Spin evolution of neutron stars in two modes: implication for millisecond pulsars

Sudip Bhattacharyya ⓘ ★

Department of Astronomy and Astrophysics, Tata Institute of Fundamental Research, Mumbai 400005, India

ABSTRACT

An understanding of spin frequency (ν) evolution of neutron stars in the low-mass X-ray binary (LMXB) phase is essential to explain the observed ν-distribution of millisecond pulsars (MSPs), and to probe the stellar and binary physics, including the possibility of continuous gravitational wave emission. Here, using numerical computations we conclude that ν can evolve in two distinctly different modes, as ν may approach a lower spin equilibrium value ($\nu_{\rm eq,per}$) for persistent accretion for a long-term average accretion rate ($\dot{M}_{\rm av}$) greater than a critical limit ($\dot{M}_{\rm av,crit}$), and may approach a higher effective spin equilibrium value ($\nu_{\rm eq,eff}$) for transient accretion for $\dot{M}_{\rm av} < \dot{M}_{\rm av,crit}$. For example, when $\dot{M}_{\rm av}$ falls below $\dot{M}_{\rm av,crit}$ for an initially persistent source, ν increases considerably due to transient accretion, which is counter-intuitive. We also find that, contrary to what was suggested, a fast or sudden decrease of $\dot{M}_{\rm av}$ to zero in the last part of the LMXB phase is not essential for the genesis of spin-powered MSPs, and neutron stars could spin-up in this $\dot{M}_{\rm av}$-decreasing phase. Our findings imply that the traditional way of ν-evolution computation is inadequate in most cases, even for initially persistent sources, and may not even correctly estimate whether ν increases or decreases.

Key words: accretion, accretion discs – methods: numerical – stars: neutron – pulsars: general – stars: rotation – X-rays: binaries.

1 INTRODUCTION

A rapidly spinning neutron star, with a measured spin frequency (ν) of $\gtrsim 100$ Hz, is typically known as a millisecond pulsar (MSP; Bhattacharya & van den Heuvel 1991). Such a star is believed to be spun up in its low-mass X-ray binary (LMXB) phase, due to the angular momentum transfer by the matter accreted from the companion donor star (Radhakrishnan & Srinivasan 1982; Alpar et al. 1982; Wijnands & van der Klis 1998; Chakrabarty & Morgan 1998; Archibald et al. 2009; Papitto et al. 2013; Bassa et al. 2014). A subset of neutron stars in the LMXB phase are observed as accretion-powered millisecond X-ray pulsars (AMXPs; Patruno & Watts 2012; Salvo & Sanna 2020, and references therein) and nuclear-powered millisecond X-ray pulsars (NMXPs; Watts 2012; Bhattacharyya 2020a, and references therein), and after this phase some of the stars manifest themselves as spin-powered MSPs (Bhattacharya & van den Heuvel 1991).

The neutron star spin evolution can happen by accretion related spin-up and spin-down torques and by electromagnetic (EM) and gravitational wave (GW) spin-down torques (e.g. Bildsten 1998; Bhattacharyya & Chakrabarty 2017; Haskell & Patruno 2017; Bhattacharyya 2017, 2020b; Chen 2020). Hence, the observed ν-value distribution of MSPs can be very useful to probe the properties and evolution of neutron stars and their binary systems, the stellar magnetospheric emission and pulsar wind, which give rise to the EM torque, and a plausible stellar ellipticty, which should give rise to the GW torque. Two notable observational aspects of this distribution

are it cuts off sharply $\sim$730 Hz (Chakrabarty et al. 2003; Ferrario & Wickramasinghe 2007; Patruno 2010) and MSPs in LMXBs appear to have overall higher ν-values than the post-LMXB phase spin-powered MSPs (Tauris 2012; Papitto et al. 2014; Patruno, Haskell & Andersson 2017). Note that one needs to understand such observed aspects of the ν-value distribution in order to use it as a tool to probe the physics of neutron stars and binaries.

The ν-value distribution is primarily caused by the spin evolution in the LMXB phase, which is generally explained in terms of the spin equilibrium frequency ($\nu_{\rm eq}$) for persistent accretion, as we briefly describe below. Note that 'persistent accretion' implies that the source does not show alternate outburst and quiescent cycles with a time-scale of months to years as observed from transients (Lin et al. 2019), but the long-term average accretion rate ($\dot{M}_{\rm av}$) can evolve with a time-scale of $\sim$ a hundred million years or more. An accreting neutron star in the LMXB phase spins up and down due to the accretion through the stellar magnetosphere. The magnetosphere can stop a thin, Keplerian disc at the magnetospheric radius (e.g. Wang 1996)

$$r_{\rm m} = \xi \left(\frac{\mu^4}{2GM\dot{M}^2} \right)^{1/7}, \tag{1}$$

where μ ($=BR^3$) is the neutron star magnetic dipole moment, B is the stellar surface dipole magnetic field, M and R are the stellar mass and radius respectively, $\dot{M}$ is the instantaneous accretion rate, and ξ is an order of unity constant. Note that, for persistent accretion, we consider $\dot{M} = \dot{M}_{\rm av}$. In the accretion phase, $r_{\rm m}$ is less than the light-cylinder radius $r_{\rm lc}$ ($=c/2\pi\nu$) and the corotation radius $r_{\rm co}$, given by

★ E-mail: sudip@tifr.res.in

used to mean 3.14.

$$r_{co} = \left(\frac{GM}{4\pi^2\nu^2} \right)^{1/3}, \qquad (2)$$

and matter and angular momentum are transferred to the neutron star. On the other hand, in the propeller phase ($r_{co} < r_m < r_{lc}$), the accreted matter is at least partially driven away from the system, and the star loses angular momentum (see Watts 2012; Bhattacharyya & Chakrabarty 2017). In the accretion phase, the star spins up and hence r_{co} decreases, and in the propeller phase, the star spins down and hence r_{co} increases. Therefore, r_{co} approaches r_m, and the neutron star reaches the spin equilibrium for $r_{co} = r_m$, with ν attaining the spin equilibrium frequency:

$$\nu_{eq} = \frac{1}{2\pi}\sqrt{\frac{GM}{r_m^3}} = \frac{1}{2^{11/14}\pi\xi^{3/2}}\left(\frac{G^5 M^5 \dot{M}^3}{\mu^6} \right)^{1/7}. \qquad (3)$$

After this, ν typically tracks ν_{eq}. But if $\dot{M}$ relatively slowly decreases to zero in the last part of the LMXB phase, and ν tracks ν_{eq} to a very small value (since, $\nu_{eq} \propto \dot{M}^{3/7}$), no spin-powered MSP would be created (e.g. Ruderman, Shaham & Tavani 1989; Lamb & Yu 2005; Tauris 2012). Therefore, it was indicated that $\dot{M}$ needs to decrease so fast in this last phase of accretion, that ν cannot track ν_{eq} (e.g. Ruderman et al. 1989). Later, it was reported with an example of numerical computation of binary stellar evolution that, while ν decreases when $\dot{M}$ drastically and rapidly decreases in the Roche lobe decoupling phase (RLDP) in the last part of the LMXB phase, it may not decrease as much as ν_{eq}, because ν may not track ν_{eq} due to a fast $\dot{M}$ decrease (Tauris 2012). It was suggested that this moderate decrease of ν perhaps explains the overall higher ν-values of MSPs in LMXBs relative to ν-values of spin-powered MSPs in the post-LMXB phase (Tauris 2012). In this Letter, we show that ν could increase due to transient accretion in the RLDP, conclude that ν should not attain a small value even if $\dot{M}$ relatively slowly decreases to zero at the end of the LMXB phase, and report complex ν-evolution possibilities and pathways.

2 TRANSIENT ACCRETION

Most neutron star LMXBs, including all AMXPs, accrete matter in a transient manner (Liu, van Paradijs & van den Heuvel 2007; Salvo & Sanna 2020). This results in alternate outburst and quiescent phases. The former phase lasts typically days to weeks when $\dot{M}$ and the observed X-ray intensity increase by several orders of magnitude from those in the latter phase, which lasts typically months to years (e.g. Yan & Yu 2015). These outbursts of accretion are believed to be caused by two instabilities: the thermal instability, for which a small increase in the accretion disc temperature T_{disc} causes an additional rise in T_{disc}, and the viscous instability, for which a little increase in $\dot{M}$ leads to a further increase in $\dot{M}$ (see Lasota 2001; Done, Gierliński & Kubota 2007, and references therein). The time-scale of the former instability is much less than the time-scale of the latter. A thermal instability could occur for $\dot{M}_{av} < \dot{M}_{av,crit}$ ($\dot{M}_{av,crit}$ is a critical $\dot{M}_{av}$), when due to the accumulation of matter, the T_{disc} value at a certain radius of an initially cold and non-ionized disc exceeds the hydrogen ionization temperature, and hence the opacity increases by a large amount, photons cannot escape easily anymore, and consequently T_{disc} increases sharply. This triggers the viscous instability, causing a large increase of $\dot{M}$, and hence an outburst happens. But if $\dot{M}_{av}$ is sufficiently high ($\dot{M}_{av} > \dot{M}_{av,crit}$), so that T_{disc} in the entire disc always exceeds the hydrogen ionization temperature, the disc should always be stable, and accretion should
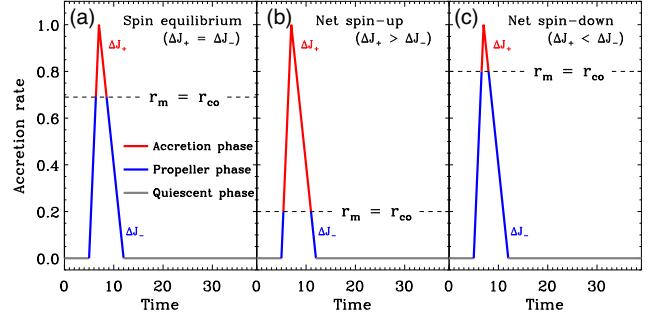

Figure 1. Schematic illustrations of an outburst cycle of a neutron star LMXB (see also Bhattacharyya & Chakrabarty 2017; Bhattacharyya 2017). The evolution of the instantaneous accretion rate $\dot{M}$, normalized by the peak accretion rate $\dot{M}_{peak}$, through three phases is shown. Here, we assume a triangular outburst profile, and the time is in an arbitrary unit. During an outburst, the magnetospheric radius r_m evolves drastically, while the corotation radius r_{co} remains almost same, and the transition to/from the accretion phase (shown in red) from/to the propeller phase (shown in blue) happens when r_m becomes equal to r_{co}. Panel (a) is for an overall spin equilibrium of the neutron star, as the total positive angular momentum (ΔJ_+) transferred in the accretion phase and the total negative angular momentum (ΔJ_-) transferred in the propeller phase are equal. Panels (b) and (c) are for a net spin-up and a net spin-down, respectively (see Section 2).

happen persistently. Therefore, an initially persistent neutron star LMXB can become a transient, whenever $\dot{M}_{av}$ falls below $\dot{M}_{av,crit}$. Particularly, this must happen in the last part (e.g. RLDP) of the LMXB phase, at the end of which accretion stops. Note that an expression of $\dot{M}_{av,crit}$, considering the X-ray irradiation of the disc, is given by (van Paradijs 1996; King, Kolb & Burderi 1996; Lasota 1997):

$$\dot{M}_{av,crit} \approx 3.2 \times 10^{15} \left(\frac{M}{M_\odot} \right)^{2/3} \left(\frac{P}{3\,h} \right)^{4/3} \text{g s}^{-1}, \qquad (4)$$

where P is the binary orbital period.

A theory and numerical computations of spin evolution for transient sources were reported in Bhattacharyya & Chakrabarty (2017, see also Bhattacharyya 2017). The main difference with persistent sources is, as $r_m \propto \dot{M}^{-2/7}$ (equation 1), during each outburst of weeks to months, r_m drastically evolves, while r_{co} ($\propto M^{1/3}\nu^{-2/3}$; equation 2) remains almost the same. Therefore, the $r_{co} = r_m$ condition is almost never satisfied for transient sources, and ν never tracks $\nu_{eq,per}$, which is the spin equilibrium frequency (equation 3) for persistent accretion (i.e. $\dot{M} = \dot{M}_{av}$). Nevertheless, an approximate or effective spin equilibrium can be achieved for transient accretion, if the total angular momentum (ΔJ_+) transferred to the neutron star in the accretion phase of an outburst is balanced by the total angular momentum (ΔJ_-) taken out from the star in the propeller phase of the same outburst (see Fig. 1a). The corresponding effective spin equilibrium frequency ($\nu_{eq,eff}$) was estimated to be

$$\nu_{eq,eff} = k\nu_{eq,peak} = \frac{k}{2^{11/14}\pi\xi^{3/2}}\left(\frac{G^5 M^5 \dot{M}_{peak}^3}{\mu^6} \right)^{1/7}, \qquad (5)$$

where $k \approx 0.85$ for a triangular outburst profile (fig. 1; Bhattacharyya & Chakrabarty 2017; Bhattacharyya 2017). Here, $\nu_{eq,peak}$ is ν_{eq} (equation 3) corresponding to $\dot{M}_{peak}$, which is the $\dot{M}$ of the peak of the outburst. When $\nu < \nu_{eq,eff}$, a relatively high r_{co} implies a sufficiently low $\dot{M}$ corresponding to $r_m = r_{co}$, which causes $\Delta J_+ > \Delta J_-$, and hence the star spins up (see Fig. 1b). Similarly, for $\nu > \nu_{eq,eff}$, the star spins down (see Fig. 1c). Therefore, while ν

approaches $\nu_{\text{eq,per}}$ for a persistent accretion, ν approaches $\nu_{\text{eq,eff}}$ for a transient accretion. Note that, since a typical $\dot{M}_{\text{peak}}/\dot{M}_{\text{av}}$ value could be $\sim$10–100 for transient sources (Burderi et al. 1999), transient accretion should spin-up neutron stars to rates several times [$\sim$2–6 for triangular outburst profiles; from equations (3) and (5)] higher than can persistent accretion (Bhattacharyya & Chakrabarty 2017).

In this Letter, we perform spin evolution computations for both persistent accretion and transient accretion using an evolving $\dot{M}_{\text{av}}$. For computations involving transient accretion, we follow the method of Bhattacharyya & Chakrabarty (2017) and Bhattacharyya (2017), which was not used for an evolving $\dot{M}_{\text{av}}$ earlier.

3 EFFECTS OF EVOLVING LONG-TERM AVERAGE ACCRETION RATE

3.1 Method

For our numerical computation of spin evolution, both due to persistent accretion and transient accretion, we follow the same method described in Bhattacharyya & Chakrabarty (2017), evolve the star for two billion years, and use reasonable parameter values for the purpose of demonstration. For example, we assume the initial values of ν and M as 1 Hz and 1.35 M$_\odot$, respectively (see Bhattacharyya & Chakrabarty 2017), and a fixed B value of 10^8 G. A fixed B-value may not be unreasonable (see Bhattacharyya & Chakrabarty 2017), and is useful to cleanly demonstrate the effect of an evolving $\dot{M}_{\text{av}}$, which is the aim of this Letter. Note that the effect of a slightly decaying B in the course of evolution would be to increase ν ($\propto B^{-6/7}$ in the equilibrium; equations 3 and 5), which does not change our conclusion.

For the spin evolution, we use following expressions of torques due to disc–magnetosphere interaction (Rappaport, Fregeau & Spruit 2004; Bhattacharyya & Chakrabarty 2017):

$$N_{\text{acc}} = \dot{M}\sqrt{GMr_{\text{m}}} + \frac{\mu^2}{9r_{\text{m}}^3}\left[2\left(\frac{r_{\text{m}}}{r_{\text{co}}}\right)^3 - 6\left(\frac{r_{\text{m}}}{r_{\text{co}}}\right)^{3/2} + 3\right] \quad (6)$$

for the accretion phase, and

$$N_{\text{prop}} = -\eta\dot{M}\sqrt{GMr_{\text{m}}} - \frac{\mu^2}{9r_{\text{m}}^3}\left[3 - 2\left(\frac{r_{\text{co}}}{r_{\text{m}}}\right)^{3/2}\right] \quad (7)$$

for the propeller phase. η is an order of unity positive constant (we use $\eta = 1$). In equations (6) and (7), the first term is the accreting material contribution, and the second term is the contribution from disc–magnetosphere interaction.

For transient accretion, we evolve a neutron star through a series of outburst and quiescent phases (see Fig. 1). Note that the outburst duty cycle (fractional duration) is $2\dot{M}_{\text{av}}/\dot{M}_{\text{peak}}$ for triangular outburst profiles (Bhattacharyya & Chakrabarty 2017). While in reality, the $\dot{M}_{\text{peak}}$-values can have a distribution, and the outburst profiles can have various irregular shapes, our simple triangular outburst profiles and a fixed $\dot{M}_{\text{peak}}$-value are useful to cleanly demonstrate the effect of an evolving $\dot{M}_{\text{av}}$. Note that Bhattacharyya (2017) showed how to compute a spin evolution for a known $\dot{M}_{\text{peak}}$ distribution, but such a detailed computation would not change our general conclusion. Besides, a different outburst profile would only imply a different k-value in equation (5) (e.g. $0.71 \lesssim k \lesssim 0.85$ for a linear rise and exponential decay, and $1 > k \gtrsim 0.85$ for a flat top; Bhattacharyya & Chakrabarty 2017), but would not change our conclusion.

Since our aim is to report general results, we do not use $\dot{M}_{\text{av}}$ evolution profiles for specific LMXB systems. Rather, we use a parametric formula (see Fig. 2 for a profile), which is useful and
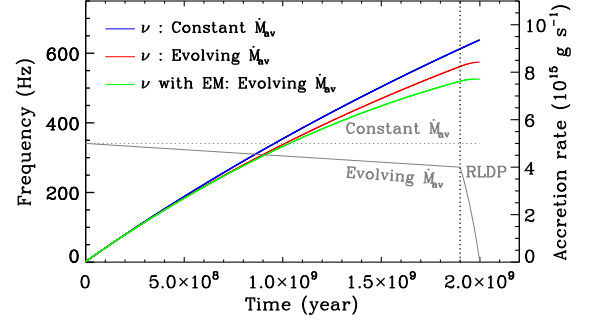

Figure 2. Numerically computed neutron star spin frequency (ν) evolution curves due to transient accretion (see Sections 3.1 and 3.2). The dotted grey horizontal line shows a constant long-term average accretion rate ($\dot{M}_{\text{av}}$) of 5×10^{15} g s^{-1}. The solid grey curve shows an evolving $\dot{M}_{\text{av}}$ with a fast decay (see Section 3.1) in the RLDP on the right of the dotted vertical line. Various ν evolution curves are for $\dot{M}_{\text{peak}} = 5 \times 10^{17}$ g s^{-1}, and based on if EM spin-down and $\dot{M}_{\text{av}}$ evolution are included not, as mentioned on the plot. This figure shows that ν can increase in the RLDP (see Section 3.2).

adequate for our purpose. One of our aims is to find how ν evolves in the last part of the LMXB phase, when $\dot{M}_{\text{av}}$ decreases to zero. This phase could significantly contribute to the creation of spin-powered MSPs and the ν-distribution (see Introduction). Following Tauris (2012), we consider a rapid $\dot{M}_{\text{av}}$ decay in this phase, referred as the RLDP (see Introduction). For this fast $\dot{M}_{\text{av}}$ decay during the last $\sim$5 per cent time of the LMXB phase (e.g. see Tauris 2012), we assume the parametric formula: $\dot{M}_{\text{av}} = a[2 - \exp(t/c)]$, where t is time, and a and c are positive constants. Besides, to study the effect of the relative values of $\dot{M}_{\text{av}}$ and $\dot{M}_{\text{av,crit}}$, we use a slow linear decay of $\dot{M}_{\text{av}}$ before the RLDP, and a constant $\dot{M}_{\text{av,crit}}$. Note that, while both $\dot{M}_{\text{av}}$ and $\dot{M}_{\text{av,crit}}$ could evolve throughout the LMXB phase in a more complex manner (e.g. Bhattacharya & van den Heuvel 1991; Chen & Podsiadlowski 2016), our simple assumption is ideal to cleanly demonstrate the main effects of $\dot{M}_{\text{av}}$ evolution. For the purpose of demonstration, we also use $\dot{M}_{\text{av,crit}} = 10^{16}$ g s^{-1}, which is reasonable considering equation (4) and measured P values of $\sim$1–10 h for AMXPs (Salvo & Sanna 2020).

3.2 Spin evolution of transient sources

First, we study how ν evolves if the neutron star accretes transiently throughout the LMXB phase (see Fig. 2). For an evolution duration of two billion years and for the examples given in this figure, ν remains much lower than $\nu_{\text{eq,eff}}$ (not shown in the figure) for $\dot{M}_{\text{peak}} = 5 \times 10^{17}$ g s^{-1}. Note that an $\dot{M}_{\text{av}} \sim 5 \times 10^{15}$–$10^{16}$ g s^{-1} is common for many neutron star LMXBs (Lamb & Yu 2005), although $\dot{M}_{\text{av}}$ could be significantly higher in the initial stage of the LMXB phase (e.g. see Tauris 2018; Chen, Liu & Wang 2020). The aim here is to find out how ν evolves in the RLDP. Fig. 2 shows that ν can increase in the RLDP, even if $\dot{M}_{\text{av}}$ decreases drastically, and regardless of the inclusion of an additional spin-down (e.g. due to the EM torque (see Bhattacharyya & Chakrabarty 2017) in the quiescent phase; see Section 1). This is not unexpected, as ν neither approaches nor tracks $\nu_{\text{eq,per}}$ during a transient accretion (see Section 2). However, we note that ν either approaches or tracks $\nu_{\text{eq,eff}}$ for such an accretion. Hence, ν either increases or decreases due to transient accretion, both in the RLDP and before the RLDP, depending on whether $\dot{M}_{\text{peak}}$ increases or decreases (see Section 2 and equation 5). Therefore, ν can decrease in the RLDP, if $\dot{M}_{\text{peak}}$ decreases. But, since $\dot{M}_{\text{peak}}$ is expected to depend on the binary orbital period P (with an estimated relation of
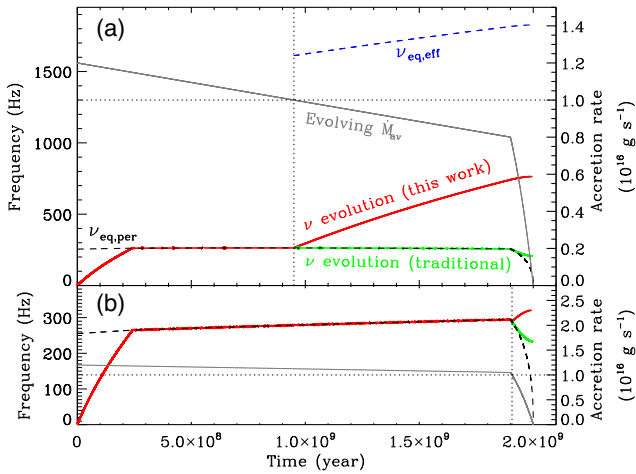
Figure 3. Numerically computed neutron star spin frequency (ν) evolution curves, where the neutron star initially accretes persistently, and then, as the long-term average accretion rate ($\dot{M}_{av}$) decreases below a critical value $\dot{M}_{av,crit}$ (here assumed to be 10^{16} g s^{-1}, shown by dotted horizontal lines), accretes transiently (see parameter values, the method and a discussion in Sections 3.1 and 3.3). Panel (a): the $\dot{M}_{av}$ curve (solid grey) falls below $\dot{M}_{av,crit}$ before the RLDP. The dotted vertical line marks the time of $\dot{M}_{av} = \dot{M}_{av,crit}$. The solid red curve shows the expected ν-evolution, initially for persistent accretion ($\dot{M}_{av} > \dot{M}_{av,crit}$), when ν first approaches and then tracks the lower spin equilibrium frequency $\nu_{eq,per}$ (black dashed curve; see Section 2), and then for transient accretion ($\dot{M}_{av} < \dot{M}_{av,crit}$), when ν approaches the higher effective spin equilibrium frequency $\nu_{eq,eff}$ (blue dashed curve; see equation 5). Note that, for a sufficiently long transient accretion phase, ν could evolve to values much higher than those observed (depending on source parameter values), unless there is an additional spin-down torque due to the GW emission (Bhattacharyya & Chakrabarty 2017). We do not include such a torque in this Letter. The solid green curve shows the traditionally computed ν-evolution for $\dot{M}_{av} < \dot{M}_{av,crit}$, where ν first tracks $\nu_{eq,per}$, and then does not track it but decreases if $\dot{M}_{av}$ rapidly falls. This panel shows the two distinctly different *modes* of ν-evolution (red curve): (i) when ν approaches and tracks $\nu_{eq,per}$, and (ii) when ν approaches $\nu_{eq,eff}$. Panel (b): similar to panel (a), but $\dot{M}_{av}$ falls below $\dot{M}_{av,crit}$ in the RLDP, and the $\nu_{eq,eff}$ curve is not shown. This panel shows that a persistent source should become a transient source at least in the RLDP, and then its ν-value increases (see Section 3.3).

$\dot{M}_{peak} \propto P^{1.79}$, Lasota 2001), and the change of P is moderate (e.g. Bhattacharya & van den Heuvel 1991), ν should not considerably decrease in the RLDP.

3.3 Spin evolution of persistent sources

How does ν evolve for a persistent source? The neutron star first spins up relatively quickly towards the spin equilibrium frequency ($\nu_{eq,per}$), and then tracks it (see Introduction and Section 2, Fig. 3). In the RLDP, if $\dot{M}_{av}$ decreases rapidly, ν decreases for a persistent source, but cannot track $\nu_{eq,per}$ anymore, which was shown by Tauris (2012), and is confirmed in Fig. 3. But in reality, a persistent source becomes a transient for $\dot{M}_{av} < \dot{M}_{av,crit}$, and consequently, ν increases, as it does not track the lower spin equilibrium frequency $\nu_{eq,per}$ anymore, and now approaches a higher effective spin equilibrium value $\nu_{eq,eff}$ (see Fig. 3, and also Section 2). Therefore, since even an initially persistent source should become a transient at least in the RLDP due to the decrease of $\dot{M}_{av}$, ν should increase in this phase (see Fig. 3b). The condition $\dot{M}_{av} < \dot{M}_{av,crit}$ can also be satisfied before the RLDP, and then ν can drastically increase in the pre-RLDP and continue to increase in the RLDP (see Fig. 3a). These are remarkably different from the currently believed spin evolution scenario, and

to the best of our knowledge, such a possibility of ν-evolution in two distinctly different *modes* (see Fig. 3) was not previously demonstrated. Moreover, while we do not show a $\dot{M}_{av} < \dot{M}_{av,crit}$ to $\dot{M}_{av} > \dot{M}_{av,crit}$ transition in Fig. 3, such a transition would usually cause a spin-down, as ν would approach a lower spin equilibrium value ($\nu_{eq,per}$).

4 DISCUSSION AND CONCLUSIONS

In this Letter, we have demonstrated, for the first time to the best of our knowledge, that the neutron star spin frequency ν can approach one of the two spin equilibrium frequencies: a lower one ($\nu_{eq,per}$) for persistent accretion and a higher one ($\nu_{eq,eff}$) for transient accretion. The former is expected to happen at a higher long-term average accretion rate ($\dot{M}_{av}$) relative to a critical value ($\dot{M}_{av,crit}$), and the latter can happen at a lower $\dot{M}_{av}$ relative to $\dot{M}_{av,crit}$. So there are two *modes* of spin evolution, and the mode leading to the higher ν occurs typically for lower $\dot{M}_{av}$ values, which is somewhat counter-intuitive. While we have shown this with examples of a sudden spin-up with a higher rate for a simple transition from $\dot{M}_{av} > \dot{M}_{av,crit}$ to $\dot{M}_{av} < \dot{M}_{av,crit}$ (see Fig. 3), an opposite transition may lead to a sudden spin-down. This gives an idea of how a complex ν-evolution curve, in which ν evolves by two alternate modes, may be caused by $\dot{M}_{av}$ and $\dot{M}_{av,crit}$ crossing each other multiple times in the LMXB phase. Even within the transient accretion regime, ν-evolution may be complex, if $\dot{M}_{peak}$ evolves (Section 3.2). This is because $\nu_{eq,eff}$ evolves with $\dot{M}_{peak}$ (equation 5), and hence ν approaches an equilibrium frequency which may change continuously in a complex way. For example, depending on the evolution of $\dot{M}_{peak}$, $\nu_{eq,eff}$ could evolve to values higher or lower than the ν-value, and hence ν could increase or decrease, respectively (see Section 2 and Fig. 1).

We have also particularly studied the spin evolution in the RLDP. As mentioned above, in the transient accretion regime, ν can spin-up or spin-down depending on the $\dot{M}_{peak}$-evolution both in the RLDP and before the RLDP. However, even if ν decreases in the RLDP, it should not decrease considerably (see Section 3.2), and such a spin-down is not due to the previously suggested (see Introduction) tracking of $\nu_{eq,per}$ by ν. On the other hand, an initially persistent source becomes a transient at least in the RLDP, as $\dot{M}_{av}$ sufficiently decreases, and hence ν should increase, as it now approaches a higher effective spin equilibrium frequency $\nu_{eq,eff}$ (see Section 3.3). Such a ν-evolution (see Fig. 3) implies that a breaking of ν from $\nu_{eq,per}$ and its subsequent decrease in the RLDP (suggested, e.g. by Tauris 2012) may not be relevant in many cases to determine the final ν-value of the LMXB phase. In fact, ν does not track or approach $\nu_{eq,per}$ anyway for transient accretion (see Section 2). Therefore, as the $\dot{M}_{av}$-evolution does not determine the ν-evolution, ν should not attain a low value (e.g. below ~100 Hz), even if $\dot{M}_{av}$ decreases relatively slowly to zero. This implies that, contrary to what was suggested earlier (see Introduction), a fast decrease of $\dot{M}_{av}$ in the last part of the LMXB phase is not essential for the creation of spin-powered MSPs.

Finally, we note that our findings open up possibilities of many pathways by which ν can evolve, and the traditional ν-evolution computation (e.g. Tauris 2012) could work only in special cases, for example, when the source remains persistent till the RLDP, and then $\dot{M}_{av}$ decreases very rapidly for $\dot{M}_{av} < \dot{M}_{av,crit}$ in the RLDP. The evolution of neutron stars via two modes due to persistent accretion and transient accretion should significantly affect ν and other observed parameter values of MSPs. Such an evolution could be studied in more detail in the near future using a considerably larger MSP sample, acquired, for example, by observations with the Square Kilometre Array (Keane et al. 2015).

DATA AVAILABILITY

REFERENCES

Alpar M. A., Cheng A. F., Ruderman M. A., Shaham J., 1982, Nature, 300, 728
Archibald A. M. et al., 2009, Science, 324, 1411
Bassa C. G. et al., 2014, MNRAS, 441, 1825
Bhattacharya D., van den Heuvel E. P. J., 1991, Phys. Rep., 203, 1
Bhattacharyya S., 2017, ApJ, 847, 2
Bhattacharyya S., 2020a, Nuclear-Powered X-ray Millisecond Pulsars
Bhattacharyya S., 2020b, MNRAS, 498, 728
Bhattacharyya S., Chakrabarty D., 2017, ApJ, 835, 4
Bildsten L., 1998, ApJ, 501, L89
Burderi L., Possenti A., Colpi M., Salvo T. D., D'Amico N., 1999, ApJ, 519, 285
Chakrabarty D. et al., 2003, Nature, 424, 42
Chakrabarty D., Morgan E. H., 1998, Nature, 394, 346
Chen W.-C., 2020, Phys. Rev. D, 102, 043020
Chen W.-C., Podsiadlowski P., 2016, ApJ, 830, 131
Chen W.-C., Liu D.-D., Wang B., 2020, ApJ, 900, L8
Done C., Gierliński M., Kubota A., 2007, ARA&A, 15, 1
Ferrario L., Wickramasinghe D., 2007, MNRAS, 375, 1009
Haskell B., Patruno A., 2017, Phys. Rev. Lett., 119, 161103
Keane E. et al., 2015, in Advancing Astrophysics with the Square Kilometre Array, Proc. Science, Pos(AAASaK)14,
King A. R., Kolb U., Burderi L., 1996, ApJ, 464, L127

Lamb F. K., Yu W., 2005, in Rasio F. A., Stairs I. H., eds, ASP Conf. Ser. Vol. 328, Binary Radio Pulsars. Astron. Soc. Pac., San Francisco, CA, p. 299
Lasota J. P., 1997, in Wickramasinghe D. T., Ferrario L., Bicknell G. V., eds, ASP Conf. Ser. Vol. 121, Accretion Phenomena and Related Outflows. Astron. Soc. Pac., p. 351
Lasota J. P., 2001, New Astron. Rev., 45, 449
Lin J., Yan Z., Han Z., Yu W., 2019, ApJ, 870, 126
Liu Q. Z., van Paradijs J., van den Heuvel E. P. J., 2007, A&A, 469, 807
Papitto A. et al., 2013, Nature, 501, 517
Papitto A., Torres D. F., Rea N., Tauris T. M., 2014, A&A, 566, A64
Patruno A., 2010, ApJ, 722, 909
Patruno A., Watts A. L., 2012, in Belloni T., Mendez M., Zhang C. M., eds, Timing Neutron Stars: Pulsations, Oscillations and Explosions. ASSL, Springer
Patruno A., Haskell B., Andersson N., 2017, ApJ, 850, 106
Radhakrishnan V., Srinivasan G., 1982, Curr. Sci., 51, 1096
Rappaport S. A., Fregeau J. M., Spruit H., 2004, ApJ, 606, 436
Ruderman M., Shaham J., Tavani M., 1989, ApJ, 336, 507
Salvo T. D., Sanna A., 2020, Accretion Powered X-ray Millisecond Pulsars, preprint (arXiv:2010.09005)
Tauris T. M., 2012, Science, 335, 561
Tauris T. M., 2018, Phys. Rev. Lett., 121, 131105
van Paradijs J., 1996, ApJ, 464, L139
Wang Y.-M., 1996, ApJ, 465, L111
Watts A. L., 2012, ARA&A, 50, 609
Wijnands R., van der Klis M., 1998, Nature, 394, 344
Yan Z., Yu W., 2015, ApJ, 805, 87

This paper has been typeset from a TeX/LaTeX file prepared by the author.