellar core is only 0.01 $M_{\odot}$ and grows by accretion. One may try to understand how low-mass stars could form in a primordial, or nearly primordial, environment if fragmentation, itself inefficient, was aided by magnetic feedback. Consider the following hypothesis. Magnetic fields are generated by a dynamo associated with magneto-rotational instabilities (MRIs) at work in protostellar or circumstellar discs. There are several necessary conditions for such a MRI dynamo to operate. First, the proton Larmor radius must be less than the disc scale-height. Secondly, there are important conditions imposed on the wavelength and on the growth rate of the MRI instability. Thirdly, there must be a seed field that exceeds the minimal field under which the MRI dynamo operates. In the following, we will examine each one of these issues, within the framework of a simple steady-state circumstellar disc model. This will allow us to obtain a critical mass of the central (proto)star above which all the conditions are fulfilled for the MRI to work.

2.1 General equations

We start by considering a three-fluid model, including free electrons, ions (protons, for simplicity) and neutral gas, of respective number densities n_e, n_i, and n_n. Setting $n \equiv n_e = n_i$ global charge neutrality implies that the electric current is $\boldsymbol{j} = -en\boldsymbol{V}$, where $\boldsymbol{V} = \boldsymbol{v}_e - \boldsymbol{v}_i$

is the electron velocity relative to ions. Similar to the procedure followed by Mestel (2003), we can combine the equation of motion of electrons with that of ions to obtain the following generalized Ohm's law equation,

$$
\boldsymbol{E} = -\frac{\boldsymbol{v} \times \boldsymbol{B}}{c} - \frac{\boldsymbol{\nabla} p_e}{n_e} + \left(\kappa_e + \kappa - \frac{\kappa_e^2}{\kappa_e + \kappa_i} \right) \frac{B}{enc} \boldsymbol{j}
$$
$$
+ \left(1 - 2F \frac{\kappa_e}{\kappa_e + \kappa_i} \right) \frac{\boldsymbol{j} \times \boldsymbol{B}}{enc} + \frac{\kappa_e}{\kappa_e + \kappa_i} \frac{F}{en} (1 - f F^{-1}) \boldsymbol{\nabla} p
$$
$$
+ \frac{F^2}{enB\kappa_i} \left[(1 - f F^{-1}) \boldsymbol{\nabla} p - \frac{\boldsymbol{j} \times \boldsymbol{B}}{c} \right] \times \boldsymbol{B}, \tag{1}
$$

where $\kappa^{-1} = \omega \tau$, $\kappa_e^{-1} = \omega \tau_e$ and $\kappa_i^{-1} = \omega \tau_i$ measure how strongly the charged particles are coupled to the magnetic field with respect to their collisional interactions. ($\omega = eB/m_e c$ and $\omega_i = eB/m_i c$ are the electron and ion gyrofrequencies, respectively, τ being the mean electron–ion collision time, and τ_e and τ_i the mean times for electron and ion collisions with neutrals.) Note that the products $\kappa_q B$ are independent of the magnetic field amplitude B. The density fraction of neutral particles is given by $F = \rho_n/(\rho_i + \rho_n)$, and their fractional contribution to the total pressure p is $f = p_n/p$. The rest of the symbols used above bear their usual meanings.

We get the magnetic field induction equation by taking the curl of equation (1). Ignoring the pressure terms, we obtain

$$
\partial_t \boldsymbol{B} = \boldsymbol{\nabla} \times [(\boldsymbol{v} + \boldsymbol{\Omega} \times \boldsymbol{r}) \times \boldsymbol{B}]
$$
$$
- \boldsymbol{\nabla} \times \left[\left(\kappa_e + \kappa - \frac{\kappa_e^2}{\kappa_e + \kappa_i} \right) \frac{B}{ne} \boldsymbol{j} \right.
$$
$$
- \left(1 - 2F \frac{\kappa_e}{\kappa_e + \kappa_i} \right) \left(\frac{\boldsymbol{j} \times \boldsymbol{B}}{en} \right)
$$
$$
\left. + \frac{F^2}{eB\kappa_i} \left(\frac{(\boldsymbol{j} \times \boldsymbol{B}) \times \boldsymbol{B}}{n} \right) \right] \tag{2}
$$

in a local Keplerian frame corotating with the disc at angular frequency Ω. The second, third and fourth terms on the right-hand side are the resistive, Hall and ambipolar diffusion terms, respectively. It is straightforward to check that we recover the usual induction equation in the limit of vanishing density of neutrals ($F, f \to 0$ and $\kappa_{e,i} \to 0$ since $\tau_{e,i} \to \infty$).

Similarly, the equation of motion of the whole plasma is

$$
\rho \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \rho \boldsymbol{\nabla}\phi + \frac{\boldsymbol{j} \times \boldsymbol{B}}{c} + \boldsymbol{F}_\nu - \rho [2\boldsymbol{\Omega} \times \boldsymbol{v} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \boldsymbol{r})], \tag{3}
$$

where $\boldsymbol{F}_\nu$ accounts for the viscous forces, and $\mathrm{d}\cdot/\mathrm{d}t = \partial_t \cdot + \boldsymbol{\Omega} \times \cdot + \boldsymbol{v} \cdot \boldsymbol{\nabla} \cdot$ is the full time derivative. Let us define the velocity departure from Keplerian motion by $\delta\boldsymbol{v} = \boldsymbol{v} - \Omega r \boldsymbol{e}_\theta$. The equation of motion then reads

$$
(\partial_t + \Omega\partial_\theta + \delta\boldsymbol{v} \cdot \boldsymbol{\nabla}) \delta\boldsymbol{v} = -2\Omega(\delta v_r \boldsymbol{e}_\theta - \delta v_\theta \boldsymbol{e}_r) + 3\Omega^2 r \boldsymbol{e}_r
$$
$$
+ \rho \boldsymbol{\nabla}\phi + \frac{\boldsymbol{j} \times \boldsymbol{B}}{c} + \boldsymbol{F}_\nu, \tag{4}
$$

and the induction equation simply becomes

$$
\partial_t \boldsymbol{B} = \boldsymbol{\nabla} \times [(\delta\boldsymbol{v} + \Omega r \boldsymbol{e}_\theta) \times \boldsymbol{B}]
$$
$$
- \boldsymbol{\nabla} \times \left[\left(\kappa_e + \kappa - \frac{\kappa_e^2}{\kappa_e + \kappa_i} \right) \frac{B}{ne} \boldsymbol{j} \right], \tag{5}
$$

where we omitted the Hall and the ambipolar terms which are of second order for initially weak magnetic fields.

Circumstellar discs are prone to gravitational instabilty. Especially in the context of Pop III star formation, Tan and Blackman pointed out that protostellar discs may develop gravitationally driven turbulence, amenable to the α formalism (e.g. Gammie 2001), that provides angular momentum transport. As shown by equations (4) and (5), gravitational instability may amplify velocity fluctuations, which in turn can, in principle, drive the growth of magnetic fields. The importance of gravito-turbulent effects depends on the ratio of the first term and the last term of the right-hand side of equation (5), which yields the actual magnetic Reynolds number. The factor multiplying the current on the right-hand side of equation (5) is

$$
\left[\kappa_e + \kappa - \frac{\kappa_e^2}{\kappa_e + \kappa_i} \right] \frac{B}{ne} = \frac{m_e c}{n\tau e^2} \left[1 + \frac{\tau}{\tau_e} \left(1 + \frac{m_e}{m_i} \frac{\tau_i}{\tau_e} \right)^{-1} \right]. \tag{6}
$$

Taking the mean collision time of charged particles with neutrals from Draine, Roberge & Dalgarno (1983), we obtain

$$
\frac{\tau_i}{\tau_e} \sim 52 \left(\frac{T}{10^4 \, \mathrm{K}} \right)^{1/2} \tag{7}
$$

and

$$
\frac{\tau}{\tau_e} \sim 1.66 \times 10^{-3} \frac{n_n}{n_i} \left(\frac{T}{10^4 \, \mathrm{K}} \right)^2, \tag{8}
$$

and the magnetic Reynolds number is then

$$
\mathcal{R}_M = \frac{\delta v r_{\mathrm{grad}}}{\eta[\ ]}, \tag{9}
$$

where the square brackets stand for the last term in equation (6), accounting for the effects of neutrals. The presence of neutral particles in the disc would enhance the dissipation effects. For initially weak magnetic fields, this would reduce the capability of gravitational instability to amplify any weak magnetic seed fields, even if velocity fluctuations would be amplified. Taking the velocity fluctuations to be of the order of the sound speed in the disc, and the gradient scale being at most of the order the disc radius, the magnetic Reynolds number reaches $\sim 10^{14}$ at $T \sim 10^4 \, \mathrm{K}$.

Based on the high Reynolds number, Tan and Blackman (2004) argue that local gravitational instability develops, leading to an alpha-driven dynamo. This could provide a mean of amplifying any initial seed fields to have a dynamical effect on primordial star formation. However, recent simulations of massive self-gravitating discs find that global gravitational instability modes dominate the energy transport (Lodato & Rice 2005). If this is correct, as Balbus & Papaloizou (1999) initially suggested, the alpha formalism would not be effective in field amplification.

Amplification of magnetic fields may, however, be driven by the action of MRI turbulence. The role of neutrals, Hall effect and ambipolar diffusion in weakly ionized discs has been investigated in several papers (e.g. Wardle 1999; Salmeron & Wardle 2003; Kunz & Balbus 2004). However, as we discuss below, one does not expect protostellar discs around forming Pop III stars to be cool enough so as to allow a dominant amount of neutral particles. In the following, we therefore consider a two-fluid disc model only.

2.2 Simple disc model

To demonstrate the potential role of magnetic fields in Pop III accretion, we use a very simple accretion disc model. We consider a thin, nearly Keplerian disc in the steady state. In that case, the accretion rate $\dot{M}$ is constant throughout the disc, and the surface density,

$$
\Sigma(r) = \int_{-\infty}^{+\infty} \rho \, \mathrm{d}z \simeq 2H(r)\rho_0(r), \tag{10}
$$

can be related to the accretion rate via

$$
\nu\Sigma = \frac{1}{3\pi} \dot{M} \left[1 - \beta \left(\frac{r_i}{r} \right)^{1/2} \right], \tag{11}
$$

where $\rho(\rho_0)$ is the disc (midplane) gas density, H is the disc height scale, ν is the viscosity, r_i is the disc inner radius and β is a parameter related to the angular momentum flux (see e.g. Spruit 2001). We rewrite the viscosity in terms of the dimensionless α viscosity (Shakura & Sunyaev 1973) as $\nu = \alpha c_S H$, where

$$c_S = \left(\frac{kT}{\mu m_p} \right)^{1/2} = f \Omega r = f \left(\frac{GM_\star}{r} \right)^{1/2} \qquad (12)$$

is the sound speed ($\mu \simeq 0.6$ is the mean atomic weight for a gas of primordial composition, and $f = H/r$ is the disc aspect ratio). We have used here the relation between the actual temperature in the disc and the virial temperature,

$$T = f^2 T_{\rm vir} \simeq 1.4 \times 10^5 f^2 \frac{M_\star}{10\,{\rm M_\odot}} \left(\frac{r}{10^3\,{\rm R_\odot}} \right)^{-1} \quad {\rm K.} \qquad (13)$$

Combining equations (10), (11) and (12), we obtain an expression for the midplane density

$$\rho_0(r) = \frac{1}{6\pi \alpha f^3} \frac{\dot{M}}{\sqrt{GM_\star r^3}} \left[1 - \beta \left(\frac{r_i}{r} \right)^{1/2} \right]. \qquad (14)$$

Furthermore, if the central star rotates at a rate $\Omega_\star < \Omega_{\rm Kep}$, one finds that $\beta = 1$, and at large radii r as compared to the disc inner radius r_i, the last term between the square brackets above can be ignored.

We are now left with the task of evaluating the aspect ratio, f. The few available models of discs around a forming Pop III star do not really agree on the disc thickness (and overall structure). Tan & McKee (2004) found a thin disc solution with $f \lesssim 0.13$ up to $r \sim 20$ au whereas Mayer & Duschl (2005) show that, in their 'fiducial model' with $\dot{M} = 10^{-4}\,{\rm M_\odot}\,{\rm yr^{-1}}$, $f \sim 0.4$ up to $r \sim 46$ au, and grows to reach unity at larger radii.

2.3 Magnetic field seeds in the disc

Various models have been proposed for the creation of cosmological magnetic fields in astrophysical plasmas in the early ages of the post-recombination Universe. Most of them rely either on thermal pressure effects (the so-called Biermann battery, Biermann 1950; for applications in cosmology, see e.g. Kulsrud et al. 1997; Hanayama et al. 2005; Krolik & Zweibel 2006) or on radiation drag (e.g. Harrison 1970; Langer, Puget & Aghanim 2003; Langer, Aghanim & Puget 2005) operating on cosmological distances. Those usually lead to relatively weak fields on large scales, and one has then to rely on amplification by contraction or dynamo and make additional assumptions (e.g. flux freezing, geometry of collapse, etc.) in order to get interesting values of fields in circumstellar discs. A particularly effective turbulent amplification mechanism that operates in a dilute plasma such as the intracluster medium or galactic halo appeals to a self-accelerating fluctuation dynamo where anisotropic pressure gradients generate plasma instabilities that result in a high Reynolds number (Schekochihin & Cowley 2006). However, the same physical processes are also at work in the disc itself, and comparatively high fields may be directly created *in situ*. We therefore do not need to have recourse to preliminary magnetic seeds generation on larger scales.

For a two component system, and with $m_e \ll \mu m_p$, combining the ion and electron equations of motion leads to the generalized Ohm's law

$$\boldsymbol{E} = \frac{\boldsymbol{B} \times \boldsymbol{v}_e}{c} + \frac{\boldsymbol{\nabla}(n_e kT)}{en_e} + \frac{\boldsymbol{F}_{\rm rad}}{e}, \qquad (15)$$

the curl of which yields the induction equation

$$\partial_t \boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{v}_e \times \boldsymbol{B} + c\boldsymbol{\nabla} \times \frac{\boldsymbol{\nabla}(n_e kT)}{en_e} + \frac{\boldsymbol{\nabla} \times \boldsymbol{F}_{\rm rad}}{e}. \qquad (16)$$

The first term on the right-hand side above corresponds to the dynamo amplification, and since we are interested here in the creation of magnetic fields starting with $B = 0$, we will ignore this term in the following. The second and last are the battery and radiation drag terms, respectively. Both can serve as sources to create a seed of magnetic fields in the disc.

As is well known, the Biermann battery term is non-zero only when the temperature and electron density gradients are not parallel, according to

$$\partial_t \boldsymbol{B} = -\frac{ck}{e} \frac{\boldsymbol{\nabla} n_e \times \boldsymbol{\nabla} T}{n_e}. \qquad (17)$$

The characteristic time-scale is the rotation time Ω, and the electron density and temperature vary over scales at least of order H and r. Therefore, we expect the field created by the battery mechanism to be of order

$$B_{\rm batt} \sim \frac{ck}{e} f^{-1} \frac{T}{\sqrt{GMr}}$$

$$\simeq 3.93 \times 10^{-12} f \left(\frac{M_\star}{10\,{\rm M_\odot}} \right)^{1/2} \left(\frac{r}{10^3\,{\rm R_\odot}} \right)^{-3/2} \quad {\rm G.} \qquad (18)$$

Turbulence might actually create density gradients on smaller scales, leading to higher field amplitudes, but by considering longer length scales, we adopt the more conservative approach and keep the lower estimate for B.

The value we obtained from the Biermann effect is just a factor of f smaller than that provided by radiation drag. Indeed, as has been shown by Balbus (1993; see also Chuzhoy 2004) through momentum conservation arguments, the typical amplitude we may expect is

$$B_{\rm rad} \sim \frac{\mu m_p c\Omega}{e}$$

$$\simeq 3.93 \times 10^{-12} \left(\frac{M_\star}{10\,{\rm M_\odot}} \right)^{1/2} \left(\frac{r}{10^3\,{\rm R_\odot}} \right)^{-3/2} \quad {\rm G} \qquad (19)$$

It so happens that this amplitude, $B_{\rm rad}$, corresponds to values of the ions Larmor radius of order the local disc radius. This means that the magnetic field is only weakly coupled to the ions. Therefore, we expect the advection of field lines by small-scale turbulence to be quite inefficient.

2.4 MRI dynamo instability

We now wish to derive a constraint on the mass of the central object by requiring the magnetic field in the disc to exceed the minimal value for the MRI to operate and drive a dynamo.

The first basic condition we consider is that the wavelength of unstable modes must exceed the particle mean free path. This condition gives the minimum field for MRI. Now in MRI (Balbus & Hawley 1998), one expects that the dominant growth mode satisfies $\lambda_{\rm mi} \approx v_A/\Omega$, where v_A is the Alfvén frequency and Ω is the disc rotation rate. Larger scales become unstable as the dynamo operates, and the maximum scale is given by the disc scale-height. This gives the maximum field strength where the dynamo saturates. For a sustained field, that does not reverse sign every rotation period, one requires a stratified disc, which allows helical turbulence to be generated. Helical turbulence results in mean field amplification via

the MRI dynamo, and it is possible that the resulting fields can attain the equipartition value (Brandenburg 2004). Now the mean free path in an ionized gas is $9T^2k^2/(\pi n_e e^4 \ln \Lambda)$ (e.g. Lang 1999), and the minimum field condition is thus

$$\frac{v_A}{\Omega} > \frac{9}{\ln \Lambda} \frac{T^2 k^2}{\pi n_e e^4}, \tag{20}$$

or

$$B > B_{mfp} = 1.75 \times 10^{-9} \alpha^{1/2} f^{11/2} \frac{20}{\ln \Lambda} \left(\frac{M_\star}{10\,M_\odot} \right)^{11/4}$$
$$\times \left(\frac{\dot{M}}{10^{-4}\,M_\odot\,yr^{-1}} \right)^{-1/2} \left(\frac{r}{10^3\,R_\odot} \right)^{-11/4} G, \tag{21}$$

where we assumed $\Omega = \Omega_{Kep}$. Note that MRI may occur even in collisionless plasmas, as shown by Quataert, Dorland & Hammett (2002). Their study, however, does not account for possible effects due to a finite Larmor radius, which we presumably cannot ignore in our case where initially weak fields imply very large Larmor radii.

The second condition we may consider is that MRI will be efficient in amplifying magnetic fields only if the amplification time-scale is shorter than the diffusion time-scale. For maximum instability scales, the growth rate is of the order of Ω, and the condition can be rewritten as

$$\frac{\eta}{\lambda_{mi}^2 \Omega} = \frac{\eta \Omega}{v_A^2} < 1, \tag{22}$$

where $\eta = 10^{13} \, (\ln \Lambda/20) \, T^{-3/2} \, cm^2 \, s^{-1}$ is the magnetic diffusivity. Assuming Keplerian rotation, this condition translates to

$$B^2 > B_{mi}^2 = 4\pi \eta \rho \sqrt{\frac{GM_\star}{r^3}}. \tag{23}$$

Using equations (13) and (14), we obtain

$$B > B_{mi} = 4.93 \times 10^{-8} \alpha^{-1/2} f^{-3} \left(\frac{\ln \Lambda}{20} \right)^{1/2} \left(\frac{M_\star}{10\,M_\odot} \right)^{-3/4}$$
$$\times \left(\frac{\dot{M}}{10^{-4}\,M_\odot\,yr^{-1}} \right)^{1/2} \left(\frac{r}{10^3\,R_\odot} \right)^{-3/4} G. \tag{24}$$

As noted by Tan & Blackman (2004) who obtained a similar result, this value of the amplitude is quite high for initial conditions in primordial star formation which starts in a protostellar environment not necessarily magnetized beforehand. However, it does not imply that MRI could not be at work in a disc surrounding the progenitor of a Pop III star. Indeed, as already stressed, the minimum value obtained above has been derived by taking into account effects of magnetic diffusivity on scales of maximal instability, which are small with respect to the disc height. This only implies that due to diffusivity, MRI for small-wavelength disturbances is *de facto* inefficient, unless the amplitude of the field is already large. Moreover, for standard values of the parameters, B_{mi} is way above B_{rad}. This means that l_{mi}, the scale of maximal instability, is much smaller than the ion Larmor radius, and therefore, as noted in the previous paragraph, MRI turbulence is unable to amplify the fields on such small scales.

Nevertheless, larger scales of order the disc (or Larmor) radius, may still remain unstable, with the growth rate in that case being proportional to the field strength itself (e.g. Balbus 1995), even if magnetic fields are dissipated on small scales. This brings us to the third and strongest condition that must be satisfied. In a differentially rotating, turbulent disc, the growth rate for MRI in the weak field limit is $\gamma \sim v_A k$, where k is the vertical wavenumber, where $v_A \ll \Omega/k$ and Ω is the local angular velocity. The growth is stabilized by

magnetic diffusivity, η, so that the dispersion relation is $\gamma + \eta k^2 \approx v_A k$ (e.g. Kitchatinov & Rüdiger 2004). Therefore, for marginal stability,

$$v_A \simeq \eta \frac{\pi}{\lambda} \tag{25}$$

with $\lambda < H = fr$. This criterion translates readily into a minimum magnetic field

$$B_{mri} = 2\pi^{3/2} \eta \rho^{1/2} f^{-1} r^{-1} \tag{26}$$

which yields

$$B_{mri} = 3.87 \times 10^{-15} \alpha^{-1/2} f^{-11/2} \frac{\ln \Lambda}{20} \left(\frac{\dot{M}}{10^{-4}\,M_\odot\,yr^{-1}} \right)^{1/2}$$
$$\times \left(\frac{M_\star}{10\,M_\odot} \right)^{-7/4} \left(\frac{r}{10^3\,R_\odot} \right)^{-1/4} G \tag{27}$$

using again equations (13) and (14) above.

2.5 Critical mass of the central object

We require now that the minimum magnetic field for the MRI to work is smaller than the field created in the disc. As we have seen, the smallest suitable field amplitude, B_{mri}, is given by the marginal instability criterion applied to scales of order the disc scale. At the same time, *in situ* magnetic field generation is likely to yield amplitudes of the order of B_{rad}. This amplitude must be higher than B_{mri}, which happens as soon as

$$M_\star > 0.46 \alpha^{-2/9} f^{-22/9} \left(\frac{\ln \Lambda}{20} \right)^{4/9} \left(\frac{\dot{M}}{10^{-4}\,M_\odot\,yr^{-1}} \right)^{2/9}$$
$$\times \left(\frac{r}{10^3\,R_\odot} \right)^{5/9} M_\odot. \tag{28}$$

The thickness of the disc has the most dramatic effect on the value of the critical mass, whereas the actual value of the viscous α parameter plays a minor role. If the disc is rather thin, with $f \sim 0.1$ for instance, then the mass spans a range of rather large values, going from 128 to 357 $M_\odot$. However, in the absence of metals, the properties of the primordial gas are likely to prevent the disc from efficiently cooling, and we expect f to be closer to unity. In that case, taking $f \sim 0.4$ as obtained by Mayer & Duschl (2005), the critical mass is much smaller, comprised between roughly 4.3 and 27.8 $M_\odot$ for α ranging from 1 to 0.01.

3 DISCUSSION

Magnetic fields have probably played a role in primordial star formation, even if starting from a medium free of magnetic fields. As we have argued, radiation drag or thermal pressure effects are able to generate magnetic seeds in the disc surrounding the central accreting stellar progenitor. Initially, those seeds are too weak to be amplified by small-scale turbulence, or even MRI effects on scales comparable to the disc scale. However, as matter accretion proceeds, the mass of the central object grows, and the gravitational potential it creates deepens, increasing the rotation velocity of the disc. Eventually, the rotation is fast enough so that long wavelength MRI modes become unstable, as the minimum magnetic field for MRI becomes smaller than the field generated in the disc. Depending on the properties of the disc, this happens once the mass of the central object reaches 4–28 $M_\odot$.

Subsequently, MRI dynamo will be at work and will amplify the magnetic field. The field can then rapidly reach dynamically

important values, and magnetically driven ejection contributes to lower the effective accretion efficiency. The actual model of magnetic winds is beyond the scope of this paper, but this suggests that feedback is likely to require magnetically driven outflows, which could occur during Pop III formation already when, as we argued above, the mass of the stellar progenitor is rather small. Note that, in their model of protostellar disc in primordial star formation, Tan & Blackman (2004) estimated the power of magnetically driven outflows. In their study, the outflow feedback effects reduce the star formation efficiency once the protostar reaches roughly $100\,M_\odot$. Incidentally, concurrent conclusions were reached by Machida et al. (2006) who simulated the collapse of a magnetized primordial cloud in rigid rotation, and the subsequent formation of a Pop III star, within the ideal magnetohydrodynamics approximation. Depending on the (high) initial value ($B_{\mathrm{init}} \gtrsim 10^{-9}\,\mathrm{G}$ for an initial cloud density $n_c \sim 10^3\,\mathrm{cm}^{-3}$) of the magnetic field, their simulations show that magnetically driven jets develop and effectively reduce the accretion rate. Further, numerical studies with higher resolution seem necessary to address both the mass ejection rate and the amplification of magnetic fields in protostellar discs for initially weaker, even vanishing, magnetic seeds.

We have argued that the interplay between the two modes of star formation, primordial, massive and conventional, involving all masses, is controlled by B and not by Z. Effective feedback requires that of order 10 per cent of the gas accretion rate is channelled into star formation, with an outflow rate that on the average must be of the order of the net star formation rate. Globally, for the star forming cloud, one expects that $\dot{M}_{\mathrm{outflow}} \sim \dot{M}_* \sim 0.1\dot{M}_{\mathrm{accretion}}$, much as is found in nearby cases of star-forming clouds.

Moreover, feedback is likely to be responsible for the turbulent support in clouds that lowers the star formation efficiency and helps to generate the conventional IMF. Thus, the onset of cloud fragmentation, due eventually but we have argued, not exclusively, to the enhanced role of cooling, would allow field amplification, angular momentum transfer, feedback and low-mass star formation.

Alternatively, suppose we accept the hypothesis that the first stars were massive objects. To avoid the empirical objections discussed above, one would have to argue that merging and coagulation of gas clumps resulted in the formation of pre-dominantly very massive objects, of characteristic mass $\gtrsim 1000\,M_\odot$, whose fate is to form intermediate mass black holes (IMBHs) with relatively low nucleosynthetic yields associated with their collapse. In this case, the seed fields may come from jets and outflows driven by spin-up of turbulent accretion disc dynamos as a consequence of accretion on to these IMBHs. One can argue that the first generation of primordial clouds, which cooled pre-dominantly via Lyman α emission, preferentially formed IMBHs, since the associated minihaloes of mass $\sim 10^7\,M_\odot$ are promising environments for forming IMBHs in view of the high core accretion rates (Zhao & Silk 2005). Accretion discs around the IMBHs provide promising sites for MRI dynamos.

Alternatively, the jets may generate magnetic fields via the Weibel instability, that although small, amounting to $\sim m_e/m_p$ of the equipartition value, could still be useful as seeds (Wiersma & Achterberg 2004). The IMBH outflows allow one to generate larger seed fields required for conventional star formation than inferred from the Biermann battery. The associated nucleosynthetic implications are modest, because IMBHs have essentially zero yield. For any reasonable IMF, there are of course associated Pop III stars.

Consider, for example, the inference from primordial star simulations that one star of mass $\sim 10^2\,M_\odot$ forms per pristine pre-galactic cloud of baryonic mass $\sim 10^7\,M_\odot$, a typical galactic precursor not dissimilar in mass to dwarf satellite galaxies. The resultant local

enrichment would amount to the order of 10^{-5} the solar value, and would be diluted by a further factor that corresponded to the fractional number of primordial star-forming clouds that formed the eventual galaxy. However, the associated population of IMBHs could amount to as much as ~ 10 per cent of the current epoch stellar mass, if accretion on to these IMBHs also provides a possible explanation of the near-infrared background excess (Madau & Silk 2005). If such IMBH-induced jets carry flux, produced by a circum-black hole accretion disc dynamo, or drive jets that generate magnetic fields vis the proton Weibel instability in the shock interaction zone, then one might easily imagine seeding the clouds with a magnetic field that more than sufficed to allow MRI to operate.

In summary, we have outlined two alternative pathways which dispense with the need for Pop III to consist exclusively of very massive stars and for the transformation from first stars into Pop II to be determined exclusively by the rise in gas and/or dust phase metallicity. MRI-dynamo generated magnetic flux in the protostellar accretion disc is the key to these possible scenarios. The nature of the seed field is the major difference. One option is that protogalactic Biermann battery-seeded MRI dynamos provide circumstellar discs with the necessary field strength to overcome high accretion rates characteristic of primordial environments. This allows, and indeed the mechanism itself requires, stars of mass below 10–$100\,M_\odot$ to form. In this case, there is no Pop III. Another possibility is that the first objects (Pop III is a misnomer since no nucleosynthetic tracers remain) were very massive ($\gtrsim 1000\,M_\odot$), and form IMBHs which expel magnetic-flux-loaded jets that have been generated by dynamo activity in the circum-IMBH disc. The MRI prostellar dynamos are seeded by jet-induced instabilities, and again, the first stars are expected to span a broad mass range. The IMF is a consequence of magnetic feedback. Of course, it may vary with epoch and/or environment, but the principal point we wish to emphasize is that it spans the conventional range of stellar masses. One could then make the transition from primordial star formation to current epoch star formation while the mean overall metallicity remained extremely low.

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