

Interacting dark energy in a closed universe

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ABSTRACT

Recent measurements of the Cosmic Microwave Anisotropies power spectra measured by the *Planck* satellite show a preference for a closed universe at more than 99 per cent confidence level (CL). Such a scenario is however in disagreement with several low redshift observables, including luminosity distances of Type Ia supernovae. Here we show that interacting dark energy (IDE) models can ease the discrepancies between *Planck* and supernovae Ia data in a closed Universe, leading to a preference for both a coupling and a curvature different from zero above the 99 per cent CL. Therefore IDE cosmologies remain as very appealing scenarios, as they can provide the solution to a number of observational tensions in different fiducial cosmologies. The results presented here strongly favour broader analyses of cosmological data, and suggest that relaxing the usual flatness and vacuum energy assumptions can lead to a much better agreement among theory and observations.

Key words: cosmic background radiation – cosmological parameters – cosmology: observations – dark energy.

1 INTRODUCTION

The recent *Planck* cosmic microwave background (CMB) power spectra, when analysed with the official *Plik* likelihood, show a clear preference for a closed Universe at more than three standard deviations (Di Valentino, Melchiorri & Silk 2019; Handley 2019; Aghanim et al. 2020b; Di Valentino, Melchiorri & Silk 2020a). This result clearly introduces a problem for the standard Λ CDM cosmological scenario, based on the inflationary prediction of a flat universe. Undetected systematics can play a role and it is obviously too soon to exclude a flat universe. For example, the authors of the *CamSpec* alternative likelihood claim a larger value of the χ^2 fit for closed universes. However, the marginalized constraints obtained from this alternative likelihood still prefer closed models at a significant level [larger than 99 per cent confidence level (CL)].

While the compatibility with a closed universe of the *Planck* data set is solid, a major problem of models with positive curvature is that they further exacerbate to at least 5.4σ (Di Valentino et al. 2019), the already 4.4σ tension on the Hubble constant between the H_0 value measured by *Planck* Aghanim et al. (2020b) in a Λ CDM model and the value obtained by the SH0ES collaboration (Riess et al. 2019; R19).¹ Furthermore, introducing a curvature in the universe free to vary implies a tension between *Planck* CMB and Baryon Acoustic Oscillation (BAO) data (Di Valentino et al. 2019; Handley 2019), and also between *Planck* CMB observations and the full-shape galaxy power spectrum (Vagnozzi et al. 2020).

The authors of Ref. Di Valentino et al. (2020a) presented the possibility of solving the tension between *Planck* and supernovae Ia luminosity distance measurements within a closed universe by including phantom dark energy, ruling out both the flatness and the cosmological constant scenario at more than 99 per cent CL, while the tension with the BAO data was still persistent.

Clearly, the tension with BAO is a major problem for these closed-phantom models. However one could argue that the BAO reconstruction from galaxy data (that is performed under the assumption of Λ CDM) could be affected by a radically different choice of the dark energy component. It is therefore interesting to consider more physically motivated dark energy models that could induce an effective phantom behaviour in the context of a closed universe.

In this manuscript, we consider a model of dark energy (DE) interacting with the dark matter (DM) as a possibility for solving the tension between *Planck* and the luminosity distance measurements within a non-flat universe. The interacting dark energy (IDE) scenarios (Pettorino 2013; Bolotin et al. 2015; Wang et al. 2016; Yang, Pan & Barrow 2018) became very promising due to an evidence of a non-null interaction (Salvatelli et al. 2014; Di Valentino, Melchiorri & Mena 2017; Kumar & Nunes 2017; Kumar, Nunes & Yadav 2019) and recently got plenty of attention as alternative scenarios for solving the Hubble constant tension (see e.g. Di Valentino et al. 2020c, 2020d; Gómez-Valent, Pettorino & Amendola 2020; Lucca & Hooper 2020; Pan, Yang & Paliathanasis 2020c; Yang et al. 2019, 2020a, 2020b) for updated results with *Planck* 2018, and references therein with older data. This happens because the flux of energy from the DM sector to the DE one within some IDE scenarios naturally provides a higher value of the Hubble constant H_0 to compensate for both the lowering of the matter energy density and the shift of the acoustic peaks in the damping tail [see also Di

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¹For a recent overview of the H_0 tension, see Di Valentino et al. (2020b).

Table 1. Flat priors on the main cosmological parameters used in this work.

Parameter	Prior
$\Omega_b h^2$	[0.005, 0.1]
$\Omega_c h^2$	[0.001, 0.99]
$100\theta_{MC}$	[0.5, 10]
τ	[0.01, 0.8]
n_s	[0.7, 1.3]
$\log [10^{10} A_s]$	[1.7, 5.0]
Ω_k	[-3, 3]
ξ	[-1, 0]

Valentino & Mena (2020) for a discussion about degeneracies in the parameters]. Therefore, it is timely to explore whether IDE scenarios can reconcile the discrepancies between *Planck* and supernovae Ia luminosity distance measurements within non-flat cosmologies.

The paper is organized as follows. In Section 2, we present the IDE model analysed here. Section 3 presents the methodology and the data sets used in this work, while in Section 4 we show the results obtained in our analysis and discuss their physical implications. Finally, Section 5 summarizes the conclusions of the present work.

2 IDE OVERVIEW

In the context of a Friedmann–Lemaître–Robertson–Walker (FLRW) universe allowing spatial curvature, we consider a very generalized cosmic scenario where the two main dark fluids of the universe, namely a pressureless DM and a DE component share an energy and/or momentum exchange mechanism in a non-gravitational way. The energy densities of the pressureless DM and DE are respectively denoted by ρ_c and ρ_x and additionally we assume that DE has a constant equation-of-state w . At the background level, the conservation equations for the pressureless DM and DE components can be decoupled into two separate equations with an inclusion of an arbitrary function, Q , known as the coupling or interacting function, as follows:

$$\dot{\rho}_c + 3\mathcal{H}\rho_c = Q, \quad (1)$$

$$\dot{\rho}_x + 3\mathcal{H}(1+w)\rho_x = -Q, \quad (2)$$

where the dot denotes the differentiation with respect to the conformal time τ , and $\mathcal{H} \equiv \dot{a}/a$ refers to the conformal Hubble rate of the FLRW universe. The function Q determines the direction of energy (and/or momentum) transfer between the dark sectors through its sign. For instance, $Q > 0$ ($Q < 0$) indicates the transfer of energy (and/or momentum) from DE (DM) to DM (DE). Once a specific function for the interaction rate Q is prescribed, by solving either analytically or numerically the continuity equations equation (1) and equation (2) the evolution of the universe will be fully determined, as all the other fluids are obeying the usual conservation equations, that is, they do not take part in the interaction process. Therefore, the interaction function plays a crucial role in the determination of the cosmic dynamics. Usually, there is no final form of the interaction function in the literature, but there are some well-known interaction functions. This is the case of the IDE model described by

$$Q = \xi \mathcal{H} \rho_x, \quad (3)$$

where ξ is the dimensionless coupling parameter which characterizes the strength of the interaction between the dark sectors. The coupling function in equation (3) was initially proposed purely from a phenomenological perspective. While most of the interaction functions are phenomenological, some recent investigations have shown that

the model in equation (3) can be derived from some multi-scalar field action (Pan, Sharov & Yang 2020a). Additionally, apart from the scalar field theory, the origin of various interaction functions can also be motivated from other existing cosmological theories with a Lagrangian description (Pan et al. 2020b). Finally, we note that the model can be analytically solved leading to the closed form expressions for ρ_c and ρ_x (Di Valentino et al. 2020d). The coincidence parameter $r = \rho_c/\rho_x$ also assumes an analytic expression and for $z \rightarrow 0$, r becomes a constant, thus, alleviating the coincidence problem.

The interaction term Q also affects the perturbation equations. In the context of linear perturbation theory, assuming the synchronous gauge, one can write down the density perturbation δ and the velocity divergence θ of the dark fluids as (Valiviita, Majerotto & Maartens 2008; Gavela et al. 2009, 2010; Di Valentino et al. 2020c):

$$\dot{\delta}_c = -\theta_c - \frac{1}{2}\dot{h} + \xi \mathcal{H} \frac{\rho_x}{\rho_c} (\delta_x - \delta_c) + \xi \frac{\rho_x}{\rho_c} \left(\frac{kv_T}{3} + \frac{\dot{h}}{6} \right), \quad (4)$$

$$\dot{\theta}_c = -\mathcal{H}\theta_c, \quad (5)$$

$$\begin{aligned} \dot{\delta}_x = & -(1+w) \left(\theta_x + \frac{\dot{h}}{2} \right) - \xi \left(\frac{kv_T}{3} + \frac{\dot{h}}{6} \right) \\ & - 3\mathcal{H}(1-w) \left[\delta_x + \frac{\mathcal{H}\theta_x}{k^2} (3(1+w) + \xi) \right], \end{aligned} \quad (6)$$

$$\dot{\theta}_x = 2\mathcal{H}\theta_x + \frac{k^2}{1+w}\delta_x + 2\mathcal{H}\frac{\xi}{1+w}\theta_x - \xi \mathcal{H}\frac{\theta_c}{1+w}. \quad (7)$$

The governing equations both at the background and perturbation levels completely determine the dynamics of the interacting universe.

Finally, we make an important comment on the early time instabilities which are associated with the interacting cosmic scenarios. As already noticed, the interaction function introduces a new free parameter ξ , which controls the energy flow between the dark sectors and along with the dark energy equation of state parameter, w , it plays a very active part in the modified perturbation equations to determine whether the interaction model leads to early time instabilities or not. This problem has been examined by several works in the past (Gavela et al. 2009, 2010) which led to the conclusion that the instability problem can be avoided if the signs of ξ and $(1+w)$ are different. Therefore, in the present article we have considered the opposite signs for ξ and $(1+w)$ in order to ensure that the underlying model does not suffer from early time instabilities, fixing the dark energy equation of state parameter $w = -0.999$.

3 OBSERVATIONAL DATA AND METHODOLOGY

We consider a baseline IDE + Ω_k model described by eight cosmological parameters (Table 1). These will be the baryon energy density $\Omega_b h^2$, the cold dark matter energy density $\Omega_c h^2$, the ratio between the sound horizon and the angular diameter distance at decoupling θ_{MC} , the reionization optical depth τ , the amplitude of the scalar primordial power spectrum A_s , the spectral index n_s , the dimensionless coupling ξ , and the curvature parameter Ω_k .

To analyse this interacting scenario with curvature and also to derive the cosmological constraints, we make use of the combinations of the most recent observational data from various sources listed below:

(i) ***Planck***: We consider as a baseline data set the latest CMB measurements provided by the final 2018 *Planck* legacy release (Aghanim et al. 2020a, 2020b).

Table 2. Observational constraints at 68 per cent CL on the independent and derived cosmological parameters arising from analyses to *Planck* observations within the Λ CDM, Λ CDM + Ω_k and IDE + Ω_k cosmologies. In the bottom line, we quote the $\ln B_{ij}$ computed with respect to the Λ CDM cosmology. The positive values are indicating a preference for the models different from the Λ CDM one.

Parameters	λ CDM	IDE	λ CDM + Ω_k	IDE + Ω_k
$\Omega_b h^2$	0.02236 ± 0.00015	0.02239 ± 0.00015	0.02260 ± 0.00017	0.02261 ± 0.00017
$\Omega_c h^2$	0.1202 ± 0.0014	<0.0634	0.1181 ± 0.0015	$0.077^{+0.035}_{-0.019}$
$100\theta_{MC}$	1.04090 ± 0.00031	$1.0458^{+0.0033}_{-0.0021}$	1.04116 ± 0.00033	$1.0437^{+0.0012}_{-0.0023}$
τ	$0.0544^{+0.0070}_{-0.0081}$	0.0541 ± 0.0076	0.0486 ± 0.0082	$0.0481^{+0.0085}_{-0.0076}$
n_s	0.9649 ± 0.0044	0.9655 ± 0.0043	0.9706 ± 0.0048	0.9708 ± 0.0047
$\ln(10^{10} A_s)$	3.045 ± 0.016	3.044 ± 0.016	3.028 ± 0.017	$3.027^{+0.017}_{-0.016}$
ξ	[0]	$-0.54^{+0.12}_{-0.28}$	[0]	<-0.385
Ω_k	[0]	[0]	$-0.044^{+0.018}_{-0.015}$	$-0.036^{+0.017}_{-0.013}$
$H_0[(\text{km s}^{-1}) \text{Mpc}^{-1}]$	67.27 ± 0.60	$72.8^{+3.0}_{-1.5}$	$54.4^{+3.3}_{-4.0}$	$58.7^{+4.1}_{-5.2}$
σ_8	0.8120 ± 0.0073	$2.3^{+0.4}_{-1.4}$	0.744 ± 0.015	$1.31^{+0.10}_{-0.54}$
Ω_m	0.3166 ± 0.0084	$0.139^{+0.034}_{-0.095}$	$0.485^{+0.058}_{-0.068}$	0.30 ± 0.11
S_8	0.834 ± 0.016	$1.30^{+0.17}_{-0.44}$	0.981 ± 0.049	$1.20^{+0.10}_{-0.22}$
$\ln B_{ij}$	–	1.2	2.3	2.5

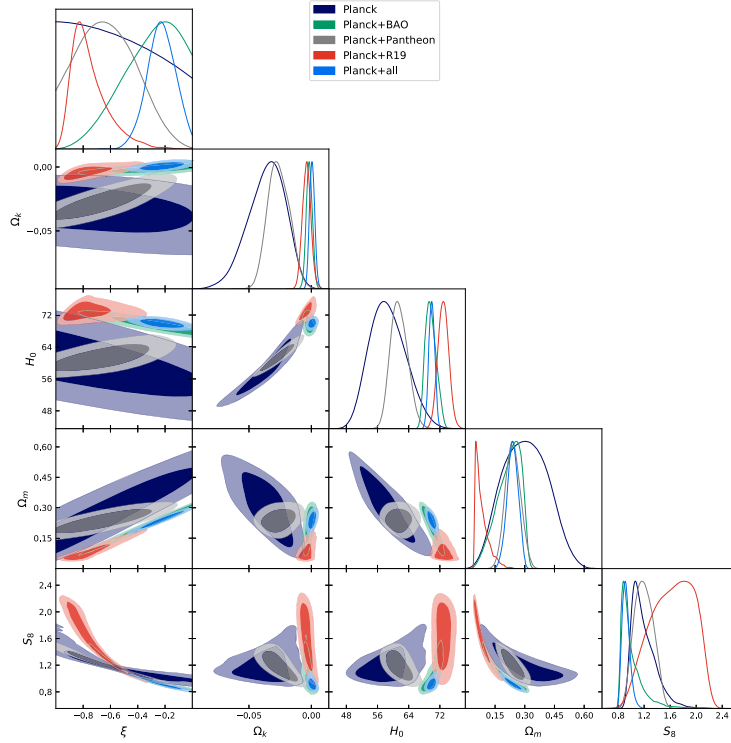


Figure 1. One-dimensional posterior distributions and two-dimensional joint contours at 68 and 95 per cent CL for the IDE + Ω_k model.

(ii) **BAO:** We include a compilation of BAO measurements from different experiments, namely 6dFGS (Beutler et al. 2011), SDSS-MGS (Ross et al. 2015), and BOSS DR12 (Alam et al. 2017) surveys, as used by the *Planck* collaboration in Aghanim et al. (2020b).

(iii) **Pantheon:** We use the 1048 data points in the redshift region $z \in [0.01, 2.3]$ of the luminosity distance data of type Ia supernovae from the Pantheon catalogue (Scolnic et al. 2018).

(iv) **R19:** We add a Gaussian prior on the Hubble constant as estimated from a reanalysis of the *Hubble Space Telescope* data using

Cepheids as calibrators by the SHOES collaboration in 2019 (Riess et al. 2019) i.e. $H_0 = 74.03 \pm 1.42 \text{ km s}^{-1} \text{ Mpc}^{-1}$ at 68 per cent CL.

For the analysis, we use our modified version of the publicly available Markov chain Monte Carlo code *CosmoMC* (Lewis, Challinor & Lasenby 2000; Lewis & Bridle 2002) package (see <http://cosmologist.info/cosmomc/>). This code supports the 2018 *Planck* likelihood (Aghanim et al. 2020a), implements an efficient sampling of the posterior distribution using the fast/slow parameter decorrelations (Lewis 2013), and has a convergence diagnostic based on the Gelman-Rubin statistics (Gelman & Rubin 1992).

Table 3. Observational constraints at 68 per cent CL on the independent and derived cosmological parameters arising from analyses to different data combinations within the IDE + Ω_k model. The $\ln B_{ij}$ are computed with respect to the Λ CDM + Ω_k cosmology for the very same data set combination. The negative values are indicating a preference for the Λ CDM + Ω_k scenario, while the positive values for the IDE + Ω_k model.

Parameters	<i>Planck</i>	<i>Planck</i> + BAO	<i>Planck</i> + Pantheon	<i>Planck</i> + R19	<i>Planck</i> + all
$\Omega_b h^2$	0.02261 ± 0.00017	0.02241 ± 0.00016	0.02258 ± 0.00016	0.02247 ± 0.00016	0.02239 ± 0.00015
$\Omega_c h^2$	$0.077^{+0.035}_{-0.019}$	$0.082^{+0.033}_{-0.015}$	$0.068^{+0.013}_{-0.018}$	<0.0253	$0.093^{+0.013}_{-0.011}$
$100\theta_{MC}$	$1.0437^{+0.0012}_{-0.0023}$	$1.04327^{+0.00009}_{-0.00022}$	$1.0442^{+0.0012}_{-0.0010}$	$1.0480^{+0.0020}_{-0.0008}$	$1.04249^{+0.00074}_{-0.00086}$
τ	$0.0481^{+0.0085}_{-0.0076}$	0.0541 ± 0.0081	0.0495 ± 0.0080	0.0534 ± 0.0079	0.0542 ± 0.0079
n_s	0.9708 ± 0.0047	0.9662 ± 0.0047	0.9701 ± 0.0046	0.9679 ± 0.0046	0.9653 ± 0.0047
$\ln(10^{10} A_s)$	$3.027^{+0.017}_{-0.016}$	3.043 ± 0.016	3.031 ± 0.017	3.040 ± 0.016	3.045 ± 0.016
ξ	<-0.385	$-0.32^{+0.31}_{-0.09}$	$-0.62^{+0.19}_{-0.25}$	$-0.75^{+0.06}_{-0.16}$	-0.23 ± 0.10
Ω_k	$-0.036^{+0.017}_{-0.013}$	-0.0016 ± 0.0024	-0.0261 ± 0.0087	-0.0038 ± 0.0034	0.0006 ± 0.0021
$H_0[(\text{km s}^{-1}) \text{Mpc}^{-1}]$	$58.7^{+4.1}_{-5.2}$	$69.7^{+1.2}_{-1.6}$	$61.6^{+2.0}_{-2.4}$	72.9 ± 1.4	69.93 ± 0.75
σ_8	$1.31^{+0.10}_{-0.54}$	$1.27^{+0.04}_{-0.46}$	$1.36^{+0.20}_{-0.31}$	$3.4^{+1.2}_{-1.4}$	$1.04^{+0.08}_{-0.15}$
Ω_m	0.30 ± 0.11	$0.219^{+0.076}_{-0.040}$	0.240 ± 0.038	$0.084^{+0.010}_{-0.039}$	0.239 ± 0.028
S_8	$1.20^{+0.10}_{-0.22}$	$1.01^{+0.04}_{-0.18}$	$1.20^{+0.14}_{-0.16}$	$1.64^{+0.41}_{-0.27}$	$0.921^{+0.043}_{-0.069}$
$\ln B_{ij}$	0.2	-1.0	3.2	5.8	-0.4

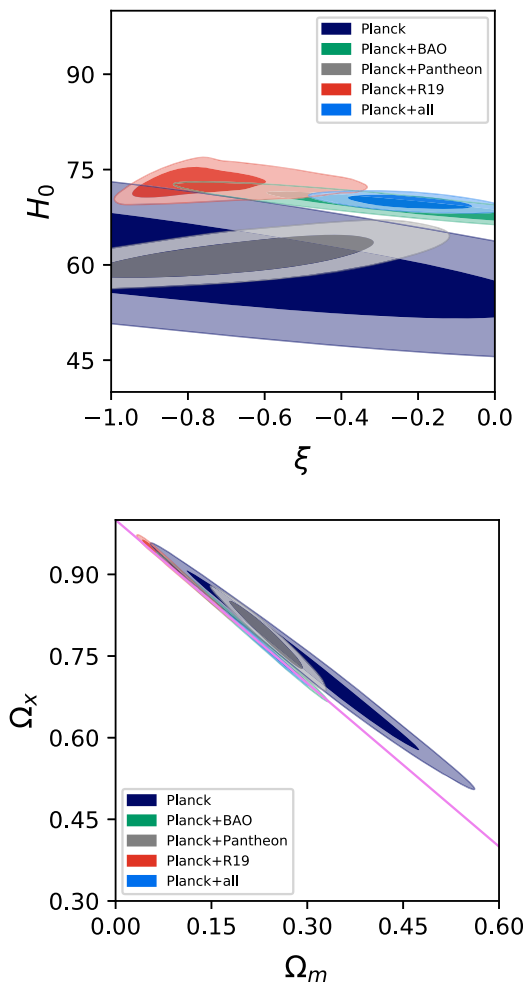


Figure 2. Two-dimensional contour plots at 68 and 95 per cent CL for the IDE + Ω_k model, showing the planes (H_0, ξ) in the top panel and (Ω_m, Ω_x) in the bottom one, where the magenta line corresponds to the flatness scenario.

4 RESULTS

We show in Table 2 a comparison of the constraints on the cosmological parameters obtained within four different scenarios: the flat Λ CDM standard model, the flat IDE model, the Λ CDM plus curvature model, and the IDE plus curvature model.

The first thing that we can notice is that the combination of the IDE model with a curvature of the universe softens the critical peculiarities we have for the two models separately (see Table 2), as for example the particular lower/higher values for H_0/Ω_m in the Λ CDM + Ω_k scenario, or the strong evidence for ξ associated to an exceptional lower amount of dark matter in the IDE model. Indeed, within the IDE + Ω_k scenario, we find for *Planck* a much more reasonable (larger) value for the matter density Ω_m (see also Fig. 1) that corresponds to a bound on $\Omega_c h^2$, instead of just obtaining an upper limit, as in the case of flat IDE cosmologies (see Table 2). A word of caution is needed here. While the values of the present matter density Ω_m may be argued to be very small and incompatible with structure formation processes, we remind the reader that within interacting cosmologies the growth of dark matter perturbations will be larger than in uncoupled models. This feature will be general for models with negative coupling and in which the energy exchange among the dark sectors is proportional to ρ_x , due to a suppression of the friction term and an enhancement of the source term in the differential growth equation (see e.g. Caldera-Cabral, Maartens & Schaefer 2009; Lopez Honorez et al. 2010). While this statement holds at the linear perturbation level, a very similar behaviour will be expected at mildly non-linear smaller scales.

Notice that the preference for a closed universe persists in the IDE + Ω_k case at more than 99 per cent CL, but with a slightly larger value for the Hubble constant, lowering the tension with R19 at 3.6σ due to the larger error bars. These two effects can be clearly noticed from the results depicted in Fig. 1 that illustrates the one- and two-dimensional posterior probability distributions for some of the most interesting parameters. Finally, the improvement in the fit for the IDE + Ω_k model is significant, as we can see from the logarithm of the Bayes factor, B_{ij} with respect to the Λ CDM model (computed using the MCEVIDENCE code Heavens et al. 2017b, Heavens et al. 2017a) that is equal to $\ln B_{ij} = 2.5$ i.e. a *definite* evidence according to the revised Jeffreys scale Kass & Raftery (1995).

We present in Table 3 the bounds on the IDE + Ω_k model for several combinations of the data sets. The most evident result is that for the IDE + Ω_k scenario *Planck* and Pantheon data sets are in excellent agreement, leading to a preference for a curvature component and a coupling different from zero with a significance above 99 per cent CL (see Fig. 2), and a Bayes factor equal to $\ln B_{ij} = 3.2$, indicating a *strong* evidence for this scenario with respect to the λ CDM + Ω_k case. However, this agreement is happening at the price of raising again the H_0 tension with R19 to the 5.2σ level, and it is also in strong disagreement with the BAO data, as can be noticed from the left panel of Fig. 2. Indeed, the *Planck* + BAO data set combination prefers a flat universe within 68 per cent CL and a coupling different from zero at about 1σ , while the *Planck* + R19 prefers a closed universe at about one standard deviation and a coupling at more than 99 per cent CL. We can, however, note that the constraints from *Planck* + R19 on the matter density and the amplitude of scalar perturbations, albeit at the linear level, are probably unrealistic even considering the word of caution stressed before. The strong improvement in the evidence of Table 3 for this case should therefore be considered with some grain of salt since the inclusion of clustering data could strongly play against this solution. In a few words, the simple Ω_k + IDE scenario considered here does not solve the Hubble tension, so we need to consider further extensions.

5 CONCLUSIONS

In this paper, we consider an extension of the standard Λ CDM model by introducing simultaneously a non-gravitational interaction between DE and DM together with a curvature component in our universe. The aim is to investigate whether the very same IDE scenario, strongly motivated for solving the H_0 tension, is also able to solve the existing tensions between high and low redshift observations within non-flat cosmologies (namely, the very low value for Hubble constant H_0 obtained within the Λ CDM + Ω_k scenario). Therefore, this is a very general scheme, which, in one hand, extends the non-interacting cosmological scenarios including the λ CDM as the base model, and, on the other hand, includes the very interesting and timely possibility of a curvature component in our universe. Such a possibility has recently been strengthened from the recent observational evidences. Our analyses confirm previous findings within the simpler non-flat λ CDM picture: *Planck* observations prefer a positive curvature of the universe at more than 99 per cent CL, but this exacerbates the Hubble constant tension at more than 5σ . While non-flat IDE scenarios provide a larger value of H_0 , the tension is still present with a significance of 3.6σ .

Nevertheless other forms for the interaction function and for the equation of state of the dark energy component could further alleviate this tension, as it is eased in the right direction due to the much lower value of Ω_m required within some family of IDE models. Interactions among the dark sectors of our universe therefore remain as a very appealing scenarios, as they can provide the solution to a number of cosmological tensions in different fiducial models.

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DATA AVAILABILITY

We used the publicly available cosmological probes, such as the CMB power spectra from *Planck* 2018, BAO data from 6dFGS, SDSS-MGS, BOSS DR12, Hubble constant measurement from the SH0ES collaboration, and the Pantheon catalogue.

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