# Type I migration in optically thick accretion dises 

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#### Abstract

We study the torque acting on a planet embedded in an optically thick accretion disc, using global two-dimensional hydrodynamic simulations. The temperature of an optically thick accretion disc is determined by the energy balance between the viscous heating and the radiative cooling. The radiative cooling rate depends on the opacity of the disc. The opacity is expressed as a function of the temperature. We find that the disc is divided into three regions that have different temperature distributions. The slope of the entropy distribution becomes steep in the inner region of the disc with high temperature and the outer region of the disc with low temperature, while it becomes shallow in the middle region with intermediate temperature. Planets in the inner and outer regions move outwards owing to the large positive corotation torque exerted on the planet by an adiabatic disc, and on the other hand, a planet in the middle region moves inwards towards the central star. Planets are expected to accumulate at the boundary between the inner and middle regions of the adiabatic disc. The positive corotation torque decreases with an increase in the viscosity of the disc. We find that the positive corotation torque acting on the planet in the inner region becomes too small to cancel the negative Lindblad torque when we include the large viscosity, which destroys the enhancement of the density in the horseshoe orbit of the planet. This leads to the inward migration of the planet in the inner region of the disc. A planet with 5 Earth masses in the inner region can move outwards in a disc with surface density of $100 \mathrm{~g} \mathrm{~cm}^{-2}$ at 1 au when the accretion rate of a disc is smaller than $2 \times 10^{-8} \mathrm{M}_{\odot} \mathrm{yr}^{-1}$.


Key words: hydrodynamics - methods: numerical - planets and satellites: formation - planetdisc interactions - protoplanetary discs.

## 1 INTRODUCTION

Planets are thought to be formed in a protoplanetary disc around a young star. A planet growing by the accretion of planetesimals exchanges angular momentum with disc gas and moves in the disc (Goldreich \& Tremaine 1979, 1980). This is called Type I migration of a planet. Type I migration is caused by the torques acting on a planet by a disc. The torque is composed of the Lindblad torque and the corotation torque. The Lindblad torque is due to the two spiral density waves excited by the planet in the disc (Ward 1986, 1997), while the corotation torque is exerted by the gas in the horseshoe region of the planet (Ward 1991; Baruteau \& Masset 2008; Paardekooper \& Mellema 2008; Paardekooper \& Papaloizou 2008). The sum of the Lindblad torque and the corotation torque determines the total torque acting on the planet.

[^0]The Lindblad torque is exerted by the disc gas at the Lindblad resonances, which are located inside and outside the orbit of the planet. The angular momentum of the planet is increased and decreased by the inner and outer Lindblad torques, respectively. The magnitude of the negative outer Lindblad torque is a little larger than that of the positive inner Lindblad torque in a disc (Ward 1986, 1997). The Lindblad torque becomes negative, leading to the inward migration of a planet.

The corotation torque exerted by the disc gas in the horseshoe orbit of a planet depends on the vortensity distribution and the entropy distribution of the disc gas. The vortensity-related corotation torque is small compared with the Lindblad torque (Masset \& Casoli 2010; Paardekooper, Baruteau \& Kley 2011), on the other hand, the entropy-related corotation torque can become larger than the magnitude of the negative Lindblad torque. Paardekooper \& Mellema (2006) showed that the corotation torque becomes positive in a disc with high opacity, reducing the migration velocity of a planet. This process was recognized and intensively studied by many researchers (Baruteau \& Masset 2008; Paardekooper \& Mellema 2008; Paardekooper \& Papaloizou 2008; Masset \& Casoli 2009; Ayliffe \& Bate 2010, 2011; Paardekooper et al. 2010, 2011;

Yamada \& Inaba 2011). The positive corotation torque cancels the negative Lindblad torque when the entropy distribution has a steep negative slope. It was also shown that a planet in an adiabatic disc with steeper negative slope starts to migrate even outwards.

The corotation torque significantly decreases in an adiabatic disc after a few libration times of a planet because the entropy distribution becomes uniform within the horseshoe region (Masset \& Casoli 2010; Paardekooper et al. 2011). Without the sustained corotation torque, only the Lindblad torque is eventually exerted on a planet in an adiabatic disc, leading to the inward migration of a planet with Earth mass in $10^{5} \mathrm{yr}$ (Ward 1997; Tanaka, Takeuchi \& Ward 2002). This is a serious problem in the core accretion model of planet formation. The lifetime of a disc, $10^{6-7} \mathrm{yr}$, is much longer than the time-scale of Type I migration, making the survival of planets in a disc difficult. However, recent radial velocity surveys of extrasolar planets show that a significant fraction of solar-type stars may harbour close-in super-Earths (Mayor et al. 2009; Lo Curto et al. 2010; Ségransan et al. 2011). Population synthesis models have great difficulties in reproducing the observed semimajor axis distribution of extrasolar planets once Type I migration is included. The reduction of the migration velocity of a planet is required (Alibert et al. 2005; Mordasini et al. 2009).

The viscosity of a disc prevents the saturation of the corotation torque (Masset \& Casoli 2010). Kley \& Crida (2008) investigated the planet-disc interactions in an optically thick accretion disc, taking into account viscous heating and radiative cooling. They found that the disc gas reaches a steady state, of which the temperature distribution has a very steep slope in a region where ice exists in a condensed form. The corotation torque always becomes positive in this region and the direction of the planet migration becomes outward. Hasegawa \& Pudritz (2011) comprehensively examined the various mechanisms to halt planet migration in a disc around a young star. They showed that the planet migration is halted at the ice line, inside of which all the ice is evaporated and solid particles are composed of rocks and metals. The slope of the temperature distribution becomes steep in the inner region. The positive corotation torque becomes large enough to make the total torque positive. On the other hand, the temperature distribution beyond the ice line is too shallow to cancel the negative Lindblad torque by the corotation torque. A planet inside and outside of the ice line is expected to move outwards and inwards, respectively. They showed that the ice line is the location to stop the planet migration in an optically thick accretion disc.

Muto \& Inutsuka (2009) showed that the torque is dependent on the magnitude of the viscosity. The viscosity modifies the density of gas near a planet and decreases the torque density. Paardekooper \& Papaloizou (2009a) also showed that the corotation torque becomes very small when they consider a disc with high viscosity. In this study, we systematically examine the total torque acting on a planet by an optically thick accretion disc. It is not clear if the positive corotation torque can always exceed the negative Lindblad torque when we include viscosity in a disc. We make global two-dimensional hydrodynamic simulations to study the effect of dissipation processes such as viscosity and radiation on the total torque.

This paper is organized as follows. In Section 2, we describe the basic equations and a disc model that we presume in this study. The dissipative terms due to viscosity and radiation are included in the basic equations. The temperature of the disc is determined by the energy balance between viscous heating and radiative cooling. The rate of radiative cooling is dependent on the opacity of the disc, which changes around the ice line. In Section 3, we show the results
of the two-dimensional hydrodynamic simulations. We show that the corotation torque decreases with an increase in viscosity. The total torque acting on the planet depends on the radiative cooling rate and the viscous heating rate. We further derive the analytical formula to relate viscosity and opacity when the total torque exerted on the planet becomes zero. We summarize the results in Section 4.

## 2 BASIC EQUATIONS AND DISC MODEL

### 2.1 Basic equations

A planet excites density waves in a disc and changes the density distribution of the disc. We examine the torque exerted on a planet by an optically thick accretion disc. A planet with 5 Earth masses rotates around a solar mass star in a fixed circular orbit. The position vector of the planet from the star is denoted by $\boldsymbol{r}_{\mathrm{p}}$. The problem is limited to a two-dimensional flow, where all physical quantities (e.g. the surface density) depend on $r$ and $\theta$, where $r$ is the distance from the star and $\theta$ is the angle between the $x$-axis and the position vector. Governing equations for the gas are the mass conservation, the Navier-Stokes equations, and the energy equation with dissipative terms due to the viscosity and radiation.

We use a cylindrical coordinate where the star is located at the centre of the coordinate. The mass of the planet is much smaller than that of the star and we neglect the indirect term. The mass conservation equation and the Navier-Stokes equations read

$$
\begin{align*}
& \frac{\partial \Sigma}{\partial t}+\frac{1}{r} \frac{\partial}{\partial r}\left(r \Sigma v_{r}\right)+\frac{1}{r} \frac{\partial}{\partial \theta}\left(\Sigma v_{\theta}\right)=0,  \tag{1}\\
& \frac{\partial}{\partial t}\left(\Sigma v_{r}\right)+\frac{1}{r} \frac{\partial}{\partial r}\left\{r\left(\Sigma v_{r}^{2}+p\right)\right\}+\frac{1}{r} \frac{\partial}{\partial \theta}\left(\Sigma v_{r} v_{\theta}\right) \\
& =\frac{\Sigma v_{\theta}^{2}}{r}+\frac{p}{r}-\Sigma \frac{\partial \Phi}{\partial r}+f_{r},  \tag{2}\\
& \frac{\partial}{\partial t}\left(\Sigma v_{\theta}\right)+\frac{1}{r} \frac{\partial}{\partial r}\left(r \Sigma v_{r} v_{\theta}\right)+\frac{1}{r} \frac{\partial}{\partial \theta}\left(\Sigma v_{\theta}^{2}+p\right) \\
& =-\frac{\Sigma v_{r} v_{\theta}}{r}-\frac{\Sigma}{r} \frac{\partial \Phi}{\partial \theta}+f_{\theta}, \tag{3}
\end{align*}
$$

where $\Sigma$ is the gas surface density, $p$ is the vertically integrated pressure, $v_{r}$ and $v_{\theta}$ are the radial and tangential velocities of the gas, and $\Phi$ is the gravitational potential. We consider a less massive disc and neglect the self-gravity of the gas. The gravitational potential of the star and the planet is given by
$\Phi=-\frac{G M_{\odot}}{r}-\frac{G M_{\mathrm{p}}}{\sqrt{r^{2}+r_{\mathrm{p}}^{2}-2 r r_{\mathrm{p}} \cos \psi+\epsilon^{2} H_{\mathrm{p}}^{2}}}$,
where $M_{\odot}$ is the mass of the sun, $M_{\mathrm{p}}$ is the mass of the planet, $\psi$ is the angle between $\boldsymbol{r}$ and $\boldsymbol{r}_{\mathrm{p}}$, and $H_{\mathrm{p}}$ is the scale height of the disc at the location of the planet. The scale height is given by
$H_{\mathrm{p}}=\frac{\sqrt{2} c_{\mathrm{p}}}{\Omega_{\mathrm{p}}}$,
where $c_{\mathrm{p}}$ and $\Omega_{\mathrm{p}}$ are, respectively, the isothermal sound velocity and the Keplerian angular velocity at $r_{\mathrm{p}}$. The smoothing length parameter, $\epsilon$, is introduced to include the effect of the scale height of a disc. It is noted that a small softening parameter leads to a strong corotation torque (Baruteau \& Masset 2008; Paardekooper et al. 2010). Yamada \& Inaba (2011) examined the torque acting on a planet with several Earth masses in an optically thin disc and found that the torques calculated by simulations with $\epsilon=0.3$ agree
with that obtained by a linear analysis. We adopt $\epsilon=0.3$ in this study. The last terms in the Navier-Stokes equations describe the radial and azimuthal components of the viscous forces:
$f_{r}=\frac{1}{r} \frac{\partial}{\partial r}\left(r \sigma_{r r}\right)+\frac{1}{r} \frac{\partial \sigma_{r \theta}}{\partial \theta}-\frac{\sigma_{\theta \theta}}{r}$
and
$f_{\theta}=\frac{1}{r^{2}} \frac{\partial}{\partial r}\left(r^{2} \sigma_{r \theta}\right)+\frac{1}{r} \frac{\partial \sigma_{\theta \theta}}{\partial \theta}$,
where $\sigma_{r r}, \sigma_{r \theta}$ and $\sigma_{\theta \theta}$ are the components of the stress tensor and are, respectively, given by
$\sigma_{r r}=2 \Sigma v \frac{\partial v_{r}}{\partial r}$,
$\sigma_{r \theta}=\Sigma \nu\left\{r \frac{\partial}{\partial r}\left(\frac{v_{\theta}}{r}\right)+\frac{1}{r} \frac{\partial v_{r}}{\partial \theta}\right\}$
and
$\sigma_{\theta \theta}=\frac{2 \Sigma v}{r}\left(\frac{\partial v_{\theta}}{\partial \theta}+v_{r}\right)$.
The gas is assumed to be ideal and the equation of state is given by
$p=\frac{\Sigma k_{\mathrm{B}} T}{\mu m_{\mathrm{H}}}$,
where $k_{\mathrm{B}}$ is the Boltzmann constant, $T$ is the mid-plane temperature, $m_{\mathrm{H}}$ is the mass of a hydrogen atom and $\mu$ is the mean molecular weight of the gas. We set up $\mu=2.34$.
The energy equation of the gas disc reads

$$
\begin{array}{r}
\frac{\partial(\Sigma e)}{\partial t}+\frac{1}{r} \frac{\partial}{\partial r}\left\{r v_{r}(\Sigma e+p)\right\}+\frac{1}{r} \frac{\partial}{\partial \theta}\left\{v_{\theta}(\Sigma e+p)\right\} \\
=v_{r} \frac{\partial p}{\partial r}+\frac{v_{\theta}}{r} \frac{\partial p}{\partial \theta}+W-Q \tag{12}
\end{array}
$$

where $e$ is the specific energy of the gas given with the pressure by
$e=\frac{p}{(\gamma-1) \Sigma}$.
In equation (13), $\gamma$ is the ratio of the specific heats at constant pressure and volume. We use $\gamma=4 / 3$ for a two-dimensional disc as given by Li et al. (2000). The energy of the gas is increased by the viscous heating term. The viscous heating term, $W$, is given by

$$
\begin{align*}
W= & \sigma_{r r} \frac{\partial v_{r}}{\partial r}+\sigma_{r \theta}\left\{r \frac{\partial}{\partial r}\left(\frac{v_{\theta}}{r}\right)+\frac{1}{r} \frac{\partial v_{r}}{\partial \theta}\right\} \\
& +\sigma_{\theta \theta}\left(\frac{1}{r} \frac{\partial v_{\theta}}{\partial \theta}+\frac{v_{r}}{r}\right) \tag{14}
\end{align*}
$$

On the other hand, the energy of the gas is decreased by the radiative cooling term of the disc, $Q$. The radiative cooling term, $Q$, is given by
$Q=2 \sigma_{\mathrm{SB}} T_{\mathrm{eff}}^{4}$,
where $\sigma_{\mathrm{SB}}$ is the Stefan-Boltzmann constant and $T_{\text {eff }}$ is the effective temperature of the disc. The radiative flux in the radial direction is neglected because it is much smaller than $Q$ by a factor of $H_{\mathrm{p}} / r_{\mathrm{p}}$. The effective temperature is related to the mid-plane temperature of the disc as (Hubeny 1990)
$T=\tau_{\text {eff }}^{1 / 4} T_{\text {eff }}$,
with
$\tau_{\mathrm{eff}}=\frac{3 \tau}{8}+\frac{\sqrt{3}}{4}+\frac{1}{4 \tau}$,
where $\tau$ is the optical depth of the disc. The optical depth of the disc is defined by
$\tau=\frac{1}{2} \xi_{\mathrm{gr}} \kappa_{0} \Sigma$,
where $\kappa_{0}$ is the best-fitting function of the opacity (Lin \& Papaloizou 1985) and $\xi_{\text {gr }}$ is the so-called grain content factor introduced by Mizuno (1980). Micron-size dust particles are the main source of the opacity. Collisions between particles might produce a large number of small dust particles, increasing the opacity of a disc (Birnstiel, Dullemond \& Brauer 2010). The opacity might decrease due to capture of small dust particles by large particles. It is valuable to study the torque acting on a planet in a disc with various values of $\xi_{\mathrm{gr}}$. We treat $\xi_{\mathrm{gr}}$ as a parameter of the simulations and consider discs with $0.1 \leq \xi_{\text {gr }} \leq 100$.
The opacity strongly depends on the temperature of the disc (Lin \& Papaloizou 1985):
$\kappa_{0}=\left\{\begin{array}{lc}5 \times 10^{-3} T & (210 \leq T<2000 \mathrm{~K}), \\ 2 \times 10^{16} T^{-7} & (170 \leq T<210 \mathrm{~K}), \\ 2 \times 10^{-4} T^{2} & (T<170 \mathrm{~K}) .\end{array}\right.$
Equation (19) was derived from the available opacity data, which incorporates small particles found in the interstellar medium. Dust particles are composed of rocks and metals in a region of the disc with $T \geq 210 \mathrm{~K}$, while ice is added to form dust particles in a region of the disc with $T<170 \mathrm{~K}$. In the transition region between the cold and the hot region, the ice is evaporating. We divide the disc into three regions: region 1 with $T \geq 210 \mathrm{~K}$, region 2 with $170 \leq$ $T<210 \mathrm{~K}$ and region 3 with $T<170 \mathrm{~K}$.
For later convenience, we write all the quantities in a nondimensional form using the unit length $r_{0}=1 \mathrm{au}$, the unit mass $M_{\odot}$ and the unit time $\Omega_{0}^{-1}$ where $\Omega_{0}$ is the Kepler frequency at 1 au . The normalized quantities are denoted by a tilde (e.g. $\tilde{r}_{\mathrm{p}}$ ).

### 2.2 Disc model

Kitamura et al. (2002) obtained images of protoplanetary discs around T Tauri stars in Taurus using the thermal emission of dust. They found that the surface density distributions of protoplanetary discs are described by a power-law distribution with a power-law index of $0-1$. The surface densities of the discs at 100 au are in a range between 0.1 and $10 \mathrm{~g} \mathrm{~cm}^{-2}$. In this study, we adopt a disc model, of which the initial condition is given by the power-law distribution
$\Sigma_{\text {ini }}=\Sigma_{0} \tilde{r}^{-\alpha}$,
where $\Sigma_{0}$ is the surface density at $r_{0}$ and is set to be $100 \mathrm{~g} \mathrm{~cm}^{-2}$.
Kitamura et al. (2002) also found disc radii expand with time, suggesting that most of the gas migrates inwards towards a central star by transporting the angular momentum outwards. We adopt an accretion disc model developed by Pringle (1981), in which the accretion rate of the gas is constant throughout a disc. The viscosity $v$ is expressed by a power-law function of the distance from the central star to satisfy the constant accretion rate of a disc as
$v=\xi_{v} v_{0} \tilde{r}^{\alpha}$,
where $\nu_{0}$ is the kinematic viscosity at $r_{0}$ and we set $\nu_{0}=4.2 \times$ $10^{15} \mathrm{~cm}^{2} \mathrm{~s}^{-1}$ and $\xi_{\mathrm{v}}$ is a viscous strength factor. The value of $v_{0}$ is approximately equivalent to 0.05 in terms of the alpha-coefficient of Shakura \& Sunyaev (1973). The accretion rate of a disc with kinematic viscosity, $v_{0}$, and the surface density given by equation (20) becomes $6.3 \times 10^{-8} \mathrm{M}_{\odot} \mathrm{yr}^{-1}$. Observations of discs


Figure 1. (a) The surface density distribution and (b) the temperature distribution of the disc with $\alpha=1.0$. The temperature distribution of the disc is determined by the balance between the viscous heating and the radiative cooling. The temperature distribution is described by power-law distributions with three different gradients. The power-law index of the temperature distribution strongly depends on the opacity of the disc. The dependence of the opacity on the temperature changes due to the evaporation of ice. The power-law indices of the temperature distribution change at 1.4 and 2.5 au .
suggest that the accretion rates of discs are $\sim 1 \times 10^{-8} \mathrm{M}_{\odot} \mathrm{yr}^{-1}$ (Hartmann et al. 1998; Kitamura et al. 2002). We treat $\xi_{\mathrm{v}}$ as another parameter in addition to $\xi_{\mathrm{gr}}$ to consider discs with various accretion rates of $0.1 \leq \xi_{\mathrm{v}} \leq 1$. The two parameters, $\xi_{\mathrm{gr}}$ and $\xi_{\mathrm{v}}$, determine the structure of a disc.

We derive the temperature distribution of an optically thick accretion disc following Kley \& Crida (2008). The temperature distribution of the disc is determined by the balance between the viscous heating term, $W$, and the radiative cooling term, $Q$. Assuming that the viscous heating is due to the Keplerian motion, we have $W$ given by
$W=\frac{9}{4} \Sigma \nu \Omega^{2}$.
Applying equations (16)-(19) to $Q$ with the assumption of $\tau \gg 1$, we obtain the temperature distribution of the disc from $W=Q$ as
$T=\left\{\begin{array}{lc}210\left(\frac{r}{r_{12}}\right)^{-(\alpha+3) / 3} & (210 \leq T<2000 \mathrm{~K}), \\ 210\left(\frac{r}{r_{12}}\right)^{-(\alpha+3) / 11} & (170 \leq T<210 \mathrm{~K}), \\ 170\left(\frac{r}{r_{23}}\right)^{-(\alpha+3) / 2} & (T<170 \mathrm{~K}),\end{array}\right.$
where $r_{12}$ and $r_{23}$ are the distances from the star to the boundary of regions 1 and 2 and that of regions 2 and 3, respectively. For later convenience, the power-law indices of the temperature distribution in the $i$ region are denoted by $-\beta_{i}$ (e.g. $\left.\beta_{1}=(\alpha+3) / 3\right)$.

Fig. 1 shows the surface density distribution and the temperature distribution of the disc with $\alpha=1.0$. It is seen from this figure that the temperature distribution has three different slopes. The


Figure 2. The boundary position between regions 1 and $2, \tilde{r}_{12}$, as a function of $\xi_{\mathrm{gr}}$ and $\xi_{\mathrm{v}}$. The solid, dashed, dot-dashed and dotted lines correspond to $\tilde{r}_{12}=2.4,1.4,1.2$ and 0.8 , respectively. We use disc models with 20 parameter sets ( $\xi_{\mathrm{gr}}, \xi_{\mathrm{v}}$ ) marked by the filled circles.
boundary between regions 1 and 2 and that between regions 2 and 3 are located at 1.4 and 2.5 au , respectively. The temperature distribution is nearly flat in region $2\left(\beta_{2}=4 / 11\right)$. The power-law indices of the temperature distributions are given by $\beta_{1}=4 / 3$ in region 1 and $\beta_{3}=2.0$ in region 3 .

The larger viscosity and/or opacity increase the temperature of the disc. The boundary positions move outwards with increases in the viscosity and/or opacity. The location of the boundary is expressed by the two parameters, $\xi_{\mathrm{gr}}$ and $\xi_{\mathrm{v}}$, and the boundary position between regions 1 and 2 is given by
$\tilde{r}_{12}=1.4\left(\frac{\xi_{\mathrm{v}} \xi_{\mathrm{gr}}}{1.0}\right)^{1 / 4}$.
The boundary position between regions 2 and 3 is given by $\tilde{r}_{23}=$ $1.8 \tilde{r}_{12}$ in the case of $\alpha=1.0$. These boundary positions were derived by Hasegawa \& Pudritz (2011) as well. It is noted that the boundary position also depends on the surface density. We plot the boundary positions of regions 1 and $2, \tilde{r}_{12}$, as a function of $\xi_{\mathrm{gr}}$ and $\xi_{\mathrm{v}}$ in Fig. 2. The solid, dashed, dot-dashed and dotted curves correspond to $\tilde{r}_{12}=2.4,1.4,1.2$ and 0.8 , respectively. It is found that the boundary position between regions 1 and 2 ranges from 0.5 to 2.5 au . The filled circles represent the parameter sets $\left(\xi_{\mathrm{gr}}, \xi_{\mathrm{v}}\right)$ used in our numerical simulations.

## 3 NUMERICAL METHOD AND SIMULATION RESULTS

A planet generates density waves in a disc inside and outside of the orbit of the planet. The inner and outer density waves exert positive and negative torques on the planet, respectively. The gravity of an outer density wave is a little stronger than that of an inner density wave because an outer density wave is closer to the planet due to the pressure gradient of a disc, leading to the negative Lindblad torque (Ward 1997). Another torque acts on the planet by a gas element in the horseshoe orbit of the planet (Baruteau \& Masset 2008; Paardekooper \& Papaloizou 2008). When a gas element in the horseshoe orbit approaches a planet, angular momentum is exchanged between the gas element and the planet. This is called the corotation torque.

The entropy of the gas is expressed as $S=p / \Sigma^{\gamma} \propto r^{\lambda}$, where $\lambda=$ ( $\gamma-1) \alpha-\beta_{i}$. The corotation torque is dependent on the entropy distribution of a disc. It was shown that the corotation torque consists
of linear and non-linear corotation torque. The non-linear corotation torque comes from the outgoing boundaries of the horseshoe region, that is, the separatrix (Masset \& Casoli 2010; Paardekooper et al. 2010). In a non-barotropic disc, the vortensity is changed after the horseshoe U-turn. The enhanced corotation torque is not due to the adiabatic compression, but comes from a singular streamline at the separatrix. Since the change of vortensity is proportional to the radial entropy gradient of a disc, a large corotation torque is induced in a disc with a large magnitude of $\lambda$. Yamada \& Inaba (2011) showed that the positive corotation torque is comparable with the negative Lindblad torque when $\lambda=-0.4$ in an adiabatic disc. The total torque acting on a planet is determined by the sum of the negative Lindblad torque and the corotation torque.
The disc we consider in Section 2.2 has a surface density distribution with $\alpha=1.0$ and a temperature distribution with $\beta_{1}=$ $4 / 3$ in region $1, \beta_{2}=4 / 11$ in region 2 and $\beta_{3}=2.0$ in region 3. The power-law indices of the entropy distribution are calculated as $\lambda=-1.0$ in region $1, \lambda=-3.0 \times 10^{-2}$ in region 2 and $\lambda=-1.7$ in region 3. Yamada \& Inaba (2011) found that the total torque acting on a planet in an adiabatic disc becomes positive when $\lambda<-0.4$. We apply this to the optically thick disc and predict that a planet migrates outwards in regions 1 and 3 while it moves inwards towards the star in region 2. Planets in regions 1 and 2 are expected to move towards the boundary of regions 1 and 2 . This might help forming larger planets. However, it is not clear whether a planet still moves outwards in region 1 even if dissipative processes are included in a disc. We make a number of numerical simulations to examine the torque acting on a planet in a disc with dissipative processes due to viscosity and radiation.

### 3.1 Numerical method

We use two-dimensional equidistant grids in $r$ and $\theta$ with a resolution of $\left(N_{r}, N_{\theta}\right)=(640,3072)$. The inner and outer radii of the disc are given by $r_{\text {min }} / r_{\mathrm{p}}=0.4$ and $r_{\text {max }} / r_{\mathrm{p}}=2.0$, respectively. The damping boundary conditions (de Val-Borro et al. 2006), where all components are relaxed towards their initial state, are used in order to reduce wave reflection from these boundaries. All the quantities in the inner and outer boundaries are always fixed to be the initial values.
We develop a two-dimensional global hydrodynamic computer program with the gravitational forces of a star and a planet and a dissipative term (Yamada \& Inaba 2011). The basic equations are solved simultaneously using the finite volume method with an operator-splitting procedure. The source terms, which include the gravity, the viscous force and the radiation, are computed with a second-order Runge-Kutta scheme. The advection terms are calculated with a second order MUSCL-Hancock scheme and an exact Riemann solver (Toro 1999; Inaba et al. 2005).
The angular momentum of the disc gas is transferred to the planet. The transfer rate of the angular momentum from the disc gas at $r$ to the planet is given by
$\Gamma_{r}=\int_{0}^{2 \pi} \Sigma \frac{\partial \Phi}{\partial \theta} r \mathrm{~d} \theta$.
By integrating the torque density over the radial distance, we obtain the total torque acting on the planet from the disc:
$\Gamma=\int_{r_{\text {min }}}^{r_{\text {max }}} \Gamma_{r} \mathrm{~d} r$.
The torque and the torque density are normalized by $\Gamma_{0}$ and $\Gamma_{0} / r_{\mathrm{p}}$, where $\Gamma_{0}$ is $\left(M_{\mathrm{p}} / M_{\odot}\right)^{2}\left(r_{\mathrm{p}} / H_{\mathrm{p}}\right)^{2} \Sigma_{\mathrm{p}} r_{\mathrm{p}}^{4} \Omega_{\mathrm{p}}^{2}$.


Figure 3. The radial distributions of the torque density acting on the planet with 5 Earth masses by the adiabatic disc at $t=25 t_{\mathrm{p}}$. The power-law indices of the temperature distributions of the disc are $\beta=4 / 3$ and $\beta=4 / 11$ in panels (a) and (b), respectively. The planets are located at 1 and 2 au in panels (a) and (b), respectively. The horizontal axis is normalized by $r_{\mathrm{p}}$. Strong corotation torque is found in panel (a), leading to the outward migration of the planet.

### 3.2 Simulation results

### 3.2.1 The torque density acting on a planet in an optically thick accretion disc

We consider two adiabatic discs that have temperature distributions with power-law indices $4 / 3$ and $4 / 11$, which correspond to that of region 1 and region 2 of the optically thick disc, respectively. The power-law index of the temperature distribution is expressed as $-\beta$. Both discs have an initial surface density with $\alpha=1.0$. Hereafter, we call the adiabatic disc with $(\alpha, \beta)=(1.0,4 / 3)$ and that with $(\alpha$, $\beta)=(1.0,4 / 11)$ disc A and disc B, respectively. The power-law index of the entropy distribution of disc A is $\lambda=-1.0$ and that of disc B is $\lambda=-3.0 \times 10^{-2}$. Yamada \& Inaba (2011) found that the corotation torque increases with a decrease in the power index of the entropy distribution in an adiabatic disc. Fig. 3 shows the radial distribution of the torque density exerted on the planet in the adiabatic discs at $t=25 t_{\mathrm{p}}$, where $t_{\mathrm{p}}$ is the rotational period of the planet. The planet is located at 1 au in disc A and 2 au in disc B. A large corotation torque is exerted on the planet in disc A due to the large entropy gradient, making the total torque positive. On the other hand, the corotation torque in disc B is too weak to cancel the negative Lindblad torque. The normalized total torques become 3.0 and -2.4 in disc A and disc B, respectively.

The half-width of the horseshoe region, $\delta x_{\mathrm{s}}$, is estimated as (Masset, D'Angelo \& Kley 2006; Paardekooper \& Papaloizou 2009b; Paardekooper et al. 2010)
$\frac{\delta x_{\mathrm{s}}}{r_{\mathrm{p}}}=2.1 \times 10^{-3} \sqrt{\left(\frac{M_{\mathrm{p}}}{M_{\mathrm{E}}}\right)\left(\frac{r_{\mathrm{p}}}{H_{\mathrm{p}}}\right)}$,


Figure 4. The half-width of the horseshoe region as a function of the planet mass. The horizontal axis is normalized by the Earth mass. An optically thick disc with an accretion rate of $\xi_{\mathrm{v}}=0.1$ is used. The numerical half-width of the horseshoe region (filled circles) is evaluated at $t=25 t_{\mathrm{p}}$. The planet is located at $\tilde{r}_{\mathrm{p}}=1.0$. The curve is drawn using the analytical expression for the half-width of the horseshoe region given by Paardekooper et al. (2010).
where $M_{\mathrm{E}}$ is the Earth mass and $H_{\mathrm{p}}$ is the scale height of the disc at $\tilde{r}=\tilde{r}_{\mathrm{p}}$ :
$\frac{H_{\mathrm{p}}}{r_{\mathrm{p}}}=4.8 \times 10^{-2}\left(\frac{\tilde{r}_{\mathrm{p}}}{1.4}\right)^{(1-\beta) / 2}$.
Masset et al. (2006) showed that the half-width of the horseshoe region of a planet scales with $M_{\mathrm{p}}^{1 / 2}$ as long as the flow around the planet remains linear. Fig. 4 shows the half-width of the horseshoe region given by the numerical simulations (the filled circles) together with the half-width of the horseshoe region given by equation (27) (the curve). Both half-widths agree well except the large planet mass. Masset et al. (2006) found the planet mass when the flow linearity breaks in a 2D simulation. The non-linearity of the flow around a planet becomes clear when the mass of the planet becomes larger than $12 M_{\mathrm{E}}\left(H_{\mathrm{p}} / 0.05 r_{\mathrm{p}}\right)^{3}$. The numerical results also start to diverge from the half-width given by equation (27) when the mass of the planet becomes larger than 10 Earth masses.

The ratios of the magnitude of the corotation torque to that of the Lindblad torque are 1.6 and 0.6 in disc A and disc B, respectively. Nearly the same Lindblad torques act on the planet in both discs. The corotation torque determines the magnitude and sign of the total torque exerted on the planet.

We perform a simulation to find the torque density exerted on a planet in an optically thick accretion disc with $\left(\xi_{\mathrm{gr}}, \xi_{\mathrm{v}}\right)=$ $(10,0.1)$. The temperature distribution is the same as that of Fig. 1. The boundary position between regions 1 and 2 is 1.4 au and that between regions 2 and 3 is 2.5 au . Fig. 5 shows the radial distributions of the torque density acting on the planets in region 1 (panel a) and in region 2 (panel b) at $t=25 t_{\mathrm{p}}$. The planets are located at 1 au in region 1 and 2 au in region 2 . The radial distribution of the torque density in region 1 is similar to that in disc A . The torque density has a local maximum and a local minimum at $\left|1-r / r_{\mathrm{p}}\right|=$ $3.3 \times 10^{-2}$. The distances from the planet to the local maximum and minimum can be approximated by the scale height of the disc in region 1, indicating that this torque corresponds to the Lindblad torque.

The flow pattern within the horseshoe region is modified by the viscosity and the vortensity is no longer conserved during the U turn. The vortensity is radially transferred by the viscosity. We compare the vortensity distributions of the inviscid disc, that is, the adiabatic disc, and the accretion disc with non-vanishing viscosity.


Figure 5. Same as Fig. 3, but the optically thick accretion disc with $\left(\xi_{\mathrm{gr}}, \xi_{\mathrm{v}}\right)=(10,0.1)$. The normalized total torque acting on the planet is 2.1 in panel (a), while it becomes -1.5 in panel (b). The planet migrates outwards in region 1 and inwards in region 2. Planets might accumulate at the boundary of the regions.

Fig. 6 shows the contour of the vortensity within the horseshoe region of the planet. It is seen from Fig. 6 that the location of the minimum vortensity moves inwards towards the central star when the viscosity is included. The viscosity plays an important role to diffuse the vortensity. In a disc with low viscosity, the corotation torque is dominated by the non-linear corotation torque. The nonlinear corotation torque decreases with increasing viscosity.

The radiation restores the entropy distribution of a disc and is important to prevent the saturation of the corotation torque. Masset \& Casoli (2010) derived the analytical expression of the corotation torque acting on a planet by a disc with viscosity and thermal diffusivity. In this study, the cooling due to the radiation is compensated by the viscous heating. The radiation only works to maintain the thermal structure of a disc. Our numerical results are compared with the analytical expression of the corotation torque given by Masset \& Casoli (2010) in Fig. 7. We remove the contribution of thermal diffusivity in the analytical expression. Our results agree well with the analytical result. In this study, the radiation affects the corotation torque exerted on a planet through the disc temperature profile. However, it is important to note that radiation significantly influences the corotation torque as pointed out by Masset \& Casoli (2010).

The time-scale for the gas to spread across the horseshoe width by the viscosity is defined by
$\tau_{\text {visc }}=\frac{\delta x_{\mathrm{s}}^{2}}{3 v}$.
We substitute equations (21) and (27) into equation (29) to estimate the viscous time-scale for the disc with $\xi_{\mathrm{v}}=0.1$ at $r_{\mathrm{p}}=1 \mathrm{au}:$
$\tau_{\text {visc }}=16\left(\frac{\xi_{\mathrm{v}}}{0.1}\right)^{-1}\left(\frac{H_{\mathrm{p}} / r_{\mathrm{p}}}{0.05}\right)^{-1}\left(\frac{M_{\mathrm{p}}}{5 M_{\mathrm{E}}}\right) \Omega_{0}^{-1}$.
We consider the pressure effect of the gas disc and obtain the turnover time along the horseshoe orbit in front of the planet at


Figure 6. Contour of the vortensity in (a) an inviscid adiabatic disc (disc A) and (b) an accretion disc with non-vanishing viscosity $\left(\xi_{\mathrm{v}}, \xi_{\mathrm{gr}}\right)=(0.1,10)$ at $t=10 t_{\mathrm{p}}$. The initial distribution of vortensity is subtracted from the vortensity for better contrast. The horizontal and vertical axes correspond to $r / r_{\mathrm{p}}$ and $\theta / \pi$, respectively. The planet with 5 Earth masses is located at 1 au .


Figure 7. The corotation torques acting on the planet with 5 Earth masses given by the analytical expression of Masset \& Casoli (2010) (solid line) and the numerical results (filled circles). The horizontal and vertical lines represent the viscous strength factor, $\xi_{\mathrm{v}}$, and the normalized corotation torque, $\tilde{\Gamma}_{\mathrm{c}}$, respectively. In deriving the numerical corotation torque, we used equation (27) for the half-width of the horseshoe region. The planet is located at 1 au.
$r_{\mathrm{p}}=1 \mathrm{au}$ (Baruteau \& Masset 2008):
$\tau_{\text {turn }}=12\left(\frac{H_{\mathrm{p}} / r_{\mathrm{p}}}{0.05}\right)^{3 / 2}\left(\frac{M_{\mathrm{p}}}{5 M_{\mathrm{E}}}\right)^{-1 / 2} \Omega_{0}^{-1}$.
The viscous time-scale is nearly equal to the turnover time in the disc with $\xi_{\mathrm{v}}=0.1$. The effect of the viscosity on the corotation torque then starts to be important. Note that the cooling time-scale by the radiation is the same with the viscous time-scale in the disc.


Figure 8. The radial distribution of the torque density acting on the planet with 5 Earth masses by region 1 of the disc with $\left(\xi_{\mathrm{gr}}, \xi_{\mathrm{v}}\right)=(1.0,1.0)$. The planet is located at 1 au . The total torque acting on the planet becomes negative, $\tilde{\Gamma}=-2.4$. Planets move inwards towards the central star even in region 1 .

The torque density near the orbit of the planet in region 2 is smaller than that in disc B. Morohoshi \& Tanaka (2003) used a local shearing box calculation and studied the gravitational interactions between a planet and an optically thin disc, taking into account energy dissipation by radiation. They found that the oval shape of the density contour profile near a planet is tilted with respect to the direction towards the central star. The asymmetry of the density structure increases the one-side torque. They showed that the density contour is less inclined with decreasing opacity. The adiabatic disc can be considered to have much larger opacity than region 2 , yielding a larger torque density near the planet.

Paardekooper et al. (2011) have derived the torque formula for the Lindblad torque and the corotation torque exerted on a planet by a disc with viscosity and thermal diffusion. We find that the corotation and Lindblad torques obtained in our simulation agree with those calculated from their torque formula within 30 per cent. The total torque is given by the sum of the Lindblad torque and the corotation torque. The total torque becomes $\tilde{\Gamma}=2.1$ in region 1 and it is slightly smaller than that in disc A, $\tilde{\Gamma}=3.0$. The density enhancement is reduced as the vortensity is smeared by the viscosity. However, the viscosity is too small to remove the corotation torque. Planets accumulate at the ice line as suggested by Hasegawa \& Pudritz (2011).

We further examine the effect of the viscosity of a disc on the torque density exerted on the planet, using a large value of the vis$\operatorname{cosity}, \xi_{\mathrm{v}}=1.0$. We utilize the same surface density distribution and the temperature distribution given by Fig. 1. Fig. 8 shows the radial distribution of the torque density exerted on the planet in region 1 at $t=25 t_{\mathrm{p}}$. The corotation torque is lost because the viscosity is large enough to smear the density enhancement. In this case, the viscous time-scale becomes much shorter than the turnover time (see equations 30 and 31). The total torque becomes negative ( $\tilde{\Gamma}=-2.4$ ), leading to the inward migration of the planet in region 1.

The density contours around the planet are shown to make clear the effect of the viscosity on the torque density. Fig. 9 shows the contours of the density fluctuations, $\Sigma\left(t=25 t_{\mathrm{p}}\right) / \Sigma_{\text {ini }}-1.0$, around the planet (a) in disc A, (b) in region 1 of the disc with $\left(\xi_{\mathrm{gr}}, \xi_{\mathrm{v}}\right)=$ $(10,0.1)$ and (c) in region 1 of the disc with $\left(\xi_{\mathrm{gr}}, \xi_{\mathrm{v}}\right)=(1.0,1.0)$. The enhancement of the density in the horseshoe orbit in panel (b) is similar to that in panel (a). On the other hand, the density in the horseshoe region is considerably reduced in panel (c) because


Figure 9. The contour plots of the surface density enhancement $\Sigma(t=$ $\left.25 t_{\mathrm{p}}\right) / \Sigma_{\text {ini }}-1.0$ around the planet with 5 Earth masses (a) in an adiabatic disc (disc A), (b) in region 1 of the disc with $\left(\xi_{\mathrm{gr}}, \xi_{\mathrm{v}}\right)=(10,0.1)$ and (c) in region 1 of the disc $\left(\xi_{\mathrm{gr}}, \xi_{\mathrm{v}}\right)=(1.0,1.0)$, respectively. The horizontal axis is normalized by $r_{\mathrm{p}}$ and the vertical axis is divided by $\pi$. The planet is located at $(r, \theta)=\left(r_{\mathrm{p}}, \pi\right)$. The density contour in panel (b) is similar to that in panel (a) because the viscosity is too small to decrease the density enhancement. On the other hand, the density enhancement is greatly reduced by the viscosity in panel (c).
of the larger viscosity. The corotation torque decreases because the viscous time-scale is much shorter than the turnover time in the case of $\xi_{\mathrm{v}}=1.0$. Moreover, the inner and outer spiral density waves are damped by the viscosity as well. This results in the small Lindblad torque.

### 3.2.2 Dependence of the total torque on the opacity and the viscosity of a disc

The corotation torque decreases in an adiabatic disc because the entropy of the gas in the horseshoe orbit of the planet tends to become uniform after the synodic period of the planet (Baruteau \& Masset


Figure 10. The time evolution of the total torque acting on the planet with 5 Earth masses in an adiabatic disc (solid curve) and in region 1 of the disc with $\left(\xi_{\mathrm{gr}}, \xi_{\mathrm{v}}\right)=(10,0.1)$ (dashed curve). In both cases, the planet is located at 1 au . The total torque decreases around $t=25 t_{\mathrm{p}}$ in the adiabatic disc due to the saturation of the corotation torque. On the other hand, a positive total torque is maintained in the disc with $\left(\xi_{\mathrm{gr}}, \xi_{\mathrm{v}}\right)=(10,0.1)$.


Figure 11. The total torques exerted on the planet by the disc with $\left(\xi_{\mathrm{gr}}, \xi_{\mathrm{v}}\right)=(1.0,1.0)$ (filled circle) and $(10,0.1)$ (filled triangle) as a function of $\tilde{r}_{\mathrm{p}}$. The boundary position between regions 1 and 2 is shown by the dotted line. In region 1 , the total torque seems to be nearly independent of $\tilde{r}_{\mathrm{p}}$. In region 2 , the total torque decreases a little with increasing $\tilde{r}_{\mathrm{p}}$.

2008; Paardekooper \& Papaloizou 2008, 2009a). This is called the saturation of the corotation torque and happens in an adiabatic disc. The corotation torque does not saturate in a viscous disc because the vortensity is transferred from the horseshoe region to the outer region of the disc by viscosity (Paardekooper \& Papaloizou 2008, 2009a). Fig. 10 shows the time evolutions of the total torque acting on the planet in the adiabatic disc (disc A) and in the optically thick accretion disc with $\left(\xi_{\mathrm{gr}}, \xi_{\mathrm{v}}\right)=(10,0.1)$ (region 1), which are represented by the solid and dotted curves, respectively. The total torque increases with time at the beginning of the simulation and reaches a steady state around $t=10 t_{p}$. The total torque decreases in the adiabatic disc by saturation; on the other hand, the total torque remains the same in the optically thick accretion disc. We do not have to worry about the saturation because we consider the planet in a disc with viscosity.

We examine the effect of the boundary of the regions of the disc on the total torque by changing the position of the planet. Fig. 11 shows the total torques acting on the planets at different positions in the optically thick accretion discs. The filled circles and triangles correspond to the total torques on the planet in discs with $\left(\xi_{\mathrm{g}}, \xi_{\mathrm{v}}\right)=$ $(1.0,1.0)$ and $(10,0.1)$, respectively. The total torque on the planet


Figure 12. The diagram of the total torques exerted on the planet with 5 Earth masses by the disc with various parameter sets of $\left(\xi_{\mathrm{gr}}, \xi_{\mathrm{v}}\right)$. The open and filled symbols correspond to the positive and negative total torques acting on the planets, respectively. We find the largest magnitude of the total torques, $|\tilde{\Gamma}|_{\max }$, from the simulations with the same boundary position of $\tilde{r}_{12}$. The total torques are plotted as circles, squares and triangles when $|\tilde{\Gamma}| \geq 0.5|\tilde{\Gamma}|_{\max }, 0.5|\tilde{\Gamma}|_{\max }>|\tilde{\Gamma}| \geq 0.1|\tilde{\Gamma}|_{\max }$ and $0.1|\tilde{\Gamma}|_{\max }>|\tilde{\Gamma}|$, respectively. The dotted and dashed curves are drawn using $\tau_{\text {visc }}=0.5 \tau_{\text {turn }}$ when $\tilde{r}_{\mathrm{p}}=0.7$ and 1.0, respectively. It is noted that the curves are truncated because $\tilde{r}_{12}$ is smaller than $\tilde{r}_{\mathrm{p}}$. The sign of the total torque changes when $\tau_{\text {visc }} \simeq 0.5 \tau_{\text {turn }}$. The planet with 5 Earth masses moves outwards in region 1 of the optically thick accretion disc with a surface density of $100 \mathrm{~g} \mathrm{~cm}^{-2}$ at 1 au when the accretion rate is smaller than $2.1 \times 10^{-8} \mathrm{M}_{\odot} \mathrm{yr}^{-1}$. The planets are then expected to accumulate at the boundary of regions 1 and 2 in the optically thick disc.
in region 1 is nearly independent of $\tilde{r}_{\mathrm{p}}$. The effect of the boundary of the regions on the total torque is very small. The total torque in region 2 decreases a little with $\tilde{r}_{\mathrm{p}}$. The analytical formula for the normalized corotation torque (Paardekooper et al. 2011) depends on the scale height and the viscous coefficient, and the corotation torque decreases with increasing $\tilde{r}_{\mathrm{p}}$ in region 2 . This agrees with our numerical results.
We perform a number of numerical simulations of gravitational interactions between the planet and the optically thick accretion discs. Fig. 12 shows the magnitude and sign of the total torques exerted on the planet in region 1 of the discs with various parameters of $\xi_{\mathrm{gr}}$ and $\xi_{\mathrm{v}}$. The planet is located at 1 au when $\tilde{r}_{12}=2.4,1.4$ and 1.2 , and at 0.7 au when $\tilde{r}_{12}=0.8$, respectively. The open and filled symbols denote the positive and negative total torques acting on the planets, respectively. We obtain the largest magnitude of the total torques, $|\tilde{\Gamma}|_{\text {max }}$, from the simulations with the same boundary position of $\tilde{r}_{12}$. The total torques are plotted as circles, squares and triangles when $|\tilde{\Gamma}| \geq 0.5|\tilde{\Gamma}|_{\max }, 0.5|\tilde{\Gamma}|_{\max }>|\tilde{\Gamma}| \geq 0.1|\tilde{\Gamma}|_{\max }$ and $0.1|\tilde{\Gamma}|_{\max }>|\tilde{\Gamma}|$, respectively. The total torque increases with decreasing $\xi_{\mathrm{gr}}$ and $\xi_{\mathrm{v}}$ and its sign changes from negative to positive.
We find a small decrease in the corotation torque when the viscous time-scale is nearly equal to the turnover time as shown in Fig. 5. The corotation torque on the planet decreases with the viscosity of the disc and vanishes when the viscous time-scale becomes much shorter than the turnover time as shown in Fig. 8. We find that the positive corotation torque cancels the negative Lindblad torque to have zero total torque when $\tau_{\text {visc }} \simeq 0.5 \tau_{\text {turn }}$. We use the temperature distribution of region 1 of the disc to find the viscous time-scale:
$\tau_{\mathrm{visc}}=1.6\left(\frac{\xi_{\mathrm{gr}}}{1.0}\right)^{-1 / 6}\left(\frac{\xi_{\mathrm{v}}}{1.0}\right)^{-7 / 6}\left(\frac{M_{\mathrm{p}}}{5 M_{\mathrm{E}}}\right) \tilde{r}_{\mathrm{p}}^{2 / 3} \Omega_{\mathrm{p}}^{-1}$.


Figure 13. Same as Fig. 12, but for a planet with (a) 3 Earth masses and (b) 7.5 Earth masses. The open and filled symbols correspond to the positive and negative total torques, respectively. The dotted curve is drawn using $\tau_{\text {visc }}=0.5 \tau_{\text {turn }}$ when $\tilde{r}_{\mathrm{p}}=1.0$. The planets with 3 and 7.5 Earth masses move outwards in region 1 of the optically thick accretion disc with a surface density of $100 \mathrm{~g} \mathrm{~cm}^{-2}$ at 1 au when the accretion rate becomes smaller than $1.2 \times 10^{-8}$ and $3.2 \times 10^{-8} \mathrm{M}_{\odot} \mathrm{yr}^{-1}$, respectively.

Using $\tau_{\text {visc }}=0.5 \tau_{\text {turn }}$, we obtain
$\xi_{\mathrm{gr}}=3.8 \times 10^{-2} \tilde{r}_{\mathrm{p}}^{11 / 5}\left(\frac{M_{\mathrm{p}}}{5 M_{\mathrm{E}}}\right)^{18 / 5} \xi_{\mathrm{v}}^{-17 / 5}$.
The dashed and dotted curves in Fig. 12 are drawn using equation (33). We consider the two positions of the planet, $\tilde{r}_{\mathrm{p}}=1.0$ and 0.7. Substituting $\tilde{r}_{\mathrm{p}}=1.0$ and 0.7 into equation (33), we draw the dashed and dotted curves, respectively. The whole region is separated into two regions by the curves. The positive total torque and the negative total torque can be found in one of the regions. We conclude that the corotation torque is moderately damped by the viscosity when $\tau_{\text {visc }} \simeq 0.5 \tau_{\text {turn }}$ and cancels the negative Lindblad torque. The opacity of the disc increases with $\xi_{\mathrm{gr}}$. Energy dissipates ineffectively in the disc with large $\xi_{\text {gr }}$, increasing the mid-plane temperature and the scale height. The viscous time-scale of the disc with large $\xi_{\mathrm{gr}}$ decreases because the width of the horseshoe region decreases with increasing scale height (see equation 27). In the disc with large $\xi_{\mathrm{gr}}$, the viscosity quickly damps the corotation torque and the negative Lindblad torque becomes dominant. Moreover, it is valuable to mention the effect of the surface density at 1 au, $\Sigma_{0}$, on equation (33). In this study, we set $\Sigma_{0}$ to be constant. Since the accretion rate and the optical depth are proportional to the surface density, $\tau_{\text {visc }}$ and $\tau_{\text {turn }}$ also increases with $\Sigma_{0}$. Hence, the coefficient of $\xi_{\mathrm{v}}$ on the right-hand side of equation (33) changes by $\Sigma_{0}$.
Additionally, one can find from equation (33) that the planet mass has a large effect on the threshold at which the total torque becomes zero. We examine the effect of the planet mass on the total torque. Fig. 13 shows the total torque exerted on the planet with 3 Earth masses and that with 7.5 Earth masses in panels (a) and (b), respectively. The planet is located at $\tilde{r}_{\mathrm{p}}=1.0$ in both cases. The half-width of the horseshoe region increases with the planet mass.

It takes a longer time to damp the density enhancement in the horseshoe region of the planet with large mass. The larger viscosity is required to reduce the corotation torque acting on the massive planet.

## 4 SUMMARY

We have studied the Type I migration of a planet in an optically thick accretion disc. The gravitational interactions between a planet and disc gas excite spiral density waves inside and outside of the orbit of the planet in the disc. The waves attract the planet gravitationally and exert torques on the planet. The negative torque by the outer density wave is a little larger than the positive torque by the inner density wave because the outer wave is closer to the planet than the inner wave due to the negative pressure gradient. The sum of the torques by the density waves (the Lindblad torque) becomes negative, leading to the inward migration of the planet (Ward 1997).

The corotation torque is very important to planetary migration because it might be able to halt the inward migration or reverse its direction (Baruteau \& Masset 2008; Paardekooper \& Papaloizou 2008). In a non-barotropic disc, it is found that the entropy-related non-linear corotation torque plays an essential role. The non-linear corotation torque comes from the density enhancement due to the generated vortensity at the outgoing separatrices (Masset \& Casoli 2010; Paardekooper et al. 2010). The entropy-related corotation torque is proportional to the radial gradient of the entropy of a disc. A disc with steep entropy gradient yields a large positive corotation torque. Yamada \& Inaba (2011) showed that the positive corotation torque cancels the negative Lindblad torque when the power-law index of the entropy distribution of the disc becomes $\lambda=-0.4$, that is, the critical power-law index of the entropy distribution.

The radiation from the central star cannot reach the mid-plane of an optically thick disc. The temperature of the mid-plane of an optically thick accretion disc is determined by the energy balance between the viscous heating and the radiative cooling. The rate of cooling by the radiation is sensitive to the opacity of the disc. Micron-size dust particles provide the sources of opacity. The opacity changes with temperature through the sublimation of ice. We utilize the opacity law frequently used in other researches and find that the disc is divided into three regions: from region 1 with the highest temperature to region 3 with the lowest temperature. The boundary positions of the regions are dependent on the viscosity and the opacity of the disc. The boundary between regions 1 and 2 is located between 0.5 and 2.5 au .

Each region has different power-law indices of the temperature and entropy distributions. The power-law indices of the entropy distribution in regions 1 and 3 are lower than the critical power-law index of -0.4 , while the power-law index in region 2 is higher. We found that the total torque exerted on the planet by the adiabatic disc becomes positive in regions 1 and 3 and becomes negative in region 2, as expected. This means that the planet moves outwards in regions 1 and 3, while it moves inwards towards the central star in region 2 . Planets might accumulate at the boundary between regions 1 and 2 in the adiabatic disc.

Dissipative processes change the magnitude of the corotation torque (Kley \& Crida 2008; Paardekooper \& Papaloizou 2008, 2009a; Yamada \& Inaba 2011). In a viscous disc, the vortensity is transferred radially as shown in Fig. 6. The viscosity is responsible for the decrease in the corotation torque. The radiation has an impact on the evolution of the entropy as well and keeps the entropy gradient within the horseshoe region. The corotation torque depends on these dissipative processes due to the viscosity and the
radiation (Masset \& Casoli 2010). The positive total torque exerted on the planet by an adiabatic disc might become negative once some dissipative processes are included. We include the viscosity and radiation into the adiabatic disc and calculate the total torque exerted on the planet in an optically thick accretion disc. We focus on the planet in regions 1 and 2 . The magnitudes of the opacity and the viscosity are expressed with the two parameters, $\xi_{\mathrm{gr}}$ and $\xi_{\mathrm{v}}$, respectively. The opacity and the viscosity increase with $\xi_{\mathrm{gr}}$ and $\xi_{\mathrm{v}}$. A large mass accretion rate of the disc is found in a disc with large $\xi_{\mathrm{v}}$. The temperature of the disc increases when the energy of the disc dissipates ineffectively (large $\xi_{\mathrm{gr}}$ ), resulting in the narrow horseshoe region. The viscosity decreases the enhancement of the density in the horseshoe region. The corotation torque gets smaller when the viscosity is larger. The total torque exerted on the planet by the optically thick accretion disc depends on the two parameters.

We have made a number of numerical simulations of gravitational interactions between the planet and the optically thick accretion disc with various parameter sets of $\xi_{\mathrm{gr}}$ and $\xi_{\mathrm{v}}$. The total torque always becomes negative in region 2 , leading to the inward migration of the planet. The total torque becomes positive in region 1, if the effect of dissipation is small. The dissipative processes work effectively and the total torque becomes zero when the time-scale for the viscosity is half of the turnover time of the planet in the horseshoe orbit, $\tau_{\text {visc }}=0.5 \tau_{\text {turn }}$. This equation is written with the parameters as $\xi_{\mathrm{v}}=$ $0.4 \tilde{r}_{\mathrm{p}}^{11 / 17}\left(M_{\mathrm{p}} / 5 M_{\mathrm{E}}\right)^{18 / 17}\left(\Sigma_{0} / 100 \mathrm{~g} \mathrm{~cm}^{-2}\right)^{7 / 17} \xi_{\mathrm{gr}}^{-5 / 17}$, considering $\Sigma_{0}$ as a parameter. In the optically thick accretion disc with $\xi_{\mathrm{gr}} \simeq 1.0$ and a surface density of $100 \mathrm{~g} \mathrm{~cm}^{-2}$ at 1 au , the accretion rate of the disc is required to be smaller than $2.1 \times 10^{-8} \mathrm{M}_{\odot} \mathrm{yr}^{-1}$ for the planet to move outwards. Our study suggests that planets with 5 Earth masses might accumulate and drive further growth of the planets at the boundary between regions 1 and 2 in the optically thick accretion disc. A small accretion rate of gas is required for small planets to move outwards.

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## APPENDIX A: EFFECT OF $\alpha$ ON THE TOTAL TORQUE ON A PLANET

We examine the dependence of the total torque on the power-law index, $\alpha$, of the surface density distribution of the disc. Fig. A1 shows the time evolutions of the total torque exerted on the planet by a disc with $\alpha=1.0$ and a disc with $\alpha=0.5$. Both discs have parameters of $\left(\xi_{\mathrm{gr}}, \xi_{\mathrm{v}}\right)=(10,0.1)$. The solid and dashed curves represent the total torques acting on the planet by the discs with $\alpha=1.0$ and $\alpha=0.5$, respectively. The total torques increase at the beginning of the simulations and eventually reach steady states in both discs. The total torque by the disc with $\alpha=0.5$ becomes approximately 1.5 times as large as that by the disc with $\alpha=1.0$.
The torque densities by the disc with $\alpha=1.0$ and by the disc with $\alpha=0.5$ are plotted as the solid and dashed curves, respectively, in Fig. A2. The inner and outer Lindblad torques increase and decrease with a decrease in $\alpha$, respectively, leading to a smaller Lindblad torque. The analytical formula for the Lindblad torque, $\tilde{\Gamma}_{\mathrm{L}}$, is given by (Paardekooper et al. 2011)
$\gamma \tilde{\Gamma}_{\mathrm{L}}=-2.5-1.7 \beta_{i}+0.1 \alpha$.
The temperature distribution of region 1 of the optically thick accretion disc has a power-law index, $\beta_{1}=(3+\alpha) / 3$. Hence, we have for the Lindblad torque in region 1
$\gamma \tilde{\Gamma}_{\mathrm{L}}=-4.2-0.47 \alpha$.


Figure A1. The time evolution of the total torque exerted on the planet with 5 Earth masses by the disc with $\left(\xi_{\mathrm{gr}}, \xi_{\mathrm{v}}\right)=(10,0.1)$. The solid and dashed curves correspond to the total torques acting on the planet by the discs with $\alpha=1.0$ and $\alpha=0.5$, respectively. The planet is located at 1 au . The total torque by the dise with $\alpha=0.5$ is larger than that by the disc with $\alpha=1.0$.


Figure A2. The torque densities exerted on the planet by the discs with $\alpha=$ 1.0 (solid curve) and $\alpha=0.5$ (dashed curve). Both discs have parameters of $\left(\xi_{\mathrm{gr}}, \xi_{\mathrm{v}}\right)=(10,0.1)$. The planet is located at 1 au .

The analytical formula suggests that the Lindblad torque increases with a decrease in $\alpha$ and agrees with numerical results. The corotation torque becomes a little larger in the disc with $\alpha=0.5$, compared with that in the disc with $\alpha=1.0$. This increase in the corotation torque is also suggested by the analytical formula for the corotation torque formula. We can derive the relation between $\xi_{\mathrm{gr}}$ and $\xi_{\mathrm{v}}$ when the total torque becomes zero, using the same procedure explained in Section 3.2. Using $\tau_{\text {visc }}=0.5 \tau_{\text {turn }}$, we have


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