

# Detecting isolated stellar-mass black holes in the absence of microlensing parallax effect

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## ABSTRACT

Gravitational microlensing can detect isolated stellar-mass black holes (BHs), which are believed to be the dominant form of Galactic BHs according to population synthesis models. Previous searches for BH events in microlensing data focused on long time-scale events with significant microlensing parallax detections. Here we show that, although BH events preferentially have long time-scales, the microlensing parallax amplitudes are so small that in most cases the parallax signals cannot be detected statistically significantly. We then identify OGLE-2006-BLG-044 to be a candidate BH event because of its long time-scale and small microlensing parallax. Our findings have implications to future BH searches in microlensing data.

**Key words:** gravitational lensing: micro – methods: data analysis – stars: black holes.

## 1 INTRODUCTION

Stars with initial masses  $\gtrsim 20 M_{\odot}$  are believed to end their lives in stellar-mass black holes (BHs). It has been estimated that the Milky Way contains  $\sim 10^8$  stellar-mass BHs (e.g. Shapiro & Teukolsky 1983). Although most massive stars are found in binaries or higher multiples, stellar evolutions frequently lead to mergers (due to common envelope evolution or gravitational radiation) or disruptions of stellar binaries, resulting in the majority of stellar-mass BHs in isolation (e.g. Belczynski, Bulik & Kluźniak 2002; Belczynski, Sadowski & Rasio 2004; Wiktorowicz et al. 2019; Olejak et al. 2020). Therefore, isolated BHs are important for our understanding of the stellar-mass BH population. In particular, a statistical knowledge of isolated BHs can help to constrain the formation channels of gravitational wave sources, such as those found by LIGO and Virgo (e.g. Abbott et al. 2016, 2019).

While many techniques exist to detect BHs in binaries, gravitational microlensing is perhaps the only viable technique to detect isolated BHs (Einstein 1936; Paczynski 1986). Starting from the widely used initial mass function and the general criterion for BH formation, Gould (2000a) estimated that  $\sim 1$  per cent of microlensing events towards the bulge direction should be due to stellar-mass BHs. Later studies that implemented more complicated physics and/or focused on specific surveys or observing strategies found consistent results (e.g. Osłowski et al. 2008; Rybicki et al. 2018; Wiktorowicz et al. 2019; Lam et al. 2020). Wiktorowicz et al. (2019) used the stellar population synthesis code, *startrack*, and estimated that there should be 14 and 26 BH microlensing events per year in surveys like OGLE-III and OGLE-IV, respectively. Lam et al. (2020) developed a new population synthesis code specifically for the searches of compact object microlensing events, *popsycle*. The authors found that with current ground-based astrometric follow-ups (e.g. Lu et al.

2016) of events from existing microlensing surveys one should expect to detect one to two BHs each year.

One issue with using microlensing to detect BHs is how to identify them from the much more abundant normal microlenses, namely, stars. In the standard microlensing model (Paczynski 1986), only the event time-scale, given by

$$t_E \equiv \frac{\theta_E}{\mu_{\text{rel}}} = \frac{\sqrt{\kappa M_L \pi_{\text{rel}}}}{\mu_{\text{rel}}}, \quad (1)$$

is related to the lens mass  $M_L$ . Here,  $\theta_E$  is the angular Einstein radius,  $\mu_{\text{rel}}$  and  $\pi_{\text{rel}}$  are the relative proper motion and relative parallax between the lens and source, respectively, and  $\kappa$  is a constant:

$$\kappa \equiv \frac{4G}{c^2 \text{au}} \approx 8.14 \frac{\text{mas}}{M_{\odot}}. \quad (2)$$

With one observable ( $t_E$ ) and three unknowns ( $M_L$ ,  $\mu_{\text{rel}}$ , and  $\pi_{\text{rel}}$ ), one cannot uniquely determine the lens mass. Statistical studies of the event time-scale distribution have been done to infer the mass function from substellar objects up to stellar-mass BHs (e.g. Sumi et al. 2011; Mróz et al. 2017), but results from this approach are very much subject to the details of the adopted Galactic model.

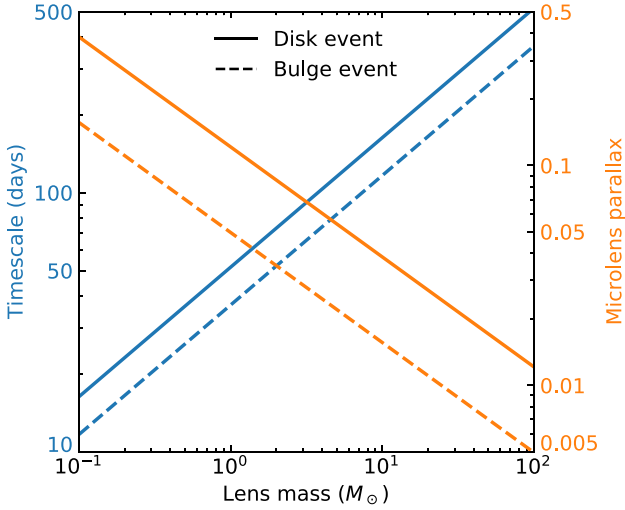
Once measured (or constrained), the microlensing parallax parameter, given by (Gould 2000b)

$$\pi_E \equiv \frac{\pi_{\text{rel}}}{\theta_E}, \quad (3)$$

can reduce the degree of freedom in the problem. As  $t_E \propto M_L^{1/2}$  whereas  $\pi_E \propto M_L^{-1/2}$  (see Fig. 1), the combination of the two can significantly reduce the statistical uncertainty in inferred lens mass, even though the mass still cannot be uniquely determined (Han & Gould 1995; Zhu et al. 2017).

With only ground-based observations,  $\pi_E$  is constrained through the annual parallax effect that originates from the orbital motion of the Earth around the Sun (Gould 1992). Such an effect is only detectable in relatively long time-scale ( $t_E \gtrsim \text{yr}/2\pi$ ) events. Luckily,

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**Figure 1.** The event time-scale  $t_E$  (blue) and the microlensing parallax parameter  $\pi_E$  (orange) as functions of lens mass for typical disc events ( $\pi_{\text{rel}} = 0.12 \text{ mas}$ ,  $\mu_{\text{rel}} = 7 \text{ mas yr}^{-1}$ ) and typical bulge events ( $\pi_{\text{rel}} = 0.02 \text{ mas}$ ,  $\mu_{\text{rel}} = 4 \text{ mas yr}^{-1}$ ), respectively. Microlensing events arisen from isolated stellar-mass BHs ( $\gtrsim 3 M_\odot$ ) should have large  $t_E$  but small  $\pi_E$ .

events due to stellar-mass BHs belong to such a category. Since the early days of microlensing searches, efforts have been taken to identify events with long time-scales and annual parallax effect and to infer masses of the foreground lenses (e.g. Agol et al. 2002; Bennett et al. 2002; Mao et al. 2002; Poindexter et al. 2005; Wyrzykowski et al. 2016). Here, we use Wyrzykowski et al. (2016) as a representative example to demonstrate the standard procedure. Starting from over 3600 microlensing events found by the OGLE-III survey (Udalski et al. 2008; Wyrzykowski et al. 2015), Wyrzykowski et al. (2016) found 59 parallax events. These were the events that showed significant ( $>50$ )  $\chi^2$  improvement in the light-curve fit after parallax parameters were included. From the standard Bayesian statistical inference, the authors identified 13 microlenses consistent with being stellar remnants.

For such massive microlenses as the isolated stellar-mass BHs, the long time-scale makes it easier to detect the annual parallax effect, but the reduced amplitude of the parallax parameter also makes the parallax effect more subtle (see Fig. 1). We show in Section 2 that the parallax effect in a truly BH event is mostly undetectable. As such, we reanalyse the OGLE-III data set with a method slightly different from that of Wyrzykowski et al. (2016) and identify one candidate BH event. This search is presented in Section 3. We discuss the results in Section 4.

## 2 ANNUAL PARALLAX EFFECT IN A BH EVENT IS UNDETECTABLE

With a single lens, the magnified source flux at any given time  $t$  is given by

$$F(t) = F_S[A(t) - 1] + F_{\text{base}}, \quad (4)$$

where the magnification  $A$  is given by

$$A(t) = \frac{u^2 + 2}{u\sqrt{u^2 + 4}}. \quad (5)$$

Here,  $F_S$  is the source flux at baseline ( $A = 1$ ),  $F_{\text{base}}$  is the total flux (i.e. source and blend) at baseline, and  $u$  is the dimensionless

separation between the source and the lens. When annual parallax is included (Gould 2004),

$$u^2 = \left( \frac{t - t_0}{t_E} + \pi_{E,N}s_N + \pi_{E,E}s_E \right)^2 + (u_0 + \pi_{E,N}s_E - \pi_{E,E}s_N)^2, \quad (6)$$

where  $t_0$  is the event peak time (in the absence of parallax effect),  $u_0$  is the impact parameter,  $t_E$  is the event time-scale in the geocentric reference frame, and  $\pi_{E,N}$  and  $\pi_{E,E}$  are the parallax components along the north and east directions, respectively. The quantities  $s_N$  and  $s_E$  are the corresponding offsets (in units of au) between Earth's actual position and the position of Earth in the absence of parallax effect. These offsets are evaluated at a fixed time very close to the peak time  $t_0$  and are thus independent of the details of the event.

The detectability of the parallax effect, namely, the deviations of  $\pi_{E,N}$  and  $\pi_{E,E}$  from zeros, can be evaluated through a Fisher matrix analysis. With the nuisance parameter  $F_{\text{base}}$  ignored, the microlensing light curve is described by the parameter set  $\theta \equiv (t_0, u_0, t_E, \pi_{E,N}, \pi_{E,E}, F_S)$ . The Fisher information matrix is then

$$\mathcal{F}_{ij} = \sum_{\{t_k\}} \frac{1}{\sigma_F^2(t_k)} \frac{\partial F(t_k)}{\partial \theta_i} \frac{\partial F(t_k)}{\partial \theta_j}. \quad (7)$$

Here,  $\sigma_F$  denotes the uncertainty in measured flux. The summation is done over a time series  $\{t_k\}$ . The inverse of the Fisher information matrix gives the covariance matrix of  $\theta$ . The detectability of the parallax effect can be quantified as

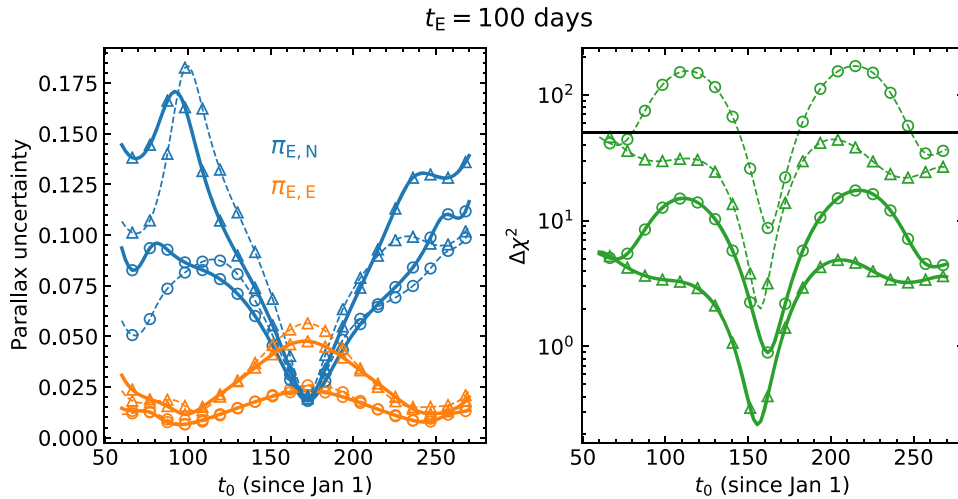
$$\Delta\chi^2 = (\theta_0 - \theta)\mathcal{F}(\theta_0 - \theta)^T, \quad (8)$$

where the no-parallax parameter set  $\theta_0$  has  $(\pi_{E,N}, \pi_{E,E}) = (0, 0)$  and other parameters the same as those in  $\theta$ .

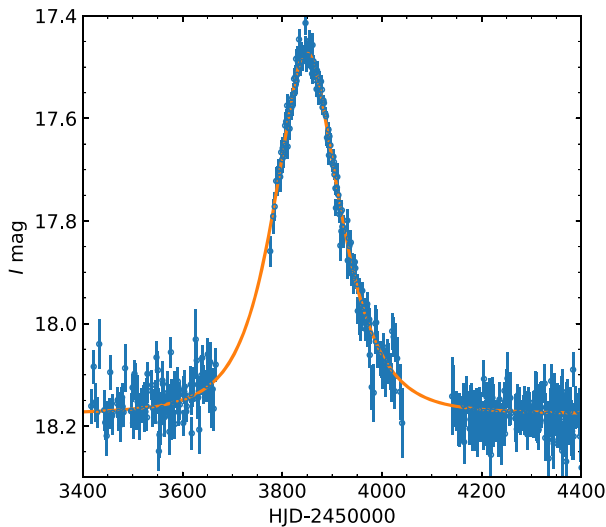
We compute  $\Delta\chi^2$  values for typical isolated BH events in an OGLE-like microlensing survey. A typical event time-scale is chosen,  $t_E = 100 \text{ d}$  (e.g. fig. 13 of Lam et al. 2020), and two sets of the parallax vector are explored,  $(\pi_{E,N}, \pi_{E,E}) = (0.1/\sqrt{2}, 0.1/\sqrt{2})$  and  $(0.03/\sqrt{2}, 0.03/\sqrt{2})$ . While the former is chosen for comparison purposes, the latter is more typical for BH events (see Fig. 1). We also fix the source magnitude to  $I = 18$  and assume no blending, which are not atypical for the OGLE-III events (Wyrzykowski et al. 2015). To mimic the OGLE-III survey, we assume that the flux noise is sky limited (i.e.  $\sigma_F = \text{constant}$  and thus  $\sigma_I \propto F^{-1}$ ) and the magnitude error  $\sigma_I = 0.04$  at  $I = 18 \text{ mag}$  source.<sup>1</sup> The cadence is chosen to be  $1 \text{ d}^{-1}$ . With two representative values of the impact parameter,  $u_0 = 0.1$  and  $0.3$ , and different values of the event peak time, we can then compute the uncertainties on the parallax components and the detectability of the parallax effect. The results are shown in Fig. 2.

As shown in Fig. 2, both components of the parallax vector show large variations with the event peak time  $t_0$ , with the eastern component,  $\pi_{E,E}$ , usually better constrained than the northern component,  $\pi_{E,N}$ . The event peak time  $t_0$  determines the orientation of the sky-projected acceleration of the Earth from the Sun relative to the Earth's trajectory. The eastern component preferentially introduces asymmetry into the microlensing light curve, whereas the northern component preferentially changes the overall magnification (e.g. Smith, Mao & Paczyński 2003). The two effects switch when the projected acceleration of the Earth from the Sun is perpendicular to Earth's trajectory (i.e. at conjunction point). Additionally, the relatively small parallax amplitudes we have chosen have a rather minor

<sup>1</sup>Such a simple assumption fails for very bright measurements ( $I \lesssim 16$ ). However, for our chosen event parameters its impact is very limited.



**Figure 2.** *Left-hand panel:* Uncertainties on the two components of the microlensing parallax vector,  $\pi_{E,N}$  (blue) and  $\pi_{E,E}$  (orange), from different event set-ups. The x-axis denotes the event peak time  $t_0$ . The parallax amplitude  $\pi_E$  is 0.1 for the dashed curves and 0.03 for the solid curves. The latter is more typical for BH events. The impact parameter  $u_0$  is 0.1 for the circles and 0.3 for the triangles. The event time-scale  $t_E$  is chosen to be 100 d and the source baseline magnitude  $I = 18$  mag. *Right-hand panel:* The resulting detectability of the parallax effect, measured in  $\Delta\chi^2$  (equation 8), for different event set-ups. The black horizontal line marks  $\Delta\chi^2 = 50$ , the threshold used in Wyrzykowski et al. (2016) for selecting BH candidate events.



**Figure 3.** Light curve of the microlensing event OGLE-2006-BLG-044. The best-fitting model is shown as orange curve.

**Table 1.** Fitting parameters of the microlensing event OGLE-2006-BLG-044. Here,  $I_b$  is the  $I$ -band magnitude of the blend object. These parameters are measured in the geocentric reference frame and the parallax reference time is set at HJD = 2453 849. The reduced  $\chi^2$  (i.e.  $\chi^2$  per degree of freedom) values are also listed. For a reference, the standard model yields  $\chi^2 = 924.8$  for 890 degrees of freedom.

	Plus solution	Minus solution
$\chi^2/\text{d.o.f.}$	921.7/888	920.8/888
$t_0 - 2450\,000$	$3848.8 \pm 0.5$	$3848.8 \pm 0.5$
$u_0$	$0.54 \pm 0.09$	$-0.58 \pm 0.06$
$t_E$ (days)	$106 \pm 12$	$99 \pm 6$
$\pi_{E,N}$	$0.06 \pm 0.05$	$-0.10 \pm 0.07$
$\pi_{E,E}$	$0.034 \pm 0.018$	$0.028 \pm 0.017$
$I_b^a$	$> 18.9$	$> 19.5$

<sup>a</sup>Derived from the 95 per cent upper limit on the blend flux.

effect on the parallax uncertainties, whereas the different impact parameters, through the larger impact on the magnifications, can lead to considerable differences in the resulting parallax uncertainties.

The main point of this exercise is to show that the microlensing parallax effect in a typical BH event (with  $t_E = 100$  d and  $\pi_E = 0.03$ ) is usually undetectable according to the criteria of Wyrzykowski et al. (2016) ( $\Delta\chi^2 > 50$ ), regardless of the exact choices of other values.

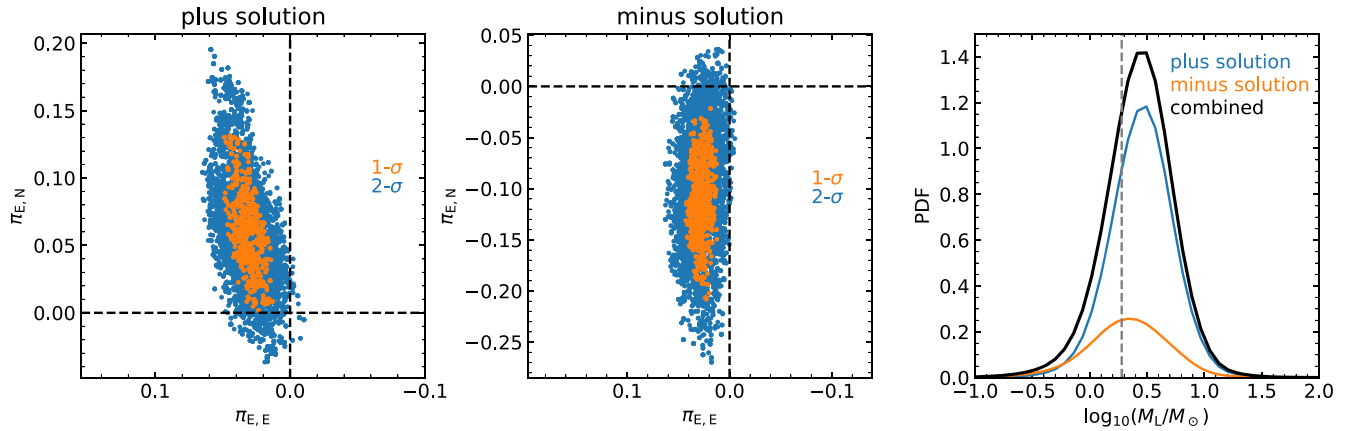
### 3 A BH CANDIDATE EVENT WITH NO PARALLAX DETECTION

Building on the conclusion from the previous section, we revisit the OGLE-III microlensing data set presented by Wyrzykowski et al. (2015) and identify one event in which the lens is possibly an isolated stellar-mass BH. The identified event did not have a statistically significant parallax signal and thus was not included in the list of candidate events in Wyrzykowski et al. (2016).

The candidate event was found through a systematic search in the OGLE-III microlensing sample. With the microlensing parameters reported by Wyrzykowski et al. (2015), we first selected events with relatively long time-scales and reasonably well-sampled light curves. These events are then fitted for microlensing parallax effect using the Markov chain Monte Carlo (MCMC) method. The parallax model is generated by the `mulensmodel` package (Poleski & Yee 2019) and the MCMC is done with the `emcee` package (Foreman-Mackey et al. 2013). The resulting Markov chains are visually inspected for convergence and the converged chains are saved for the mass inference. Two degenerate solutions due to the ecliptic degeneracy (Skowron et al. 2011) are identified. They are labelled ‘plus’ (for  $u_0 > 0$ ) and ‘minus’ (for  $u_0 < 0$ ) solutions, respectively.

Our method to infer the lens mass is similar to that of Zhu et al. (2017) but with some modifications. In a short summary, the method derives the probability distribution of the lens mass in a Bayesian framework (e.g. Batista et al. 2011)

$$P(\log M_L) \propto \int \frac{d^4\Gamma}{dt_E d \log M_L d^2 \tilde{v}_{\text{hel}}} \mathcal{L}(t_E, \tilde{v}_{\text{hel}}) dt_E d^2 \tilde{v}_{\text{hel}}, \quad (9)$$



**Figure 4.** Posterior distributions of the parallax vector for the plus (left) and minus (middle) solutions. The dashed lines indicate the zero parallax positions. The probability distributions of the lens mass are shown in the right-hand panel. Contributions from individual solutions are indicated. The grey dashed vertical line indicates  $M_L = 1.9 M_\odot$ , the 95 per cent upper limit on the lens mass if the lens is luminous.

where

$$\frac{d^4 \Gamma}{dt_E d \log M_L d^2 \tilde{v}_{\text{hel}}} = 4n_{L,*} D_L^4 f_v(\tilde{v}_{\text{hel}}) \frac{d\xi(M_L)}{d \log M_L} \frac{\mu_{\text{rel}}^3}{\tilde{v}_{\text{hel}}} \quad (10)$$

represents the prior information combining the lens mass and lens kinematics. Here,  $n_{L,*}$  is the number density of the lens at a given location,  $f_v(\tilde{v}_{\text{hel}})$  is the probability distribution of the transverse velocity vector  $\tilde{v}_{\text{hel}}$ ,  $d\xi/d \log M_L$  is the lens mass function, and  $\mathcal{L}(t'_E, \tilde{v}_{\text{hel}})$  is the likelihood distribution of the microlensing parameters from the light-curve modelling. Note that we use  $t'_E$  for the event time-scale measured in the heliocentric frame, which can be determined for a given parameter set  $\theta$ . The integral in equation (9) is done with a summation over the entire Markov chain.

For nearly all of the events modelled here, the amplitude of the parallax vector is statistically consistent with zero and thus the direction, which is also the direction of  $\tilde{v}_{\text{hel}}$ , is nearly unconstrained. To prevent the prior mass function from driving the posterior to very unusual lens kinematics, we use a log-flat distribution as the lens mass function,  $d\xi/d \log M_L \propto \text{constant}$ . Such a mass function can produce unbiased lens mass probability distribution, as is demonstrated in Appendix A. The upper limit of the mass function is extended up to  $100 M_\odot$ . Other parts of the Galactic model are detailed in Zhu et al. (2017).

The best candidate we identified is OGLE-2006-BLG-044.<sup>2</sup> The light curve is shown in Fig. 3 and our best-fitting parameters are given in Table 1. As seen from Table 1 as well as illustrated in the left-hand and middle panels of Fig. 4, the parallax parameters of this event are consistent with zero at the  $2\sigma$  level. If the lens is luminous, it would contribute to the blend flux. In other words, the blend flux provides an upper limit on the mass of a luminous lens. With the distance modulus (14.58) and the extinction ( $A_I = 2.56$ ) given by the OGLE survey at this particular line of sight (Nataf et al. 2013), we can convert the apparent  $I$ -band magnitude of the blend object into the absolute magnitude  $M_I$ . With the stellar mass–absolute magnitude relations from Pecaut, Mamajek & Bubar (2012) and Pecaut & Mamajek (2013), we then find that the mass of the luminous lens cannot be higher than  $1.6$  or  $1.9 M_\odot$  for the minus and plus solutions, respectively. Both are the 95 per cent upper limit. This

calculation has assumed that the lens is behind all the dust and very close to the source, which is assumed to be in the bulge. The mass limit for a luminous lens can only be lower if the lens gets closer.

As seen from Fig. 4, although the parallax is not statistically significantly detected in OGLE-2006-BLG-044, the inferred lens mass distribution suggests that the lens is relatively massive and probably (with 66 per cent probability) dark. If  $3 M_\odot$  is taken as the upper limit of any neutron star, then there is a 39 per cent probability that the lens in OGLE-2006-BLG-044 is a stellar-mass BH.

A similar Bayesian analysis is performed on the lens distance. The median value of the lens distance probability distribution is found to be 3.2 kpc and the 68 per cent confidence interval is 2.0–4.9 kpc. Therefore, the lens of OGLE-2006-BLG-044 is likely in the disc.

## 4 DISCUSSION

Gravitational microlensing can detect dark objects such as stellar-mass BHs. Compared to other techniques, microlensing is unique in its sensitivity to isolated BHs, which are thought to be the dominant form of stellar-mass BHs according to population synthesis models.

Previous searches for BH events in microlensing data have focused on long time-scale events with significant parallax signals. Here we show that, while BH events have indeed preferentially long time-scale, their microlensing parallax amplitudes are so small that the parallax signal becomes undetectable in the most cases.

Following this new finding, we then looked into the public OGLE-III microlensing data base and identified a new BH candidate event, OGLE-2006-BLG-044. Although the parallax signal was not detected at any statistically significant level, the long time-scale and the small parallax amplitude together suggest that the lens is probably dark and has a 39 per cent probability of being a stellar-mass BH.

Our results have implications to future searches and statistical analysis of Galactic BHs in the microlensing data set.

## ACKNOWLEDGEMENTS

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<sup>2</sup>This event is also labelled as OGLEIII-ULENS-1643 in the Wyrzykowski et al. (2015) catalogue.

## DATA AVAILABILITY

No new data were generated or analysed in support of this research.

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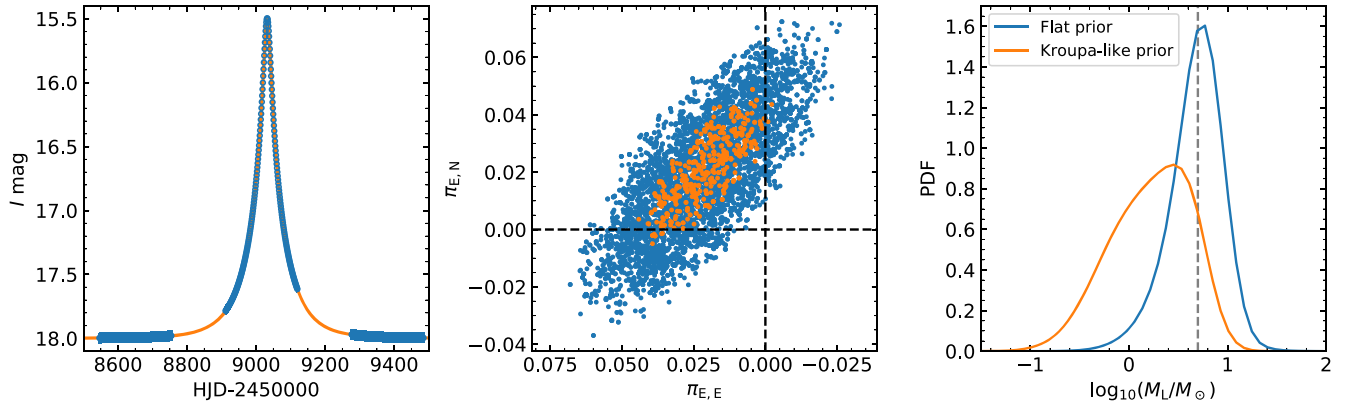
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## APPENDIX A: THE CHOICE OF MASS FUNCTION PRIOR

When the lens mass is inferred in the Bayesian analysis, a flat mass function ( $d\xi/d\log M_L$  is constant) is used. Such a mass function is far from being representative of the current-day stellar mass function, but it provides an unbiased estimate of the lens mass in our analysis.

We use a simulated microlensing event to demonstrate this point. An event light curve was generated with  $t_0 = 2459\,032$  (i.e. 2020 July 1),  $t_E = 100$  d,  $u_0 = 0.1$ , and  $\pi_{E,N} = \pi_{E,E} = 0.03/\sqrt{2}$ , with the data sampling and precision chosen to best match that of OGLE-III (see Section 2). See the left-hand panel of Fig. A1 for an illustration of the simulated light curve. Galactic microlensing events with the chosen  $t_E$  and  $\pi_E$  are overwhelmingly dominated by BH events, as can be seen clearly in fig. 13 of Lam et al. (2020). The simulated event was then processed in the same way as the OGLE-III events in this study to produce the constraints on the parallax parameters (middle panel of Fig. A1) and the lens mass probability distribution (right-hand panel of Fig. A1). The inferred lens mass distribution peaks at the most probable lens mass (i.e. BH lenses).

For a comparison, a different lens mass function was used to derive the lens mass probability distribution and the result was also shown in the right-hand panel of Fig. A1. This Kroupa-like mass function (Kroupa & Weidner 2003), with its form  $d\xi/d\log M_L \propto M_L^{-1.7}$ , gives more weights to the low-mass lenses, making it a better match to the current-day stellar mass function (although it is still far from perfect). However, the lens mass probability distribution inferred with the use of this more realistic mass function would suggest that the lens is most likely a normal star. The use of this Kroupa-like mass function, or any kind of realistic stellar mass function, tends to downplay the importance of the information from the parallax constraint (or measurement) and therefore leads to an underestimated lens mass probability distribution.



**Figure A1.** A simulated BH event light curve (left-hand panel), the constraint on the microlensing parallax parameters (middle panel), and the inferred lens mass probability distribution (right-hand panel). We use  $t_E = 100$  d,  $u_0 = 0.1$ , and  $\pi_{E,N} = \pi_{E,E} = 0.03/\sqrt{2}$  in the light-curve simulation. Events with the chosen  $t_E$  and  $\pi_E$  are overwhelmingly dominated by BH lenses (e.g. fig. 13 of Lam et al. 2020). The meanings of the data points in the middle panel are the same as those in Fig. 4. When inferring the lens mass distribution, we have used two different mass priors. A flat prior can successfully recover the most likely mass of the lens, with the vertical dashed line marking  $M_L = 5 M_\odot$ . A steep mass function that better matches the current stellar mass function,  $d\xi/d \log M_L \propto M_L^{-1.7}$ , which we call a Kroupa-like prior (Kroupa & Weidner 2003), will lead to an underestimated lens mass probability distribution.

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