Localization of the Standard Model via the Higgs mechanism and a finite electroweak monopole from non-compact five dimensions

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We propose a minimal and self-contained model in non-compact flat five dimensions that localizes the Standard Model (SM) on a domain wall. Localization of gauge fields is achieved by condensation of the Higgs field via a Higgs-dependent gauge kinetic term in the 5D Lagrangian. The domain wall connecting vacua with unbroken gauge symmetry drives the Higgs condensation, which provides both electroweak symmetry breaking and gauge field localization at the same time. Our model predicts higher-dimensional interactions $|H|^{2n}(F_{\mu\nu})^2$ in the low-energy effective theory. This leads to two expectations: One is a new tree-level contribution to $H \to \gamma\gamma$ ($H \to gg$) decay whose signature will be testable in future LHC experiments. The other is a finite electroweak monopole that may be accessible to the MoEDAL experiment. Interactions of the translational Nambu–Goldstone boson are shown to satisfy a low-energy theorem.
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1. Introduction

The hypothesis that our 4D world is embedded in higher-dimensional spacetime has been a hot topic in high-energy physics for decades. Indeed, many mysteries of the Standard Model (SM) can be explained in this way. In particular, the discovery of D-branes in superstring theories [1] has intensified the research into brane-world scenarios more than anything else. The seminal works [2–5] provided the basic templates for further studies.

The biggest advantage of models in extra dimensions is the ability to utilize the *geometry* of the extra dimensions. A conventional setup, common among the extra-dimensional models, is that extra dimensions are prepared as a compact manifold/orbifold. Namely, our 4D spacetime is treated differently compared with extra dimensions.

In order to make things more natural, we can harness the *topology* of extra dimensions in addition to the geometry. The idea is quite simple and dates back to the early 1980s [6]; namely, the seed of dynamical creation of branes in extra dimensions is a spontaneous symmetry breaking giving rise to a topologically stable soliton/defect on which our 4D world is localized. The topology ensures not

only the stability of the brane but also the presence of chiral matters localized on the brane [6,7]. In addition, the graviton can be trapped [8–13]. Thus, topological solitons provide a natural framework bridging the gap between extra dimensions and four dimensions.

In contrast, localizing massless gauge bosons, especially non-Abelian gauge bosons, is quite difficult. A great deal of work has been published so far [14–37]. However, each of these has some advantages and disadvantages and there seems to be only a small amount of universal understanding. Then, a new mechanism utilizing a field-dependent gauge kinetic term (field-dependent permeability),

$$-\beta(\phi_i)^2 F_{MN} F^{MN} \quad (M, N = 0, 1, 2, 3, 4), \tag{1.1}$$

where ϕ_i are scalar fields, came out in Ref. [38]. This is a semiclassical realization of the confining phase [2,39–44] rather than the Higgs phase outside the solitons. The authors have continuously studied brane-world models with topological solitons by using Eq. (1.1) [45–51]. Let us highlight several results: We investigated the geometric Higgs mechanism, which is the conventional Higgs mechanism driven by the positions of multiple domain walls in an extra dimension in Ref. [49]. Then we proposed a model in which the brane world on five domain walls naturally gives an $SU(5)$ grand unified theory in Ref. [50]. Furthermore, we have clarified how to derive a low-energy effective theory on the solitons in the models with a nontrivial gauge kinetic term (1.1) by extending the R_ξ gauge in any spacetime dimensions D [51]. Another group also recently studied the SM in a similar model with β^2 taken as a given background in $D = 5$ [52,53]. They have also discussed phenomenology involving Nambu–Goldstone (NG) bosons for broken translation.

In this paper, we propose a minimal and self-contained model in non-compact flat five dimensions that localizes the SM on a domain wall. A striking difference from the previous works [45–51] is that we do not need extra scalar fields ϕ_i, which were introduced only for localizing gauge fields via Eq. (1.1). Instead, we put the SM Higgs in that role. As a consequence, localization of massless/massive gauge fields and the electroweak symmetry breaking have the same origin. In other words, the Higgs field is an active player in five dimensions with a new role as a localizing agent of gauge fields on the domain wall, in addition to the conventional roles giving masses to gauge bosons and fermions. Since our model does not need extra scalar fields ϕ_i, it is not only very economical in terms of field content but we are also free from the possible concern that ϕ_i would have an undesirable impact on the low-energy physics. We also study the translational NG boson $Y(x^\mu)$. Due to a low-energy theorem, it should have a derivative coupling with all other particles including Kaluza–Klein (KK) particles. We find a new vertex $\bar{\psi}^{(KK)} \gamma^\mu \partial_\mu Y \psi^{(SM)}$ that provides a new diagram for the production of KK quarks $\psi^{(SM)} + \psi^{(SM)} \rightarrow \psi^{(KK)} + \psi^{(KK)}$ in the LHC experiment. This should be a dominant production process compared to the usual gluon fusion, and can easily violate experimental bounds. To avoid this, we will set the fundamental 5D energy scale sufficiently large, providing that all the KK modes are supermassive. However, surprisingly, the Higgs-dependent gauge kinetic term (1.1) can naturally leave masses of localized lightest particles to be of the order of the SM energy scale. Thus, all KK particles and the NG boson have no impact on the low-energy physics. Nevertheless, as a consequence of Eq. (1.1), regardless of the extra particles, our model still has a new experimental signature in the $H \rightarrow \gamma\gamma$ ($H \rightarrow gg$) decay channel at tree level, which will be testable in future LHC experiments. Furthermore, we point out that the localization via Eq. (1.1) yields higher-dimensional interactions $|H|^{2n}(F_{\mu\nu})^2$ in the low-energy effective theory and it provides a natural reason to have a finite electroweak monopole solution. Its mass has been previously estimated [70,71] as $\lesssim 5.5$ TeV, so that it could be pair-produced at the

LHC and accessible to the MoEDAL experiment [72,73]. Thus, our model can pay the price for an electroweak monopole.

The paper is organized as follows. In Sect. 2, we explain all the essential ingredients in a simple toy model of Abelian–Higgs-scalar model in $D = 5$. We explain how a domain wall drives condensation of the Higgs field and at the same time localizes massless/massive gauge bosons and also chiral fermions. Phenomenological viability, the translational zero mode, and the relevance of the $H \to \gamma\gamma$ decay channel are addressed in Sect. 3. We present a realistic model localizing the SM in Sect. 4 and discuss the finite electroweak monopole in Sect. 5. Our results are summarized and discussed in Sect. 6. Appendix A is devoted to defining the mode expansion on a stable background. Mode expansion and effective potentials on an unstable background are described in Appendix B. Some formulae for KK fermion pair production by NG boson exchange are given in Appendix C.

2. Localization via Higgs mechanism

In order to illustrate a novel role of the Higgs mechanism besides the conventional roles of giving masses to gauge fields and chiral fermions in a gauge-invariant manner, let us consider a simple Abelian–Higgs-scalar model in $D = 5$ flat spacetime as a toy model. The following arguments are quite universal so that it is straightforward to apply them to non-Abelian gauge theories, such as the SM, which we discuss in Sect. 4, and also to models with $D \geq 5$ [62].

A simple Abelian–Higgs-scalar model in $D = 5$ reads:[1]

$$\mathcal{L} = -\beta(\mathcal{H})^2 \mathcal{F}_{MN}^2 + |\mathcal{D}_M \mathcal{H}|^2 + (\partial_M \mathcal{T})^2 - V$$
$$+ i\bar{\Psi}\Gamma_M \mathcal{D}^M \Psi + i\bar{\tilde{\Psi}}\Gamma_M \partial^M \tilde{\Psi} + \left(\eta \mathcal{T}\bar{\Psi}\Psi - \tilde{\eta}\mathcal{T}\bar{\tilde{\Psi}}\tilde{\Psi} + \chi \mathcal{H}\bar{\Psi}\tilde{\Psi} + \text{h.c.}\right), \quad (2.1)$$

$$V = \Omega^2 |\mathcal{H}|^2 + \lambda^2 \left(|\mathcal{H}|^2 + \mathcal{T}^2 - v^2\right)^2, \quad (2.2)$$

with $\mathcal{F}_{MN} = \partial_M \mathcal{A}_N - \partial_N \mathcal{A}_M$. Here $\mathcal{T}$ is a real scalar field, and $\mathcal{H}$ is the Higgs field that interacts with $\mathcal{A}_M$ not only via the covariant derivative $\mathcal{D}_M \mathcal{H} = \partial_M \mathcal{H} + iq_H \mathcal{A}_M \mathcal{H}$, but also through a non-minimal gauge kinetic term with the field-dependent function β^2 defined by

$$\beta(\mathcal{H})^2 = \frac{|\mathcal{H}|^2}{4\mu^2}. \quad (2.3)$$

The covariant derivative of the charged fermion field is defined by $\mathcal{D}_M \Psi = \partial_M \Psi + iq_f \mathcal{A}_M \Psi$. $\tilde{\Psi}$ is a neutral fermion. The bosonic part of the model has Z_2 symmetry $\mathcal{T} \to -\mathcal{T}$. The mass dimensions of the fields and parameters are summarized as $[\mathcal{H}] = [\mathcal{T}] = \frac{3}{2}$, $[\mathcal{A}_M] = 1$, $[\Psi] = [\tilde{\Psi}] = 2$, $[\mu] = [\Omega] = [\lambda^{-2}] = [\eta^{-2}] = [\tilde{\eta}^{-2}] = [\chi^{-2}] = [v^{\frac{2}{3}}] = 1$, and $[\beta] = \frac{1}{2}$. The 5D Gamma matrix Γ^M is related to the 4D one as $\Gamma^\mu = \gamma^\mu$ and $\Gamma^5 = i\gamma^0 \gamma^1 \gamma^2 \gamma^3 = i\gamma^5$.

There are two discrete vacua $\mathcal{T} = \pm v$ with $\mathcal{H} = 0$. The vacua break the Z_2 symmetry but preserve $U(1)$ gauge symmetry, which is necessary to localize the massless $U(1)$ gauge field on a domain wall [38,45–51]. Therefore, the Higgs mechanism does not take place in the vacua.

However, spontaneous breaking of the Z_2 symmetry gives rise to a topologically stable domain wall, connecting these two discrete vacua. Depending on the values of the parameters, the following

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[1] The bosonic part is a simple extension of the well studied model [63–65] in which the Higgs field $\mathcal{H}$ is replaced by a real scalar field.

stable domain-wall solutions are obtained:

$$\mathcal{T}_0{}' = v \tanh \lambda v y, \quad \mathcal{H}_0 = 0, \qquad (\lambda v \leq \Omega), \tag{2.4}$$

$$\mathcal{T}_0 = v \tanh \Omega y, \quad \mathcal{H}_0 = \bar{v} \operatorname{sech} \Omega y, \qquad (\lambda v > \Omega), \tag{2.5}$$

with $\bar{v} = \sqrt{v^2 - \Omega^2/\lambda^2}$ and $y = x^4$. We are not interested in the former solution (2.4) since $U(1)$ is unbroken everywhere and the gauge field is not dynamical due to $\beta^2 = 0$. On the other hand, as we will show below, the latter solution (2.5) localizes the $U(1)$ gauge field by $\beta^2 \propto \operatorname{sech}^2 \Omega y$. When the Higgs is neutral ($q_H = 0$), the lightest mode of the localized gauge field is precisely massless [49–51] whereas, as we will see, it becomes massive when the Higgs is charged ($q_H \neq 0$).

To understand the mechanism for the localized massless gauge field becoming massive, let us compute the low-energy effective potential for the effective Higgs field in four dimensions in the parameter region

$$0 < \epsilon^2 \ll 1, \quad \epsilon^2 \equiv \frac{\lambda^2 \bar{v}^2}{\Omega^2} = \frac{\lambda^2 v^2 - \Omega^2}{\Omega^2}. \tag{2.6}$$

From the linearized field equation around the background of the domain-wall solution (2.5), we find that there is a mass gap of order Ω, and two discrete modes much lighter than the mass gap. The lowest mode is exactly the massless Nambu–Goldstone (NG) boson corresponding to spontaneously broken translation symmetry along the y direction. Its interactions with all other effective fields are generally suppressed by inverse powers of the large mass scale, whose characteristics will be discussed in Sects. 3.2 and 4. Disregarding the NG boson, we retain only one light boson, whose wave function is well approximated by the same functional form as the background solution $\mathcal{H}_0(y)$ in Eq. (2.5). When $\lambda \bar{v} = 0$, this wave function gives the zero mode exactly, corresponding to the condensation mode at the critical point $\lambda v = \Omega$, where the $\mathcal{H}$ field begins to condense. After $\mathcal{H}$ condenses, this mode becomes slightly massive above the critical point (2.6) with a mass of order $\lambda \bar{v}$, whose wave function receives small corrections suppressed by powers of ϵ (including an admixture of fluctuations of $\mathcal{T}$). Combining the background solution and the fluctuation, we introduce the following effective field $H(x)$ (a quasi-modulus) corresponding to the Higgs field in the low-energy effective field theory:

$$\mathcal{H}(x, y) = \sqrt{\frac{\Omega}{2}} H(x) \operatorname{sech} \Omega y. \tag{2.7}$$

Inserting this ansatz into the Lagrangian and integrating over y, we obtain the effective action as

$$\mathcal{L}_{\mathrm{Higgs}}(H) = |D_\mu H|^2 - V_H, \quad V_H = \lambda_2^2 |H|^2 + \frac{\lambda_4^2}{2} |H|^4, \tag{2.8}$$

$$\lambda_2^2 = -\frac{4\lambda^2 \bar{v}^2}{3}, \quad \lambda_4^2 = \frac{2\lambda^2 \Omega}{3}, \tag{2.9}$$

where the effective gauge field in the covariant derivative D_μ is more precisely defined below; see Eq. (2.17). The possible corrections suppressed by powers of ϵ^2 can be systematically computed as described in Appendix A. This is just a conventional Higgs Lagrangian that catches all the essential features. First, note that the sign of the quadratic term is determined by $\bar{v}^2 = v^2 - \Omega^2/\lambda^2$. When $\bar{v}^2 = 0$ ($v\lambda = \Omega$), the Higgs is massless, corresponding to the condensation zero mode in Eq. (2.7). When $\bar{v}^2 < 0$ ($v\lambda < \Omega$), the vacuum expectation value (VEV) is $\langle H \rangle = 0$. Thus, we reproduce the
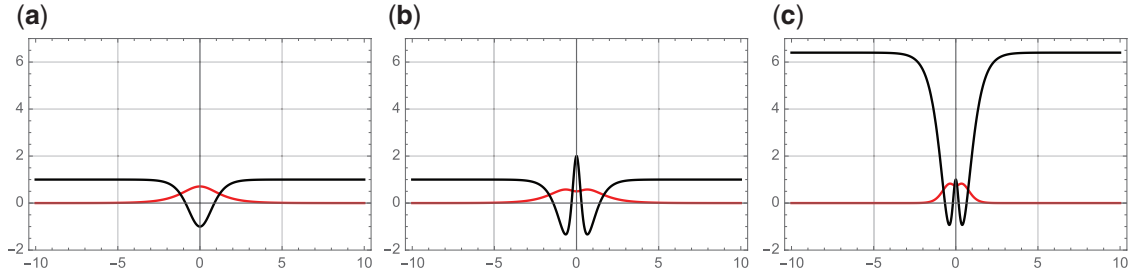
Fig. 1. The black lines show the Schrödinger potentials $V_S = \beta''/\beta$ for β^2 given in Eq. (2.3) (a), in Eq. (3.17) (b), and in Eq. (5.2) (c). The potential of (c) is multiplied by 0.1 for clarity. The horizontal axis is Ωy. The red curves show the corresponding zero-mode wave functions.

solution (2.4). On the other hand, when $\bar{v}^2 > 0$ ($v\lambda > \Omega$), we have a nonzero VEV for the effective Higgs field $H(x)$:

$$\langle H \rangle = \sqrt{\frac{2}{\Omega}}\,\bar{v} \equiv \frac{v_h}{\sqrt{2}}, \tag{2.10}$$

which correctly gives the solution (2.5). Note that the VEV v_h can also be obtained directly from the 5D field $\mathcal{H}$ as

$$\frac{v_h^2}{2} = \int_{-\infty}^{\infty} dy\, \mathcal{H}_0^2 = \frac{2\bar{v}^2}{\Omega}. \tag{2.11}$$

The mass of the physical Higgs boson can be read from Eq. (2.8) as

$$m_h^2 = \frac{8}{3}\lambda^2\bar{v}^2 = \frac{8}{3}\Omega^2\epsilon^2, \tag{2.12}$$

which is of order ϵ^2 as we expected. Thus, the y-dependent Higgs condensation $\mathcal{H}_0(y)$ of Eq. (2.5) in $D = 5$ that is driven by the domain wall $\mathcal{T}_0(y)$ connecting two unbroken vacua indeed gives the Higgs mechanism through Eq. (2.8). To complete the picture, we next calculate the mass of gauge bosons. We will assume $v\lambda > \Omega$ in the rest of the paper, so that the solution (2.5) always applies.

To figure out the spectrum of the gauge field, first of all, we use canonical normalization $A_M = 2\beta\mathcal{A}_M$. The linearized equation of motion for A_μ in the generalized R_ξ gauge [51,62] is

$$\left\{ \eta^{\mu\nu}\Box - \left(1 - \frac{1}{\xi}\right)\partial^\mu\partial^\nu + \eta^{\mu\nu}\left(-\partial_y^2 + \frac{(\partial_y^2\beta)}{\beta} + 2q_H^2\mu^2\right)\right\} A_\nu = 0. \tag{2.13}$$

Thus, the Kaluza–Klein (KK) spectrum is identical to eigenvalues of a 1D quantum mechanical problem with the Schrödinger potential $V_S = (\partial_y^2\beta)/\beta + 2q_H^2\mu^2$. Figure 1(a) shows the corresponding Schrödinger potential. The eigenvalues m_n^2 and eigenfunctions $\phi_n(y)$ can be easily obtained [51]. There is a unique bound state:

$$\phi_0(y) = \frac{\sqrt{2}\bar{v}}{v_h}\operatorname{sech}\Omega y, \quad m_0^2 = 2q_H^2\mu^2. \tag{2.14}$$

No other bound states exist and a continuum of scattering modes parametrized by the momentum k corresponds to the eigenvalues $m_k^2 = k^2 + \Omega^2 + q_H^2\mu^2$. Thus, the mass gap between the unique bound state ϕ_0 and the higher KK modes is of order Ω (under the assumption $\Omega \gg \mu$), which is the

inverse width of the domain wall. In terms of the original field $\mathcal{A}_\mu$, the lightest massive gauge boson $A_\mu^{(0)}(x)$ is given by

$$\mathcal{A}_\mu = \frac{A_\mu}{2\beta} = \frac{\mu\phi_0}{\mathcal{H}_0} A_\mu^{(0)}(x) + \cdots = \frac{\sqrt{2}\mu}{v_h} A_\mu^{(0)}(x) + \cdots , \qquad (2.15)$$

where the ellipses stand for the heavy continuum modes. The mass of the lightest massive gauge boson is

$$m_A = m_0 = \sqrt{2}\, q_H \mu. \qquad (2.16)$$

One can show that the fifth gauge field $\mathcal{A}_y$ has no physical degrees of freedom [51].

Having Eq. (2.15) at hand, we are now able to read the effective gauge coupling constant. By plugging Eqs. (2.7) and (2.15) into Eq. (2.1) and integrating it over y, we have the kinetic term for H as

$$\int_{-\infty}^{\infty} dy \, |\mathcal{D}_\mu \mathcal{H}|^2 = \left| \left(\partial_\mu + iq_H \frac{\sqrt{2}\mu}{v_h} A_\mu^{(0)} \right) H \right|^2 + \cdots = |D_\mu H|^2 + \cdots , \qquad (2.17)$$

where the ellipses stand for the massive modes. Thus, the effective 4D gauge coupling reads

$$e = \frac{\sqrt{2}\mu}{v_h}. \qquad (2.18)$$

Combining Eq. (2.18) with Eq. (2.16), and Eq. (2.10) with Eq. (2.12), we see that what happens here is perfectly consistent with the ordinary Higgs mechanism:

$$m_A = q_H e v_h, \quad m_h = \lambda_4 v_h. \qquad (2.19)$$

Finally, let us investigate the domain-wall fermions Ψ and $\tilde{\Psi}$ [6,7]. In the region (2.6) together with a phenomenological condition explained later, our parameters should satisfy the following inequality:

$$\frac{\eta v}{\Omega} \simeq \frac{\tilde{\eta} v}{\Omega} \simeq 1 \gg \frac{\chi \bar{v}}{\Omega}. \qquad (2.20)$$

Then we can treat the Yukawa term $\mathcal{H}\bar{\Psi}\tilde{\Psi}$ in Eq. (2.1) as a perturbation. In order to study the unperturbed Dirac equation, we decompose 5D fermions as

$$\Psi = \sum_n \left(f_L^{(n)}(y) \psi_L^{(n)}(x) + f_R^{(n)}(y) \psi_R^{(n)}(x) \right), \qquad (2.21)$$

where $\psi_L^{(n)}$ and $\psi_R^{(n)}$ are left-handed ($\gamma^5 \psi_L = -\psi_L$) and right-handed ($\gamma^5 \psi_R = \psi_R$) spinors in four dimensions,

$$i\partial\!\!\!/ \psi_L^{(n)} = M_n \psi_R^{(n)}, \quad i\partial\!\!\!/ \psi_R^{(n)} = M_n \psi_L^{(n)}, \qquad (2.22)$$

and the mode functions $f_L^{(n)}$ and $f_R^{(n)}$ satisfy

$$Q f_L^{(n)} + M_n f_R^{(n)} = 0, \quad Q^\dagger f_R^{(n)} + M_n f_L^{(n)} = 0, \qquad (2.23)$$

with $Q = \partial_y + \eta \mathcal{T}_0$, and $Q^\dagger = -\partial_y + \eta \mathcal{T}_0$. Assuming the 5D Yukawa coupling to satisfy $\eta > 0$, we find a unique zero mode

$$f_L^{(0)}(y) = N_{L,0} \left(\cosh \Omega y \right)^{-\frac{\eta v}{\Omega}}, \quad f_R^{(0)}(y) = 0, \quad M_0 = 0, \tag{2.24}$$

where $N_{L,0}$ is a normalization constant. The number of excited bound KK states corresponds to $n = \lfloor \frac{\eta v}{\Omega} \rfloor$ ($\lfloor \, \rfloor$ is the floor function). For example, the first excited bound state exists when $\frac{\eta v}{\Omega} \geq 1$ and its wave function and mass are given by

$$f_L^{(1)} = N_{L,1} \sinh \Omega y \left(\cosh \Omega y \right)^{-\frac{\eta v}{\Omega}}, \quad f_R^{(1)} = N_{R,1} Q f_L^{(1)}, \quad M_1^2 = \left(2 \frac{\eta v}{\Omega} - 1 \right) \Omega^2. \tag{2.25}$$

The mass gap between the zero mode and the KK modes is again of order Ω for the parameter region given in Eq. (2.20). The analysis for $\tilde{\Psi}$ can be done similarly by replacing η with $\tilde{\eta}$ and by exchanging L and R.

The interaction between the lightest massive gauge boson $A_\mu^{(0)}$ and the fermionic zero mode $\psi_L^{(0)}$ is obtained as

$$\int_{-\infty}^{\infty} dy \, i\bar{\Psi}\Gamma^\mu \mathcal{D}_\mu \Psi = i\bar{\psi}_L^{(0)} \gamma^\mu \left(\partial_\mu + i q_f \frac{\sqrt{2}\mu}{v_h} A_\mu^{(0)} \right) \psi_L^{(0)} + \cdots, \tag{2.26}$$

where the ellipses stand for the massive modes. Notice that the gauge coupling is the same as in Eq. (2.18). We have to emphasize that the effective gauge coupling e is the same for any localized fields. Universality is ensured by the fact that the wave function of the lightest mode of $\mathcal{A}_\mu$ is always constant.

We can also easily derive an effective Yukawa coupling as follows:

$$\int_{-\infty}^{\infty} dy \, \chi \mathcal{H} \bar{\Psi} \tilde{\Psi} \supset \chi \bar{v} \tau(b, \tilde{b}) \bar{\psi}_L^{(0)} \tilde{\psi}_R^{(0)}, \quad b \equiv \frac{\eta v}{\Omega}, \ \tilde{b} \equiv \frac{\tilde{\eta} v}{\Omega}, \tag{2.27}$$

with a dimensionless constant $\tau(b, \tilde{b}) = \frac{\Gamma(\frac{1+b+\tilde{b}}{2})}{\Gamma(\frac{2+b+\tilde{b}}{2})} \sqrt{\frac{\Gamma(b+\frac{1}{2})\Gamma(\tilde{b}+\frac{1}{2})}{\Gamma(b)\Gamma(\tilde{b})}}$, where $\Gamma(x)$ is the gamma function. Thus the Yukawa coupling in four dimensions reads

$$\chi_4 = \frac{\tau(b, \tilde{b}) \chi \bar{v}}{v_h} \simeq \chi \sqrt{\Omega}, \tag{2.28}$$

where we assume that $\tau(b, \tilde{b})$ is of order one because of $b \simeq \tilde{b} \simeq 1$.

Before closing this section, let us comment on the Higgs field. The Higgs condensation occurs at the 5D level leading to the localization of the massless/massive gauge bosons in our model. A new feature of our Higgs mechanism is that the order parameter $\mathcal{H}$ induced by the domain wall is position-dependent. As a consequence, the effective Higgs field is localized and only the massive physical Higgs boson h remains in the low-energy physics. In contrast, if one uses other neutral scalar fields ϕ_i to localize the gauge fields [45–51], one has to prepare another trick to localize the Higgs fields too. For example, in recent papers [52,53], the kinetic term of the Higgs field is not minimal but multiplied by a function $\beta^2(\phi)$. In such models, the Higgs field (massive Higgs boson and massless NG boson) is localized on the domain wall and Higgs condensation occurs in the low-energy effective theory. Namely, the Higgs field plays no active role at the 5D level.

3. Phenomenological implications

3.1. Mass scales

In order to have a phenomenologically viable model, we need to explain the observed mass m_A of a gauge boson, vacuum expectation value v_h of the 4D Higgs field, and mass m_h of the physical Higgs boson. These observables are necessary and sufficient to fix the parameters of the gauge-Higgs sector of the SM (gauge coupling e, and quadratic and quartic couplings of Higgs scalars). We can regard all these masses to be of order 10^2 GeV, taking the 4D gauge coupling[2] e and Higgs quartic coupling λ_4 to be roughly of order unity.[3] On the other hand, we have four parameters, Ω, v, λ, μ, in the bosonic part of the 5D Lagrangian (2.1). It is convenient to take Ω as the fundamental mass scale of the high-energy microscopic theory. Three other parameters can be put into two mass scales, $\mu, \lambda\bar{v}$, and one dimensionless combination $\lambda_4^2 = 2\lambda^2\Omega/3$ in Eq. (2.9), where $\bar{v} = \sqrt{v^2 - (\Omega/\lambda)^2}$. From Eqs. (2.10), (2.12), and (2.16), the masses of the low-energy effective theory are given in terms of parameters of the 5D theory as

$$m_A = \sqrt{2}q_H\mu, \quad v_h = \frac{2}{\sqrt{\Omega}}\bar{v}, \quad m_h = \sqrt{\frac{8}{3}}\lambda\bar{v}. \tag{3.1}$$

Fitting these masses to experimentally observed values, we still have one mass scale Ω completely free. Therefore we can choose the energy scale Ω of the 5D theory as large as we wish, leaving a phenomenologically viable model at low energies.

For instance, if we choose the ratio of the high-energy scale and SM scale to be parametrized as

$$\epsilon^2 = \frac{\lambda^2\bar{v}^2}{\Omega^2} \sim 10^{-2a} \ll 1, \tag{3.2}$$

we find the scale of parameters in the model as

$$\lambda\bar{v} \sim 10^2 \text{ Gev} \ll \lambda v \sim \Omega \sim 10^{2+a} \text{ GeV}, \tag{3.3}$$

implying $\lambda \sim 10^{-1-a/2}$ GeV$^{-1/2}$, $v \sim 10^{3+3a/2}$ GeV$^{3/2}$, and $\bar{v} \sim 10^{3+a/2}$ GeV$^{3/2}$. This large mass gap allows us to use the low-energy effective field theory retaining only light fields with a mass of order $\lambda\bar{v}$ or less. In order to achieve this hierarchy, we need fine-tuning of the parameters $\lambda\bar{v} \ll \Omega$, as in Eq. (3.3).

For the fermionic sector, we require Eqs. (2.20) and (2.28). Therefore, we have $\eta \sim \tilde{\eta} \sim 10^{-1-a/2}$ GeV$^{-1/2}$. In order to obtain appropriate values of the 4D Yukawa couplings, for instance, for the top Yukawa coupling to be of order one, we need the 5D Yukawa coupling as

$$\chi \simeq \frac{\chi_{4,\text{top}}}{\sqrt{\Omega}} \sim 10^{-1-\frac{a}{2}} \text{ GeV}^{-\frac{1}{2}}. \tag{3.4}$$

Thus, the 5D Yukawa couplings η, $\tilde{\eta}$, and χ are naturally set to be the same order. Note that this also justifies Eq. (2.20). To understand the hierarchy of lighter fermion masses, we can use the usual mechanism of splitting the position of localized fermions as explained briefly in Sect. 4.

In summary, to have the SM at low energy, all the dimensionful parameters in the 5D Lagrangian are set to be of the same order as

$$\Omega \sim \lambda^{-2} \sim v^{\frac{2}{3}} \sim \eta^{-2} \sim \tilde{\eta}^{-2} \sim \chi^{-2} \sim 10^{2+a} \text{ GeV}. \tag{3.5}$$

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[2] Here we have just one gauge coupling, because of our simplification of $U(1)$ instead of the $SU(2) \times U(1)$ gauge group.

[3] Actually, they are somewhat less than unity experimentally, in conformity with the perturbativity of SM.

We need fine-tuning for two small parameters of mass dimension: $\lambda \bar{v}, \mu \sim 10^2$ GeV. An estimate of the lower bound for the parameter $\Omega \sim 10^{2+a}$ GeV will be discussed in Sect. 4 using constraints from the LHC data.

3.2. Translational zero mode

Here we study interactions of the translational Nambu–Goldstone (NB) mode, and their impact on low-energy phenomenology. The symmetry principle gives low-energy theorems, dictating that the NG bosons interact with corresponding symmetry currents as derivative interactions (no interaction at the vanishing momentum of NG bosons). Hence their interactions are generally suppressed by powers of large mass scale. In order to understand the interactions of the NG bosons, it is most convenient to consider the moduli approximation[76] where the moduli are promoted to fields in the low-energy effective Lagrangian. Let us consider a general theory with a number of fields[4] $\phi^i(x, y)$ admitting a solution (soliton) of the field equation, which we take as a background. When the theory is translationally invariant, the position Y of the soliton is a moduli. It is contained in the solution as $\phi^i(x, y - Y)$. In the modulus approximation, we promote the moduli parameter Y to a field $Y(x)$ slowly varying in the world volume of the soliton. We call this moduli field $Y(x)$ an NG field.[5] By introducing the NG boson decay constant f_Y to adjust the mass dimension of the NG field to the canonical value $[Y(x)] = 1$, we obtain

$$\mathcal{L}_{\mathrm{NG}} = \int dy\, \mathcal{L}\left(\phi\left(x, y - \frac{1}{f_Y}Y(x)\right)\right). \tag{3.6}$$

The precise value of the decay constant f_Y is determined by requiring the kinetic term of NG boson to be canonical as illustrated in the subsequent explicit calculation. By integrating over y, we can obtain the effective interaction of the NG field. One should note that the constant part Y of the NG field $Y(x)$ is nothing but the position of the wall, which can be absorbed into the integration variable y by a shift $y \to y - Y$ because of the translational invariance. Hence the constant Y disappears from the effective action after the y-integration is done. This fact guarantees that $Y(x)$ must always appear in the low-energy effective theory with derivatives, i.e., $\partial_\mu Y(x)$. Let us examine how this fact fixes the interactions of the NG particle in the effective Lagrangian to produce the low-energy theorem. The derivative ∂_μ can only come from the derivative term in the original action $\mathcal{L}$, giving terms linear in the NG particle $Y(x)$ as

$$\mathcal{L}_{\mathrm{NG}} = -\int dy\, \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi^i} \frac{\partial \phi^i}{\partial y} \frac{\partial_\mu Y}{f_Y} + \cdots = -\frac{1}{f_Y} \partial_\mu Y(x) \left[\int dy\, T^{\mu y}\right] + \cdots, \tag{3.7}$$

where the energy–momentum tensor T^{MN} of matter in five dimensions is given by

$$T^{MN} = \frac{\partial \mathcal{L}}{\partial \partial_M \phi^i} \partial^N \phi^i - \eta^{MN} \mathcal{L}. \tag{3.8}$$

This is the low-energy theorem of the NG particle for spontaneously broken translation. Thus we find that there are no nonderivative interactions that remain at the vanishing momentum of NG bosons, including KK particles. For instance, the possible decay amplitude of a KK fermion into an ordinary

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[4] In our concrete model, we have fields such as $\mathcal{A}_M, \mathcal{T}, \mathcal{H}, \Psi$, and $\tilde{\Psi}$.

[5] This definition is, in general, a nonlinear field redefinition of the effective field that arises in the mode analysis of fluctuation fields, such as in Appendix A.

fermion and an NG boson should vanish at zero momentum of the NG boson and will be suppressed by inverse powers of large mass scale such as Ω. In this way, we can compute the effective action of the NG field in powers of the derivative ∂_μ. Usually we retain up to second order in derivatives, but higher-derivative corrections can be obtained systematically with some effort [77].

Let us compute the effective Lagrangian of the NG field $Y(x)$ more explicitly by using the moduli approximation in our model as

$$\mathcal{T} = v \tanh\left(\Omega y - \frac{1}{f_Y}Y(x)\right), \quad \mathcal{H} = \sqrt{\frac{\Omega}{2}}H(x)\,\mathrm{sech}\left(\Omega y - \frac{1}{f_Y}Y(x)\right). \tag{3.9}$$

The wall position moduli in wave functions of fermions must also be promoted to the NG field $Y(x)$, i.e.,

$$f_{L,R}^{(n)}(y) \to f_{L,R}^{(n)}\left(y - \frac{1}{f_Y}Y(x)\right), \tag{3.10}$$

although we only retain the zero mode given in Eq. (2.24) in order to obtain a low-energy effective Lagrangian for light particles. Plugging these ansatzes into the 4D kinetic terms of $\mathcal{T}$, $\mathcal{H}$, and Ψ and integrating over y, we obtain the effective Lagrangian containing the NG field. The requirement of canonical normalization of the NG field $Y(x)$ fixes the decay constant f_Y as

$$f_Y = \frac{2\sqrt{2}\,v}{\sqrt{3\Omega}}. \tag{3.11}$$

We finally obtain the effective Lagrangian for low-energy particles as:

$$\mathcal{L}_{\mathrm{NG}} = \frac{1}{2}\partial_\mu Y \partial^\mu Y \left(1 + \frac{\Omega}{2v^2}|H|^2\right). \tag{3.12}$$

A few features can be noted. First of all, the NG bosons have only derivative interactions, as required by the above general consideration. Secondly, the derivative interaction produces higher-dimensional operators coupled to NG bosons. The required mass parameter in the coefficient of the interaction term is given by the high-energy scale as $\Omega/(2v^2) \sim 1/\Omega^2$. Therefore the interaction is suppressed by a factor of $(\mathrm{momentum})/\Omega$. Thirdly, the interaction linear in the NG particle in Eq. (3.7) happens to be absent in this model. This is a result of a selection rule in our model.[6] The Lagrangian (2.1) and the background solution (2.5) allows us to assign generalized parity under the reflection symmetries $y \to -y$, as a conserved quantum number to all modes including KK modes. Since the NG boson has odd parity, whereas all other low-energy particles including fermions have even parity, we end up in the quadratic interaction for the NG boson $Y(x)$, as given in Eq. (3.12). The parity quantum number under $y \to -y$ may not be conserved in more general models, and can have a nonvanishing interaction linear in $\partial_\mu Y(x)$ given in Eq. (3.7).

Only when we take into account the heavy KK modes [53] do we have interactions linear in $\partial_\mu Y$. For example, including the lightest KK fermion given in Eq. (2.25) ($b = \frac{\eta v}{\Omega} > 1$ in order to have a discrete state) we obtain a vertex

$$\int_{-\infty}^{\infty} dy\, i\bar{\Psi}\Gamma_M \mathcal{D}^M \Psi \supset i\alpha \frac{\sqrt{\Omega}}{v}\partial_\mu Y \left(\bar{\psi}_L^{(1)}\gamma^\mu \psi_L^{(0)} - \bar{\psi}_L^{(0)}\gamma^\mu \psi_L^{(1)}\right), \tag{3.13}$$

---

[6] Note that a nonderivative coupling $Y\bar{\psi}_L^{(0)}\psi_R^{(1)}$ from $\mathcal{T}\bar{\Psi}\Psi$ was recently studied in Ref. [53]. However, the symmetry principle of the NG boson for translation does not allow coupling without the derivative ∂_μ.

where α is a dimensionless constant of order one defined by $\alpha \equiv \frac{\sqrt{3}}{4} \frac{b}{\sqrt{b-1}} \frac{B(b+\frac{1}{2},b-\frac{1}{2})}{B(b+1,b-1)}$, where $B(x,y)$ is the beta function.[7] The above interaction gives the decay process $\psi_L^{(1)} \to Y \psi_L^{(0)}$. $\tilde{\Psi}$ yields similar interactions between Y and $\psi_R^{(n)}$. Although the NG boson amplitudes are generally suppressed by the ratio p_μ / Ω with the large mass scale Ω, it can give a significant decay rate in the case of two-body decay like here. Moreover, this type of vertex provides a new diagram for the production of KK quarks $\psi_{L,R}^{(1)}$ out of quarks $\psi_{L,R}^{(0)}$ in the colliding nucleons via the NG boson exchange

$$\psi_i^{(0)} + \psi_j^{(0)} \to \psi_i^{(1)} + \psi_j^{(1)} \quad (i,j = L,R) \tag{3.14}$$

in the LHC experiment. This should be the dominant production mechanism because of the large momentum fraction of quarks as given by their distribution function inside nucleons. The production process (3.14) tells us the lower bound of the KK quark masses. We will estimate it in Sect. 4 where the Standard Model is embedded in our framework.

3.3. $h \to \gamma\gamma$

As explained above, our model provides a domain wall inside which all the SM particles are localized. All the KK modes are separated by the mass gap $\Omega \sim 10^{2+a}$ GeV. Furthermore, for the minimal β^2 as given in Eq. (2.3), there are no additional localized KK modes of the gauge fields [51]. At first sight, one might wonder if the low-energy theory would be distinguishable from the conventional SM if a is sufficiently large. However, a significant difference between these two theories is an additional interaction between the Higgs boson and the gauge bosons due to the field-dependent gauge kinetic term. For illustration, suppose that $\mathcal{A}_M$ is the electromagnetic gauge field and the Higgs boson is neutral with $q_H = 0$. Nevertheless, the field-dependent gauge kinetic term yields an interaction between the photon and the neutral Higgs boson. This mechanism is valid also for the physical Higgs boson in our model. To see this, let us consider the fluctuation of the physical Higgs boson $h(x)$ by perturbing H in Eq. (2.7) about $H = v_h$:

$$\mathcal{H} = \bar{v} \left(1 + \frac{\sqrt{2}h(x)}{v_h} \right) \text{sech}\,\Omega y. \tag{3.15}$$

Then the first term of Eq. (2.1) yields

$$-\int_{-\infty}^{\infty} dy \, |\beta|^2 (\mathcal{F}_{MN})^2 = -\frac{1}{4} \left(1 + 2\frac{\sqrt{2}h}{v_h} + \frac{2h^2}{v_h^2} \right) (F_{\mu\nu}^{(0)})^2. \tag{3.16}$$

Thus, there is a new tree-level amplitude for $h \to \gamma\gamma$. In the SM, the Higgs boson decays into two photons mediated by top or W bosons at the one-loop level. The operator of interest is $c\frac{h}{v_h}(F_{\mu\nu}^{(0)})^2$, whose coefficient is bounded by the LHC measurement as $c \sim 10^{-3}$ [66,67]. However, our simplest model has $c = \frac{1}{2}$, so is strongly excluded experimentally.

3.4. Generalized models

To have a phenomenologically acceptable $h \to \gamma\gamma$ decay amplitude, we can modify the field-dependent gauge kinetic term as, e.g.,

$$\beta^2(\mathcal{H}) = \frac{1}{2\mu^2} \left(|\mathcal{H}|^2 - \frac{3}{4} \frac{|\mathcal{H}|^4}{\bar{v}^2} \right). \tag{3.17}$$

---

[7] Note that $\alpha \to 0$ as $b \to 1$.

The background configuration of the Higgs field $\mathcal{H} = \mathcal{H}_0(y)$ remains the same as in Eq. (2.5) since the $\beta^2 \mathcal{F}_{MN}^2$ term does not contribute to the background solution. The reason for selecting this specific modification will be explained below soon. Before that, however, let us mention that the modification comes with a price. The linearized equation of motion in the generalized R_ξ gauge for the gauge field with a generic β reads [51,62]

$$\left\{ \eta^{\mu\nu}\Box - \left(1 - \frac{1}{\xi}\right)\partial^\mu\partial^\nu + \eta^{\mu\nu}\left(-\partial_y^2 + \frac{(\partial_y^2\beta)}{\beta} + q_H^2\frac{\mathcal{H}_0^2}{2\beta^2}\right)\right\}A_\nu = 0. \qquad (3.18)$$

Then, determining the physical spectrum corresponds to solving the eigenvalue problem

$$\left(-\partial_y^2 + \frac{(\partial_y^2\beta)}{\beta} + q_H^2\frac{\mathcal{H}_0^2}{2\beta^2}\right)\phi_n = m_n^2\phi_n. \qquad (3.19)$$

If β^2 is quadratic in $\mathcal{H}$ as was the case in Eq. (2.3), the third term on the left-hand side is constant. Therefore, the problem is of the same complexity as if $q_H = 0$. On the other hand, when β^2 is not purely quadratic, the eigenvalue problem is essentially different from that of $-\partial_y^2 + \frac{(\partial_y^2\beta)}{\beta}$. Figure 1(b) shows the corresponding Schrödinger potential. In the case of Eq. (3.17), the Schrödinger equation in terms of the dimensionless coordinate $z = \Omega y$ is given by

$$\left(-\partial_z^2 + \frac{\partial_z^2\beta_0}{\beta_0} + q_H^2\frac{\mu^2}{\Omega^2}\frac{2}{1 - \frac{3}{4}\frac{\mathcal{H}_0^2}{\bar{v}^2}}\right)\phi_n = \frac{m_n^2}{\Omega^2}\phi_n, \quad \beta_0 = \beta(\mathcal{H}_0). \qquad (3.20)$$

Note that this is independent of $\bar{v}$ because of $\mathcal{H}_0 = \bar{v}\,\text{sech}\,z$. Although we cannot solve this exactly, we can still solve this problem perturbatively for $\Omega \gg \mu$ by treating the third term on the left-hand side as a small correction. The lowest eigenfunction and eigenvalue are approximately given by

$$\phi_0 = \frac{\mu\sqrt{2\Omega}}{\bar{v}}\beta_0, \quad m_0^2 \simeq 2q_H^2\mu^2\int dy\,\phi_0^2\left(1 - \frac{3}{4}\text{sech}^2\Omega y\right)^{-1} = 2q_H^2\mu^2. \qquad (3.21)$$

This is just the same as Eq. (2.14), and, therefore, the mass of the lightest massive gauge boson is of order μ, which justifies our assumption $\Omega \gg \mu$. Since the situation is almost the same as in the simplest model, we have $v_h^2/2 = \int dy\,\mathcal{H}_0^2 = 2\bar{v}^2/\Omega$, and the effective gauge coupling is $e \sim \mu/v_h \sim 1$. Thus the modified model defined by Eq. (3.17) provides the SM at low energies in the same manner as the simplest model does.

Now, let us turn to the problem of $h \to \gamma\gamma$. So we set $q_H = 0$ and Eq. (3.21) becomes the exact wave function of the massless photon. As before, we put $\mathcal{H}$ given in Eq. (3.15) into the gauge kinetic term $-\beta^2 \mathcal{F}_{MN}^2$ with β^2 given in Eq. (3.17). Then, we find

$$-\int_{-\infty}^{\infty} dy\,\beta^2(\mathcal{F}_{\mu\nu})^2 = \left[-\frac{1}{4} + \frac{2h^2}{v_h^2} + \mathcal{O}\left(\frac{h^3}{v_h^3}\right)\right](F_{\mu\nu}^{(0)})^2. \qquad (3.22)$$

As we see, the term $h(F_{\mu\nu}^{(0)})^2$ does not exist. Therefore, the modified model is compatible with the bound given by the current experimental measurement of $h \to \gamma\gamma$.

If the factor in front of the quartic term of Eq. (3.17) deviates slightly from $\frac{3}{4}$, the term $h(F_{\mu\nu}^{(0)})^2$ comes back with a tiny factor. We can compare the contribution of this tree-level term to $h \to \gamma\gamma$ with those mediated by top/W-boson loop in the SM. If a sizable discrepancy is found in the future

experiments in the $h \to \gamma\gamma$ channel compared with the SM prediction, it could be a signature of our model.

Of course, the modification in Eq. (3.17) is just an example. There are other modifications that forbid the $h \to \gamma\gamma$ process at tree level. For instance, in addition to $h\gamma\gamma$, one can eliminate other higher-dimensional interactions such as the $hh\gamma\gamma$ vertex by appropriately choosing β^2.

The above consideration holds for another similar process of $h \to gg$ (two gluons). An experimental signature should be the decay of the physical Higgs particle to hadronic jets. Moreover, it will affect the production rate of physical Higgs particles from hadron collisions.

Recently, another interesting signature was proposed from the localized heavy KK modes of gauge bosons and fermions [52,53], although the presence and/or the number of localized KK modes is more dependent on the details of the models. Our model has the same signatures too but they are subdominant in our model since they are one-loop effects of the supermassive KK modes.

4. The Standard Model

Let us briefly describe how our mechanism works in the SM. The minimal 5D Lagrangian is

$$
\mathcal{L} = -\beta(\mathcal{H})^2 \left[\left(\mathcal{G}_{MN}^a \right)^2 + \left(\mathcal{W}_{MN}^i \right)^2 + \mathcal{B}_{MN}^2 \right] + |\mathcal{D}_M \mathcal{H}|^2 + (\partial_M \mathcal{T})^2 - V
$$
$$
+ i\bar{U}\Gamma^M \mathcal{D}_M U + i\bar{Q}\Gamma^M \mathcal{D}_M Q + \eta_R \left(\mathcal{T} - m \right) \bar{U}U - \eta_L \mathcal{T}\bar{Q}Q + \chi \bar{Q}\mathcal{H}U + \text{h.c.}, \quad (4.1)
$$
$$
V = \Omega^2 |\mathcal{H}|^2 + \lambda^2 \left(\mathcal{T}^2 + |\mathcal{H}|^2 - v^2 \right)^2, \quad (4.2)
$$

where $\mathcal{G}_{MN}$, $\mathcal{W}_{MN}$, and $\mathcal{B}_{MN}$ are the field strengths of the $SU(3)_C$, $SU(2)_W$, and $U(1)_Y$ gauge fields, respectively. More explicitly, they are given by $\mathcal{W}_{MN} = \partial_M \mathcal{W}_N - \partial_N \mathcal{W}_M + iq [\mathcal{W}_M, \mathcal{W}_N]$, and so on. The Higgs field $\mathcal{H}$ is an $SU(2)_W$ doublet with the covariant derivative $\mathcal{D}_M \mathcal{H} = \left(\partial_M + \frac{i}{2}q\mathcal{W}_M + \frac{i}{2}q'\mathcal{B}_M \right) \mathcal{H}$, with q and q' being 5D gauge couplings for $SU(2)_W$ and $U(1)_Y$ relative to that of $SU(3)_C$. We will assume $\tan\theta_{\rm w} = q'/q$ to reproduce the SM at low energy. The fermions Q and U are doublet and singlet of $SU(2)_W$, respectively. Flavor indices for U, Q, and the couplings are implicit.

As before, there are two discrete vacua $\mathcal{T} = \pm v$ and $\mathcal{H} = 0$. The background domain-wall solution in the parameter region $\lambda v > \Omega$ is given by

$$
\mathcal{T}_0 = v \tanh \Omega y, \quad \mathcal{H}_0 = \begin{pmatrix} 0 \\ \bar{v}\,\text{sech}\,\Omega y \end{pmatrix}. \quad (4.3)
$$

The Higgs doublet $H(x)$ in the 4D effective theory is found in $\mathcal{H}$ as is done in Eq. (2.7). The Higgs potential is identical to that in Eq. (2.8). One can show that the upper component and the imaginary part of the lower component are localized NG bosons and are absorbed by the W and Z bosons. Indeed, the spectra of $W_\mu^\pm = 2\beta \mathcal{W}_\mu$ and $Z_\mu = 2\beta \mathcal{Z}_\mu$ are determined by the 1D Schrödinger problems

$$
-\partial_y^2 + \frac{(\partial_y^2 \beta)}{\beta} + \frac{q^2}{4} \frac{\mathcal{H}_0^2}{2\beta^2}, \quad -\partial_y^2 + \frac{(\partial_y^2 \beta)}{\beta} + \frac{q^2}{4\cos^2\theta_{\rm w}} \frac{\mathcal{H}_0^2}{2\beta^2}. \quad (4.4)
$$

The details of the derivation will be given elsewhere [62]. On the other hand, the photon $A_\mu = 2\beta \mathcal{A}_\mu$ and gluon $G_\mu = 2\beta \mathcal{G}_\mu$ are determined by $-\partial_y^2 + \frac{(\partial_y^2 \beta)}{\beta}$. Therefore, the lightest modes $\phi_0 \propto \beta$ of the photon and gluon are exactly massless. The results so far are independent of β^2. To be concrete, let
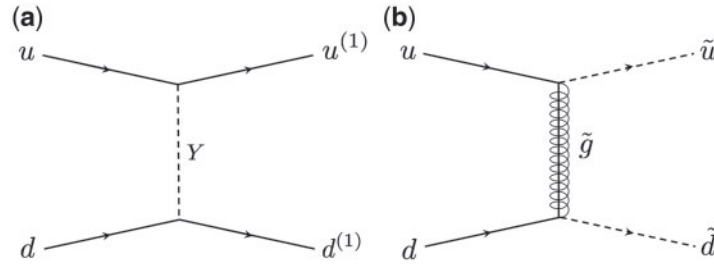
Fig. 2. Feynman diagrams for the processes (*a*) $ud \to u^{(1)}d^{(1)}$ and (*b*) $ud \to \tilde{u}\tilde{d}$.

us choose the simplest function $\beta^2 = |\mathcal{H}|^2/4\mu^2$. Then the effective $SU(2)_W$ gauge couplings and the electric charge are given by

$$g = \frac{\sqrt{2}q\mu}{v_h}, \quad g' = \frac{\sqrt{2}q'\mu}{v_h}, \quad e = \frac{qq'}{\sqrt{q^2+q'^2}}\frac{\sqrt{2}\mu}{v_h} = \frac{gg'}{\sqrt{g^2+g'^2}}, \tag{4.5}$$

where v_h is given in Eq. (2.10). The masses of W and Z are easily read from Eq. (4.4) as

$$m_W^2 = \frac{q^2\mu^2}{2} = \frac{g^2 v_h^2}{4}, \quad m_Z^2 = \frac{q^2\mu^2}{2\cos^2\theta_w} = \frac{g^2 v_h^2}{4\cos^2\theta_w}. \tag{4.6}$$

For the fermions, we assume $\eta_L > 0$ and $\eta_R > 0$. Then the left-handed fermion from Q is localized at the zero of $\mathcal{T}$, while the right-handed fermion from U is localized at the zero of $\mathcal{T} - m$. The Yukawa term $\chi\bar{Q}\mathcal{H}U$ is responsible for giving nonzero masses to the localized chiral fermions, which are necessarily exponentially small for $m \neq 0$ since the left- and right-handed fermions are split in space. By distinguishing parameters such as m for different generations, as was done in many models with extra dimensions [74,75], the hierarchical Yukawa coupling can be naturally explained in our model.

This way, the SM particles are correctly localized on the domain wall in our framework.

Before closing, we evaluate the lower bound of the KK quark mass by using the KK quark production process in Eq. (3.14) via Nambu–Goldstone boson exchange. If we take initial quarks of different flavors for simplicity, we have only the single Feynman diagram depicted in Fig. 2(a). In the process (3.14) followed by $\psi_{L,R}^{(1)} \to Y\psi_{L,R}^{(0)}$, the final state contains two SM fermion jets and some missing energy of the NG boson Y, whose signature is similar to squark pair production, where a squark decays into the partner SM quark and a gluino or neutralino in the simplified supersymmetric models [54–56]. In most of the kinematical regions, the dominant process for squark pair production is given by the Feynman diagram depicted in Fig. 2(b). Since both processes involve the same valence quark distribution functions, we can compare these cross sections directly to obtain an order-of-magnitude estimate of the lower bound for the KK quark mass using the analysis for the squark mass bound. As shown in Appendix C, the differential cross section $\frac{d\sigma}{dt}$ of Eq. (3.14) producing a pair of the first KK fermions with mass M_1 is given by summing the contributions from initial states of different chiralities (LL, RR, LR, RL) as

$$\frac{d\sigma}{dt}(ud \to u^{(1)}d^{(1)}) = \frac{\alpha^4}{576\pi^2}\frac{\Omega^2}{v^4}\frac{(1-\beta_{M_1}\cos\theta)^2(1-\beta_{M_1}^2)}{(\beta_{M_1}^2+1-2\beta_{M_1}\cos\theta)^2}, \tag{4.7}$$

where $\beta_{M_1} = \sqrt{1-\frac{M_1^2}{E^2}}$, E is the center-of-mass energy of incoming particles, and θ is the scattering angle. We ignore the masses of the SM quarks and all the parameters are taken to be common for the different quarks just for simplicity. We can assume $v \approx \Omega^{3/2}$ and $M_1 \approx \Omega$ for simplicity.

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The squark production $ud \to \tilde{u}\tilde{d}$ cross section [57–59] is

$$\frac{d\sigma}{dt}(ud \to \tilde{u}\tilde{d}) = \frac{g_s^4}{288\pi} \frac{1 + \beta_{m_{\tilde{q}}}^2 \cos^2\theta + (m_{\tilde{g}2} - m_{\tilde{q}}^2)/E^2}{(2E^2(1 - \beta_{m_{\tilde{q}}}\cos\theta) + m_{\tilde{g}}^2 - m_{\tilde{q}}^2)^2}, \tag{4.8}$$

with $\beta_{m_{\tilde{q}}} = \sqrt{1 - \frac{m_{\tilde{q}}^2}{E^2}}$. The $SU(3)_C$ gauge coupling and gluino mass are denoted as g_s and $m_{\tilde{g}}$, and a common mass $m_{\tilde{q}}$ is assumed for squarks of different flavors and chiralities.

To obtain the bound for the production of heavy particles, we can expect that the cross section near threshold ($\beta = 0$) is a good guide for the order-of-magnitude estimate. Both differential cross sections become constants without angular dependence at the threshold, and their ratio is given as

$$\frac{\frac{d\sigma}{dt}(ud \to u^{(1)}d^{(1)})|_{E=M_1}}{\frac{d\sigma}{dt}(ud \to \tilde{u}\tilde{d})|_{E=m_{\tilde{q}}}} = \frac{1}{2\pi} \frac{\alpha^4}{g_s^4} \frac{m_{\tilde{g}}^2 m_{\tilde{q}}^2}{\Omega^4} \left(1 + \frac{m_{\tilde{q}}^2}{m_{\tilde{g}}^2}\right)^2. \tag{4.9}$$

The simplified analysis for squark production gives $m_{\tilde{q}} > 1.5$ TeV, assuming $m_{\tilde{q}} = m_{\tilde{g}}$ [60,61]. The identical bound for the KK fermion mass $M_1 \sim \Omega > 1.5$ TeV is obtained for $2\alpha^4/(\pi g_s^4) \approx 1$. Since $\Omega = 10^{2+a}$ GeV, we have the lower bound for a as $a \gtrsim 1$. If the coupling α of the KK fermion is larger than g_s, we obtain a larger lower bound for its mass. To determine how much larger requires a more detailed analysis of the data.

5. Finite electroweak monopoles

The SM has a point magnetic monopole, which is the so-called Cho–Maison (CM) monopole [68]. It is different from either a Dirac monopole or a Nambu electroweak monopole [69]. Unfortunately, its mass diverges due to a singularity at the center of the monopole. Cho, Kim, and Yoon (CKY) [70] have proposed a modification of the SM in four dimensions that includes the field-dependent gauge kinetic term as $\mathcal{L} \in -\frac{\epsilon(|H|/v_h)}{4}(B_{\mu\nu})^2$. In order to have the conventional SM at the electroweak vacuum $|H| = v_h$, the normalization should be fixed as $\epsilon(|H| \to v_h) = 1$. It was found that this modification makes the CM monopole regular if $\epsilon \sim |H|^n$ with $n > 4 + 2\sqrt{3} \simeq 7.46$ as $|H| \to 0$. However, it has recently been pointed out by Ellis, Mavromatos, and You (EMY) [71] that the original CKY model is incompatible with LHC measurements of the Higgs boson $H \to \gamma\gamma$. They have proposed generalizations of the CKY model that are compatible with the LHC measurements. Their conclusion is that the monopole mass is $\lesssim 5.5$ TeV, so it could be pair-produced at the LHC and accessible to the MoEDAL experiment [72,73].

Neither CKY nor EMY discuss the underlying rationale for their modifications to the SM. In contrast, our 5D model has a clear motivation for the field-dependent gauge kinetic term, which is the domain-wall-induced Higgs mechanism. For example, one of EMY's proposals is [71]

$$\epsilon_1 = 5\left(\frac{H}{v_h}\right)^8 - 4\left(\frac{H}{v_h}\right)^{10}. \tag{5.1}$$

This can be derived from our model with

$$\beta^2 = \frac{|\mathcal{H}|^2}{\mu^2}\left(10\frac{|\mathcal{H}|^6}{\bar{v}^6} - 9\frac{|\mathcal{H}|^8}{\bar{v}^8}\right). \tag{5.2}$$

The background solution is still $\mathcal{H}_0 = \bar{v}\,\text{sech}\,\Omega y$. Figure 1(c) shows the corresponding Schrödinger potential. Then the wave function of the massive $U(1)_Y$ gauge field reads $\phi_0 \simeq$

$\sqrt{\frac{35\Omega}{64\bar{v}^8}\left(10\mathcal{H}_0^8 - 9\frac{\mathcal{H}_0^{10}}{\bar{v}^2}\right)}$. As before, we identify the 4D Higgs field $H(x)$ as $\mathcal{H} = \bar{v}\frac{H(x)}{v_h}$ sech Ωy with $v_h = \sqrt{\frac{2}{\Omega}}\bar{v}$. We find EMY's model from the five dimensions via the domain wall and the Higgs mechanism as

$$-\int_{-\infty}^{\infty} dy\, \beta^2 (\mathcal{B}_{\mu\nu})^2 = -\int_{-\infty}^{\infty} dy\, \beta^2 \frac{\phi_0^2}{4\beta_0^2}(B_{\mu\nu}^{(0)})^2 = -\frac{\epsilon_1}{4}(B_{\mu\nu}^{(0)})^2, \tag{5.3}$$

where we ignored contributions from the massive KK modes.

Note that β^2 modifies not only the gauge kinetic term of $U(1)_Y$ but also that of $SU(2)_W$. An electroweak monopole in such a theory also has a finite mass [78].

CKY have claimed that discovery of an electroweak monopole is a real final test for the SM [70]. For us, it is not only the topological test of the SM but would also give constraints for restricting the β^2 factor of the 5D theory.

6. Conclusions and discussion

We proposed a minimal model in flat non-compact five dimensions that realizes the SM on a domain wall. In our approach, the key ingredients for achieving this result are the following: (i) the spacetime is 5D, (ii) there is an extra scalar field $\mathcal{T}$ that is responsible for the domain wall, (iii) there is a field-dependent gauge kinetic term as a function of the absolute square of the Higgs field.

In our model, all spatial dimensions are treated on the same footing at the beginning. The effective compactification of the fifth dimension happens as a result of the domain-wall formation breaking the Z_2 symmetry spontaneously. The presence of the domain wall automatically localizes chiral fermions [6,7]. The key feature of our model is that the Higgs-dependent gauge kinetic term drives the localization of SM gauge bosons and the electroweak symmetry breakdown *at the same time*. The condensation of the SM Higgs field inside the wall for $\Omega < \lambda v$ can be understood as follows. As we let the parameter Ω decrease across λv, we find that a massless mode emerges at the critical point $\Omega = \lambda v$, which becomes tachyonic below the critical point and condenses until a new stable configuration is formed. It is interesting to observe that this thought-process is analogous to a second-order phase transition if we regard the parameter Ω as temperature.

Contrary to the conventional wisdom in domain-wall model-building, where the formation of the domain wall happens separately from the Higgs condensation to break electroweak symmetry, in our model we succeeded in combining both mechanisms and keeping the Higgs field active even in five dimensions. In other words, our model is very economical in terms of field content. Naively, one may expect that this means that the domain-wall mass scale must coincide with the SM scale, but surprisingly, that does not have to be so. As we have argued in Sect. 3 all light modes are separated from all KK modes by the mass scale Ω, which is of order 10^{2+a} GeV, where a can be large at the cost of only mild fine-tuning. We found a natural bound $a \gtrsim 1$ in Sect. 4. The reason why this separation of scales happens naturally is that we are near the critical point of the domain-wall-induced Higgs condensation. In short, our model can be viewed as an enrichment of the conventional domain-wall model-building toolbox by a new instrument, which is the domain-wall-induced condensation where the Higgs field plays the role of a position-dependent order parameter.

In addition to the conceptual advantages listed above, we investigated a new interaction $h\gamma\gamma$ (and hgg) coming from Eq. (1.1). This should be bounded by the LHC measurement [66,67]; therefore it gives a constraint to β^2. However, a small deviation from the exactly vanishing amplitude $h\gamma\gamma$

from tree-level coupling is allowed, which could be a testable signature in future experiments at the LHC. This possibility of the tree-level coupling of $h\gamma\gamma$ is a new signature of our model of domain-wall-induced Higgs condensation and gauge field localization. This feature is in contrast to similar models of gauge field localization without the active participation of the Higgs field in the localization mechanism [52,53]. For instance, these models generally give only loop effects of KK particles, instead of the tree-level $h\gamma\gamma$ coupling. Therefore we can have a testable signature of $h\gamma\gamma$ even if there are no low-lying KK particles, unlike these models. Furthermore, our 5D model explains the higher-dimensional interaction as in Eq. (5.1) that allows the existence of a finite electroweak monopole, whereas previous studies have failed to provide the origin of such higher-dimensional operators [70,71]. The monopole mass was estimated [70,71] as $\lesssim 5.5$ TeV, so that it can be pair-produced at the LHC and accessible to the MoEDAL experiment [72,73]. If an electroweak monopole is found, it will provide indirect evidence for the extra dimensions and the domain wall. Our domain-wall model can account for the hierarchical Yukawa coupling in the SM from the position difference of localized wave functions of matter, as was done in many models with extra dimensions [74,75].

If we introduce the other scalar fields ϕ_i to localize the gauge field and the Higgs field via $\beta(\phi_i)$ as in Eq. (1.1), they would have an impact on the low-energy physics like $\phi_i \rightarrow hh$, $\phi_i \rightarrow \gamma\gamma$, and $\phi_i \rightarrow gg$. Therefore, we have to be very cautious about including the extra scalar fields ϕ_i. Our model is free from this kind of concern, which is some of the important progress achieved in this work.

Although we have not explained it in detail, the absence of an additional light scalar boson from A_y is one of the important properties of our model [51–53]. Moreover, the fact that the localization of gauge fields via Eq. (1.1) automatically ensures the universality of gauge charges is also important.

In summary, the particle contents appearing in the low-energy effective theory on the domain wall are identical to those in the SM. All the KK modes can be sufficiently separated from the SM particles as long as we set $\Omega \sim 10^{2+a}$ GeV to be sufficiently large. Nevertheless, our model is distinguishable from the SM by the new tree-level decay $h \rightarrow \gamma\gamma$ ($h \rightarrow gg$) and a finite electroweak monopole. A possible concern in our model is the additional massless particle $Y(x)$, which is inevitable because it is the NG mode for spontaneously broken translational symmetry. However, thanks to the low-energy theorems, all the interactions including $Y(x)$ must appear with derivatives $\partial_\mu Y(x)$. Consequently, they are suppressed by the large mass scale Ω and have practically no impact on phenomena at energies much lower than the large mass scale Ω. The KK quark pair production via NG particle exchange gives a lower bound for Ω that is larger than 1.5 TeV. Larger Ω requires severer fine-tuning, but is safer phenomenologically, whereas smaller Ω requires less fine-tuning and can be disproved more easily by experimental data.

Let us discuss the possible effects of radiative corrections in our low-energy effective theory. The particle content of effective theory below the mass scale Ω is identical to the SM except for the NG boson $Y(x)$ for translation. The higher-dimensional operators of NG boson interactions are suppressed by powers of the large mass Ω. Hence they do not contribute to phenomena at energies much below the scale Ω, in the spirit of the effective Lagrangian approach. The only possible exception is the Higgs coupling of gauge fields expressed by higher-dimensional operators with the small mass scale μ in the gauge kinetic function. This coupling of Higgs boson and gauge fields such as in Eq. (3.22) is given by the Higgs vacuum expectation value v_h. We need to assume that the higher-dimensional coupling of Higgs boson and gauge fields is fine-tuned to that value when the Higgs vacuum expectation value is fine-tuned to a value much smaller than Ω. With this assumption, we expect that the radiative corrections to quantities such as the physical Higgs

boson mass should be essentially the same as the nonsupersymmetric SM. For instance, we need to implement supersymmetry if we wish to make the fine-tuning less severe in our model.

Models with warped spacetime [4,5] exhibit features similar to our model, except that the usual assumption of a delta-function-like brane in models with warped spacetime is replaced by a smooth localized energy density (fat brane) in our model. Previously we have studied BPS domain-wall solutions embedded into 4- and 5D supergravity [11–13]. These solutions are quite similar to the BPS domain-wall solutions in our present model. From these examples, we expect that our model can be coupled to gravity giving a fat brane embedded into warped spacetime. The resulting model should give physics in warped spacetime with a finite wall width. We expect that the phenomenology of our model will not be affected too much as long as we consider phenomena at energies below the gravitational (Planck) scale.

Finally, our model offers an interesting problem for the study of the cosmological evolution of the universe. Let us restrict ourselves to the region of temperature around the scale $\lambda \bar{v} \sim 10^2$ GeV, where an analysis using the effective potential is applicable. As we calculate explicitly in Appendixes A and B, we find that the effective potential computed on a stable background with $\langle H \rangle = v_h/\sqrt{2}$ is slightly different from that computed on an unstable background with vanishing Higgs $\langle H \rangle = 0$. More explicitly, only the quadratic term has a different coefficient λ_2^2: it changes from $-4(\lambda \bar{v})^2/3$ at $\langle H \rangle = v_h/\sqrt{2}$ to $-(\lambda \bar{v})^2$. We can understand this phenomenon as follows. The definition of the effective Higgs field depends on the background solution on which we expand the quantum fluctuation. The off-shell extrapolation of the effective potential computed on a particular background is different from that computed on a different background. Consequently, even though the extrapolated effective potential can give the position of another neighboring stationary point correctly, the curvature (mass squared) around it need not reproduce the value of mass squared computed on that point, since the background is different. This feature is in contrast to ordinary local field theory, and perhaps can be interpreted as a composite nature of fluctuation fields on solitons. The coefficient λ_2^2 is directly related to the transition temperature of phase transition during the cosmological evolution. At zero temperature, our effective potential (2.9) calculated on the background of $\langle H \rangle = v_h/\sqrt{2}$ is valid, since we assume $\Omega < \lambda v$. As we heat up the universe starting from this situation, finite temperature effects come in to raise the effective potential for nonzero values of the Higgs field. Eventually, around a certain temperature of order $\lambda \bar{v}$, we will find a phase transition to the phase without Higgs condensation, namely $SU(2) \times U(1)$ gauge symmetry restoration. To estimate this transition temperature, we need to study the change of effective potential during this process. As we noted, the coefficient λ_2^2 is likely to change gradually from $-4(\lambda \bar{v})^2/3$ to $-(\lambda \bar{v})^2$. Therefore we need to take account of the change of λ_2^2 besides the finite temperature effects. This is an interesting new challenge to determine the transition temperature in this kind of model. We leave this issue for future study.

Acknowledgements

Appendix A.   Mode equations on the stable BPS solution

Here we define mode expansions for Higgs and other fields in order to compute the low-energy effective action in four dimensions. We need to choose a solution of field equations as a background on which we expand fluctuation fields. Since we are interested in the parameter region (2.6), we should choose the stable BPS solution in Eq. (2.5). With this background, we define fluctuation fields δT and $\delta H_R, \delta H_I$ as

$$\mathcal{T} = v \tanh \Omega y + \frac{\delta \mathcal{T}}{\sqrt{2}}, \quad \mathcal{H} = \frac{\bar{v}}{\cosh \Omega y} + \frac{\delta \mathcal{H}_R + i \delta \mathcal{H}_I}{\sqrt{2}}. \tag{A.1}$$

The quadratic part of the bosonic Lagrangian is given by means of Hamiltonians K_{TR}, K_I:

$$\mathcal{L}^{(2)} = \mathcal{L}_{TR}^{(2)} + \mathcal{L}_I^{(2)}, \tag{A.2}$$

$$\mathcal{L}_{TR}^{(2)} = \frac{1}{2} \Phi^T [-\partial_\mu \partial^\mu - K_{T,R}] \Phi, \quad \Phi^T = (\delta \mathcal{T}, \delta \mathcal{H}_R),$$

$$K_{TR} = -\partial_y^2 \mathbf{1}_2 + \begin{pmatrix} 2\lambda^2 (\mathcal{H}_0^2 + 3\mathcal{T}_0^2 - v^2) & 4\lambda^2 \mathcal{T}_0 \mathcal{H}_0 \\ 4\lambda^2 \mathcal{T}_0 \mathcal{H}_0 & \Omega^2 + 2\lambda^2 (3\mathcal{H}_0^2 + \mathcal{T}_0^2 - v^2) \end{pmatrix}, \tag{A.3}$$

$$\mathcal{L}_I^{(2)} = \frac{1}{2} \delta \mathcal{H}_I [-\partial_\mu \partial^\mu - K_I] \delta \mathcal{H}_I,$$

$$K_I = -\partial_y^2 + \Omega^2 + 2\lambda^2 (\mathcal{H}_0^2 + \mathcal{T}_0^2 - v^2). \tag{A.4}$$

Once we obtain the eigenfunctions of these Hamiltonians, we can obtain mode expansions of the 5D fields into KK towers of effective fields, such as

$$\Phi_i(x, y) = \sum_{n=0}^{\infty} \phi_n(x) u_i^{(n)}(y), \quad i = T, R, \tag{A.5}$$

where the nth eigenstate generally has components in both 5D fields $\delta \mathcal{T}$ and $\delta \mathcal{H}_R$, since they have the coupled Hamiltonian K_{TR}. The label of eigenstates n also contains continuum states.

Since $\delta \mathcal{H}_I$ will be absorbed by the gauge boson by the Higgs mechanism, we will consider only the coupled linearized field equation for $\delta \mathcal{T}$ and $\delta \mathcal{H}_R$. Since the coupled equation is difficult to solve exactly, we solve it starting from the $\lambda \bar{v} = 0$ case as a perturbation series in powers of the small parameter $\epsilon^2 = (\lambda \bar{v} / \Omega)^2$.

At $\lambda \bar{v} = 0$, the Hamiltonian K_{TR} becomes diagonal and the $\mathcal{T}$ and $\mathcal{H}_R$ linearized field equations decouple:

$$K_T = -\partial_y^2 + 4\Omega^2 - \frac{6\Omega^2}{\cosh^2 \Omega y}, \tag{A.6}$$

$$K_R = -\partial_y^2 + \Omega^2 - \frac{2\Omega^2}{\cosh^2 \Omega y}. \tag{A.7}$$

The eigenvalues of the Hamiltonian give mass squared m^2 of the corresponding effective fields.

In the parameter region (2.6), we find two discrete bound states for δT, and a continuum of states with the threshold at $(m_T^{(2)})^2 = (2\Omega)^2$:

$$u_T^{(0)}(y) = \frac{\sqrt{3\Omega}}{2} \frac{1}{\cosh^2 \Omega y}, \quad (m_T^{(0)})^2 = 0, \tag{A.8}$$

$$u_T^{(1)}(y) = \sqrt{\frac{3\Omega}{2}} \frac{\tanh \Omega y}{\cosh \Omega y}, \quad (m_T^{(1)})^2 = 3(\Omega)^2. \tag{A.9}$$

We recognize that the massless mode is precisely the Nambu–Goldstone boson for spontaneously broken translation. For the fluctuation $\delta \mathcal{H}_R$, we find that there is only one discrete bound state below the threshold at Ω^2:

$$u_R^{(0)}(y) = \frac{\sqrt{\Omega}}{2} \frac{1}{\cosh \Omega y}, \quad (m_R^{(0)})^2 = 0. \tag{A.10}$$

This is the massless particle at the critical point where condensation of $\mathcal{H}_R$ starts. It is not an accident that the functional form of this mode function is identical to the condensation of $\mathcal{H}_R$ in Eq. (2.5). This mode will become a massive physical Higgs particle when we switch on the perturbation $(\lambda\bar{v})^2 > 0$.

We can now systematically compute the perturbative corrections in powers of the small parameter ϵ. The lowest-order correction to the eigenvalue can be obtained by taking the expectation value of the perturbation Hamiltonian in terms of the lowest-order wave function. Therefore we obtain the mass eigenvalue of the physical Higgs particle up to the leading order:

$$(m_h)^2 = \int dy u_R^{(0)}(y)[(K_{TR})_{22} - K_R]u_R^{(0)}(y) = \frac{8}{3}(\lambda\bar{v})^2. \tag{A.11}$$

This result agrees with the result of the analysis using the effective potential (2.8). In fact, we can reproduce the effective potential by evaluating the cubic and quartic terms in the fluctuation field $\delta\mathcal{H}_R$. With the perturbation theory, we can compute corrections to the Higgs mass to any desired order of ϵ,

$$V_H = -\frac{4(\lambda\bar{v})^2}{3}|H|^2 + \frac{\lambda^2\Omega}{3}|H|^4, \tag{A.12}$$

in agreement with Eq. (2.8).

Appendix B. Mode equations on the unstable BPS solution

We can choose another BPS solution (2.4) as background, which becomes stable in the parameter region $\Omega < \lambda v$. We define a small fluctuation around this background as

$$\mathcal{T} = v \tanh \lambda v y + \delta\mathcal{T}'/\sqrt{2}, \quad \mathcal{H} = (\delta\mathcal{H}_R' + i\delta\mathcal{H}_I')/\sqrt{2}. \tag{B.1}$$

The linearized field equation, in this case, is decoupled with the Hamiltonian K_T', K_R', K_I' as

$$K_T' = -\partial_y^2 + 4(\lambda v)^2 - \frac{6(\lambda v)^2}{\cosh^2 \lambda v y}, \tag{B.2}$$

$$K_R' = K_I' = -\partial_y^2 + \Omega^2 - \frac{2(\lambda v)^2}{\cosh^2 \lambda v y}. \tag{B.3}$$

We find exact mode functions in this case. We find two discrete bound states for $\delta\mathcal{T}'$ and a continuum of states with the threshold at $(m_T'^{(2)})^2 = (2\lambda v)^2$:

$$u_T'^{(0)}(y) = \frac{\sqrt{3}\lambda v}{2} \frac{1}{\cosh^2 \lambda vy}, \quad (m_T'^{(0)})^2 = 0, \tag{B.4}$$

$$u_T'^{(1)}(y) = \sqrt{\frac{3\lambda v}{2}} \frac{\tanh \Omega y}{\cosh \lambda vy}, \quad (m_T'^{(1)})^2 = 3(\lambda v)^2. \tag{B.5}$$

The massless mode gives an exact NG boson mode function in this case. For the fluctuation $\delta\mathcal{H}_R'$, we find that there is only one discrete bound state below the threshold at Ω^2:

$$u_R'^{(0)}(y) = \frac{\sqrt{\lambda v}}{2} \frac{1}{\cosh \lambda vy}, \quad (m_R'^{(0)})^2 = -(\lambda \bar{v})^2. \tag{B.6}$$

This is precisely the tachyonic mode at the unstable background solution. We note that the value of (negative) mass squared is different from the corresponding value $-4(\lambda\bar{v})^2/3$ of the off-shell extension to $H = 0$ of the effective potential computed on the stable BPS solution in Eq. (2.9). This is due to the fact that a different background solution gives a different spectrum of fluctuations, even though they are qualitatively similar.

Once the exact mode function is obtained, on the background of the unstable solution, we only need to insert the following ansatz into the 5D Lagrangian and integrate over y, in order to obtain the effective potential of the effective Higgs field $H'(x)$:

$$\mathcal{T} = v \tanh \lambda vy, \quad \mathcal{H} = H'(x)\sqrt{\frac{\lambda v}{2}} \frac{1}{\cosh \lambda vy}. \tag{B.7}$$

After integrating over y, we obtain the effective action as

$$\mathcal{L}_{\text{Higgs}}(H') = |D_\mu H'|^2 - V_{H'}, \quad V_{H'} = -(\lambda\bar{v})^2|H'|^2 + \frac{\lambda^2 \Omega}{3}|H'|^4. \tag{B.8}$$

The quadratic term agrees with the mass squared eigenvalue of the mode equation of fluctuations. It is interesting to observe that the coefficient of the quadratic term is different from that computed on the stable BPS solution as background, although the quartic term is identical.

Appendix C.   Cross section for KK fermion pair production by NG boson exchange

Here we calculate the differential cross section (4.7). First we consider the process $u_L d_L \rightarrow u_L^{(1)} d_L^{(1)}$, whose Feynman diagram is shown in Fig. 2(a). The amplitude is given in terms of spinor wave functions u_{uL} and u_{dL} of incoming SM fermions, and $u_{u^{(1)}L}$ and $u_{d^{(1)}L}$ of outgoing KK quarks as

$$i\mathcal{M} = \frac{\alpha^2 \Omega}{v^2} \frac{i}{t} (\bar{u}_{u^{(1)}L}(k_1)i(\not{p}_1 - \not{k}_1)u_{uL}(p_1))(\bar{u}_{d^{(1)}L}(k_2)i(\not{p}_1 - \not{k}_1)u_{dL}(p_2)), \tag{C.1}$$

with $t = (p_1 - k_1)^2$. We approximate SM quarks to be massless, and assume the same vertex couplings α for $uu^{(1)}Y$ and $dd^{(1)}Y$ for simplicity, although they can be different since fermion wave functions for $u, u^{(1)}$ and $d, d^{(1)}$ are in general different. The squared amplitude is

$$|\mathcal{M}|^2 = \frac{4\alpha^4 \Omega^2}{v^4 t^2} \{2(p_1 \cdot (p_1 - k_1))(k_1 \cdot (p_1 - k_1)) - (p_1 \cdot k_1)t\}$$

$$\times \{2(p_2 \cdot (p_1 - k_1))(k_2 \cdot (p_1 - k_1)) - (p_2 \cdot k_2)t\}$$

$$= \frac{4\alpha^4 \Omega^2}{v^4} \frac{E^4(1 - \beta \cos\theta)^2(1 - \beta^2)}{(\beta^2 + 1 - 2\beta \cos\theta)^2}, \tag{C.2}$$

which leads to the differential cross section

$$\frac{d\sigma}{dt}(u_L d_L \to u_L^{(1)} d_L^{(1)}) = \frac{\alpha^4}{576\pi^2 s} \frac{\Omega^2}{v^4} \frac{E^2(1 - \beta_{M_1} \cos\theta)^2(1 - \beta_{M_1}^2)}{(\beta_{M_1}^2 + 1 - 2\beta_{M_1} \cos\theta)^2}, \tag{C.3}$$

with $s = 4E^2$. Other combinations of initial quark chiralities RR, LR, RL are found to give identical differential cross sections. Hence we find Eq. (4.7).

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