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Erratum: Relaxation of spherical stellar systems

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This is an erratum to the paper 'Relaxation of spherical stellar systems' (2019, MNRAS, 490, 478–490). Figs 11 and 13 of Lau & Binney (2019) (hereafter LB19) included plots of action-space velocities computed from N-body models. Unfortunately, the plotted velocities are incorrect. The mean action-space velocity is the flux **F** divided by the DF $f(\mathbf{J})$. The flux of an N-body model is

$$\mathbf{F}(\mathbf{J},t)=\sum_\alpha \dot{\mathbf{J}}_\alpha \delta\left[\mathbf{J}-\left(\mathbf{J}_{0\alpha}+\int_0^t \mathrm{d}t'\,\dot{\mathbf{J}}_\alpha\right)\right], \tag{1}$$

where $\mathbf{J}_0\alpha$ is the location of the αth particle at time zero and $\dot{\mathbf{J}}_\alpha(t)$ is its velocity. One should plot this discontinuous flux averaged through the cells of a grid. It is also expedient to average the flux through a time interval $\delta t = 2\epsilon$. Thus one should plot

$$\begin{aligned}\overline{\mathbf{F}} &\equiv \frac{1}{2\epsilon}\int_{t-\epsilon}^{t+\epsilon}\mathrm{d}t\int \mathrm{d}^3\mathbf{J}\,\mathbf{F}(\mathbf{J},t)\\ &=\frac{1}{2\epsilon}\sum_\alpha[\mathbf{J}_\alpha(t+\epsilon)-\mathbf{J}_\alpha(t-\epsilon)]. \end{aligned}\tag{2}$$

It's important here that the sum is over the particles that are in the cell at time t. Given that data are available only at discrete time intervals, ϵ cannot be made very small and many of these particles will not be in the cell at $t\pm\epsilon$. Unfortunately the paper showed plots computed from $\overline{\mathbf{F}}=\sum_\alpha[\mathbf{J}_\alpha(t+\epsilon)-\mathbf{J}_\alpha(t)]/\epsilon$ for the particles in the cell at time t. That is, the plots show the mean of the velocity in the *next* time interval. Outside a cluster's core this tends to be dominated by the first order diffusion coefficient. Indeed, the second-order diffusion tensor $\langle\Delta_i\Delta_j\rangle$ drives diffusion away from the origin of action space, while the first-order coefficient $\langle\Delta_i\rangle$, which embodies dynamical friction, pushes them back. Particles currently in the upper half of the energy distribution are likely to have recently picked up energy in an encounter and can be expected soon to regress to the mean energy through the action of $\langle\Delta_i\rangle$. When the time-centred difference (2) is used, the boost in $\dot{\mathbf{J}}$ from $\langle\Delta_i\Delta_j\rangle$ that is responsible for a star being in quite an energetic bin at time t is correctly reflected in $\overline{\mathbf{F}}$, while the lagging difference used by LB19 is biased towards the effect of $\langle\Delta_i\rangle$. We were alerted to this problem when Douglas Heggie pointed out the similarity between the central panel of fig. 11 in LB19 and the upper panel of fig. B1 in Hamilton et al. (2018), which shows the first-order contribution to **F** from the classical local-scattering approximation of Chandrasekhar.

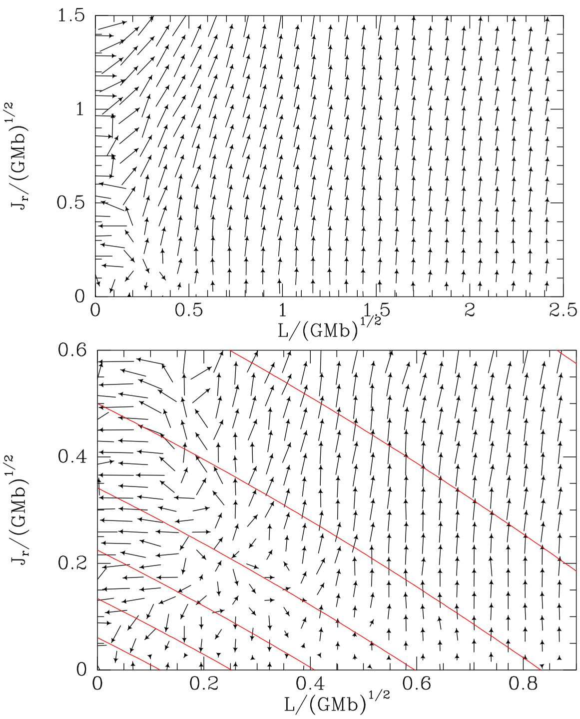

Figure 1. The mean velocities of stars through action space determined from N-body realisations of an isochrone cluster. The upper panel is what should have been the centre panel of LB19 while the lower panel is the correct version of fig. 13 of that paper.

The upper and lower panels of Fig. 1 are, respectively, the correct versions of the centre panel of fig. 11 and of fig. 13 in LB19. These plots resemble the classically computed velocities shown in the bottom panel of LB19's fig. 11 quite closely, the differences being strongly concentrated around the line $L = 0$.

REFERENCES

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