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BRS Invariance of Unoriented Open-Closed String Field Theory

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We present the full action for the unoriented open-closed string field theory which is based on $\alpha = p^+$ HIKKO type vertices. The BRS invariance of the action is proved up to terms which are expected to cancel the anomalous one-loop contributions. This implies that the system is invariant under the gauge transformations with open and closed string field parameters up to the anomalies.

§1. Introduction

In a previous paper,[1] which we refer to henceforth as I, we constructed a consistent string field theory (SFT) for an unoriented open-closed string mixed system to quadratic order in the string fields and proved invariance under the gauge transformation with a closed string field parameter. It was pointed out that the infinity cancellation between the disk and projective plane amplitudes[2)-8)] plays an essential role for the gauge invariance of the theory. This, in particular, implies that any *oriented* string field theory containing an open string, where there is no projective plane amplitude contribution, cannot be a consistent theory at least on a flat background.[9)-15),7)] For the case of a light-cone gauge SFT, this implies the violation of Lorentz invariance.

In this paper, we continue this task and present the full action for this unoriented open-closed string field theory which is an $\alpha = p^+$ HIKKO type theory[16)] based on light-cone style vertices. (Recall that the $\alpha = p^+$ HIKKO type theory is a HIKKO type theory in which the unphysical string length parameter α is set equal to the physical + component of string momentum p^μ; i.e., $\alpha = 2p^+$ for open string and $\alpha = p^+$ for closed string. Hence, at the price of losing manifest covariance, this theory is free from the problem of overcounting the moduli at loop levels, which the original HIKKO theory[30),31)] suffered from.) The BRS invariance of the action is thoroughly proved, up to the terms which are expected to cancel the anomalous one-loop contributions.

The SFT action for such an open-closed string mixed system is known in the case of an *oriented* string in the light-cone gauge[17)-20)] and it has five types of interaction terms, open 3- and 4-string vertices $V_3^{\rm o}$, $V_4^{\rm o}$, a closed 3-string vertex $V_3^{\rm c}$, an open-closed transition vertex U and an open-open-closed vertex U_Ω. In the present case of unoriented strings, two additional quadratic interaction terms become newly allowed.

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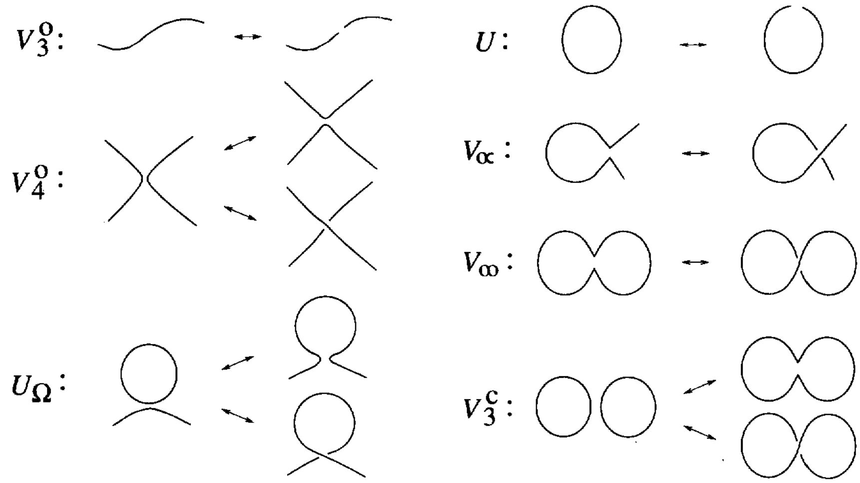


Fig. 1. Seven interaction vertices.

They are self-intersection interactions $V_\propto$ for open string and V_∞ for closed string,[21] as studied in detail in I. Intuitively, the string interactions are of only two types if viewed locally on the string world sheet; one is the joining-splitting type interaction typically appearing in $V_3^{\rm o}$, and the other is the rearrangement interaction typically appearing in $V_3^{\rm c}$. If this is true, these seven vertices already exhaust all the possible interaction terms. They are depicted in Fig. 1. From our previous work, we naturally expect that the full action of the present system is given by

$$
\begin{aligned}
S = &-\frac{1}{2}\langle\Psi|\,\tilde{Q}_{\rm B}^{\rm o}\Pi\,|\Psi\rangle - \frac{1}{2}\langle\Phi|\,\tilde{Q}_{\rm B}^{\rm c}(b_0^-\mathcal{P}\Pi)\,|\Phi\rangle \\
&+\frac{g}{3}\langle V_3^{\rm o}(1,2,3)|\,|\Psi\rangle_{321} + x_4\frac{g^2}{4}\langle V_4^{\rm o}(1,2,3,4)|\,|\Psi\rangle_{4321} + x_\propto\hbar\frac{g^2}{2}\langle V_\propto(1,2)|\,|\Psi\rangle_{21} \\
&+x_{\rm c}\hbar^{1/2}\frac{g^2}{3!}\langle V_3^{\rm c}(1^{\rm c},2^{\rm c},3^{\rm c})|\,|\Phi\rangle_{321} + x_\infty\hbar\frac{g^2}{2}\langle V_\infty(1^{\rm c},2^{\rm c})|\,|\Phi\rangle_{21} \\
&+x_u\hbar^{1/2}g\,\langle U(1,2^{\rm c})|\,|\Phi\rangle_2|\Psi\rangle_1 + x_\Omega\hbar^{1/2}\frac{g^2}{2}\langle U_\Omega(1,2,3^{\rm c})|\,|\Phi\rangle_3|\Psi\rangle_{21}\,, \qquad (1\cdot1)
\end{aligned}
$$

where x_4, $x_\propto$, $x_{\rm c}$, x_∞, x_u and x_Ω are coupling constants (relative to the open 3-string coupling constant g), and we have explicitly shown the power of $\hbar$ (as a loop expansion parameter)[19),22)] for each interaction term for clarity, although we will suppress them henceforth. For notation and conventions, we follow our previous paper, I. The open and closed string fields are denoted by $|\Psi\rangle$ and $|\Phi\rangle$, respectively. Both of these are Grassmann *odd*. The multiple products of string fields are denoted for brevity as

$$
|\Psi\rangle_{n\cdots21} \equiv |\Psi\rangle_n\cdots|\Psi\rangle_2\,|\Psi\rangle_1\,. \qquad (1\cdot2)
$$

The BRS charges $\tilde{Q}_{\rm B}$ with tilde here, introduced in I, are given by the usual BRS charges $Q_{\rm B}$ plus counterterms for the 'zero intercept' proportional to the squared string length parameter α^2:

$$
\tilde{Q}_{\rm B}^{\rm o} = Q_{\rm B}^{\rm o} + \lambda_{\rm o}g^2\alpha^2c_0\,, \qquad \tilde{Q}_{\rm B}^{\rm c} = Q_{\rm B}^{\rm c} + \lambda_{\rm c}g^2\alpha^2c_0^+\,. \qquad (1\cdot3)
$$

The ghost zero-modes for the closed string are defined by

$$c_0^+ \equiv (c_0 + \bar{c}_0)/2\,, \qquad c_0^- \equiv c_0 - \bar{c}_0\,,$$
$$b_0^+ \equiv b_0 + \bar{b}_0\,, \qquad b_0^- \equiv (b_0 - \bar{b}_0)/2\,. \tag{1·4}$$

The string fields are always accompanied by the unoriented projection operator Π, which is given by using the twist operator Ω in the form $\Pi = (1+\Omega)/2$, where Ω for the open string case means also taking transposition of the matrix index. The closed string is further accompanied by the projection operator $\mathcal{P}$, projecting out the $L_0 - \bar{L}_0 = 0$ modes

$$\mathcal{P} \equiv \int_0^{2\pi} \frac{d\theta}{2\pi} \exp i\theta(L_0 - \bar{L}_0), \tag{1·5}$$

and the corresponding anti-ghost zero-mode factor $b_0^- = (b_0 - \bar{b}_0)/2$.

The main purpose of this paper is to prove the BRS invariance of the action (1·1) and to determine the coupling constants x_4, $x_\propto$, x_c, x_∞, x_u and x_Ω. As is well known already in the light-cone gauge SFT, however, the open-closed mixed system suffers from an anomaly [18),19),17)] and thus the system is not BRS invariant as long as we consider the tree action (1·1) alone. Ideally, we should also discuss the anomalous loop diagram contributions here. But, since the BRS invariance proof is a bit too long already at the 'tree level', we are obliged to defer the anomaly discussion to a forthcoming paper. Therefore, here we do the following. First we classify the terms appearing in the BRS transform $\boldsymbol{\delta}_{\rm B}S$ of the action into groups according to the numbers of the external open and closed string fields and the power of the coupling constant g. BRS invariance implies that those terms should cancel each other separately in each group. Cancellation always occur between a pair of the configurations in which the interactions at two interaction points take place in opposite order. Then we can see which groups of the terms become the counterterms for the anomalous loop diagrams; namely, those 'loop' groups contain the terms for which the configurations become loop diagrams if the order of the two interactions is interchanged. For all the other groups (which we call 'tree' groups), we prove successively that the cancellations between such pairs of configurations indeed occur and the terms in each group in $\boldsymbol{\delta}_{\rm B}S$ completely cancel out.

The paper is organized as follows. In §2 we explain in some detail how the SFT vertices are constructed, since the signs are very important to demonstrate cancellation for proving the BRS invariance. In §3 we calculate the BRS transformation $\boldsymbol{\delta}_{\rm B}S$ of the action in a systematic way and classify the terms appearing into the groups mentioned above. Section 4 is the main part of this paper, where we present the BRS invariance proof of our action in a manner as explained above. The final section 5 is devoted to summary. In Appendix A we summarize the general rule for obtaining the BRS and gauge transformation laws from the action with a precise treatment of the statistics of the open and closed string fields. In Appendix B we explain how the "Generalized Gluing and Resmoothing Theorem" (GGRT) proved by LeClair, Peskin and Preitschopf [23)] and the present authors [24)] for the pure open string system case is made applicable to the present open-closed mixed system.

§2. Vertices

To discuss the BRS invariance of the action (1·1), we must show the cancellations between various pairs of terms, as we will do in later sections. Therefore it is very important to define the vertices correctly, including their signs and the weights. Fortunately, the definition of the vertices in the manner of LeClair, Peskin and Preitschopf (LPP),[23] is very powerful and convenient also for this purpose. Each vertex of our string field theory is defined in the form of a product of the LPP vertex corresponding to a specified way of gluing of strings and the anti-ghost factors corresponding to the moduli parameters (interaction points) of the vertices. The GGRT[23),24] for the LPP vertices makes it possible to treat the weights of the terms without recourse to detailed expressions for the LPP vertices, and the signs can be traced neatly by the anti-ghost factors contained in our SFT vertices.

Taking these into account, we give a definition of our SFT vertices in this section. For clarity, by taking the open 4-string vertex $\langle V_4^{\rm o}(1,2,3,4)|$ as a concrete example, we first explain in some detail how our SFT vertices are constructed.

The corresponding LPP vertex $\langle v_4^{\rm o}|$ is uniquely given once we know how the participating strings are glued. In our case of $\alpha = p^+$ HIKKO type theory,[16] the gluing is specified by using the string length parameters α_r as well as by the moduli parameter σ_0 specifying the interaction point. Hence, for a given set of the α parameters, $(\alpha_1, \alpha_2, \alpha_3, \alpha_4)$, the corresponding LPP vertex is denoted as

$$\langle v_4^{{\rm o}\,(\alpha_1,\alpha_2,\alpha_3,\alpha_4)}(1,2,3,4;\sigma_0)|\,, \tag{2·1}$$

and is defined by referring to the conformal field theory Green function:

$$\begin{aligned}\langle v_4^{{\rm o}\,(\alpha_1,\alpha_2,\alpha_3,\alpha_4)}(1,2,3,4;\sigma_0)|\,\mathcal{O}_4^{(4)}\,|0\rangle_4\,\mathcal{O}_3^{(3)}\,|0\rangle_3\,\mathcal{O}_2^{(2)}\,|0\rangle_2\,\mathcal{O}_1^{(1)}\,|0\rangle_1 \\ = \prod_{r=1}^{4}\left(\frac{dZ_r}{dw_r}\right)^{d_{\mathcal{O}_r}}\cdot\Big\langle \mathcal{O}_4(Z_4)\mathcal{O}_3(Z_3)\mathcal{O}_2(Z_2)\mathcal{O}_1(Z_1)\Big\rangle.\end{aligned} \tag{2·2}$$

We call this type of vertex with string length parameters specified a 'specific LPP vertex', as distinguished from a 'generic LPP vertex' introduced below. This open 4-string vertex exists only for sets of the α parameters $(\alpha_1, \alpha_2, \alpha_3, \alpha_4)$ with alternating signs, $(+,-,+,-)$ and $(-,+,-,+)$. The string configuration is explicitly depicted in Fig. 2 for the case of $\text{sign}(\alpha_1,\alpha_2,\alpha_3,\alpha_4) = (+,-,+,-)$. An important property of such a specific LPP vertex is

$$\begin{aligned}\langle v_4^{{\rm o}\,(\alpha_1,\alpha_2,\alpha_3,\alpha_4)}(1,2,3,4;\sigma_0)| &= \langle v_4^{{\rm o}\,(\alpha_2,\alpha_3,\alpha_4,\alpha_1)}(2,3,4,1;\sigma_0)| \\ &= \langle v_4^{{\rm o}\,(\alpha_3,\alpha_4,\alpha_1,\alpha_2)}(3,4,1,2;\sigma_0)|\,,\end{aligned} \tag{2·3}$$

etc. Namely, once the vertex type is fixed (now, $\langle v_4^{\rm o}|$), the specific LPP vertex is uniquely given by specifying which string (i.e., Fock space label) $r = 1, 2, \cdots$ corresponds to which length parameter α_r. The order of the arguments is irrelevant aside from the cyclic ordering among open strings (and totally irrelevant for closed strings). This property is apparently trivial, since those LPP vertices correspond

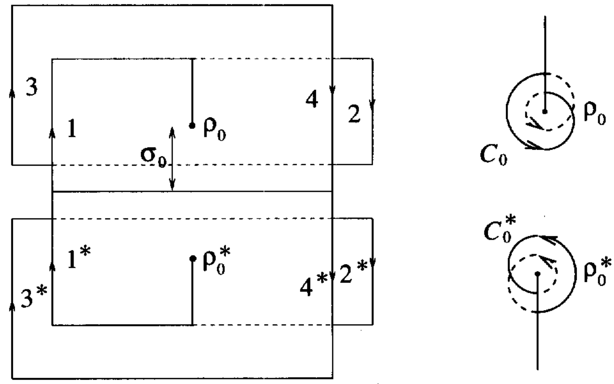


Fig. 2. The ρ plane of the open 4-string vertex $V_4^{\rm o}$. The integration contours C_0 and C_0^* used in Eq. (2·9) for defining b_{ρ_0} and $b_{\rho_0^*}$ are also shown.

to the same mapping of the unit disks $|w_r| \leq 1$ of strings r into the complex plane z. But the equality including the overall sign factor is not so trivial in fact, and therefore we demonstrate it from the definition (2·2):

$$\begin{aligned}
&\langle v_4^{{\rm o}\,(\alpha_2,\alpha_3,\alpha_4,\alpha_1)}(2,3,4,1;\sigma_0)|\, \mathcal{O}_1^{(1)}\,|0\rangle_1\, \mathcal{O}_4^{(4)}\,|0\rangle_4\, \mathcal{O}_3^{(3)}\,|0\rangle_3\, \mathcal{O}_2^{(2)}\,|0\rangle_2 \\
&= \prod_{r=1}^{4}\left(\frac{dZ_r}{dw_r}\right)^{d_{\mathcal{O}_r}} \cdot \Big\langle \mathcal{O}_1(Z_1)\mathcal{O}_4(Z_4)\mathcal{O}_3(Z_3)\mathcal{O}_2(Z_2)\Big\rangle \\
&= (-1)^{|1|(|2|+|3|+|4|)} \prod_{r=1}^{4}\left(\frac{dZ_r}{dw_r}\right)^{d_{\mathcal{O}_r}} \cdot \Big\langle \mathcal{O}_4(Z_4)\mathcal{O}_3(Z_3)\mathcal{O}_2(Z_2)\mathcal{O}_1(Z_1)\Big\rangle \\
&= (-1)^{|1|(|2|+|3|+|4|)} \langle v_4^{{\rm o}\,(\alpha_1,\alpha_2,\alpha_3,\alpha_4)}(1,2,3,4;\sigma_0)|\, \mathcal{O}_4^{(4)}|0\rangle_4\, \mathcal{O}_3^{(3)}|0\rangle_3\, \mathcal{O}_2^{(2)}|0\rangle_2\, \mathcal{O}_1^{(1)}|0\rangle_1 \\
&= \langle v_4^{{\rm o}\,(\alpha_1,\alpha_2,\alpha_3,\alpha_4)}(1,2,3,4;\sigma_0)|\, \mathcal{O}_1^{(1)}\,|0\rangle_1\, \mathcal{O}_4^{(4)}\,|0\rangle_4\, \mathcal{O}_3^{(3)}\,|0\rangle_3\, \mathcal{O}_2^{(2)}\,|0\rangle_2\,. \qquad (2\cdot4)
\end{aligned}$$

Here $|r|$ is the statistics index of the operator $\mathcal{O}_r$ which is 0 (1) if $\mathcal{O}_r$ is bosonic (fermionic). Note that this simple property results from the fact that the Fock state $\mathcal{O}_r^{(r)}\,|0\rangle_r$ of string r and the conformal field theory operator $\mathcal{O}_r$ obeys the same statistics thanks to the *convention* that the $SL(2;C)$ vacuum $|0\rangle$ is Grassmann *even.*

We now define a 'generic LPP vertex' $\langle v_4^{\rm o}(1,2,3,4;\sigma_0)|$ by integrating over the length parameters α_r as follows:

$$\langle v_4^{\rm o}(1,2,3,4;\sigma_0)| = \int \prod_{r=1}^{4} d\alpha_r\, \delta\left(\sum_r \alpha_r\right) \langle v_4^{{\rm o}\,(\alpha_1,\alpha_2,\alpha_3,\alpha_4)}(1,2,3,4;\sigma_0)|\,. \qquad (2\cdot5)$$

This generic LPP vertex enjoys the cyclic symmetry property because of Eq. (2·3):

$$\langle v_4^{\rm o}(1,2,3,4;\sigma_0)| = \langle v_4^{\rm o}(2,3,4,1;\sigma_0)| = \langle v_4^{\rm o}(3,4,1,2;\sigma_0)| = \langle v_4^{\rm o}(4,1,2,3;\sigma_0)|\,. \qquad (2\cdot6)$$

Henceforth the term 'LPP vertex' will always mean this generic LPP vertex. Although the integration is performed over the length parameters α_r in this generic LPP vertex, only a single specific LPP vertex is picked up if it is contracted with the specific external string states $\mathcal{O}_r^{(r)}\,|0\rangle_r$; usually, the external state operator $\mathcal{O}_r^{(r)}$ takes the form $\mathcal{O}_r^{(r)} = \hat{\mathcal{O}}_r^{(r)} \exp(ip_r X^{(r)})$, so that the state

$$\mathcal{O}_r^{(r)}\,|0\rangle_r = \hat{\mathcal{O}}_r^{(r)} e^{ip_r X^{(r)}}\,|0\rangle_r = \hat{\mathcal{O}}_r^{(r)}\,|p\rangle_r \tag{2·7}$$

carries definite momentum $p_r = (\boldsymbol{p}_r, p_r^-, p_r^+)$ and hence a definite string length parameter $\alpha_r = 2p_r^+$ ($\alpha_r = p_r^+$ for closed string). Since the specific LPP vertex $\langle v_4^{\mathrm{o}\,(\alpha_1,\alpha_2,\alpha_3,\alpha_4)}(1,2,3,4;\sigma_0)|$ is constructed on the bra state $\prod_r {}_r\langle\alpha_r|$ and the bra and ket states carrying different values of α_r are orthogonal to each other, a single specific LPP vertex can survive.

Finally, the vertex $\langle V_4^{\mathrm{o}}(1,2,3,4)|$ used in the string field theory can now be defined by

$$\langle V_4^{\mathrm{o}}(1,2,3,4)| = \int_{\sigma_i}^{\sigma_f} d\sigma_0\, \langle v_4^{\mathrm{o}}(1,2,3,4;\sigma_0)|\, b_{\sigma_0} \prod_r \Pi^{(r)}, \tag{2·8}$$

where σ_i and σ_f denote the initial and final points of the moduli σ_0 (interaction point), Π is the unoriented projection operator, and b_{σ_0} is the anti-ghost factor associated with the quasi-conformal deformation of the Riemann surface corresponding to the change of the moduli σ_0,[25)-27)] which, in this 4-string vertex case, is explicitly given by

$$b_{\sigma_0} = \left(\frac{d\rho_0}{d\sigma_0}\right) b_{\rho_0} + \left(\frac{d\rho_0^*}{d\sigma_0}\right) b_{\rho_0^*}, \qquad b_{\rho_0} = \oint_{C_0} \frac{d\rho}{2\pi i} b(\rho), \quad b_{\rho_0^*} = \oint_{C_0^*} \frac{d\rho}{2\pi i} b(\rho), \tag{2·9}$$

where C_0 denotes the closed contour encircling the interaction point ρ_0 on the ρ-plane, and C_0^* and ρ_0^* are their mirrors (see Fig. 2). More generally speaking, this anti-ghost factor b_{σ_0} is characterized by the property that its BRS transform,

$$T_{\sigma_0} \equiv \{\, b_{\sigma_0},\, Q_{\mathrm{B}} \,\} = \left(\frac{d\rho_0}{d\sigma_0}\right) T_{\rho_0} + \left(\frac{d\rho_0^*}{d\sigma_0}\right) T_{\rho_0^*}, \qquad T_{\rho_0} = \oint_{C_0} \frac{d\rho}{2\pi i} T(\rho), \tag{2·10}$$

(where $T(\rho)$ is the energy-momentum tensor), is a generator of the infinitesimal transformation for the change of the moduli σ_0;[28),29)] i.e., $\langle v_4^{\mathrm{o}}(1,2,3,4;\sigma_0)|\, T_{\sigma_0} = (d/d\sigma_0)\, \langle v_4^{\mathrm{o}}(1,2,3,4;\sigma_0)|$. We, therefore, have the following important relation using the BRS invariance of the LPP vertex, $\langle v_4^{\mathrm{o}}(1,2,3,4;\sigma_0)|\, Q_{\mathrm{B}} = 0$ with $Q_{\mathrm{B}} \equiv \sum_r Q_{\mathrm{B}}^{(r)}$:

$$\begin{aligned} \langle v_4^{\mathrm{o}}(1,2,3,4;\sigma_0)|\, b_{\sigma_0} Q_{\mathrm{B}} &= \langle v_4^{\mathrm{o}}(1,2,3,4;\sigma_0)|\, \{\, b_{\sigma_0},\, Q_{\mathrm{B}} \,\} \\ &= \langle v_4^{\mathrm{o}}(1,2,3,4;\sigma_0)|\, T_{\sigma_0} = \frac{d}{d\sigma_0} \{\, \langle v_4^{\mathrm{o}}(1,2,3,4;\sigma_0)| \,\}\,. \end{aligned} \tag{2·11}$$

We come to another important point here: How do we define the moduli σ_0 explicitly? We can use as σ_0 the value of the sigma coordinate $\sigma_0^{(r)}$ of any one of the participating strings, r. (For convenience sake, we define the coordinate $\sigma_0^{(r)}$ as $\sigma_0^{(r)} \equiv |\alpha_r|\,\mathrm{Im}\ln w_r$, although the original sigma coordinate of string r is $\sigma_r = \mathrm{Im}\ln w_r$, so that the distance measured by $\sigma_0^{(r)}$ is equal to that on the ρ-plane in magnitude independently of r.) But the point is that the directions in which $\sigma_0^{(r)}$

increase are opposite if the signs α_r are opposite, which causes a sign change to the definition (2·8). Indeed, if we take two adjacent strings 1 and 2 which carry opposite signs of α_r, for instance, and suppose that the points $\sigma_0^{(1)}$ and $\sigma_0^{(2)}$ correspond to the same point on the ρ-plane, then the neighboring points $\sigma_0^{(1)}+\varepsilon$ and $\sigma_0^{(2)}-\varepsilon$ represent the same point. The contribution of this infinitesimal region to the integral in the SFT vertex (2·8) has opposite sign. Indeed, since $d\sigma_0^{(1)}=-d\sigma_0^{(2)}$ and hence

$$b_{\sigma_0^{(1)}}=\left(\frac{d\rho_0}{d\sigma_0^{(1)}}\right)b_{\rho_0}=-\left(\frac{d\rho_0}{d\sigma_0^{(2)}}\right)b_{\rho_0}=-b_{\sigma_0^{(2)}}, \tag{2·12}$$

we have

$$\int_{\sigma_0^{(1)}}^{\sigma_0^{(1)}+\varepsilon}d\sigma_0^{(1)}b_{\sigma_0^{(1)}}=\int_{\sigma_0^{(2)}}^{\sigma_0^{(2)}-\varepsilon}(-d\sigma_0^{(2)})(-b_{\sigma_0^{(2)}})=-\int_{\sigma_0^{(2)}-\varepsilon}^{\sigma_0^{(2)}}d\sigma_0^{(2)}b_{\sigma_0^{(2)}}. \tag{2·13}$$

Because of this, we generally have the relation

$$\int_{\sigma_i^{(r)}}^{\sigma_f^{(r)}}d\sigma_0^{(r)}\,\langle v_4^{\rm o}(1,2,3,4;\sigma_0^{(r)})|\,b_{\sigma_0^{(r)}}=\mathrm{sign}(\alpha_r\alpha_s)\int_{\sigma_i^{(s)}}^{\sigma_f^{(s)}}d\sigma_0^{(s)}\,\langle v_4^{\rm o}(1,2,3,4;\sigma_0^{(s)})|\,b_{\sigma_0^{(s)}}. \tag{2·14}$$

So we must specify which string's $\sigma_0^{(r)}$ coordinate is used for the moduli in the definition of the SFT vertex (2·8). We sometimes use notation like

$$\langle V_4^{\rm o}(1,\overset{\downarrow}{2},3,4)|=\int_{\sigma_i^{(2)}}^{\sigma_f^{(2)}}d\sigma_0^{(2)}\,\langle v_4^{\rm o}(1,2,3,4;\sigma_0^{(2)})|\,b_{\sigma_0^{(2)}}\prod_r\Pi^{(r)}, \tag{2·15}$$

to denote explicitly which string's $\sigma_0^{(r)}$ coordinate is used by putting a down arrow on the string label. However, we take the *convention* that SFT vertices *with the down arrow omitted* always mean that we use the $\sigma_0^{(r)}$ coordinate of the open string which appears as the *first argument* in the SFT vertex; namely,

$$\langle V_4^{\rm o}(1,2,3,4)|\;=\;\langle V_4^{\rm o}(\overset{\downarrow}{1},2,3,4)|\;=\int_{\sigma_i^{(1)}}^{\sigma_f^{(1)}}d\sigma_0^{(1)}\,\langle v_4^{\rm o}(1,2,3,4;\sigma_0^{(1)})|\,b_{\sigma_0^{(1)}}\prod_r\Pi^{(r)}. \tag{2·16}$$

With this convention, the 4-string SFT vertex properly satisfies the *anti-cyclic* symmetry

$$\langle V_4^{\rm o}(1,2,3,4)|\;=-\,\langle V_4^{\rm o}(2,3,4,1)|\;=+\,\langle V_4^{\rm o}(3,4,1,2)|\;=-\,\langle V_4^{\rm o}(4,1,2,3)|\,, \tag{2·17}$$

because of the alternating sign property of $(\alpha_1,\alpha_2,\alpha_3,\alpha_4)$ and the cyclic symmetry of LPP vertex. Note that this property should hold in any case as long as we take the convention that the open string field $|\Psi\rangle$ is Grassmann *odd*. This is because the SFT vertex appears in the action in the form $\langle V_4^{\rm o}(1,2,3,4)|\,|\Psi\rangle_4\,|\Psi\rangle_3\,|\Psi\rangle_2\,|\Psi\rangle_1$, the string label r is just a dummy there and so we should have

$$\langle V_4^{\rm o}(1,2,3,4)|\,|\Psi\rangle_{4321}=\;\langle V_4^{\rm o}(2,3,4,1)|\,|\Psi\rangle_{1432}=-\,\langle V_4^{\rm o}(2,3,4,1)|\,|\Psi\rangle_{4321}\,. \tag{2·18}$$

In a similar fashion to this V_4 example, we can define all the vertices appearing in the action (1·1). The quadratic vertices U, $V_\propto$ and V_∞ are defined explicitly in I. For the other cubic interaction vertices, it has long been known how the strings are glued (see, e.g., Refs. 30) ~ 33)). For clarity, we here cite the expressions for all the seven vertices following our way of construction and notation. The cubic interaction vertices $V_3^{\rm o}$ for open and $V_3^{\rm c}$ for closed strings and the open-closed transition vertex U have no moduli parameters:

$$\begin{aligned}\langle V_3^{\rm o}(1,2,3)| &= \langle v_3^{\rm o}(1,2,3)| \prod_{r=1,2,3} \Pi^{(r)}\,, \\ \langle V_3^{\rm c}(1^{\rm c},2^{\rm c},3^{\rm c})| &= \langle v_3^{\rm c}(1^{\rm c},2^{\rm c},3^{\rm c})| \prod_{r=1^{\rm c},2^{\rm c},3^{\rm c}} (b_0^- \mathcal{P}\Pi)^{(r)}\,, \\ \langle U(1,2^{\rm c})| &= \langle u(1,2^{\rm c})|\, (b_0^- \mathcal{P})^{(2^{\rm c})} \prod_{r=1,2^{\rm c}} \Pi^{(r)}\,. \end{aligned} \tag{2·19}$$

Note that the anti-ghost and projection factors $(b_0^- \mathcal{P}\Pi)$ for the closed strings, each being Grassmann odd, are always multiplied in the order as appearing in the argument of the vertex. The open-open-closed vertex U_Ω and closed intersection vertex V_∞, as well as the open quartic interaction vertex $V_4^{\rm o}$, have one moduli parameter specifying the interaction point:

$$\begin{aligned}\langle V_4^{\rm o}(1,2,3,4)| &= \int_{\sigma_i}^{\sigma_f} d\sigma_0^{(1)}\, \langle v_4^{\rm o}(1,2,3,4;\sigma_0^{(1)})|\, b_{\sigma_0^{(1)}} \prod_{r=1}^{4} \Pi^{(r)}\,, \\ \langle U_\Omega(1,2,3^{\rm c})| &= \int_{\sigma_i}^{\sigma_f} d\sigma_0^{(1)}\, \langle u_\Omega(1,2,3^{\rm c};\sigma_0^{(1)})|\, b_{\sigma_0^{(1)}} \Pi^{(1)}\Pi^{(2)} (b_0^- \mathcal{P}\Pi)^{(3^{\rm c})}\,, \\ \langle V_\infty(1^{\rm c},2^{\rm c})| &= \int_0^{\alpha_1\pi/2} d\sigma_0^{(1)} \langle v_\infty(1^{\rm c},2^{\rm c};\sigma_0^{(1)})|\, b_{\sigma_0^{(1)}} \prod_{r=1^{\rm c},2^{\rm c}} (b_0^- \mathcal{P}\Pi)^{(r)}. \end{aligned} \tag{2·20}$$

Finally the open intersection vertex $V_\propto$ has two moduli parameters corresponding to its two interaction points:

$$\langle V_\propto(1,2)| = \int_{0\le\sigma_1^{(1)}\le\sigma_2^{(1)}\le\pi\alpha_1} d\sigma_1 d\sigma_2 \langle v_\propto(1,2;\sigma_1^{(1)},\sigma_2^{(1)})|\, b_{\sigma_1^{(1)}} b_{\sigma_2^{(1)}} \prod_{r=1,2} \Pi^{(r)}\,. \tag{2·21}$$

§3. BRS Transformation

Once the action is given, there is now a standard procedure for obtaining the BRS and gauge transformations.[34)-37)] In this procedure, if the action is invariant under the BRS transformation, the invariance under the gauge transformation automatically follows. Although this procedure is in principle well-known, details such as signs are by no means trivial in this case of a mixed system of open and closed strings. For this reason, in Appendix A we explain the details of this procedure by developing a concise notation which can be easily translated into the present bra-ket notation. Following this procedure, we calculate in this section the BRS transformation of our action in a systematic way.

Let us write the action (1·1) in the generic form

$$
\begin{aligned}
S &= S^{\rm o}_{(2)} + S^{\rm c}_{(2)} + \sum_i S_{(i)} \\
S^{\rm o}_{(2)} &= -\frac{1}{2}\langle\Psi|\,\tilde{Q}^{\rm o}_{\rm B}\Pi\,|\Psi\rangle = -\frac{1}{2}\langle R^{\rm o}(1,2)|\,\tilde{Q}^{{\rm o}(2)}_{\rm B}\Pi^{(2)}|\Psi\rangle_{21}\,, \\
S^{\rm c}_{(2)} &= -\frac{1}{2}\langle\Phi|\,\tilde{Q}^{\rm c}_{\rm B}(b_0^-\mathcal{P}\Pi)\,|\Phi\rangle = +\frac{1}{2}\langle R^{\rm c}(1^{\rm c},2^{\rm c})|\,\tilde{Q}^{{\rm c}(2)}_{\rm B}(b_0^-\mathcal{P}\Pi)^{(2)}|\Phi\rangle_{2^{\rm c}1^{\rm c}}\,, \\
S_{(i)} &= \frac{g_{(i)}}{c(i)!\,o(i)}\Big\langle V_{(i)}(J_1,\cdots,J_{o(i)};I^{\rm c}_1,\cdots,I^{\rm c}_{c(i)})\Big|\,|\Phi\rangle_{I^{\rm c}_{c(i)}\cdots I^{\rm c}_1}|\Psi\rangle_{J_{o(i)}\cdots J_1} \\
&= \frac{g_{(i)}}{c(i)!\,o(i)}\Big\langle V_{(i)}(\boldsymbol{J}^1_{o(i)};\boldsymbol{I}^1_{c(i)})\Big|\,|\Phi\rangle_{\boldsymbol{I}_1^{c(i)}}|\Psi\rangle_{\boldsymbol{J}_1^{o(i)}}\,,
\end{aligned}
\tag{3·1}
$$

where $c(i)$ and $o(i)$ are the numbers of the closed and open string fields, respectively, appearing in the i-th type vertex $\langle V_{(i)}|$, and vectors like $\boldsymbol{J}^1_{o(i)}$ and $\boldsymbol{I}_1^{c(i)}$ are abbreviations for the ordered sets of indices $(J_1,\cdots,J_{o(i)})$ and $(I^{\rm c}_{c(i)},\cdots,I^{\rm c}_1)$. According to Eq. (A·19) in Appendix A, the BRS transformation $\boldsymbol{\delta}_{\rm B}$ of the (ket) string field is given by the differentiation of the action S with respect to the bra string field. Using the rule of differentiation explained in Appendix A and, in particular, noting that $\delta/\delta\langle\Psi|$ is Grassmann even and $\delta/\delta\langle\Phi|$ is Grassmann odd, and using $(\delta/\delta_a\langle\Psi|)\,|\Psi\rangle_b = |R^{\rm o}(a,b)\rangle$ and $(\delta/\delta_{a^{\rm c}}\langle\Phi|)\,|\Phi\rangle_{b^{\rm c}} = |R^{\rm c}(a^{\rm c},b^{\rm c})\rangle$, we find the following BRS transformation law for open and closed string fields, respectively:

$$
\begin{aligned}
\boldsymbol{\delta}^{\rm open}_{\rm B}|\Psi\rangle_a &= \frac{\delta}{\delta_a\langle\Psi|}S = -\tilde{Q}^{\rm o}_{\rm B}\Pi|\Psi\rangle_a + \sum_j \boldsymbol{\delta}^{\rm open}_{{\rm B}(j)}|\Psi\rangle_a \\
\boldsymbol{\delta}^{\rm open}_{{\rm B}(j)}|\Psi\rangle_a &\equiv \frac{\delta}{\delta_a\langle\Psi|}S_{(j)} = \frac{g_{(j)}}{c(j)!}\Big\langle V_{(j)}(\boldsymbol{L}^1_{o(j)-1},b;\boldsymbol{K}^1_{c(j)})\Big|\,|\Phi\rangle_{\boldsymbol{K}_1^{c(j)}}|R^{\rm o}(a,b)\rangle\,|\Psi\rangle_{\boldsymbol{L}_1^{o(j)-1}}, \\
\boldsymbol{\delta}^{\rm closed}_{\rm B}b_0^-\,|\Phi\rangle_{a^{\rm c}} &= \frac{\delta}{\delta_{a^{\rm c}}\langle\Phi|}S = -\tilde{Q}^{\rm c}_{\rm B}(b_0^-\mathcal{P}\Pi)|\Phi\rangle_{a^{\rm c}} + \sum_j \boldsymbol{\delta}^{\rm closed}_{{\rm B}(j)}b_0^-\,|\Phi\rangle_{a^{\rm c}} \\
\boldsymbol{\delta}^{\rm closed}_{{\rm B}(j)}b_0^-\,|\Phi\rangle_{a^{\rm c}} &\equiv \frac{\delta}{\delta_{a^{\rm c}}\langle\Phi|}S_{(j)} = (b_0^-c_0^-)^{(a^{\rm c})}\frac{\delta}{\delta_{a^{\rm c}}\langle\Phi|}S_{(j)} \\
&= -b_0^{-(a^{\rm c})}\frac{g_{(j)}}{(c(j)-1)!o(j)}\Big\langle V_{(j)}(\boldsymbol{L}^1_{o(j)};\boldsymbol{K}^1_{c(j)-1},\check{b}^{\rm c})\Big|\,|R^{\rm c}(a^{\rm c},b^{\rm c})\rangle\,|\Phi\rangle_{\boldsymbol{K}_1^{c(j)-1}}|\Psi\rangle_{\boldsymbol{L}_1^{o(j)}}.
\end{aligned}
\tag{3·2}
$$

Here we have used the fact that $(\delta/\delta_{a^{\rm c}}\langle\Phi|)S_{(j)}$ always contains the anti-ghost factor $b_0^{-(a^{\rm c})}$ from the structure of our vertices, so that when we multiply it by the factor of $(b_0^-c_0^-)^{(a^{\rm c})}$ we effectively obtain 1 since $b_0^-c_0^-b_0^- = b_0^-$. Moreover, the $b_0^{-(b^{\rm c})}$ factor contained in the vertex

$$
\Big\langle V_{(j)}(\boldsymbol{L}^1_{o(j)};\boldsymbol{K}^1_{c(j)-1},b^{\rm c})\Big| \equiv \Big\langle V_{(j)}(\boldsymbol{L}^1_{o(j)};\boldsymbol{K}^1_{c(j)-1},\check{b}^{\rm c})\Big|\,b_0^{-(b^{\rm c})}
\tag{3·3}
$$

has been eliminated together with $c_0^{-(a^{\rm c})}$ by using the equality

$$
(b_0^-c_0^-)^{(a^{\rm c})}b_0^{-(b^{\rm c})}\,|R^{\rm c}(a^{\rm c},b^{\rm c})\rangle = (b_0^-c_0^-)^{(a^{\rm c})}b_0^{-(a^{\rm c})}\,|R^{\rm c}(a^{\rm c},b^{\rm c})\rangle = b_0^{-(a^{\rm c})}\,|R^{\rm c}(a^{\rm c},b^{\rm c})\rangle\,.
\tag{3·4}
$$

$\boldsymbol{\delta}^{\rm open}_{{\rm B}\,(j)}$ and $\boldsymbol{\delta}^{\rm closed}_{{\rm B}\,(j)}$ are the parts of the BRS transformation coming from the $S_{(j)}$ term of the action. The free part of the BRS transformation, $\boldsymbol{\delta}^{\rm open}_{{\rm B}\,(2)}|\Psi\rangle_a \equiv -\tilde{Q}^{\rm o}_{\rm B}\Pi|\Psi\rangle_a$ and $\boldsymbol{\delta}^{\rm closed}_{{\rm B}\,(2)}b_0^-|\Phi\rangle_{a^{\rm c}} \equiv -\tilde{Q}^{\rm c}_{\rm B}(b_0^-\mathcal{P}\Pi)|\Phi\rangle_{a^{\rm c}}$, on the action S can be easily calculated to yield

$$
\begin{aligned}
&(\boldsymbol{\delta}^{\rm open}_{{\rm B}\,(2)}+\boldsymbol{\delta}^{\rm closed}_{{\rm B}\,(2)})S \\
&= +\langle R^{\rm o}(1,2)|\,\tilde{Q}^{{\rm o}(2)}_{\rm B}\tilde{Q}^{{\rm o}(2)}_{\rm B}\Pi\,|\Psi\rangle_{21} + \langle R^{\rm c}(1^{\rm c},2^{\rm c})|\,\tilde{Q}^{{\rm c}(2^{\rm c})}_{\rm B}\tilde{Q}^{{\rm c}(2^{\rm c})}_{\rm B}(b_0^-\mathcal{P}\Pi)^{(2^{\rm c})}\,|\Phi\rangle_{2^{\rm c}1^{\rm c}} \\
&\quad + \sum_i (-)^{o(i)+c(i)+1}\frac{g_{(i)}}{c(i)!o(i)}\left\langle V_{(i)}(\boldsymbol{J}^1_{o(i)};\boldsymbol{I}^1_{c(i)})\right| \sum_{a=I,J} Q^{(a)}_{\rm B}|\Phi\rangle_{\boldsymbol{I}_1^{c(i)}}|\Psi\rangle_{\boldsymbol{J}_1^{o(i)}}\,, \qquad (3\cdot5)
\end{aligned}
$$

where use has been made of the commutativity of $\tilde{Q}_{\rm B}$ with the projection operators Π and $\mathcal{P}$. Note that $\sum\tilde{Q}_{\rm B}$ acting on $\langle V_{(i)}|$ has become $\sum Q_{\rm B}$. This holds since the difference between them $\sum\lambda\alpha^2c_0$ vanishes generally on the vertex $\langle V_{(i)}|$:

$$
\begin{aligned}
&\left\langle V_{(i)}(\boldsymbol{J}^1_{o(i)};\boldsymbol{I}^1_{c(i)})\right|\left(\sum_{k=1}^{c(i)}\lambda_{\rm c}\alpha^2_{I_k^{\rm c}}c_0^{+\,(I_k^{\rm c})}+\sum_{k=1}^{o(i)}\lambda_{\rm o}\alpha^2_{J_k}c_0^{(J_k)}\right) \\
&= \left\langle V_{(i)}(\boldsymbol{J}^1_{o(i)};\boldsymbol{I}^1_{c(i)})\right|\lambda_{\rm o}\oint_{C_{\rho_0}}\frac{d\rho}{2\pi i}c(\rho)=0. \qquad (3\cdot6)
\end{aligned}
$$

Here $\lambda_{\rm c}=2\lambda_{\rm o}$ has been used and C_{ρ_0} is a closed contour encircling all the interaction points on the ρ plane. The presence of the anti-ghost factors b_{ρ_0} sitting at the interaction points ρ_0 is potentially dangerous, since they yield poles $\langle b(z_0)c(z)\rangle = 1/(z_0-z)$ on the z plane. But when going to the z plane, $\oint_{C_{\rho_0}}(d\rho/2\pi i)c(\rho)$ becomes $\oint_{C_{z_0}}(dz/2\pi i)(d\rho/dz)^2c(z)$, and $(d\rho/dz)^2$ contains double zeros, $(z_0-z)^2$, there. Therefore the integrand is regular even at interaction points and hence vanishes. Note that the squares of the tilded BRS operators in Eq. (3·5) become $\tilde{Q}^{\rm o}_{\rm B}\tilde{Q}^{\rm o}_{\rm B}=\lambda_{\rm o}\alpha^2g^2\{Q^{\rm o}_{\rm B},c_0\}$ and $\tilde{Q}^{\rm c}_{\rm B}\tilde{Q}^{\rm c}_{\rm B}=\lambda_{\rm c}\alpha^2g^2\{Q^{\rm c}_{\rm B},c_0^+\}$ for the open and closed string cases, respectively, by using the nilpotency of the usual $Q_{\rm B}$ as well as of c_0.

Now we calculate the j-th open BRS transformation part $\boldsymbol{\delta}^{\rm open}_{{\rm B}\,(j)}$ of the i-th action term $S_{(i)}$. Noting that $\boldsymbol{\delta}_{{\rm B}\,(j)}$ and $|R^{\rm o}(a,b)\rangle$ are Grassmann odd and that the Grassmann even-oddness of $\langle V_{(j)}|$ is $(-1)^{o(j)+c(j)}$, we find

$$
\begin{aligned}
\boldsymbol{\delta}^{\rm open}_{{\rm B}\,(j)}S_{(i)} &= \frac{g_{(i)}}{c(i)!}\left\langle V_{(i)}(a,\boldsymbol{J}^2_{o(i)};\boldsymbol{I}^1_{c(i)})\right|\,|\Phi\rangle_{\boldsymbol{I}_1^{c(i)}}|\Psi\rangle_{\boldsymbol{J}_2^{o(i)}}\left(-\boldsymbol{\delta}^{\rm open}_{{\rm B}\,(j)}|\Psi\rangle_a\right) \\
&= -\frac{g_{(i)}}{c(i)!}\left\langle V_{(i)}(a,\boldsymbol{J}^2_{o(i)};\boldsymbol{I}^1_{c(i)})\right|\,|\Phi\rangle_{\boldsymbol{I}_1^{c(i)}}|\Psi\rangle_{\boldsymbol{J}_2^{o(i)}} \\
&\qquad\times\frac{g_{(j)}}{c(j)!}\left\langle V_{(j)}(\boldsymbol{L}^1_{o(j)-1},b;\boldsymbol{K}^1_{c(j)})\right|\,|\Phi\rangle_{\boldsymbol{K}_1^{c(j)}}\,|R^{\rm o}(a,b)\rangle\,|\Psi\rangle_{\boldsymbol{L}_1^{o(j)-1}} \\
&= (-)^{1+o(j)+c(j)}\frac{g_{(i)}g_{(j)}}{c(i)!c(j)!}\left\langle V_{(j)}(\boldsymbol{L}^1_{o(j)-1},b;\boldsymbol{K}^1_{c(j)})\right|\left\langle V_{(i)}(a,\boldsymbol{J}^2_{o(i)};\boldsymbol{I}^1_{c(i)})\right| \\
&\qquad\times|\Phi\rangle_{\boldsymbol{I}_1^{c(i)}}|\Psi\rangle_{\boldsymbol{J}_2^{o(i)}}|\Phi\rangle_{\boldsymbol{K}_1^{c(j)}}\,|R^{\rm o}(a,b)\rangle\,|\Psi\rangle_{\boldsymbol{L}_1^{o(j)-1}} \\
&= (-)^{1+o(j)+c(j)}\frac{g_{(i)}g_{(j)}}{c(i)!c(j)!}\left\langle V_{(j)}(\boldsymbol{L}^1_{o(j)-1},b;\boldsymbol{K}^1_{c(j)})\right|\left\langle V_{(i)}(a,\boldsymbol{J}^2_{o(i)};\boldsymbol{I}^1_{c(i)})\right|
\end{aligned}
$$

Table I. Coefficients $C_{ji}^{\rm open}$ of $\boldsymbol{\delta}_{{\rm B}\,(j)}^{\rm open} S_{(i)}$.

		(i)	$V_3^{\rm o}(g)$	$U(x_u g)$	$V_4^{\rm o}(x_4 g^2)$	$V_\propto(x_\propto g^2)$	$U_\Omega(x_\Omega g^2)$
		$o(i)+1$	$4\equiv 0$	$2\equiv 0$	$5\equiv 1$	$3\equiv 1$	$3\equiv 1$
		$c(i)$	0	1	0	0	1
(j)	$c(j)+1$	$o(j)$					
$V_3^{\rm o}(g)$	1	$3\equiv 1$	$-g^2$	$+x_u g^2$	$+x_4 g^3$	$+x_\propto g^3$	$-x_\Omega g^3$
$U(x_u g)$	$2\equiv 0$	1	$-x_u g^2$	$+x_u^2 g^2$	$-x_u x_4 g^3$	$-x_u x_\propto g^3$	$+x_u x_\Omega g^3$
$V_4^{\rm o}(x_4 g^2)$	1	$4\equiv 0$	$+x_4 g^3$	$-x_4 x_u g^3$	$-x_4^2 g^4$	$-x_4 x_\propto g^4$	$+x_4 x_\Omega g^4$
$V_\propto(x_\propto g^2)$	1	$2\equiv 0$	$+x_\propto g^3$	$-x_\propto x_u g^3$	$-x_\propto x_4 g^4$	$-x_\propto^2 g^4$	$+x_\propto x_\Omega g^4$
$U_\Omega(x_\Omega g^2)$	$2\equiv 0$	$2\equiv 0$	$+x_\Omega g^3$	$-x_\Omega x_u g^3$	$+x_\Omega x_4 g^4$	$+x_\Omega x_\propto g^4$	$-x_\Omega^2 g^4$

Table II. Coefficients $C_{ji}^{\rm closed}$ of $\boldsymbol{\delta}_{{\rm B}\,(j)}^{\rm closed} S_{(i)}$.

		(i)	$U(x_u g)$	$V_\infty(x_\infty g^2)$	$V_3^{\rm c}(\frac{1}{2!}x_{\rm c} g^2)$	$U_\Omega(\frac{1}{2}x_\Omega g^2)$
		$o(i)+1$	$2\equiv 0$	1	1	$3\equiv 1$
(j)	$c(j)$	$o(j)$				
$\check{U}\,(x_u g)$	1	1	$-x_u^2 g^2$	$+x_u x_\infty g^2$	$+\frac{1}{2!}x_u x_{\rm c} g^3$	$+\frac{1}{2}x_u x_\Omega g^3$
$\check{V}_\infty\,(x_\infty g^2)$	$2\equiv 0$	0	$+x_\infty x_u g^3$	$+x_\infty^2 g^4$	$+\frac{1}{2!}x_\infty x_{\rm c} g^4$	$+\frac{1}{2}x_\infty x_\Omega g^4$
$\check{V}_3^{\rm c}\,(\frac{1}{2!}x_{\rm c} g^2)$	$3\equiv 1$	0	$+\frac{1}{2!}x_{\rm c} x_u g^3$	$-\frac{1}{2!}x_{\rm c} x_\infty g^4$	$-\frac{1}{2!2!}x_{\rm c}^2 g^4$	$-\frac{1}{2!2}x_{\rm c} x_\Omega g^4$
$\check{U}_\Omega\,(\frac{1}{2}x_\Omega g^2)$	1	$2\equiv 0$	$+\frac{1}{2}x_\Omega x_u g^3$	$-\frac{1}{2}x_\Omega x_\infty g^4$	$-\frac{1}{2!2}x_\Omega x_{\rm c} g^4$	$-\frac{1}{4}x_\Omega^2 g^4$

$$\times(-)^{c(i)+o(i)-1+c(j)}\,|R^{\rm o}(a,b)\rangle\,|\Phi\rangle_{\boldsymbol{I}_1^{c(i)}}(-)^{c(j)(o(i)-1)}|\Phi\rangle_{\boldsymbol{K}_1^{c(j)}}|\Psi\rangle_{\boldsymbol{J}_2^{o(i)}}|\Psi\rangle_{\boldsymbol{L}_1^{o(j)-1}}$$

$$= C_{ji}^{\rm open}\left\langle V_{(j)}(\boldsymbol{L}^1_{o(j)-1}, a; \boldsymbol{K}^1_{c(j)})\right|\left\langle V_{(i)}(b, \boldsymbol{J}^2_{o(i)}; \boldsymbol{I}^1_{c(i)})\right|\,|R^{\rm o}(a,b)\rangle$$
$$\times|\Phi\rangle_{\boldsymbol{I}_1^{c(i)},\boldsymbol{K}_1^{c(j)}}|\Psi\rangle_{\boldsymbol{J}_2^{o(i)},\boldsymbol{L}_1^{o(j)-1}}, \qquad (3\cdot7)$$

with the final coefficient given by

$$C_{ji}^{\rm open} = (-)^{(c(j)+1)(o(i)+1)+o(j)+c(i)}\frac{g_{(i)}g_{(j)}}{c(i)!c(j)!}\,. \qquad (3\cdot8)$$

Note that, in going to the last line, we have exchanged the arguments a and b of $|R^{\rm o}(b,a)\rangle$ using the anti-symmetry property, $|R^{\rm o}(b,a)\rangle = -\,|R^{\rm o}(a,b)\rangle$. This was done for later convenience in applying the GGRT. In the same way we can find the j-th closed BRS transformation part $\boldsymbol{\delta}_{{\rm B}\,(j)}^{\rm closed}$ of the i-th action term $S_{(i)}$:

$$\boldsymbol{\delta}_{{\rm B}\,(j)}^{\rm closed}S_{(i)} = C_{ji}^{\rm closed}\left\langle V_{(j)}(\boldsymbol{L}^1_{o(j)}; \boldsymbol{K}^1_{c(j)-1}, a^{\rm c})\right|\left\langle V_{(i)}(\boldsymbol{J}^1_{o(i)}; b^{\rm c}, \boldsymbol{I}^2_{c(i)})\right|\,|R^{\rm c}(a^{\rm c},b^{\rm c})\rangle$$
$$\times|\Phi\rangle_{\boldsymbol{I}_2^{c(i)},\boldsymbol{K}_1^{c(j)-1}}|\Psi\rangle_{\boldsymbol{J}_1^{o(i)},\boldsymbol{L}_1^{o(j)}} \qquad (3\cdot9)$$

with the coefficient

$$C_{ji}^{\rm closed} = (-)^{c(j)(o(i)+1)+o(j)}\frac{g_{(i)}g_{(j)}}{(c(i)-1)!(c(j)-1)!o(i)o(j)}\,. \qquad (3\cdot10)$$

The resultant coefficients $C_{ji}^{\rm open}$ and $C_{ji}^{\rm closed}$ for $\boldsymbol{\delta}_{{\rm B}\,(j)}^{\rm open}S_{(i)}$ and $\boldsymbol{\delta}_{{\rm B}\,(j)}^{\rm closed}S_{(i)}$ are summarized in Tables I and II, respectively.

It can be easily shown that $\boldsymbol{\delta}_{\mathrm{B}\,(j)}S_{(i)} = \boldsymbol{\delta}_{\mathrm{B}\,(i)}S_{(j)}$ both for $\boldsymbol{\delta}^{\mathrm{open}}_{\mathrm{B}\,(j)}$ and $\boldsymbol{\delta}^{\mathrm{closed}}_{\mathrm{B}\,(j)}$, so that we need retain only half of the terms $\boldsymbol{\delta}_{\mathrm{B}\,(j)}S_{(i)}$ for $i \neq j$ by using the coefficients multiplied by 2. We thus can write the explicit form for the full BRS transformation $\boldsymbol{\delta}_{\mathrm{B}}S$ of the action by arranging the terms with the same number of open and closed external states and the same powers of g as given below: there are 6 vertices containing open string fields and 5 vertices containing closed string fields (including the 2-point kinetic terms), so that $6 \times 7/2 + 5 \times 6/2 = 36$ terms appear in total. ($\langle U|\sum Q_{\mathrm{B}}$ and $\langle U_{\Omega}|\sum Q_{\mathrm{B}}$ terms below should be counted as two terms each since they contain both $Q^{\mathrm{o}}_{\mathrm{B}}$ and $Q^{\mathrm{c}}_{\mathrm{B}}$.) We have the following:

$\mathrm{O}(g)$:

$$\text{(T1)}\qquad \tfrac{2}{3}\,\langle V^{\mathrm{o}}_3(1,2,3)|\left(Q^{(1)}_{\mathrm{B}}+Q^{(2)}_{\mathrm{B}}+Q^{(3)}_{\mathrm{B}}\right)|\Psi\rangle_{321} \tag{3·11}$$

$$\text{(T2)}\qquad -2x_u\,\langle U(1,2^{\mathrm{c}})|\left(Q^{(1)}_{\mathrm{B}}+Q^{(2^{\mathrm{c}})}_{\mathrm{B}}\right)|\Phi\rangle_{2^{\mathrm{c}}}\,|\Psi\rangle_1 \tag{3·12}$$

$\mathrm{O}(g^2)$:

$$\text{(T3)}\qquad \tfrac{2}{3}x_{\mathrm{c}}\,\langle V^{\mathrm{c}}_3(1^{\mathrm{c}},2^{\mathrm{c}},3^{\mathrm{c}})|\left(Q^{(1)}_{\mathrm{B}}+Q^{(2)}_{\mathrm{B}}+Q^{(3)}_{\mathrm{B}}\right)|\Phi\rangle_{3^{\mathrm{c}}2^{\mathrm{c}}1^{\mathrm{c}}} \tag{3·13}$$

$$\text{(T4)}\qquad \Big[-\tfrac{1}{2}x_4\,\langle V^{\mathrm{o}}_4(1,2,3,4)|\left(Q^{(1)}_{\mathrm{B}}+Q^{(2)}_{\mathrm{B}}+Q^{(3)}_{\mathrm{B}}+Q^{(4)}_{\mathrm{B}}\right) - \langle V^{\mathrm{o}}_3(1,2,a)|\,\langle V^{\mathrm{o}}_3(b,3,4)|\,|R^{\mathrm{o}}(a,b)\rangle\Big]|\Psi\rangle_{4321} \tag{3·14}$$

$$\text{(T5)}\qquad \Big[x_{\Omega}\,\langle U_{\Omega}(1,2,3^{\mathrm{c}})|\left(Q^{(1)}_{\mathrm{B}}+Q^{(2)}_{\mathrm{B}}+Q^{(3^{\mathrm{c}})}_{\mathrm{B}}\right) -2x_u\,\langle U(a,3^{\mathrm{c}})|\,\langle V^{\mathrm{o}}_3(b,1,2)|\,|R^{\mathrm{o}}(a,b)\rangle\Big]|\Phi\rangle_{3^{\mathrm{c}}}\,|\Psi\rangle_{21} \tag{3·15}$$

$$\text{(L1)}\qquad \Big[-x_{\propto}\,\langle V_{\propto}(1,2)|\left(Q^{(1)}_{\mathrm{B}}+Q^{(2)}_{\mathrm{B}}\right) -x_u^2\,\langle U(1,\check{a}^{\mathrm{c}})|\,\langle U(2,b^{\mathrm{c}})|\,|R^{\mathrm{c}}(a^{\mathrm{c}},b^{\mathrm{c}})\rangle +\lambda_{\mathrm{o}}\alpha_2^2\,\langle R^{\mathrm{o}}(1,2)|\,\{Q^{(2)}_{\mathrm{B}},c_0{}^{(2)}\}\Big]|\Psi\rangle_{21} \tag{3·16}$$

$$\text{(T6)}\qquad \Big[-x_{\infty}\,\langle V_{\infty}(1^{\mathrm{c}},2^{\mathrm{c}})|\left(Q^{(1)}_{\mathrm{B}}+Q^{(2)}_{\mathrm{B}}\right) +x_u{}^2\,\langle U(a,1^{\mathrm{c}})|\,\langle U(b,2^{\mathrm{c}})|\,|R^{\mathrm{o}}(a,b)\rangle +\lambda_{\mathrm{c}}\alpha^2_{2^{\mathrm{c}}}\,\langle R^{\mathrm{c}}(1^{\mathrm{c}},2^{\mathrm{c}})|\,\{Q^{(2^{\mathrm{c}})}_{\mathrm{B}},c_0{}^{+(2^{\mathrm{c}})}\}(b_0^-\mathcal{P})^{(2^{\mathrm{c}})}\Big]|\Phi\rangle_{2^{\mathrm{c}}1^{\mathrm{c}}} \tag{3·17}$$

$\mathrm{O}(g^3)$:

$$\text{(T7)}\qquad +2x_4\,\langle V^{\mathrm{o}}_3(1,2,a)|\,\langle V^{\mathrm{o}}_4(b,3,4,5)|\,|R^{\mathrm{o}}(a,b)\rangle\,|\Psi\rangle_{54321} \tag{3·18}$$

$$\text{(T8)}\qquad \Big[2x_{\Omega}\,\langle U_{\Omega}(1,a,4^{\mathrm{c}})|\,\langle V^{\mathrm{o}}_3(b,2,3)|\,|R^{\mathrm{o}}(a,b)\rangle -2x_ux_4\,\langle U(a,4^{\mathrm{c}})|\,\langle V^{\mathrm{o}}_4(b,1,2,3)|\,|R^{\mathrm{o}}(a,b)\rangle\Big]|\Phi\rangle_{4^{\mathrm{c}}}\,|\Psi\rangle_{321} \tag{3·19}$$

$$\text{(T9)}\qquad \Big[x_ux_{\mathrm{c}}\,\langle U(1,\check{a}^{\mathrm{c}})|\,\langle V^{\mathrm{c}}_3(b^{\mathrm{c}},2^{\mathrm{c}},3^{\mathrm{c}})|\,|R^{\mathrm{c}}(a^{\mathrm{c}},b^{\mathrm{c}})\rangle -2x_{\Omega}x_u\,\langle U_{\Omega}(1,a,2^{\mathrm{c}})|\,\langle U(b,3^{\mathrm{c}})|\,|R^{\mathrm{o}}(a,b)\rangle\Big]|\Phi\rangle_{3^{\mathrm{c}}2^{\mathrm{c}}}\,|\Psi\rangle_1 \tag{3·20}$$

$$\text{(L2)}\qquad \Big[2x_{\propto}\,\langle V_{\propto}(1,a)|\,\langle V^{\mathrm{o}}_3(b,2,3)|\,|R^{\mathrm{o}}(a,b)\rangle$$

$$+x_\Omega x_u \langle U_\Omega(1,2,\check{a}^{\rm c})|\ \langle U(3,b^{\rm c})|\ |R^{\rm c}(a^{\rm c},b^{\rm c})\rangle \Big] |\Psi\rangle_{321} \qquad (3\cdot21)$$

(T10) $$\Big[-2x_u x_\propto \langle U(a,1^{\rm c})|\ \langle V_\propto(b,2)|\ |R^{\rm o}(a,b)\rangle$$
$$+2x_u x_\infty \langle U(2,\check{a}^{\rm c})|\ \langle V_\infty(b^{\rm c},1^{\rm c})|\ |R^{\rm c}(a^{\rm c},b^{\rm c})\rangle \Big] |\Phi\rangle_{1^{\rm c}} |\Psi\rangle_2 \qquad (3\cdot22)$$

$O(g^4)$:

(T11) $$-{x_4}^2 \langle V_4^{\rm o}(1,2,3,a)|\ \langle V_4^{\rm o}(b,4,5,6)|\ |R^{\rm o}(a,b)\rangle |\Psi\rangle_{654321} \qquad (3\cdot23)$$

(T12) $$2x_\Omega x_4 \langle U_\Omega(1,a,5^{\rm c})|\ \langle V_4^{\rm o}(b,2,3,4)|\ |R^{\rm o}(a,b)\rangle |\Phi\rangle_{5^{\rm c}} |\Psi\rangle_{4321} \qquad (3\cdot24)$$

(T13) $$-\tfrac{1}{4}{x_{\rm c}}^2 \langle V_3^{\rm c}(1^{\rm c},2^{\rm c},\check{a}^{\rm c})|\ \langle V_3^{\rm c}(b^{\rm c},3^{\rm c},4^{\rm c})|\ |R^{\rm c}(a^{\rm c},b^{\rm c})\rangle |\Phi\rangle_{4^{\rm c}3^{\rm c}2^{\rm c}1^{\rm c}} \qquad (3\cdot25)$$

(T14) $$\Big[-{x_\Omega}^2 \langle U_\Omega(1,a,3^{\rm c})|\ \langle U_\Omega(b,2,4^{\rm c})|\ |R^{\rm o}(a,b)\rangle$$
$$-\tfrac{1}{2}x_\Omega x_{\rm c} \langle U_\Omega(1,2,\check{a}^{\rm c})|\ \langle V_3^{\rm c}(b^{\rm c},3^{\rm c},4^{\rm c})|\,|R^{\rm c}(a^{\rm c},b^{\rm c})\rangle \Big] |\Phi\rangle_{4^{\rm c}3^{\rm c}} |\Psi\rangle_{21} \qquad (3\cdot26)$$

(L3) $$\Big[-2x_\propto x_4 \langle V_\propto(1,a)|\ \langle V_4^{\rm o}(b,2,3,4)|\ |R^{\rm o}(a,b)\rangle$$
$$-\tfrac{1}{4}{x_\Omega}^2 \langle U_\Omega(1,2,\check{a}^{\rm c})|\ \langle U_\Omega(3,4,b^{\rm c})|\ |R^{\rm c}(a^{\rm c},b^{\rm c})\rangle \Big] |\Psi\rangle_{4321} \qquad (3\cdot27)$$

(T15) $$-x_{\rm c} x_\infty \langle V_3^{\rm c}(1^{\rm c},2^{\rm c},\check{a}^{\rm c})|\ \langle V_\infty(b^{\rm c},3^{\rm c})|\ |R^{\rm c}(a^{\rm c},b^{\rm c})\rangle |\Phi\rangle_{3^{\rm c}2^{\rm c}1^{\rm c}} \qquad (3\cdot28)$$

(T16) $$\Big[2x_\Omega x_\propto \langle U_\Omega(1,a,3^{\rm c})|\ \langle V_\propto(b,2)|\ |R^{\rm o}(a,b)\rangle$$
$$-x_\Omega x_\infty \langle U_\Omega(1,2,\check{a}^{\rm c})|\ \langle V_\infty(b^{\rm c},3^{\rm c})|\ |R^{\rm c}(a^{\rm c},b^{\rm c})\rangle \Big] |\Phi\rangle_{3^{\rm c}} |\Psi\rangle_{21} \qquad (3\cdot29)$$

(L4) $$-{x_\propto}^2 \langle V_\propto(1,a)|\ \langle V_\propto(b,2)|\ |R^{\rm o}(a,b)\rangle |\Psi\rangle_{21} \qquad (3\cdot30)$$

(L5) $${x_\infty}^2 \langle V_\infty(1^{\rm c},\check{a}^{\rm c})|\ \langle V_\infty(b^{\rm c},2^{\rm c})|\ |R^{\rm c}(a^{\rm c},b^{\rm c})\rangle |\Phi\rangle_{2^{\rm c}1^{\rm c}} \qquad (3\cdot31)$$

§4. BRS invariance

The light-cone gauge string field theory for an open-closed mixed system has long been known to have an anomaly which breaks Lorentz invariance at the one-loop level.[18)-20)] This anomaly is present even in an oriented string system, and the existence of an open-closed transition interaction U is required to cancel it. In our framework of $\alpha = p^+$ HIKKO[16)] type unoriented open-closed string theory, this is reflected in the fact that the BRS (and gauge) invariance suffers from an anomaly.

The five terms labeled (L1) $\sim$ (L5) in Eqs. (3·16), (3·21), (3·27), (3·30) and (3·31), do not vanish by themselves and will be cancelled by the anomalous contributions of one-loop diagrams.*) More naturally, we should state this oppositely: if we started with a theory possessing only the $V_3^{\rm o}$, $V_4^{\rm o}$ and $V_3^{\rm c}$ interactions, then the

*) In I, we erroneously claimed that the (L5) term, $\langle V_\infty|\,\langle V_\infty|\,|R^{\rm c}\rangle$, cancels out totally by itself. Actually, the configuration shown in (L5) in Fig. 3, which corresponds to the case where the two crosscap cuts overlap, does not cancel and needs the loop counterterm.

theory is safely BRS invariant at tree level. At the quantum level, however, the BRS invariance is violated by some anomalous one-loop diagrams, and the other interaction vertices U, U_Ω and $V_\propto, V_\infty$ are required to be introduced to cancel these anomalies. Their coupling strengths are found to be of orders

$$U,\ U_\Omega \sim O(\hbar^{1/2}), \qquad V_\propto,\ V_\infty \sim O(\hbar^1) \tag{4·1}$$

in $\hbar$ as a loop expansion parameter, as already shown in the action (1·1).[22] Therefore, the non-vanishing nature of the (L1) $\sim$ (L5) terms is by no means unwelcome. Rather, it gives the very reason for existence of the interaction vertices U, U_Ω and $V_\propto, V_\infty$.

In this paper we confine ourselves to proving the BRS invariance only at the tree level and defer the proof of cancellations between the anomalous one-loop contributions and the (L1) $\sim$ (L5) terms to a forthcoming paper. We, therefore, do not discuss the five terms (L1) $\sim$ (L5) in any detail. But here, let us just see what types of one-loop diagrams the five terms (L1) $\sim$ (L5) will cancel. This is shown in Fig. 3. There examples of string configurations which need loop counterterms are given. In each diagram, there are two interaction points, and depending on which interaction of the two takes place earlier than the other, two different intermediate states appear for a given set of initial and final states; the upper paths correspond to the 'tree' terms appearing in (L1) $\sim$ (L5), respectively, and the lower paths to the loop diagrams. (Whether the path corresponds to a loop or tree diagram can be judged by considering whether the momenta of the intermediate states are uniquely determined when those of the initial and final states are given.) Look at the first configuration (L1), for example. The upper path represents a possible configuration for the term $\langle U(1,\overset{\vee}{a}{}^{\mathrm{c}})|\ \langle U(2,b^{\mathrm{c}})|\ |R^{\mathrm{c}}(a^{\mathrm{c}},b^{\mathrm{c}})\rangle$ in Eq. (3·16), and the same configuration of the initial and final strings can be realized by choosing the other intermediate state shown in the lower path of the figure, which corresponds to the one-loop (non-planar but orientable) diagram constructed by using open 3-string vertices $\langle V_3^{\mathrm{o}}|$ twice. This (L1) example is just the same as the Lorentz anomaly in the case of the light-cone gauge string field theory mentioned above.

In this section, we prove that the theory has BRS symmetry at the tree level if the parameters λ_{c}, λ_{o}, x_u, $x_\propto$, x_∞, x_Ω, x_4 and x_{c} in the action satisfy

$$\lambda_{\mathrm{c}} = 2\lambda_{\mathrm{o}} = -\lim_{\epsilon\to 0}\frac{32nx_u^2}{\epsilon^2}, \tag{4·2}$$

$$x_\infty = nx_u^2 = 4\pi i x_\propto, \tag{4·3}$$

$$x_4 = 1, \tag{4·4}$$

$$x_u = x_\Omega, \tag{4·5}$$

$$x_{\mathrm{c}} = 8\pi i x_\Omega\,, \tag{4·6}$$

where Eqs. (4·2) and (4·3) are the relations derived in I.

The order g terms (T1) and (T2) in Eqs. (3·11), (3·12) and the order g^2 term (T3) in Eq. (3·13) vanish due to the BRS invariance of the vertices V_3^{o}, U and V_3^{c}, respectively; neither of these has moduli parameters, and hence each is essentially

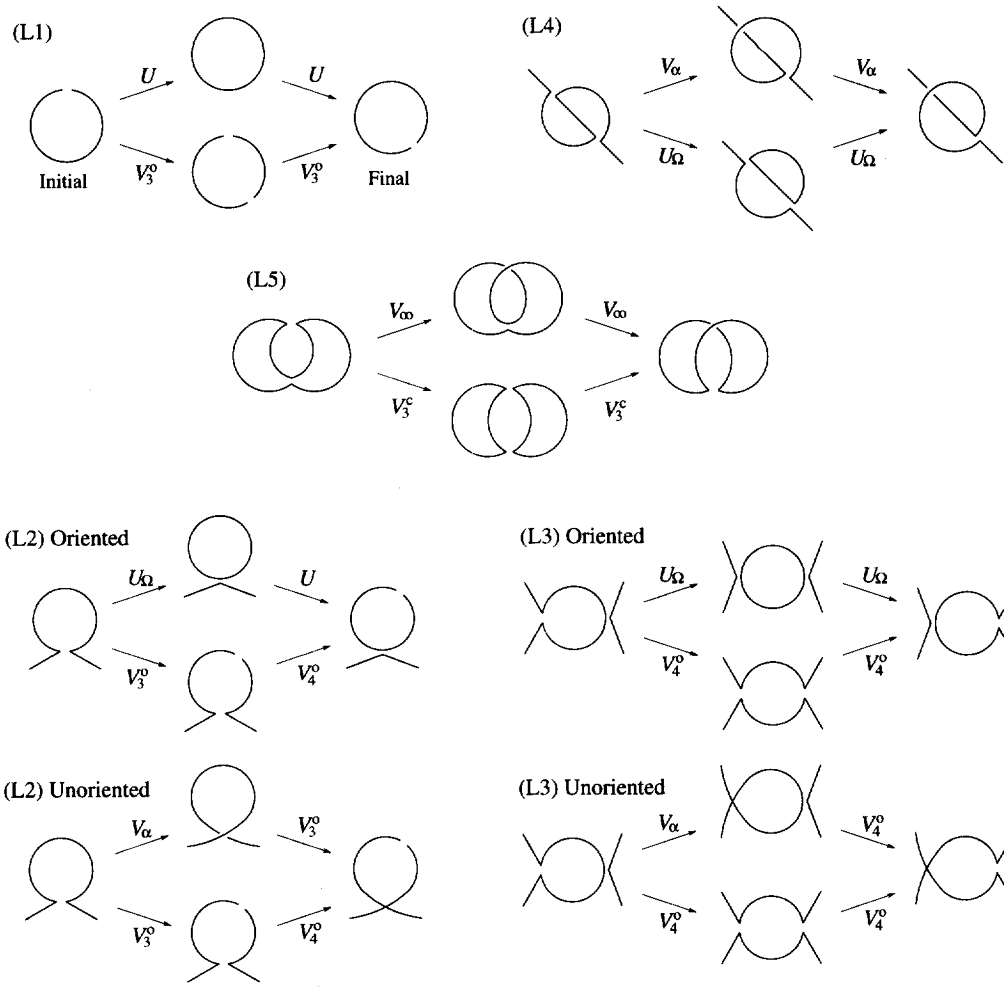


Fig. 3. Configurations requiring loop counterterms. The upper paths correspond to the 'tree' terms appearing in (L1) $\sim$ (L5), respectively, and the lower paths to the loop diagrams.

identical to the corresponding LPP vertex which is manifestly BRS invariant by construction. The terms (T6) and (T10) in Eqs. (3·17) and (3·22), containing only the quadratic interaction vertices U, $V_\propto$ and V_∞, were proved to vanish in I if the coupling relations (4·2) and (4·3) are satisfied. Thus we now discuss the remaining eleven terms, (T4), (T5), (T7) $\sim$ (T9) and (T11) $\sim$ (T16), successively and show that they indeed vanish.

In this section we often use the GGRT, which was originally proved in Refs. 23) and 24) for the simplest cases of purely open string system. But here we need more general formulas. Indeed we have various types of vertices containing closed strings also and must treat the contractions of such vertices by the closed string reflector $|R^{\rm c}(a^{\rm c}, b^{\rm c})\rangle$ as well as by the open one $|R^{\rm o}(a, b)\rangle$. We, however, show in the Appendix B that almost the same form of GGRT formulas actually hold for all the cases we need. So we use such formulas freely in the following.

4.1. $O(g^2)$ invariance

4.1.1. T4 terms

The cancellation of the two terms in (T4) in Eq. (3·14) has long been known, since the first proof by HIKKO in Ref. 30). However, here we prove this again to demonstrate how much the proof is simplified by the use of our present machinery of the LPP vertex. This will determine x_4 to be 1 in the present definition of the vertex.

The second term of (T4) corresponds to the gluing of two 3-string LPP vertices $\langle v_3^{\rm o}|$. For this simplest gluing, we have the GGRT formula

$$\langle v_3^{\rm o}(1,2,a)|\,\langle v_3^{\rm o}(b,3,4)|\,|R^{\rm o}(a,b)\rangle = \langle \tilde{v}_4^{\rm o}(1,2,3,4)|\,. \qquad (4\cdot7)$$

Here $\langle \tilde{v}_4^{\rm o}(1,2,3,4)|$ is a generic LPP vertex for four open-strings, given by an integration of the specific LPP vertex $\langle \tilde{v}_4^{{\rm o}\,(\alpha_1,\alpha_2,\alpha_3,\alpha_4)}(1,2,3,4)|$ over the string length parameters $\alpha_1, \alpha_2, \alpha_3$ and α_4. The specific vertex $\langle \tilde{v}_4^{{\rm o}\,(\alpha_1,\alpha_2,\alpha_3,\alpha_4)}|$ represents the LPP vertices which correspond to various ways of gluing the four open strings depending on the set of the values of the parameters $\alpha_1, \alpha_2, \alpha_3$ and α_4. The possible 4-string configurations which can be realized by gluing two 3-string vertices for all possible choices of string length parameters fall into three types, (a), (b) and (c), drawn in Fig. 4.

Let us consider the type (a) configuration first, and name the four strings there 1, 2, 3, and 4, as drawn in (a-1) in Fig. 5. But this configuration can be realized in two ways by using two 3-string vertices, as drawn in (a-1) and (a-2) in Fig. 5, where the dashed lines denote the intermediate strings a and b which are glued together by $|R^{\rm o}(a,b)\rangle$. Thus this vertex $\langle \tilde{v}_4^{{\rm o}(\alpha_1,\alpha_2,\alpha_3,\alpha_4)}(1,2,3,4)|$ appears twice in the second term in (T4); namely, one term corresponding to the configuration (a-1) is contained

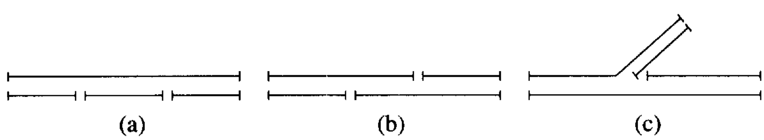


Fig. 4. The possible three types of configurations obtained by gluing two open 3-string vertices.

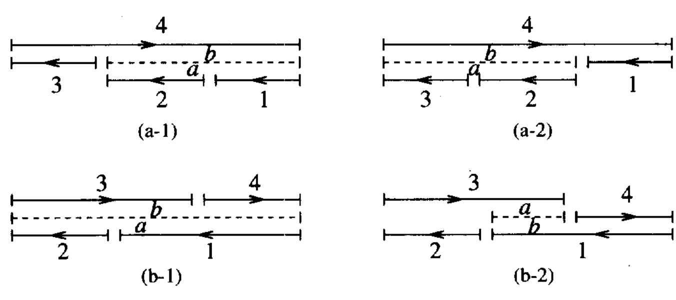


Fig. 5. Two ways of gluing to realize the type (a) and (b) configurations, respectively. The dashed lines denote the intermediate strings.

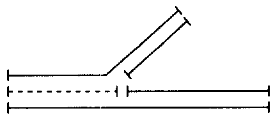


Fig. 6. A unique way for drawing an intermediate string in (c).

in the second term of (T4) in the form

$$-\langle v_3^{\rm o(\alpha_1,\alpha_2,\alpha_a)}(1,2,a)|\,\langle v_3^{\rm o(\alpha_b,\alpha_3,\alpha_4)}(b,3,4)|\,|R^{\rm o}(a,b)\rangle\,|\Psi\rangle_{4321}$$
$$= -\langle \tilde{v}_4^{\rm o\,(\alpha_1,\alpha_2,\alpha_3,\alpha_4)}(1,2,3,4)|\,|\Psi\rangle_{4321}\,, \tag{4·8}$$

and the other term corresponding to (a-2) in the form

$$-\langle v_3^{\rm o(\alpha_2,\alpha_3,\alpha_a)}(2,3,a)|\,\langle v_3^{\rm o(\alpha_b,\alpha_4,\alpha_1)}(b,4,1)|\,|R^{\rm o}(a,b)\rangle\,|\Psi\rangle_{1432}$$
$$= -\langle \tilde{v}_4^{\rm o\,(\alpha_2,\alpha_3,\alpha_4,\alpha_1)}(2,3,4,1)|\,|\Psi\rangle_{1432} = +\langle \tilde{v}_4^{\rm o(\alpha_1,\alpha_2,\alpha_3,\alpha_4)}(1,2,3,4)|\,|\Psi\rangle_{4321}\,, \tag{4·9}$$

where we have used the GGRT (4·7), the cyclic symmetry of the LPP 4-string vertex $\langle\tilde{v}_4^{\rm o}|$ similar to Eq. (2·3), and $|\Psi\rangle_{1432} = -|\Psi\rangle_{4321}$, because of the Grassmann odd property of the open string fields $|\Psi\rangle$. We see that these two terms have *opposite signs* and cancel each other. Consequently, we have proven that the second term in (T4) actually contains no terms $\propto \langle\tilde{v}_4^{\rm o(\alpha_1,\alpha_2,\alpha_3,\alpha_4)}(1,2,3,4)|$ corresponding to the type (a) configuration. Similarly, the terms corresponding to the type (b) configuration, realized in two ways, (b-1) and (b-2) in Fig. 5, can be seen to cancel out in the second term in (T4). (Actually, the same Eqs. (4·8) and (4·9) apply to the (b-1) and (b-2) terms, respectively, if we name the strings as drawn in Fig. 5.)

Thus, now the only remaining terms are those corresponding to the type (c) configuration in Fig. 4, called the 'horn diagram' by HIKKO.[30] In contrast to the previous types (a) and (b), this configuration is realized by using two 3-string vertices in a *unique* way, as drawn in Fig. 6. Therefore the terms of this configuration must be cancelled by new contribution other than the second term in (T4). This is just given by the first term in (T4), as we now see.

The first term in (T4) can be rewritten as

$$-\frac{x_4}{2}\langle V_4^{\rm o}(1,2,3,4)|\sum_r Q_{\rm B}^{(r)}|\Psi\rangle_{4321} = -\frac{x_4}{2}\int_{\sigma_i}^{\sigma_f} d\sigma_0\,\frac{d}{d\sigma_0}\Big(\langle v_4^{\rm o}(1,2,3,4;\sigma_0)|\Big)\,|\Psi\rangle_{4321}$$
$$= -\frac{x_4}{2}\Big(\langle v_4^{\rm o}(1,2,3,4;\sigma_f)| - \langle v_4^{\rm o}(1,2,3,4;\sigma_i)|\Big)\prod_r \Pi^{(r)}\,|\Psi\rangle_{4321}\,, \tag{4·10}$$

using the property (2·11) that the BRS charge $Q_{\rm B}$ acts on the SFT vertex as a differential operator with respect to the moduli parameter. Here we have omitted the unoriented projection operators $\prod_{r=1}^4 \Pi^{(r)}$ for brevity, which we shall do also henceforth without notice unless they become important. We immediately recognize that the appearing surface terms, $\langle v_4^{\rm o}(\sigma_0)|$ at the end-points $\sigma_0 = \sigma_f$ and σ_i, just realize the same string-configurations as the horn diagram, as depicted in Fig. 7. Figure 7 is drawn assuming that the string 4 has the maximal string length $|\alpha|$

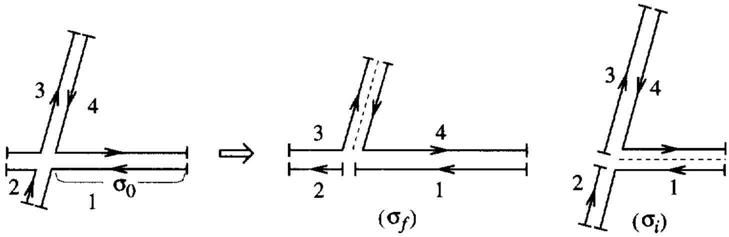


Fig. 7. Two configurations of the 4-string vertex $\langle v_4^{\rm o}(\sigma_0)|$ realized at the end-points $\sigma_0 = \sigma_f$ and σ_i. The dotted lines indicate the intermediate string in the case that they are realized by gluing two 3-string vertices.

among the four. Note that these specific configurations with $|\alpha_4|$ being maximum are contained four times for each with $\sigma_0 = \sigma_f$ and σ_i in Eq. (4·10), since the labels $1 \sim 4$ are dummy there and the vertex $\langle v_4^{\rm o}|$ is cyclic symmetric. On the other hand, in the second term of (T4) using two 3-string vertices, the (c) terms corresponding to these horn diagram configurations appear twice for each of σ_f and σ_i; indeed, in view of Fig. 7, we have the following two terms for the configuration (σ_f), (omitting the superscripts like $(\alpha_2, \alpha_3, \alpha_a)$ of the vertices for brevity here and henceforth),

$$\begin{aligned} -\langle v_3^{\rm o}(2,3,a)|\,\langle v_3^{\rm o}(b,4,1)|\,|R^{\rm o}(a,b)\rangle\,|\Psi\rangle_{1432} &= +\langle v_4^{\rm o}(1,2,3,4;\sigma_f)|\,|\Psi\rangle_{4321}\,, \\ -\langle v_3^{\rm o}(4,1,a)|\,\langle v_3^{\rm o}(b,2,3)|\,|R^{\rm o}(a,b)\rangle\,|\Psi\rangle_{3214} &= +\langle v_4^{\rm o}(1,2,3,4;\sigma_f)|\,|\Psi\rangle_{4321}\,, \end{aligned} \quad (4·11)$$

by using the GGRT (4·7) and $|\Psi\rangle_{1432} = -|\Psi\rangle_{4321}$, etc., and, similarly, two terms of $-\langle v_4^{\rm o}(1,2,3,4;\sigma_i)|\,|\Psi\rangle_{4321}$ for the configuration (σ_i).*) Therefore, the terms corresponding to these horn diagram configurations cancel between the first and second terms in (T4), Eq. (3·14), if the 4-string coupling constant x_4 satisfies

$$\left(-\frac{x_4}{2}\right) \times 4 + (+1) \times 2 = 0 \quad \Rightarrow \quad x_4 = 1. \quad (4·12)$$

4.1.2. T5 terms

The vanishing nature of the (T5) terms in Eq. (3·15) can be proved in a very similar manner as in the previous case.

The second term of (T5) has three possible configurations, as depicted in Fig. 8. The type (b) configuration can be realized by gluing the two vertices $\langle U|$ and $\langle V_3^{\rm o}|$ again in two ways, as drawn in Fig. 9, and appear in the second term in (T5) in the forms

$$\begin{aligned} \langle U(a,1^c)|\,\langle V_3^{\rm o}(b,2,3)|\,|R^{\rm o}(a,b)\rangle\,|\Phi\rangle_{1^c}\,|\Psi\rangle_{32} &= \langle \tilde v(2,3,1^c)|\,|\Phi\rangle_{1^c}\,|\Psi\rangle_{32}\,, \\ \langle U(a,1^c)|\,\langle V_3^{\rm o}(b,3,2)|\,|R^{\rm o}(a,b)\rangle\,|\Phi\rangle_{1^c}\,|\Psi\rangle_{23} &= \langle \tilde v(3,2,1^c)|\,|\Phi\rangle_{1^c}\,|\Psi\rangle_{23}\,, \end{aligned} \quad (4·13)$$

respectively. Here $\langle \tilde v(2,3,1^c)|$ denotes the LPP vertex for one closed and two open strings resulting from this gluing. This vertex is cyclic symmetric with respect to

*) Note that, although the first equation here in (4·11) and the previous Eq. (4·9) look the same, they actually represent different quantities corresponding to different regions of string length parameters; here the string fields $|\Psi\rangle_i$ $(i = 1,2,3,4)$ have alternating signs of string length parameters, i.e., $\{\alpha_1, \alpha_3 > 0,\ \alpha_2, \alpha_4 < 0\}$ or $\{\alpha_1, \alpha_3 < 0,\ \alpha_2, \alpha_4 > 0\}$, while, in Eq. (4·9), this is the case in the region $\{\alpha_1, \alpha_2, \alpha_3 > 0,\ \alpha_4 < 0\}$ or $\{\alpha_1, \alpha_2, \alpha_3 < 0,\ \alpha_4 > 0\}$.

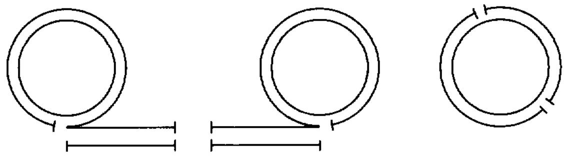


Fig. 8. Two types of configurations for $\langle U| \langle V_3^{\rm o}| |R^{\rm o}\rangle$.

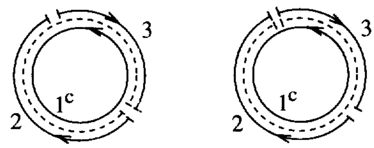


Fig. 9. Two ways of gluing yielding the configuration (b) in Fig. 8.

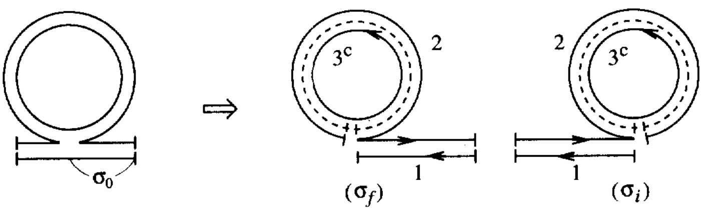


Fig. 10. Two configurations of $\langle u_\Omega(\sigma_0)|$ realized at the end-points $\sigma_0 = \sigma_f$ and σ_i.

the two open string arguments ($\langle\tilde{v}(2,3,1^{\rm c})| = \langle\tilde{v}(3,2,1^{\rm c})|$) since the matrix indices of the two open strings are contracted between the two. Since $|\Psi\rangle_{23} = -|\Psi\rangle_{32}$, the two terms in (4·13) clearly cancel each other.

Remaining are the terms of type (a) configurations, which are again to be cancelled by the first term in (T5). The first term of (T5) is rewritten as follows in the same way as in Eq. (4·10):

$$\begin{aligned}
&x_\Omega \langle U_\Omega(1,2,3^{\rm c})|\, Q_{\rm B}\, |\Phi\rangle_{3^{\rm c}} |\Psi\rangle_{21} \\
&= -x_\Omega \int_{\sigma_i}^{\sigma_f} \langle u_\Omega(1,2,3^{\rm c};\sigma_0)|\,\{\, b_{\sigma_0},\ Q_{\rm B}\,\}(b_0^- \mathcal{P}\Pi)^{(3^{\rm c})} |\Phi\rangle_{3^{\rm c}} |\Psi\rangle_{21} \\
&= -x_\Omega\Big\{ \langle u_\Omega(1,2,3^{\rm c};\sigma_f)| - \langle u_\Omega(1,2,3^{\rm c};\sigma_i)| \Big\}(b_0^- \mathcal{P}\Pi)^{(3^{\rm c})} |\Phi\rangle_{3^{\rm c}} |\Psi\rangle_{21}\,. \qquad (4\cdot14)
\end{aligned}$$

These surface terms $\langle u_\Omega(1,2,3^{\rm c};\sigma_0)|$ with $\sigma_0 = \sigma_f$ and σ_i have the same string-configurations as the type (a) as drawn in Fig. 10. Each of these terms with the string lengths specified and satisfying $|\alpha_2| > |\alpha_1|$ appears twice in this Eq. (4·14). The corresponding (a) terms in the second term in (T5) are given, by the help of GGRT, as

$$\begin{aligned}
&-2x_u \langle U(a,3^{\rm c})|\, \langle V_3^{\rm o}(b,2,1)|\, |R^{\rm o}(a,b)\rangle\, |\Phi\rangle_{3^{\rm c}} |\Psi\rangle_{12} \\
&\qquad = -2x_u \langle u_\Omega(2,1,3^{\rm c};\sigma_f)|\,(b_0^- \mathcal{P}\Pi)^{(3^{\rm c})} |\Phi\rangle_{3^{\rm c}} |\Psi\rangle_{12}\,, \\
&-2x_u \langle U(a,3^{\rm c})|\, \langle V_3^{\rm o}(b,1,2)|\, |R^{\rm o}(a,b)\rangle\, |\Phi\rangle_{3^{\rm c}} |\Psi\rangle_{21} \\
&\qquad = -2x_u \langle u_\Omega(1,2,3^{\rm c};\sigma_i)|\,(b_0^- \mathcal{P}\Pi)^{(3^{\rm c})} |\Phi\rangle_{3^{\rm c}} |\Psi\rangle_{21}\,, \qquad (4\cdot15)
\end{aligned}$$

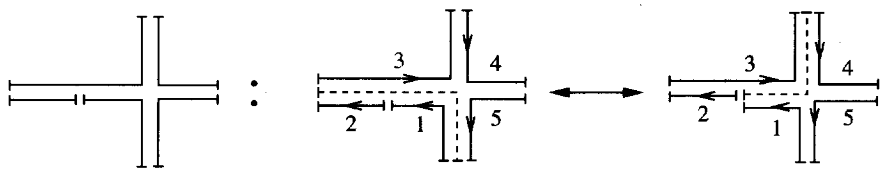


Fig. 11. The unique configuration for $\langle V_3^{\rm o}|\,\langle V_4^{\rm o}|\,|R^{\rm o}\rangle$ and two ways of gluing realizing it.

where we have used the fact that both $\langle V_3^{\rm o}|$ and $|R^{\rm o}\rangle$ are Grassmann odd. Noting that $\langle u_\Omega(2,1,3^{\rm c};\sigma_0)| = \langle u_\Omega(1,2,3^{\rm c};\sigma_0)|$ and $|\Psi\rangle_{21} = -|\Psi\rangle_{12}$, we see that these terms in (4·14) and (4·15) exactly cancel each other if

$$-x_\Omega \times 2 + 2x_u = 0 \quad \Rightarrow \quad x_u = x_\Omega. \tag{4·16}$$

4.2. $O(g^3)$ invariance

4.2.1. T7 term

In this case the generic configuration is unique and is of the type drawn in Fig. 11. Let us name the five strings $1 \sim 5$ in a cyclic order as shown there. Then, for any single configuration with a definite set of string length parameters $(\alpha_1,\cdots,\alpha_5)$, there are always two ways to realize it by gluing the vertices $\langle V_4^{\rm o}|$ and $\langle V_3^{\rm o}|$, as shown in Fig. 11. The term corresponding to the (a) diagram is contained in (T7), Eq. (3·18), in the form

$$\begin{aligned}
&\langle V_3^{\rm o}(1,2,a)|\,\langle V_4^{\rm o}(b,3,4,5)|\,|R^{\rm o}(a,b)\rangle\,|\Psi\rangle_{54321} \\
&= +\,\langle V_3^{\rm o}(1,2,a)|\,\langle V_4^{\rm o}(b,3,\overset{\downarrow}{4},5)|\,|R^{\rm o}(a,b)\rangle\,|\Psi\rangle_{54321} \\
&= +\int_{\sigma_i^{(4)}}^{\sigma_f^{(4)}} d\sigma_0^{(4)}\,\langle v_3^{\rm o}(1,2,a)|\,\langle v_4^{\rm o}(b,3,4,5;\sigma_0^{(4)})|\,b_{\sigma_0^{(4)}}\,|R^{\rm o}(a,b)\rangle\,|\Psi\rangle_{54321} \\
&= -\int_{\sigma_i^{(4)}}^{\sigma_f^{(4)}} d\sigma_0^{(4)}\,\langle \tilde v(12345;\sigma_0^{(4)})|\,b_{\sigma_0^{(4)}}\,|\Psi\rangle_{54321}\,,
\end{aligned} \tag{4·17}$$

where $\overset{\downarrow}{4}$ means that the anti-ghost factor $b_{\sigma_0^{(4)}}$ with string-4 moduli $\sigma_0^{(4)}$ is used as explained in Eq. (2·15), and the identity (2·14) has been used. $\langle \tilde v(12345;\sigma_0^{(4)})|$ is the LPP vertex for the five strings resulting from this gluing. Note that a sign change has occurred for the last expression since we have changed the order of $b_{\sigma_0^{(4)}}$ and $|R^{\rm o}\rangle$ before applying the GGRT. The term corresponding to the (b) diagram is, on the other hand, contained in (T7) in the form

$$\begin{aligned}
&\langle V_3^{\rm o}(2,3,a)|\,\langle V_4^{\rm o}(b,4,5,1)|\,|R^{\rm o}(a,b)\rangle\,|\Psi\rangle_{15432} \\
&= -\,\langle V_3^{\rm o}(2,3,a)|\,\langle V_4^{\rm o}(b,\overset{\downarrow}{4},5,1)|\,|R^{\rm o}(a,b)\rangle\,|\Psi\rangle_{15432} \\
&= -\int_{\sigma_i^{(4)}}^{\sigma_f^{(4)}} d\sigma_0^{(4)}\,\langle v_3^{\rm o}(2,3,a)|\,\langle v_4^{\rm o}(b,4,5,1;\sigma_0^{(4)})|\,b_{\sigma_0^{(4)}}\,|R^{\rm o}(a,b)\rangle\,|\Psi\rangle_{54321}
\end{aligned}$$

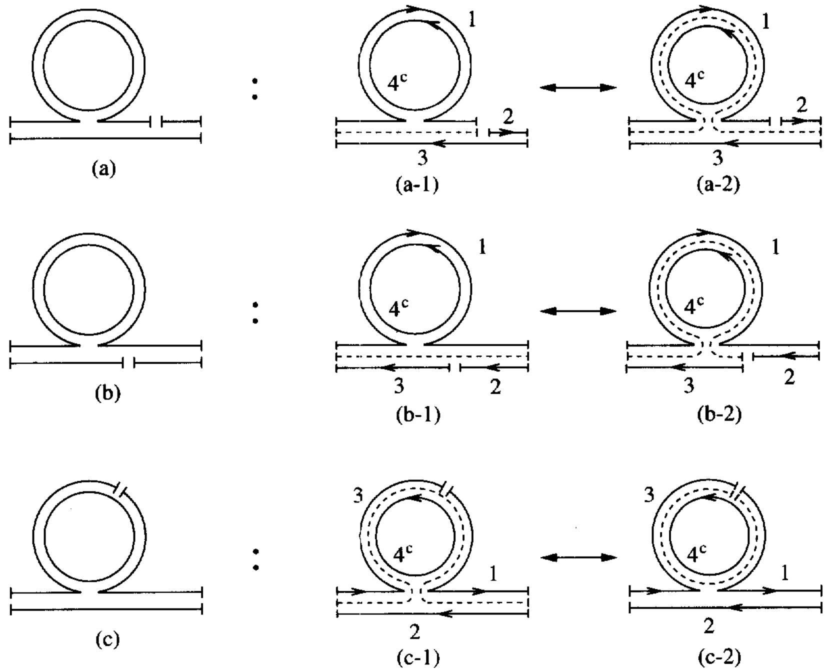


Fig. 12. Three configurations for (T8) terms.

$$= + \int_{\sigma_i^{(4)}}^{\sigma_f^{(4)}} d\sigma_0^{(4)} \langle \tilde{v}(12345;\sigma_0^{(4)})|\, b_{\sigma_0^{(4)}}\, |\Psi\rangle_{54321}\,, \qquad (4\cdot18)$$

where the cyclic symmetry property of the LPP vertex $\langle \tilde{v}(12345;\sigma_0^{(4)})|$ and the property $|\Psi\rangle_{15432} = +|\Psi\rangle_{54321}$ have been used. The negative sign here has appeared, since the identity (2·14) implies

$$\langle V_4^{\rm o}(\overset{\downarrow}{b},4,5,1)| = -\langle V_4^{\rm o}(b,\overset{\downarrow}{4},5,1)|\,. \qquad (4\cdot19)$$

Thus the two contributions (4·17) and (4·18) cancel each other, proving that (T7) vanishes.

4.2.2. T8 terms

Generic configurations resulting from the contraction of two vertices $\langle U_\Omega|$ and $\langle V_3^{\rm o}|$, or $\langle U|$ and $\langle V_4^{\rm o}|$, fall into three types, (a), (b) and (c), depicted in Fig. 12. Each of these is realized in two ways, as shown in Fig. 12; only the (c-2) diagram is given by gluing $\langle U|$ and $\langle V_4^{\rm o}|$, and all the others are given by gluing $\langle U_\Omega|$ and $\langle V_3^{\rm o}|$. As in the previous cases, cancellations occur between the two ways of gluing in each pair. Denoting the LPP vertex resultant from this gluing by $\langle \tilde{v}(123,4^{\rm c})|$ generically, the pair of (a-1) and (a-2) is contained in (T8) in the following form:

(a-1) :

$$2x_\Omega\, \langle U_\Omega(1,a,4^{\rm c})|\ \langle V_3^{\rm o}(b,2,3)|\ |R^{\rm o}(a,b)\rangle\, |\Phi\rangle_{4^{\rm c}}\, |\Psi\rangle_{321}$$

$$= 2x_\Omega \int d\sigma_0^{(1)} \langle u_\Omega(1,a,4^{\rm c};\sigma_0^{(1)})|\, b_{\sigma_0^{(1)}} (b_0^- \mathcal{P})^{(4^{\rm c})} \langle v_3^{\rm o}(b,2,3)|\, |R^{\rm o}(a,b)\rangle\, |\Phi\rangle_{4^{\rm c}}\, |\Psi\rangle_{321}$$

$$= 2x_\Omega \int d\sigma_0^{(1)} \langle \tilde{v}(123, 4^c)| \, b_{\sigma_0^{(1)}} (b_0^- \mathcal{P})^{(4^c)} \, |\Phi\rangle_{4^c} \, |\Psi\rangle_{321} \,,$$

(a-2) :

$$2x_\Omega \langle U_\Omega(3, a, 4^c)| \; \langle V_3^o(b,1,2)| \; |R^o(a,b)\rangle \, |\Phi\rangle_{4^c} \, |\Psi\rangle_{213}$$

$$= 2x_\Omega \int d\sigma_0^{(3)} \langle u_\Omega(3, a, 4^c; \sigma_0^{(3)})| \, b_{\sigma_0^{(3)}} (b_0^- \mathcal{P})^{(4^c)} \langle v_3^o(b,1,2)| \, |R^o(a,b)\rangle \, |\Phi\rangle_{4^c} \, |\Psi\rangle_{213}$$

$$= 2x_\Omega \int d\sigma_0^{(3)} \langle \tilde{v}(123, 4^c)| \, b_{\sigma_0^{(3)}} (b_0^- \mathcal{P})^{(4^c)} \, |\Phi\rangle_{4^c} \, |\Psi\rangle_{213} \,. \tag{4·20}$$

Although the states have the same sign, $|\Psi\rangle_{213} = + |\Psi\rangle_{321}$, the anti-ghost factors have opposite signs, $\int d\sigma_0^{(3)} b_{\sigma_0^{(3)}} = - \int d\sigma_0^{(1)} b_{\sigma_0^{(1)}}$, since the directions in which $\sigma_0^{(3)}$ and $\sigma_0^{(1)}$ increase must be opposite in order to keep a common configuration, similarly to Eq. (2·14) in the open 4-string vertex case. Thus (a-1) and (a-2) cancel each other. The same equations (4·20) also apply to the (b-1) and (b-2) diagrams if we name the strings as shown in Fig. 12, so that the (b) configuration also cancels out. Cancellation between (c-1) and (c-2), on the other hand, occurs if the condition

$$x_\Omega = x_u x_4 \,, \tag{4·21}$$

holds. Indeed, the (c-1) diagram is contained in (T8) in the form

$$2x_\Omega \langle U_\Omega(2, a, 4^c)| \; \langle V_3^o(b,3,1)| \; |R^o(a,b)\rangle \, |\Phi\rangle_{4^c} \, |\Psi\rangle_{132}$$

$$= 2x_\Omega \int d\sigma_0^{(2)} \langle u_\Omega(2, a, 4^c; \sigma_0^{(2)})| \, b_{\sigma_0^{(2)}} (b_0^- \mathcal{P})^{(4^c)} \langle v_3^o(b,3,1)| \, |R^o(a,b)\rangle \, |\Phi\rangle_{4^c} \, |\Psi\rangle_{132}$$

$$= 2x_\Omega \int d\sigma_0^{(2)} \langle \tilde{v}(123, 4^c)| \, b_{\sigma_0^{(2)}} (b_0^- \mathcal{P})^{(4^c)} \, |\Phi\rangle_{4^c} \, |\Psi\rangle_{321} \,, \tag{4·22}$$

while (c-2) is contained in (T8) in the form

$$-2x_u x_4 \langle U(a, 4^c)| \; \langle V_4^o(b,1,2,3)| \; |R^o(a,b)\rangle \, |\Phi\rangle_{4^c} \, |\Psi\rangle_{321}$$

$$= -2x_u x_4 \langle U(a, 4^c)| \; \langle V_4^o(b,1,\overset{\downarrow}{2},3)| \; |R^o(a,b)\rangle \, |\Phi\rangle_{4^c} \, |\Psi\rangle_{321}$$

$$= -2x_u x_4 \int d\sigma_0^{(2)} \langle u(a, 4^c)| \, (b_0^- \mathcal{P})^{(4^c)} \langle v_4^o(b,1,2,3; \sigma_0^{(2)})| \, b_{\sigma_0^{(2)}} |R^o(a,b)\rangle \, |\Phi\rangle_{4^c} \, |\Psi\rangle_{321}$$

$$= -2x_u x_4 \int d\sigma_0^{(2)} \langle \tilde{v}(123, 4^c)| \, b_{\sigma_0^{(2)}} (b_0^- \mathcal{P})^{(4^c)} \, |\Phi\rangle_{4^c} \, |\Psi\rangle_{321} \,. \tag{4·23}$$

Here use has been made of the Grassmann oddness of $\langle v_4^o|$ and $|R^o\rangle$. The required condition (4·21) is actually satisfied by the relations $x_4 = 1$ and $x_u = x_\Omega$ already determined in Eqs. (4·12) and (4·16).

4.2.3. T9 terms

The generic configurations obtained by contracting the vertex $\langle U|$ with $\langle V_3^c|$ or $\langle U_\Omega|$ are of two types, (a) and (b), depicted in Fig. 13. Again each of them is realized in two ways, as shown in Fig. 13. As in the previous cases, cancellations occur between (a-1) and (a-2), and (b-1) and (b-2). Denoting the LPP vertex corresponding to the glued configuration (a) in Fig. 13 by $\langle \tilde{v}(1, 2^c 3^c; \sigma_0)|$, the pair of diagrams (a-1)

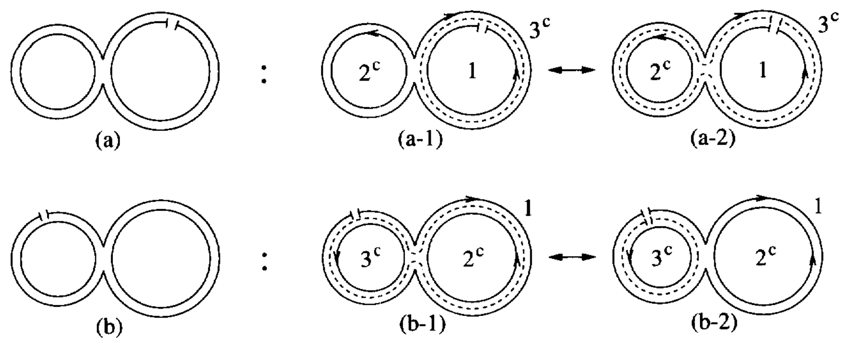


Fig. 13. Two configurations exist for (T9) terms.

and (a-2) are contained in (T9) in the following form:

(a-1) :

$$
\begin{aligned}
&x_u x_c \langle U(1,\overset{\vee}{a}{}^{c})|\,\langle V_3^c(b^c,2^c,3^c)|\,|R^c(a^c,b^c)\rangle\,|\Phi\rangle_{3^c2^c}\,|\Psi\rangle_1\\
&= x_u x_c \langle u(1a^c)|\langle v_3^c(b^c2^c3^c)|\,b_0^{-(b^c)}\int\frac{d\theta}{2\pi}e^{i\theta(L-\bar{L})^{(b^c)}}\prod_{r=2,3}(b_0^-\mathcal{P})^{(r^c)}\,|R^c(a^cb^c)\rangle\,|\Phi\rangle_{3^c2^c}|\Psi\rangle_1\\
&= x_u x_c\int\frac{d\theta}{2\pi}\langle u(1a^c)|\langle v_3^c(b^c2^c3^c)|\,e^{i\theta(L-\bar{L})^{(b^c)}}|R^c(a^cb^c)\rangle\,b_0^{-(b^c)}\prod_{r=2,3}(b_0^-\mathcal{P})^{(r^c)}\,|\Phi\rangle_{3^c2^c}|\Psi\rangle_1\\
&= x_u x_c\int\frac{d\theta}{2\pi}\langle\tilde{v}(1,2^c3^c;\theta)|\,b_0^{-(b^c)}\prod_{r=2,3}(b_0^-\mathcal{P})^{(r^c)}\,|\Phi\rangle_{3^c2^c}\,|\Psi\rangle_1\,,
\end{aligned}
\tag{4·24}
$$

(a-2) :

$$
\begin{aligned}
&-2x_\Omega x_u\langle U_\Omega(1,a,2^c)|\,\langle U(b,3^c)|\,|R^o(a,b)\rangle\,|\Phi\rangle_{3^c2^c}\,|\Psi\rangle_1\\
&= -2x_\Omega x_u\int d\sigma_0\,\langle u_\Omega(1a2^c;\sigma_0)|\,b_{\sigma_0}(b_0^-\mathcal{P})^{(2^c)}\langle u(b3^c)|\,(b_0^-\mathcal{P})^{(3^c)}|R^o(ab)\rangle\,|\Phi\rangle_{3^c2^c}\,|\Psi\rangle_1\\
&= +2x_\Omega x_u\int d\sigma_0\,\langle u_\Omega(1a2^c;\sigma_0)|\langle u(b3^c)||R^o(ab)\rangle\,b_{\sigma_0}(b_0^-\mathcal{P})^{(2^c)}(b_0^-\mathcal{P})^{(3^c)}|\Phi\rangle_{3^c2^c}\,|\Psi\rangle_1\\
&= +2x_\Omega x_u\int d\sigma_0\,\langle\tilde{v}(1,2^c3^c;\sigma_0)|\,b_{\sigma_0}\prod_{r=2,3}(b_0^-\mathcal{P})^{(r^c)}\,|\Phi\rangle_{3^c2^c}\,|\Psi\rangle_1\,.
\end{aligned}
\tag{4·25}
$$

Here some of the commas in the string arguments of the vertices have been omitted for brevity, and we have used the Grassmann oddness of $|R^o\rangle$ and the GGRT. The resultant LPP vertices for the glued configurations (a-1) and (a-2) are clearly the same (see Fig. 14 at $\tau=0$):

$$
\langle\tilde{v}(1,2^c3^c;\theta)| = \langle\tilde{v}(1,2^c3^c;\sigma_0)| \qquad \text{for} \quad \alpha_{b^c}\theta=\sigma_0\,. \tag{4·26}
$$

We, therefore, have only to compare the anti-ghost factors $b_0^{-(b^c)}$ and b_{σ_0} appearing in Eqs. (4·24) and (4·25), respectively. This comparison is actually very similar to that performed in I for the cancellation of (T10) between $\langle U|\,\langle V_\propto|\,|R^o\rangle$ and $\langle U|\,\langle V_\infty|\,|R^o\rangle$. For this purpose, consider the ρ-plane diagrams drawn in Fig. 14, where the figures represent the configurations which reduce to the present ones (a-1) and (a-2), respectively, as the time interval τ goes to zero. First, as done in I, the anti-ghost

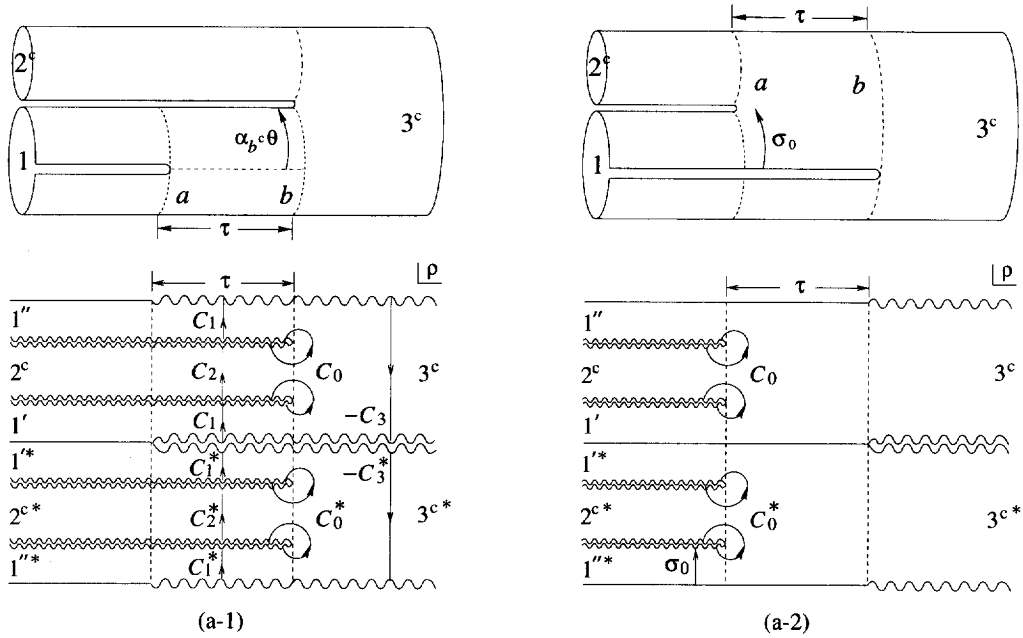


Fig. 14. ρ planes for the diagrams which reduce to (a-1) and (a-2) in Fig. 13 at $\tau = 0$.

factor $b_0^{-(b^c)}$ is replaced by $\widetilde{b}_0^{-(b^c)} \equiv b_0^{-(b^c)} + (\alpha_{b^c}/\alpha_{2^c}) b_0^{-(2^c)} + (\alpha_{b^c}/\alpha_{3^c}) b_0^{-(3^c)}$. This is possible since $b_0^{-(2^c)}$ and $b_0^{-(3^c)}$ are zero in front of the factors $\prod_{r=2,3} (b_0^- \mathcal{P})^{(r^c)}$ present in Eq. (4·24). Then, note that

$$b_0^{-(r^c)} \equiv \frac{1}{2}(b_0^{(r)} - \bar{b}_0^{(r)}) = \frac{1}{2}\left(\oint \frac{d\rho_r}{2\pi i} b^{(r)}(\rho_r) - \text{a.h.}\right)$$
$$= \frac{1}{2}\left(\oint \frac{d\rho}{2\pi i}\frac{d\rho}{d\rho_r} b(\rho) - \text{a.h.}\right) = \frac{1}{2}\alpha_r \left(\oint \frac{d\rho}{2\pi i} b(\rho) - \text{a.h.}\right), \quad (4·27)$$

where we have used the fact that the ρ coordinate is identified with $\rho = \alpha_r \rho_r + \text{const}$ in the region of string r. Hence $\widetilde{b}_0^{-(b^c)}$ can be seen to reduce to the following expression by making a deformation of the integration contour:

$$\widetilde{b}_0^{-(b^c)} = \frac{1}{2}\alpha_{b^c}\left(\oint_{C_1+C_2-C_3} - \oint_{C_1^*+C_2^*-C_3^*}\right)\frac{d\rho}{2\pi i} b(\rho)$$
$$= -\frac{1}{2}\alpha_{b^c}\left(\oint_{C_0} - \oint_{C_0^*}\right)\frac{d\rho}{2\pi i} b(\rho) \equiv -\frac{1}{2}\alpha_{b^c}\left(b_{\rho_0} - b_{\rho_0^*}\right). \quad (4·28)$$

The contours C_i and C_i^* here are shown in Fig. 14.

On the other hand, the anti-ghost factor b_{σ_0} appearing in Eq. (4·25) can be written in the form

$$b_{\sigma_0} = \left(\frac{d\rho_0}{d\sigma_0}\right)\oint_{C_0}\frac{d\rho}{2\pi i} b(\rho) + \left(\frac{d\rho_0^*}{d\sigma_0}\right)\oint_{C_0^*}\frac{d\rho}{2\pi i} b(\rho) = i\left(b_{\rho_0} - b_{\rho_0^*}\right), \quad (4·29)$$

using $d\rho_0/d\sigma_0 = i$ and $d\rho_0^*/d\sigma_0 = -i$. Therefore, (4·24) and (4·25) cancel each other if

$$x_u x_c \int_0^{2\pi} \frac{d\theta}{2\pi} \left(-\frac{1}{2}\alpha_{b^c}\right) \cdots = -2ix_\Omega x_u \int_0^{\alpha_1\pi} d\sigma_0 \cdots , \tag{4·30}$$

since the integrands are the same for $\alpha_{b^c}\theta = \sigma_0$ by Eq. (4·26). If we note the relation $\alpha_{b^c} = \alpha_1/2$ (since the strings b^c and 1 are closed and open strings, respectively, in the (a-1) case), we see that the integration regions on both sides in Eq. (4·30) coincide ($\alpha_{b^c}\int_0^{2\pi} d\theta = \int_0^{\alpha_1\pi} d(\alpha_{b^c}\theta) = \int_0^{\alpha_1\pi} d\sigma_0$), and thus the cancellation is complete if

$$-\frac{1}{4\pi}x_c = -2ix_\Omega \quad \Rightarrow \quad x_c = 8\pi i x_\Omega . \tag{4·31}$$

The cancellation between (b-1) and (b-2) is also seen in quite the same way, and thus (T9) has been proved to vanish.

4.3. $O(g^4)$ *invariance*

4.3.1. T11 term

This (T11) term has already been proved to vanish by HIKKO.[30] The configuration obtained by contracting two $\langle V_4^{\rm o}|$ vertices is unique and is of the type drawn in Fig. 15. Name the six open strings $1 \sim 6$ in a cyclic order as shown there. For any single configuration with a definite set of string length parameters $(\alpha_1, \cdots, \alpha_6)$, there are always two ways to realize it by gluing the vertices $\langle V_4^{\rm o}|$ and $\langle V_4^{\rm o}|$, as shown in Fig. 15. The terms corresponding to the two configurations (a) and (b) are contained in (T11) in the following forms, respectively:

$$\begin{aligned}
&\langle V_4^{\rm o}(1,2,3,a)|\,\langle V_4^{\rm o}(b,4,5,6)|\,|R^{\rm o}(a,b)\rangle\,|\Psi\rangle_{654321} \\
&= (-)\,\langle V_4^{\rm o}(1,\overset{\downarrow}{2},3,a)|\,(-)\,\langle V_4^{\rm o}(b,4,5,\overset{\downarrow}{6})|\,|R^{\rm o}(a,b)\rangle\,|\Psi\rangle_{654321} \\
&= +\int d\sigma_0^{(2)} d\sigma_0^{(6)} \langle v_4^{\rm o}(123a;\sigma_0^{(2)})|\, b_{\sigma_0^{(2)}} \langle v_4^{\rm o}(b456;\sigma_0^{(6)})|\, b_{\sigma_0^{(6)}} |R^{\rm o}(ab)\rangle\,|\Psi\rangle_{654321}, \\
&\langle V_4^{\rm o}(2,3,4,a)|\,\langle V_4^{\rm o}(b,5,6,1)|\,|R^{\rm o}(a,b)\rangle\,|\Psi\rangle_{165432} \\
&= \langle V_4^{\rm o}(\overset{\downarrow}{2},3,4,a)|\,(+)\,\langle V_4^{\rm o}(b,5,\overset{\downarrow}{6},1)|\,|R^{\rm o}(a,b)\rangle\,(-)\,|\Psi\rangle_{654321} \\
&= -\int d\sigma_0^{(2)} d\sigma_0^{(6)} \langle v_4^{\rm o}(234a;\sigma_0^{(2)})|\, b_{\sigma_0^{(2)}} \langle v_4^{\rm o}(b561;\sigma_0^{(6)})|\, b_{\sigma_0^{(6)}} |R^{\rm o}(ab)\rangle\,|\Psi\rangle_{654321}.
\end{aligned} \tag{4·32}$$

Note that the minus sign in the last expression has come from $|\Psi\rangle_{165432} = -|\Psi\rangle_{654321}$, giving the opposite signs for the two terms. Apply the GGRT to both terms there. Then, clearly, they yield the same LPP vertex $\langle\tilde{v}(123456;\sigma_0^{(2)},\sigma_0^{(6)})|$, maintaining opposite overall signs, so that they turn out to cancel each other.

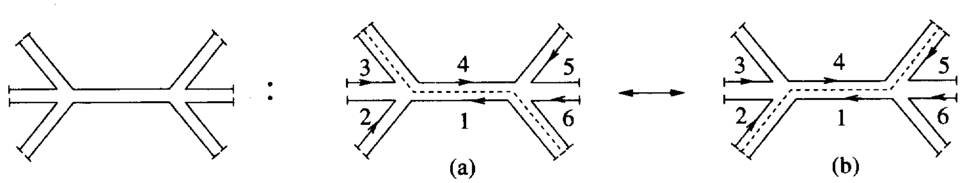


Fig. 15. The unique configuration for the (T11) term.

4.3.2. T12 term

There appear two distinct configurations when contracting the vertices $\langle U_\Omega|$ and $\langle V_4^{\rm o}|$, which are the types (a) and (b) given in Fig. 16. Again these are each realized in two ways, as drawn in Fig. 16. The terms of the diagrams (a-1) and (a-2) are contained in (T12) in the following forms, respectively:

$$\begin{aligned}
&\langle U_\Omega(1,a,5^{\rm c})|\,\langle V_4^{\rm o}(b,2,3,4)|\,|R^{\rm o}(a,b)\rangle\,|\Phi\rangle_{5^{\rm c}}\,|\Psi\rangle_{4321}\\
&\quad= \langle u_\Omega(1a5^{\rm c};\sigma_0^{(1)})|\,b_{\sigma_0^{(1)}}\,\langle v_4^{\rm o}(b234;\sigma_0^{(3)})|\,b_{\sigma_0^{(3)}}\,|R^{\rm o}(ab)\rangle\,|\Phi\rangle_{5^{\rm c}}\,|\Psi\rangle_{4321}\,, \qquad (4\cdot33)\\
&\langle U_\Omega(3,a,5^{\rm c})|\,\langle V_4^{\rm o}(b,4,1,2)|\,|R^{\rm o}(a,b)\rangle\,|\Phi\rangle_{5^{\rm c}}\,|\Psi\rangle_{2143}\\
&\quad= \langle u_\Omega(3a5^{\rm c};\sigma_0^{(3)})|\,b_{\sigma_0^{(3)}}\,\langle v_4^{\rm o}(b412;\sigma_0^{(1)})|\,b_{\sigma_0^{(1)}}\,|R^{\rm o}(ab)\rangle\,|\Phi\rangle_{5^{\rm c}}\,|\Psi\rangle_{2143}\,. \qquad (4\cdot34)
\end{aligned}$$

Here the common integration symbols $\int d\sigma_0^{(1)}d\sigma_0^{(3)}$ have been suppressed, and use has been made of $\langle V_4^{\rm o}(b,2,3,4)| = +\langle V_4^{\rm o}(b,2,\overset{\downarrow}{3},4)|$. In this case, the states are the same ($|\Psi\rangle_{4321} = |\Psi\rangle_{2143}$), and the GGRT gives a common LPP vertex for the glued configuration (a), but the orders of the anti-ghost factor $b_{\sigma_0^{(1)}}b_{\sigma_0^{(3)}}$ are opposite for the two terms. They thus cancel each other. For the case of the (b) configuration, (b-1) term is given by the same Eq. (4·33) while (b-2) by

$$\begin{aligned}
&\langle U_\Omega(2,a,5^{\rm c})|\,\langle V_4^{\rm o}(b,3,4,1)|\,|R^{\rm o}(a,b)\rangle\,|\Phi\rangle_{5^{\rm c}}\,|\Psi\rangle_{1432}\\
&\quad= \langle u_\Omega(2a5^{\rm c};\sigma_0^{\prime(2)})|\,b_{\sigma_0^{\prime(2)}}\,\langle v_4^{\rm o}(b341;\sigma_0^{(3)})|\,(-b_{\sigma_0^{(3)}})\,|R^{\rm o}(ab)\rangle\,|\Phi\rangle_{5^{\rm c}}\,|\Psi\rangle_{1432}\,, \qquad (4\cdot35)
\end{aligned}$$

with the symbol $\int d\sigma_0^{\prime(2)}d\sigma_0^{(3)}$ suppressed again. Comparing the diagrams (b-1) and (b-2), we see that the interaction point $\sigma_0^{\prime(2)}$ here of $\langle u_\Omega(2,a,5^{\rm c};\sigma_0^{\prime(2)})|$ is the same as that of $\langle u_\Omega(1,a,5^{\rm c};\sigma_0^{(1)})|$ in Eq. (4·33), so that $b_{\sigma_0^{\prime(2)}} = -b_{\sigma_0^{(1)}}$ (directions are opposite). Therefore the anti-ghost factors are the same for each, $b_{\sigma_0^{\prime(2)}}(-b_{\sigma_0^{(3)}}) = b_{\sigma_0^{(1)}}b_{\sigma_0^{(3)}}$, but the states have opposite signs, $|\Psi\rangle_{1432} = -\,|\Psi\rangle_{4321}$. Thus (T12) is also proved to vanish.

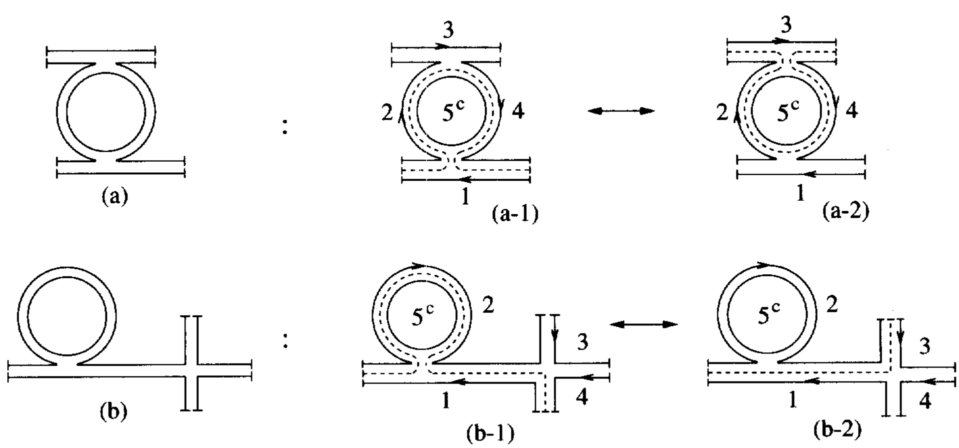


Fig. 16. Two configurations for the (T12) term.

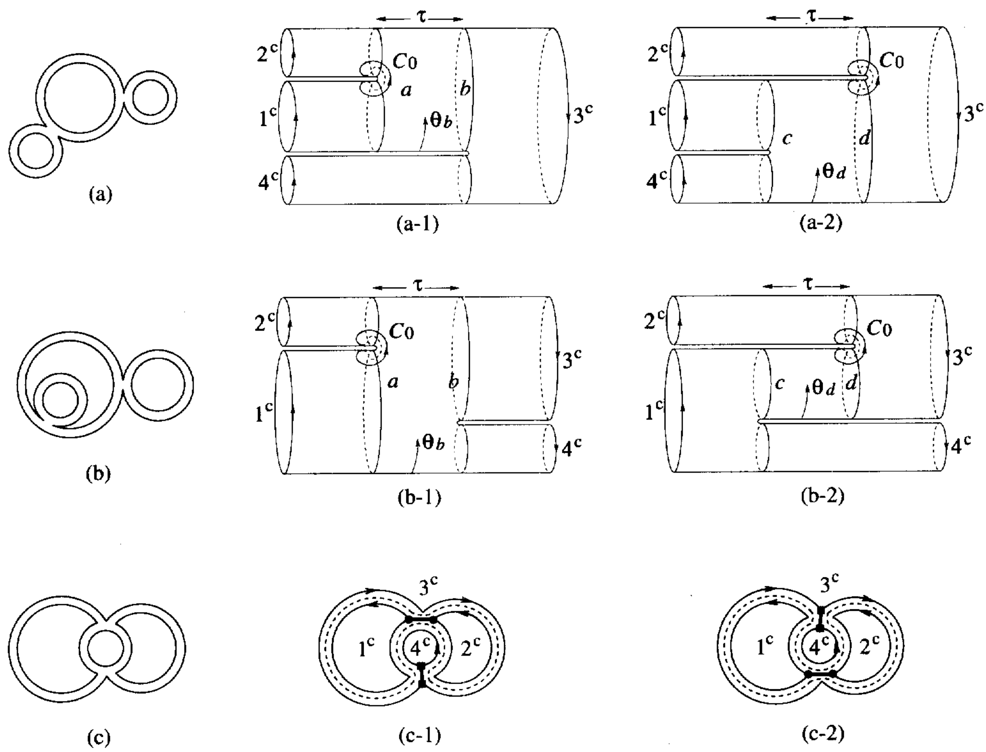


Fig. 17. Three configurations for (T13) term. In (c-1) and (c-2), the bonds connecting the solid dots and solid squares denote the 1^c-2^c and 3^c-4^c interaction points, respectively.

4.3.3. T13 term

This term was already analyzed intensively and shown to vanish by HIIKKO.[31] So let us show this fact in our terminology briefly.

The generic configurations obtained by gluing two closed 3-string vertices $\langle V_3^c|$ fall into three types, (a) $\sim$ (c), in Fig. 17. Each of these is realized in two ways of gluing, as shown there. The (a-1) and (a-2) terms for (a) (and (b-1) and (b-2) for (b) as well, if the strings are named as shown in Fig. 17) are contained in the (T13) term, (3·25), in the following forms, respectively:

$$\begin{aligned}&\langle V_3^c(1^c,2^c,\overset{\vee}{a^c})|\,\langle V_3^c(b^c,3^c,4^c)|\,|R^c(a^c,b^c)\rangle\,|\Phi\rangle_{4^c3^c2^c1^c}\\&=\langle v_3^c(1^c,2^c,a^c)|\,\langle v_3^c(b^c,3^c,4^c)|\,b_0^{-(b^c)}\int\frac{d\theta_b}{2\pi}e^{i\theta_b(L-\bar{L})^{(b^c)}}\,|R^c(a^c,b^c)\rangle\\&\quad\times\prod_{r=1^c,2^c,3^c,4^c}(b_0^-\mathcal{P})^{(r^c)}\,|\Phi\rangle_{4^c3^c2^c1^c}\,,\end{aligned}\tag{4·36}$$

$$\begin{aligned}&\langle V_3^c(4^c,1^c,\overset{\vee}{c^c})|\,\langle V_3^c(d^c,2^c,3^c)|\,|R^c(c^c,d^c)\rangle\,|\Phi\rangle_{3^c2^c1^c4^c}\\&=\langle v_3^c(4^c,1^c,c^c)|\,\langle v_3^c(d^c,2^c,3^c)|\,b_0^{-(d^c)}\int\frac{d\theta_d}{2\pi}e^{i\theta_d(L-\bar{L})^{(d^c)}}\,|R^c(c^c,d^c)\rangle\\&\quad\times\prod_{r=4^c,1^c,2^c,3^c}(b_0^-\mathcal{P})^{(r^c)}\,|\Phi\rangle_{3^c2^c1^c4^c}\,.\end{aligned}\tag{4·37}$$

The external states and the $b_0^{-(r^c)}$ factors associated with them are common as a whole to these two terms; $\prod_{r=1^c,2^c,3^c,4^c}(b_0^-\mathcal{P})^{(r^c)} = -\prod_{r=4^c,1^c,2^c,3^c}(b_0^-\mathcal{P})^{(r^c)}$ and $|\Phi\rangle_{4^c3^c2^c1^c} = -|\Phi\rangle_{3^c2^c1^c4^c}$. Therefore the sign difference should come from $b_0^{-(b^c)}$ and $b_0^{-(d^c)}$. By reasoning similar to the (T9) (a-1) case, these anti-ghost factors can be converted into

$$b_0^{-(b^c)} \Rightarrow +\frac{1}{2}\alpha_{b^c}(b_{\rho_0} - \bar{b}_{\rho_0^*}), \qquad b_0^{-(d^c)} \Rightarrow -\frac{1}{2}\alpha_{d^c}(b_{\rho_0} - \bar{b}_{\rho_0^*}), \tag{4·38}$$

in the presence of $b_0^{-(1^c)}b_0^{-(2^c)}$ and of $b_0^{-(2^c)}b_0^{-(3^c)}$, respectively, where $b_{\rho_0} = \oint_{C_0}(d\rho/2\pi)\, b(\rho)$ is the anti-ghost factor corresponding to the shift of the 1^c-2^c string interaction points drawn in Fig. 17 (and $\bar{b}_{\rho_0^*}$ is its anti-holomorphic counterpart). On the other hand, comparing the diagrams, (a-1) with (a-2) and (b-1) with (b-2), we easily see that these pairs of diagrams realize the same glued configurations when the twisting angles θ_b and θ_d of the intermediate closed strings satisfy the relation

$$\alpha_b\theta_b = -\alpha_d\theta_d \tag{4·39}$$

if the origins of the θ are chosen suitably. Therefore, to keep the same glued configurations, the directions in which θ_b and θ_d increase are opposite, and the opposite signs (as well as their weights) of $b_0^{-(b^c)}$ and $b_0^{-(d^c)}$ in Eq. (4·38) reflect this fact. Note that the (a) configuration, by definition, corresponds to the twisting angle regions $-\alpha_1\pi \le \alpha_b\theta_b \le \alpha_1\pi$ and $-\alpha_1\pi \le \alpha_d\theta_d \le \alpha_1\pi$ for (a-1) and (a-2) diagrams, respectively, and that the (b) configuration corresponds to the twisting angle regions $-(\alpha_1 - |\alpha_4|)\pi \le \alpha_b\theta_b \le (\alpha_1 - |\alpha_4|)\pi$ and $-(\alpha_1 - |\alpha_4|)\pi \le \alpha_d\theta_d \le (\alpha_1 - |\alpha_4|)\pi$ for (b-1) and (b-2) diagrams, respectively. We thus have shown that (a-1) and (a-2) terms, and (b-1) and (b-2) as well, cancel each other between these integration regions.

If the twisting angle θ_b in the (b-1) diagram exceeds the above limit and falls into the region $(\alpha_1-|\alpha_4|)\pi \le \alpha_b\,|\theta_b| \le (\alpha_2+|\alpha_3|)\pi$ (note that $(\alpha_1-|\alpha_4|)+2\alpha_2 = \alpha_2+|\alpha_3|$), then the (b-1) diagram turns into the (c) configuration. The (c-1) and (c-2) diagrams in Fig. 17 correspond to positive and negative θ_b, respectively, in this region. Then it is clear from the figure that the 1^c-2^c string interaction point of (c-1) corresponds to the 3^c-4^c string interaction point of (c-2) and vice versa. But, if the anti-ghost factor $b_0^{-(b^c)}$ is expressed by b_{ρ_0} of the 3^c-4^c interaction point, it has an extra minus sign relative to the above 1^c-2^c interaction point case (see the (b-1) diagram), which again reflects the fact that the directions in which θ_b and $-\theta_b$ increase (realizing the same configuration) are opposite. Thus (c-1) and (c-2) are also seen to cancel each other.

4.3.4. T14 terms

There are 5 relevant configurations in this case, (a) $\sim$ (e), shown in Fig. 18, each of which is realized in two ways, as drawn there. The terms (a-1) and (a-2) for (a), and (b-1) and (b-2) for (b) as well by naming the strings as shown in Fig. 18, are contained in the first term in (T14) in the forms

$$\langle U_\Omega(1,a,3^c)|\,\langle U_\Omega(b,2,4^c)|\,|R^o(a,b)\rangle\,|\Phi\rangle_{4^c3^c}\,|\Psi\rangle_{21}$$

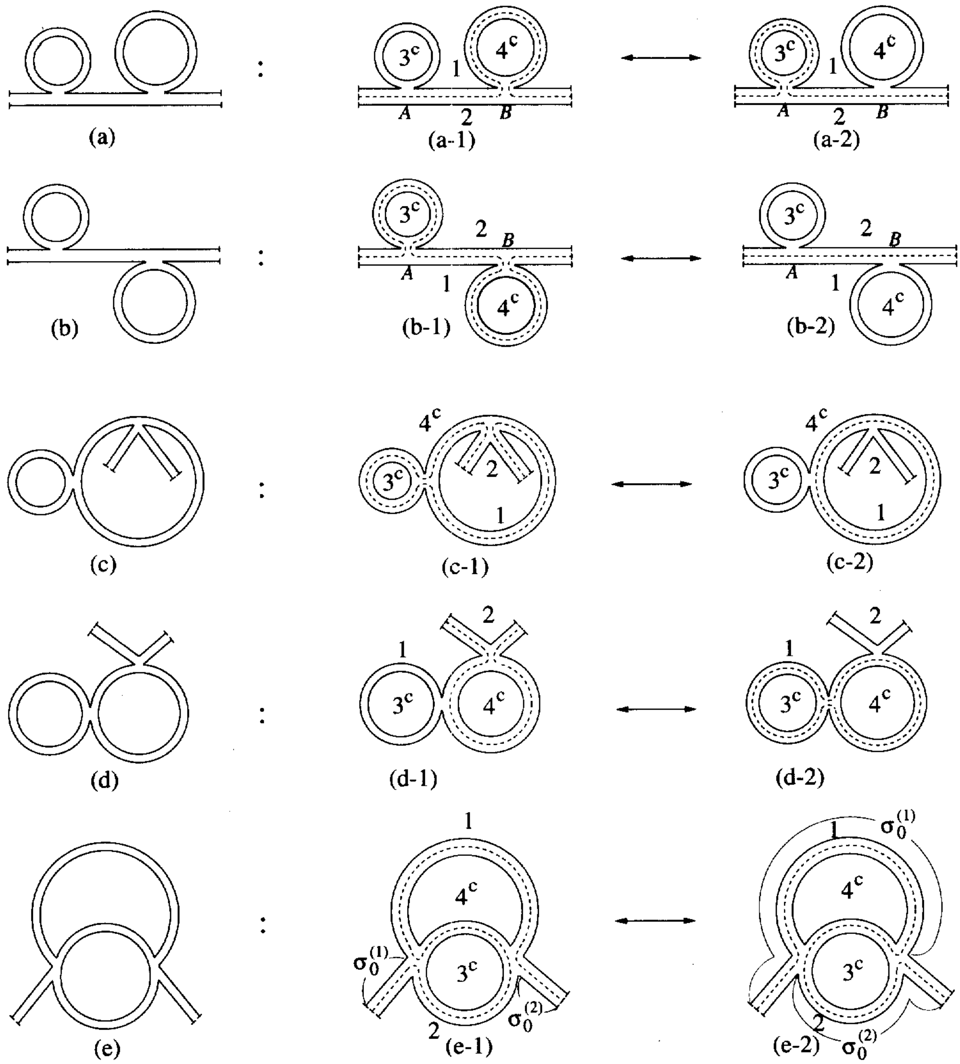


Fig. 18. Five configurations for (T14) terms.

$$= \langle u_\Omega(1a3^c;\sigma_{0A}^{(1)})|\, b^A_{\sigma_0^{(1)}}(-)\,\langle u_\Omega(b24^c;\sigma_{0B}^{(2)})|\, b^B_{\sigma_0^{(2)}}\,|R^o(ab)\rangle\,|\Phi\rangle_{4^c3^c}\,|\Psi\rangle_{21}\,,$$

$$\langle U_\Omega(2,a,3^c)|\,\langle U_\Omega(b,1,4^c)|\,|R^o(a,b)\rangle\,|\Phi\rangle_{4^c3^c}\,|\Psi\rangle_{12}$$

$$= \langle u_\Omega(2a3^c;\sigma_{0A}^{(2)})|\, b^A_{\sigma_0^{(2)}}(-)\,\langle u_\Omega(b14^c;\sigma_{0B}^{(1)})|\, b^B_{\sigma_0^{(1)}}\,|R^o(ab)\rangle\,|\Phi\rangle_{4^c3^c}\,|\Psi\rangle_{12}\,, \quad (4\cdot 40)$$

respectively, where the integration symbols $\int d\sigma_0^{(1)} d\sigma_0^{(2)}$ have been suppressed for brevity, $\langle U_\Omega(b,r,4^c)| = (-)\,\langle U_\Omega(b,\overset{\downarrow}{r},4^c)|$ with $r=1$ and 2 have been used, and the labels A and B have been attached to the anti-ghost factors to distinguish the two interaction points appearing in Fig. 18. So despite the appearance, the anti-ghost factors have the same signs for the two terms ($b^A_{\sigma_0^{(1)}} b^B_{\sigma_0^{(2)}} = b^A_{\sigma_0^{(2)}} b^B_{\sigma_0^{(1)}}$), since we have $b^A_{\sigma_0^{(1)}} = -b^A_{\sigma_0^{(2)}}$ and $b^B_{\sigma_0^{(2)}} = -b^B_{\sigma_0^{(1)}}$. The states, on the other hand, have opposite signs ($|\Psi\rangle_{21} = -|\Psi\rangle_{12}$), and hence the (a) and (b) terms vanish in (T14).

The cancellations in the other two cases of (c) and (d) are those between contractions $\langle U_\Omega|\,\langle U_\Omega|\,|R^{\rm o}\rangle$ and $\langle U_\Omega|\,\langle V_3^{\rm c}|\,|R^{\rm c}\rangle$. The terms (c-1) and (c-2) (and (d-1) and (d-2) as well, if the strings are named as shown in Fig. 18) are contained in the first and second terms in (T14) in the forms

$$\begin{aligned}
&-x_\Omega^2\,\langle U_\Omega(1,a,3^{\rm c})|\,\langle U_\Omega(b,2,4^{\rm c})|\,|R^{\rm o}(a,b)\rangle\,|\Phi\rangle_{4^{\rm c}3^{\rm c}}\,|\Psi\rangle_{21}\\
&= -x_\Omega^2\int d\sigma_0^{(1)}d\sigma_0^{(2)}\,\langle u_\Omega(1,a,3^{\rm c};\sigma_0^{(1)})|\,b_{\sigma_0^{(1)}}(b_0^-\mathcal{P})^{(3^{\rm c})}\\
&\qquad\times(-)\,\langle u_\Omega(b,2,4^{\rm c};\sigma_0^{(2)})|\,b_{\sigma_0^{(2)}}(b_0^-\mathcal{P})^{(4^{\rm c})}\,|R^{\rm o}(a,b)\rangle\,|\Phi\rangle_{4^{\rm c}3^{\rm c}}\,|\Psi\rangle_{21}\\
&= +x_\Omega^2\int d\sigma_0^{(2)}d\sigma_0^{(1)}\,\langle \tilde v(123^{\rm c}4^{\rm c};\sigma_0^{(2)},\sigma_0^{(1)})|\,b_{\sigma_0^{(2)}}b_{\sigma_0^{(1)}}\prod_{r=3,4}(b_0^-\mathcal{P})^{(r^{\rm c})}|\Phi\rangle_{4^{\rm c}3^{\rm c}}|\Psi\rangle_{21}\,,
\end{aligned} \tag{4·41}$$

$$\begin{aligned}
&-\frac{1}{2}x_\Omega x_{\rm c}\,\langle U_\Omega(1,2,\check{a}^{\rm c})|\,\langle V_3^{\rm c}(b^{\rm c},3^{\rm c},4^{\rm c})|\,|R^{\rm c}(a^{\rm c},b^{\rm c})\rangle\,|\Phi\rangle_{4^{\rm c}3^{\rm c}}\,|\Psi\rangle_{21}\\
&= -\frac{1}{2}x_\Omega x_{\rm c}(-)\int d\sigma_0^{(2)}\,\langle u_\Omega(1,2,a;\sigma_0^{(2)})|\,b_{\sigma_0^{(2)}}\\
&\qquad\times\langle v_3^{\rm c}(b^{\rm c},3^{\rm c},4^{\rm c})|\,b_0^{-(b^{\rm c})}\int\frac{d\theta}{2\pi}e^{i\theta(L-\bar L)^{(b^{\rm c})}}\prod_{r=3,4}(b_0^-\mathcal{P})^{(r^{\rm c})}\,|R^{\rm c}(a^{\rm c},b^{\rm c})\rangle\,|\Phi\rangle_{4^{\rm c}3^{\rm c}}\,|\Psi\rangle_{21}\\
&= +\frac{1}{2}x_\Omega x_{\rm c}\int d\sigma_0^{(2)}\frac{d\theta}{2\pi}\,\langle \tilde v(123^{\rm c}4^{\rm c};\sigma_0^{(2)},\theta)|\,b_{\sigma_0^{(2)}}b_0^{-(b^{\rm c})}\prod_{r=3,4}(b_0^-\mathcal{P})^{(r^{\rm c})}|\Phi\rangle_{4^{\rm c}3^{\rm c}}|\Psi\rangle_{21}\,,
\end{aligned} \tag{4·42}$$

respectively, where care has been taken with regard to the signs, and identities similar to (2·14) have been used for $\langle U_\Omega|$. Here the LPP glued vertices denote

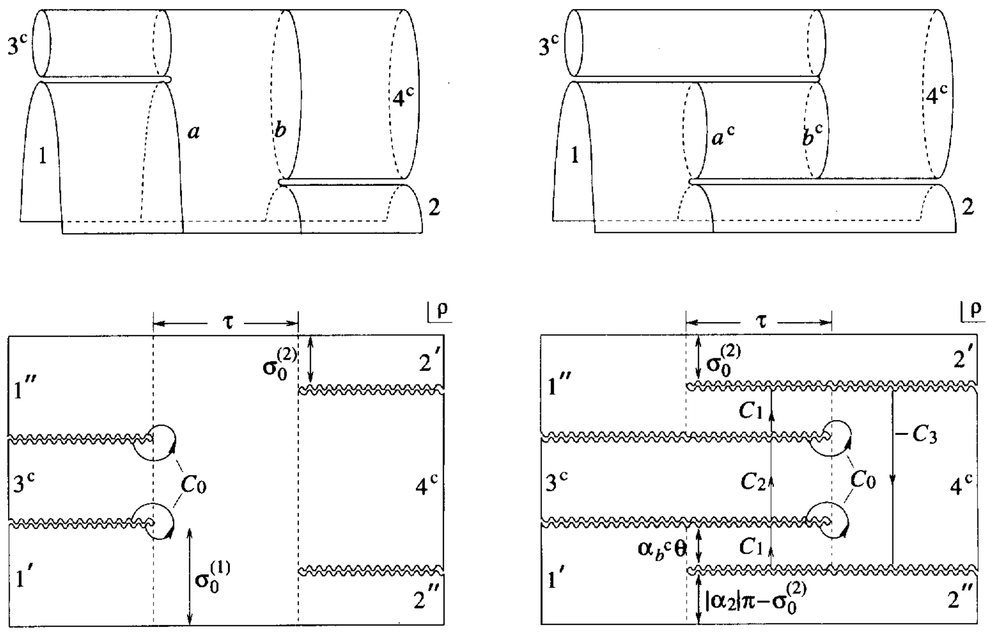
$$\begin{aligned}
\langle \tilde v(123^{\rm c}4^{\rm c};\sigma_0^{(2)},\sigma_0^{(1)})| &\equiv \langle u_\Omega(1a3^{\rm c};\sigma_0^{(1)})|\,\langle u_\Omega(b24^{\rm c};\sigma_0^{(2)})|\,|R^{\rm o}(ab)\rangle\\
\langle \tilde v(123^{\rm c}4^{\rm c};\sigma_0^{(2)},\theta)| &\equiv \langle u_\Omega(12a^{\rm c};\sigma_0^{(2)})|\,\langle v_3^{\rm c}(b^{\rm c}3^{\rm c}4^{\rm c})|\,e^{i\theta(L-\bar L)^{(b^{\rm c})}}\,|R^{\rm c}(a^{\rm c}b^{\rm c})\rangle\,.
\end{aligned} \tag{4·43}$$

By drawing the ρ plane diagram corresponding to the present configurations (c-1) and (c-2) as shown in Fig. 19, we see that the glued configurations, and hence these LPP vertices, coincide when $\sigma_0^{(1)}$ and θ satisfy the relation

$$\langle \tilde v(123^{\rm c}4^{\rm c};\sigma_0^{(2)},\theta)| = \langle \tilde v(123^{\rm c}4^{\rm c};\sigma_0^{(2)},\sigma_0^{(1)})| \qquad \text{for}\quad \alpha_{b^{\rm c}}\theta+|\alpha_2|\,\pi-\sigma_0^{(2)}=\sigma_0^{(1)}\,. \tag{4·44}$$

Thus, in view of Eqs. (4·41) and (4·42), we must again compare the anti-ghost factors $b_{\sigma_0^{(1)}}$ and $b_0^{-(b^{\rm c})}$ appearing there. This is actually the same situation as encountered in the (T9) case above. Indeed, if we compare the ρ plane diagrams, Fig. 19 for the present case and Fig. 14 for the (T9) case, we can see an exact parallel. Therefore, from Eqs. (4·28) and (4·29), we have the equality

$$b_0^{-(b^{\rm c})} = -\frac{1}{2}\alpha_{b^{\rm c}}\left(b_{\rho_0}-b_{\rho_0^*}\right) = +\frac{1}{2}i\alpha_{b^{\rm c}}b_{\sigma_0^{(1)}}\,. \tag{4·45}$$



(c-1) (c-2)

Fig. 19. ρ planes for the (c-1) and (c-2) diagrams in Fig. 18.

and we also see that the full region of (c-2) with $0 \le \theta < 2\pi$ corresponds to a part of the region of (c-1) with $\sigma_0^{(1)}$:

$$|\alpha_2|\,\pi - \sigma_0^{(2)} \le \sigma_0^{(1)} \le |\alpha_1|\,\pi - \sigma_0^{(2)}. \tag{4·46}$$

(The same is true also for the configuration (d): the full region of (d-2) with $0 \le \theta < 2\pi$ corresponds to a part of (d-1) in $|\alpha_2|\,\pi - \sigma_0^{(2)} \le \sigma_0^{(1)} \le |\alpha_1|\,\pi - \sigma_0^{(2)}$.) Thus the two terms in these regions in Eq. (4·41) cancel each other if

$$+x_\Omega^2 \int_{|\alpha_2|\pi-\sigma_0^{(2)}}^{|\alpha_1|\pi-\sigma_0^{(2)}} d\sigma_0^{(1)} = -\frac{1}{2}x_\Omega x_{\rm c}\int_0^{2\pi}\frac{d\theta}{2\pi}\frac{1}{2}i\alpha_{b^{\rm c}} \tag{4·47}$$

holds. That is, since $\alpha_{b^{\rm c}}d\theta = d\sigma_0^{(1)}$, we find

$$x_\Omega = -ix_{\rm c}\frac{1}{8\pi} \quad\Rightarrow\quad x_{\rm c} = 8\pi i x_\Omega. \tag{4·48}$$

This is the same condition as Eq. (4·31).

What happens, then, to the configuration (c-1) or (d-1) if $\sigma_0^{(1)}$ goes outside the region of Eq. (4·46)? Consider the case (c-1) first. A little inspection of the diagram (c-1) in Fig. 18 (or in Fig. 19) shows that it yields the configurations (e-1) and (e-2) for the regions

$$\text{(e-1)}: \qquad \sigma_0^{(1)} \le |\alpha_1|\,\pi - \sigma_0^{(2)} \le \sigma_0^{(1)} + |\alpha_3^{\rm c}|\,2\pi$$

$$\Rightarrow \quad (|\alpha_1| - 2\,|\alpha_3^c|)\pi \le \sigma_0^{(1)} + \sigma_0^{(2)} \le |\alpha_1|\,\pi\,,$$

$$\text{(e-2):} \qquad |\alpha_1|\,\pi - \sigma_0^{(1)} \le \sigma_0^{(2)} \le |\alpha_1|\,\pi - \sigma_0^{(1)} + |\alpha_3^c|\,2\pi$$

$$\Rightarrow \quad |\alpha_1|\,\pi \le \sigma_0^{(1)} + \sigma_0^{(2)} \le (|\alpha_1| + 2\,|\alpha_3^c|)\pi \qquad (4{\cdot}49)$$

and the configurations (b-1) and (b-2) for the remaining regions, $\sigma_0^{(1)} \le (|\alpha_1| - 2\,|\alpha_3^c|)\pi - \sigma_0^{(2)}$ and $\sigma_0^{(1)} \ge (|\alpha_1| + 2\,|\alpha_3^c|)\pi - \sigma_0^{(2)}$. Consideration of the configuration (d-1) shows that the whole region outside (4·46) yields (a-1) and (a-2). The configurations (a-1) and (a-2), as well as (b-1) and (b-2), have been shown to cancel with each other already in the above.

Let us now show that the two configurations (e-1) and (e-2) also cancel each other. We already know from Eq. (4·49) that the two configurations (e-1) and (e-2) come from the single term (4·41) in different regions of the two parameters $(\sigma_0^{(1)},\ \sigma_0^{(2)})$. Moreover, from the diagrams (e-1) and (e-2) in Fig. 18, they clearly give the same glued configuration, (e), when

$$(\sigma_0^{(1)},\ \sigma_0^{(2)}) \ \text{in (e-1)} \qquad \leftrightarrow \qquad (|\alpha_2|\,\pi - \sigma_0^{(2)},\ |\alpha_1|\,\pi - \sigma_0^{(1)}) \ \text{in (e-2)}, \qquad (4{\cdot}50)$$

which indeed gives a one-to-one mapping between the two regions (4·49) for (e-1) and (e-2). And thus the anti-ghost factors also have the correspondence

$$(b_{\sigma_0^{(1)}},\ b_{\sigma_0^{(2)}}) \ \text{in (e-1)} \qquad \leftrightarrow \qquad (-b_{\sigma_0^{(2)}},\ -b_{\sigma_0^{(1)}}) \ \text{in (e-2)}, \qquad (4{\cdot}51)$$

where the minus signs come from the fact that the directions in which $\sigma_0^{(r)}$ increase are opposite for the two cases. Therefore, the anti-ghost factor $b_{\sigma_0^{(2)}} b_{\sigma_0^{(1)}}$ in Eq. (4·41) is the same but has opposite order for the two configurations (e-1) and (e-2), so that they exactly cancel each other.

4.3.5. T15 term

When contracting $\langle V_3^c|$ and $\langle V_\infty|$, there appear two glued configurations, (a) and (b), as drawn in Fig. 20. For the former configuration (a), it is easy to see the cancellation between the two ways of gluing (a-1) and (a-2) giving a common configuration: they appear in the (T15) term (3·28) in the following forms, respectively:

$$\begin{aligned}
&\langle V_3^c(1^c, 2^c, \overset{\vee}{a^c})|\ \langle V_\infty(b^c, 3^c)|\ |R^c(a^c, b^c)\rangle\, |\Phi\rangle_{3^c2^c1^c} \\
&= \int \frac{d\theta_b}{2\pi}\, \langle v_3^c(1^c, 2^c, a^c)|\ \langle v_\infty(b^c, 3^c; \sigma_0)|\, b_{\sigma_0} b_0^{-(b^c)} e^{i\theta_b (L-\bar{L})^{(b^c)}} (b_0^- \mathcal{P})^{(3^c)} \\
&\qquad \times |R^c(a^c, b^c)\rangle \prod_{r=1,2} (b_0^- \mathcal{P})^{(r^c)}\, |\Phi\rangle_{3^c2^c1^c} \\
&= \int \frac{d\theta_b}{2\pi}\, \langle \tilde{v}(1^c2^c3^c; \sigma_0, \theta_b)|\, b_{\sigma_0} b_0^{-(b^c)} \prod_{r=1,2,3} (b_0^- \mathcal{P})^{(r^c)}\, |\Phi\rangle_{3^c2^c1^c}\,, \qquad (4{\cdot}52)
\end{aligned}$$

$$\begin{aligned}
&\langle V_3^c(2^c, 3^c, \overset{\vee}{c^c})|\ \langle V_\infty(d^c, 1^c)|\ |R^c(c^c, d^c)\rangle\, |\Phi\rangle_{1^c3^c2^c} \\
&= \int \frac{d\theta_d}{2\pi}\, \langle v_3^c(2^c, 3^c, c^c)|\ \langle v_\infty(d^c, 1^c; \sigma_0)|\, b_{\sigma_0} b_0^{-(d^c)} e^{i\theta_d (L-\bar{L})^{(d^c)}} (b_0^- \mathcal{P})^{(1^c)}
\end{aligned}$$

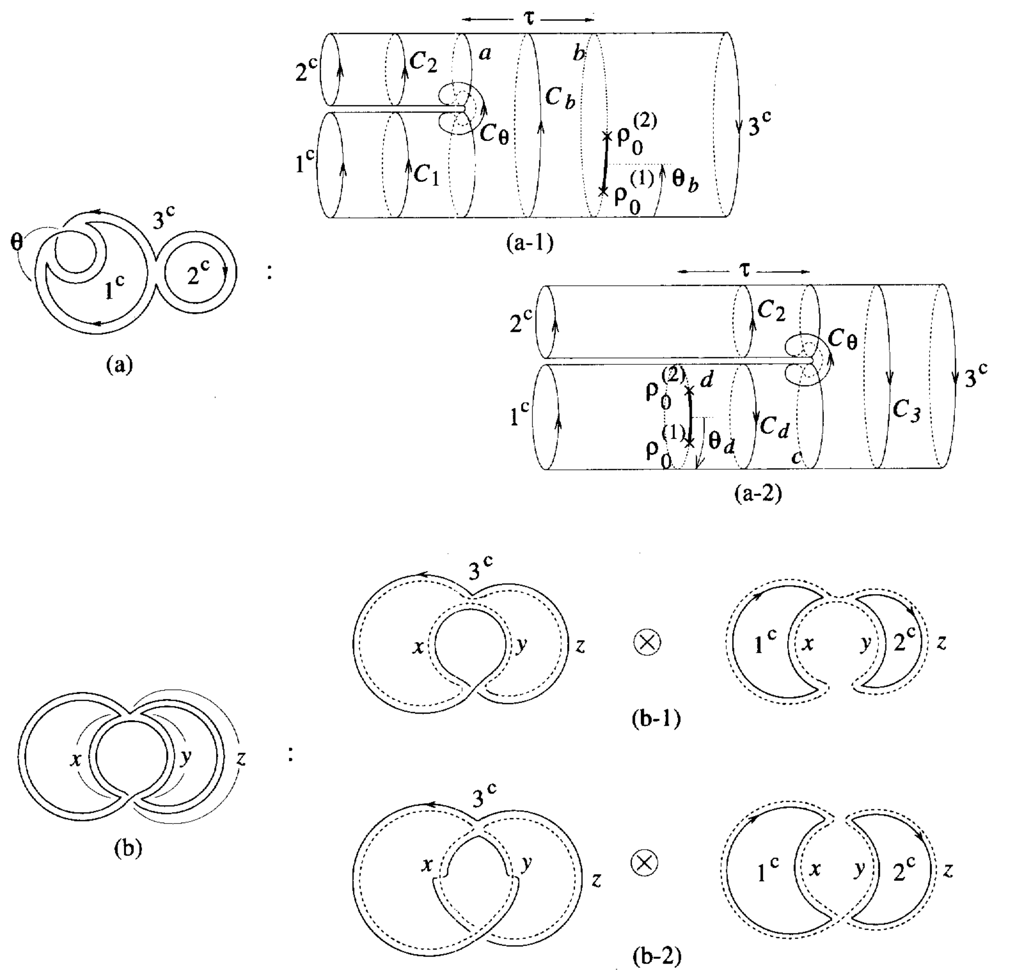


Fig. 20. Two configurations for (T15) term. The diagrams (b-1) and (b-2) are drawn by decomposing the process into the initial-to-intermediate and intermediate-to-final transition parts, for clarity.

$$\times\,|R^{\rm c}(c^{\rm c},d^{\rm c})\rangle \prod_{r=2,3} (b_0^- \mathcal{P})^{(r^{\rm c})}\,|\Phi\rangle_{1^{\rm c}3^{\rm c}2^{\rm c}}$$

$$= \int \frac{d\theta_d}{2\pi}\,\langle \tilde{v}(1^{\rm c}2^{\rm c}3^{\rm c};\sigma_0,\theta_d)|\,b_{\sigma_0} b_0^{-(d^{\rm c})} \prod_{r=1,2,3} (b_0^- \mathcal{P})^{(r^{\rm c})}\,|\Phi\rangle_{3^{\rm c}2^{\rm c}1^{\rm c}}\,. \qquad (4\cdot53)$$

Here the LPP vertices for the glued configurations are defined by

$$\langle \tilde{v}(1^{\rm c}2^{\rm c}3^{\rm c};\sigma_0,\theta_b)| \equiv \langle v_3^{\rm c}(1^{\rm c},2^{\rm c},a^{\rm c})|\,\langle v_\infty(b^{\rm c},3^{\rm c};\sigma_0)|\,e^{i\theta_b(L-\bar{L})^{(b^{\rm c})}}\,|R^{\rm c}(a^{\rm c},b^{\rm c})\rangle\,, \qquad (4\cdot54)$$

and similarly for $\langle \tilde{v}(1^{\rm c}2^{\rm c}3^{\rm c};\sigma_0,\theta_d)|$. For the common configuration (a), the anti-ghost factor b_{σ_0} coming from the $\langle V_\infty|$ vertex is common to the two terms (4·52) and (4·53). Therefore we have only to compare the anti-ghost factors $b_0^{-(b^{\rm c})}$ and $b_0^{-(d^{\rm c})}$. By the same method as used for the (T9) term near Eq. (4·28), they can be replaced by

$$b_0^{-(b^c)} \Rightarrow \frac{1}{2}\alpha_{b^c}\left(\oint_{C_b+C_1+C_2} -\text{a.h.}\right)\frac{d\rho}{2\pi i}b(\rho) = +\frac{1}{2}|\alpha_{3^c}|\left(b_{\rho_0} - b_{\rho_0^*}\right),$$

$$b_0^{-(d^c)} \Rightarrow \frac{1}{2}\alpha_{d^c}\left(\oint_{C_d+C_2+C_3} -\text{a.h.}\right)\frac{d\rho}{2\pi i}b(\rho) = -\frac{1}{2}\alpha_{1^c}\left(b_{\rho_0} - b_{\rho_0^*}\right), \quad (4·55)$$

where the contours C_b, C_d and C_i $(i = 1,2,3)$ are drawn in Fig. 20 and $b_{\rho_0} = \oint_{C_\theta}(d\rho/2\pi i)b(\rho)$ with the contour C_θ encircling the 3-closed-string interaction point $\rho_0 = i\alpha_1\pi$. Here we have used the fact that $\alpha_{b^c} = |\alpha_3| > 0$ and $\alpha_{d^c} = -\alpha_1 < 0$ (or $\alpha_{b^c} \cdot \alpha_{d^c} < 0$, more generally). Because of this, the anti-ghost factors $b_0^{-(b^c)}$ and $b_0^{-(d^c)}$, with integration measures $d(|\alpha_3|\,\theta_b)$ and $d(\alpha_1\theta_d)$, respectively, are equal but have opposite signs. This reflects the fact that the intermediate closed strings in the two configurations (a-1) and (a-2) must be twisted in opposite directions (by amounts $|\alpha_3\theta_b| = |\alpha_1\theta_d|$), in order to keep the common glued configuration as seen in Fig. 20. Thus the two terms (a-1) and (a-2), (4·52) and (4·53), cancel each other.

Note, however, that this cancellation occurs between (a-1) in the restricted region

$$-\alpha_1\pi + \sigma_0 \le |\alpha_3|\,\theta_b \le \alpha_1\pi - \sigma_0 \quad (4·56)$$

and (a-2) in the full region $-\pi \le \theta_d < \pi$. If the twisting angle θ_b comes into the regions

$$R_+ : \quad \alpha_1\pi - \sigma_0 \le |\alpha_3|\,\theta_b \le \alpha_1\pi + \sigma_0\,,$$
$$R_- : \quad -(\alpha_1\pi - \sigma_0) \ge |\alpha_3|\,\theta_b \ge -(\alpha_1\pi + \sigma_0)\,, \quad (4·57)$$

then the cross cap occupying the region $\text{Im}\rho \in [\,|\alpha_3|\,\theta_b - \sigma_0,\ |\alpha_3|\,\theta_b + \sigma_0\,]$ on the ρ plane overlaps with the 3-closed-string interaction point $\rho_0 = \pm i\alpha_1\pi$, and the resultant configurations become of types (b-2) and (b-1) in Fig. 20, respectively. One easily recognizes (b-1) to be the configuration in the R_- region, but (b-2), at first sight, might not look like the configuration in the R_+ region. However, if one redraws the (b-2) diagram in Fig. 20 by exchanging the places of the two handles y and z, then the self-intersecting point of string 3 originally present at the bottom comes to the top in the diagram, and it can be recognized to be really the configuration in the R_+ region.

From the (b-1) (or (a-1)) diagram in Fig. 20, the lengths of the handles x and y of the (b-1) diagram in the region R_- are found to be

$$|x^-| = \alpha_1\pi + \sigma_0^- + |\alpha_3|\,\theta_b^-, \qquad |y^-| = -\,|\alpha_3|\,\theta_b^- + \sigma_0^- - \alpha_1\pi\,, \quad (4·58)$$

where we have put the superscript "$-$" on θ_b and σ_0 in this R^- case to distinguish it from the R^+ case below. Note that $\theta_b^- < 0$ in this region R_-. Similarly, taking account of the exchange of the y and z handles explained above, the lengths of the handles x and y of the (b-2) diagram in the region R_+ are found to be

$$|x^+| = \alpha_1\pi + \sigma_0^+ - |\alpha_3|\,\theta_b^+, \qquad |y^+| = 2\alpha_2\pi - (|\alpha_3|\,\theta_b^+ - \alpha_1\pi + \sigma_0^+)\,. \quad (4·59)$$

Hence, in order for the (b-1) and (b-2) to give the same glued configuration, these lengths must coincide ($|x^-| = |x^+|$ and $|y^-| = |y^+|$), from which we find the correspondence

$$\rho_{0-}^{(1)} = \rho_{0+}^{(2)} - 2i\,|\alpha_3|\,\pi \qquad \rho_{0-}^{(2)} = -\rho_{0+}^{(1)}\,,$$
$$\text{with} \qquad \rho_{0\pm}^{(1)} \equiv i(|\alpha_3|\,\theta_b^{\pm} - \sigma_0^{\pm}), \qquad \rho_{0\pm}^{(2)} \equiv i(|\alpha_3|\,\theta_b^{\pm} + \sigma_0^{\pm})\,. \qquad (4\cdot60)$$

Here $\rho_0^{(1)} = i(|\alpha_3|\,\theta_b - \sigma_0)$ and $\rho_0^{(2)} = i(|\alpha_3|\,\theta_b + \sigma_0)$ are the coordinates of the two endpoints of the cross cap on the ρ plane. However, we should note that the LPP vertices with these parameter sets (σ_0^+, θ_b^+) and (σ_0^-, θ_b^-) are *not* equal. This is because the orientation of string 2^c has been reversed in the above exchange process of the x and y handles, and so the precise relationship between the LPP vertices for these two configurations is given by

$$\Big\langle \tilde{v}(1^c2^c3^c; \sigma_0^+, \theta_b^+)\Big| = \Big\langle \tilde{v}(1^c2^c3^c; \sigma_0^-, \theta_b^-)\Big|\,\Omega^{(2^c)}\,, \qquad (4\cdot61)$$

with the understanding that the parameters (σ_0^+, θ_b^+) and (σ_0^-, θ_b^-) are related to each other by Eq. (4·60).

Since both (b-1) and (b-2) come from the same (a-1) term, we have now only to compare the relative sign of the the anti-ghost factor $b_{\sigma_0} b_0^{-(b^c)}$ in Eq. (4·52) for the two cases of $R^{\pm}$ with the angle relations (4·60). As was performed explicitly in I for the glued vertex $\langle\hat{U}|\,\langle V_\infty|\,|R^c\rangle$, the anti-ghost factor $b_{\sigma_0} b_0^{-(b^c)}$ can be replaced by $b_{\rho_0^{(1)}} b_{\rho_0^{(2)}}$ up to an irrelevant proportionality factor (independent of θ and σ_0), where $b_{\rho_0^{(1)}}$ and $b_{\rho_0^{(2)}}$ are the anti-ghost factors corresponding to the shifts of the two end-points $\rho_0^{(1)}$ and $\rho_0^{(2)}$ of the cross cap (see the diagram (a-1) in Fig. 20). This can be easily understood: $b_0^{-(b^c)}$ is essentially the anti-ghost factor for the shift of θ_b, and hence $b_0^{-(b^c)} \propto (b_{\rho_0^{(1)}} + b_{\rho_0^{(2)}})$. Also b_{σ_0} is the anti-ghost factor for the shift of σ_0, and hence $b_{\sigma_0} \propto (b_{\rho_0^{(1)}} - b_{\rho_0^{(2)}})$. Thus the product gives $\propto b_{\rho_0^{(1)}} b_{\rho_0^{(2)}}$. Coming back to the comparison of the anti-ghost factor $b_{\sigma_0} b_0^{-(b^c)}$, from the angle relations (4·60), we are tempted to immediately write

$$\begin{cases} b_{\rho_{0-}^{(1)}} = +b_{\rho_{0+}^{(2)}} \\ b_{\rho_{0-}^{(2)}} = -b_{\rho_{0+}^{(1)}} \end{cases} \quad \Rightarrow \quad b_{\rho_{0-}^{(1)}} b_{\rho_{0-}^{(2)}} = +b_{\rho_{0+}^{(1)}} b_{\rho_{0+}^{(2)}}\,. \qquad (4\cdot62)$$

But these are not quite correct. This is because the ρ planes for the two cases R^+ and R^- are equal only under the twist operation $\Omega^{(2^c)}$, as noted in Eq. (4·61). Therefore, the precise form of Eq. (4·62) reads

$$\begin{cases} \Omega^{(2^c)^{-1}} b_{\rho_{0-}^{(1)}} \Omega^{(2^c)} = +b_{\rho_{0+}^{(2)}} \\ \Omega^{(2^c)^{-1}} b_{\rho_{0-}^{(2)}} \Omega^{(2^c)} = -b_{\rho_{0+}^{(1)}} \end{cases} \quad \Rightarrow \quad \Omega^{(2^c)^{-1}} b_{\rho_{0-}^{(1)}} b_{\rho_{0-}^{(2)}} \Omega^{(2^c)} = +b_{\rho_{0+}^{(1)}} b_{\rho_{0+}^{(2)}}\,. \qquad (4\cdot63)$$

These hold in the presence of the factor $\prod_{r=1^c,2^c,3^c} (b_0^- \mathcal{P})^{(r^c)}$. Indeed, these relations can be directly confirmed by comparing the integration contours (which take the shape of an '8' across the cross cap cut) defining the anti-ghost factors $b_{\rho_{0\pm}^{(i)}}$ $(i=1,2)$ on the two ρ planes for R^+ and R^- cases. (See Figs. 21 and 22 to confirm the equality

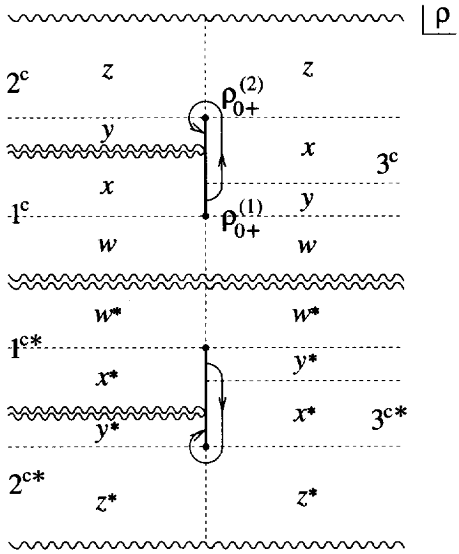


Fig. 21. Integration contour for $b_{\rho_{0+}^{(2)}}$ on the ρ plane of $\langle\tilde{v}(1^c2^c3^c;\sigma_0^+,\theta_b^+)|$.

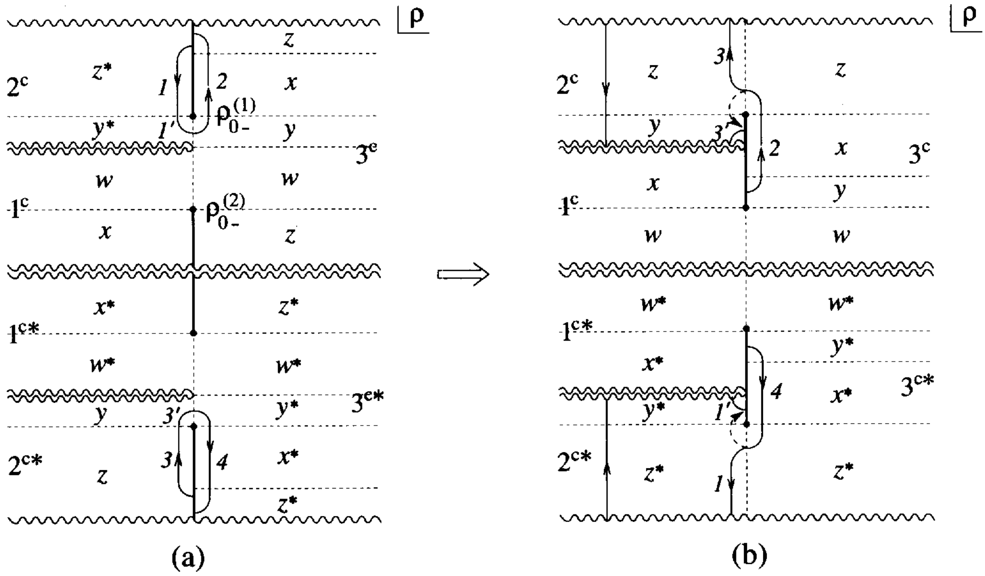


Fig. 22. (a) Integration contour for $b_{\rho_{0-}^{(1)}}$ on the ρ plane of $\langle\tilde{v}(1^c2^c3^c;\sigma_0^-,\theta_b^-)|$. Going to (b), the ρ plane is rearranged first by twisting the closed strings 1^c and 3^c by an amount $|x|$ (σ length of the x region), and then by exchanging the regions $x+y \leftrightarrow y^*+x^*$ of string 2^c (i.e., acting with $\Omega^{(2^c)}$). The resultant plane (b) becomes the ρ plane of $\langle\tilde{v}(1^c2^c3^c;\sigma_0^-,\theta_b^-)|\,\Omega^{(2^c)}$. The integration contour for $b_{\rho_{0-}^{(1)}}$ on this plane is seen to coincide with that of $b_{\rho_{0+}^{(2)}}$ in Fig. 21 by deformation after adding $b_0^{-(2^c)}$.

$\Omega^{(2^c)}{}^{-1} b_{\rho_{0-}^{(1)}} \Omega^{(2^c)} = +b_{\rho_{0+}^{(2)}}$, for instance.) Hence, together with Eq. (4·61), we obtain

$$\left\langle \tilde{v}(1^c 2^c 3^c; \sigma_0^+, \theta_b^+) \right| b_{\rho_{0+}^{(1)}} b_{\rho_{0+}^{(2)}} = + \left\langle \tilde{v}(1^c 2^c 3^c; \sigma_0^-, \theta_b^-) \right| b_{\rho_{0-}^{(1)}} b_{\rho_{0-}^{(2)}} \Omega^{(2^c)}. \qquad (4·64)$$

There appears no relative minus sign, unlike the cases considered to this point. However, we should note that $b_0^{(2)}$ is odd under $\Omega^{(2^c)}$ (i.e., $\Omega^{(2^c)}{}^{-1} b_0^{(2)} \Omega^{(2^c)} = -b_0^{(2)}$), so that we have a relative minus sign:

$$\left\langle \tilde{v}(1^c 2^c 3^c; \sigma_0^+, \theta_b^+) \right| b_{\rho_{0+}^{(1)}} b_{\rho_{0+}^{(2)}} \prod_{r=1,2,3} (b_0^- \mathcal{P})^{(r^c)}$$
$$= - \left\langle \tilde{v}(1^c 2^c 3^c; \sigma_0^-, \theta_b^-) \right| b_{\rho_{0-}^{(1)}} b_{\rho_{0-}^{(2)}} \prod_{r=1,2,3} (b_0^- \mathcal{P})^{(r^c)} \Omega^{(2^c)}. \qquad (4·65)$$

The $\Omega^{(2^c)}$ on the right-hand side disappears in the actual vertex, since unoriented projection operators $\Pi^{(r)} = (1 + \Omega^{(r)})/2$ are acting on each external string. We thus have shown the cancellation of (b-1) and (b-2) terms, finishing the proof for the complete cancellation of (T15) terms.

4.3.6. T16 terms

The generic configurations resultant from the contraction of the two vertices $\langle U_\Omega|$ and $\langle V_\propto|$, or $\langle U_\Omega|$ and $\langle V_\infty|$, fall into four types, (a) $\sim$ (d), depicted in Fig. 23. Only (b-2) is given by gluing $\langle U_\Omega|$ and $\langle V_\infty|$, and all the others are by gluing $\langle U_\Omega|$ and $\langle V_\propto|$. As always, cancellations occur between the two ways of gluing for a given type configuration.

The type (a) configuration is realized by the (a-1) diagram at $\tau = 0$ in Fig. 23 in the restricted regions with $0 \le \sigma_1^{(b)} \le \sigma_2^{(b)} \le \sigma_0^{(1)}$ or $2\alpha_{2^c}\pi + \sigma_0^{(1)} \le \sigma_1^{(b)} \le \sigma_2^{(b)} \le |\alpha_{3^c}|\pi$, and by the (a-2) diagram in the region satisfying $0 \le \sigma_1^{(d)} \le \sigma_2^{(d)} \le \sigma_0^{(3)}$ or $\sigma_0^{(3)} \le \sigma_1^{(d)} \le \sigma_2^{(d)} \le \alpha_1\pi$. The terms (a-1) and (a-2) are contained in the first term in (T16) in the forms

$$\langle U_\Omega(1, a, 2^c)|\, \langle V_\propto(b, 3)|\, |R^o(a, b)\rangle\, |\Phi\rangle_{2^c}\, |\Psi\rangle_{31}$$
$$= \int d\sigma_0^{(1)} d\sigma_1^{(b)} d\sigma_2^{(b)}\, \langle u_\Omega(1, a, 2^c; \sigma_0^{(1)})|\, b_{\sigma_0^{(1)}} (b_0^- \mathcal{P})^{(2^c)}$$
$$\times \langle v_\propto(b, 3; \sigma_1^{(b)}, \sigma_2^{(b)})|\, b_{\sigma_1^{(b)}} b_{\sigma_2^{(b)}}\, |R^o(a, b)\rangle\, |\Phi\rangle_{2^c}\, |\Psi\rangle_{31}\,, \qquad (4·66)$$

$$\langle U_\Omega(3, c, 2^c)|\, \langle V_\propto(d, 1)|\, |R^o(c, d)\rangle\, |\Phi\rangle_{2^c}\, |\Psi\rangle_{13}$$
$$= \int d\sigma_0^{(3)} d\sigma_1^{(d)} d\sigma_2^{(d)}\, \langle u_\Omega(3, c, 2^c; \sigma_0^{(3)})|\, b_{\sigma_0^{(3)}} (b_0^- \mathcal{P})^{(2^c)}$$
$$\times \langle v_\propto(d, 1; \sigma_1^{(d)}, \sigma_2^{(d)})|\, b_{\sigma_1^{(d)}} b_{\sigma_2^{(d)}}\, |R^o(c, d)\rangle\, |\Phi\rangle_{2^c}\, |\Psi\rangle_{13}\,. \qquad (4·67)$$

From the diagrams (a-1) and (a-2) at $\tau = 0$ in Fig. 23, we see that the directions in which σ increases are opposite for strings b and d, and also for strings 1 and 3, so that we have $b_{\sigma_1^{(d)}} = -b_{\sigma_2^{(b)}}$, $b_{\sigma_2^{(d)}} = -b_{\sigma_1^{(b)}}$ and $b_{\sigma_0^{(3)}} = -b_{\sigma_0^{(1)}}$. Thus the products of anti-ghost factors have the same sign ($b_{\sigma_0^{(1)}} b_{\sigma_1^{(b)}} b_{\sigma_2^{(b)}} = b_{\sigma_0^{(3)}} b_{\sigma_1^{(d)}} b_{\sigma_2^{(d)}}$), but the

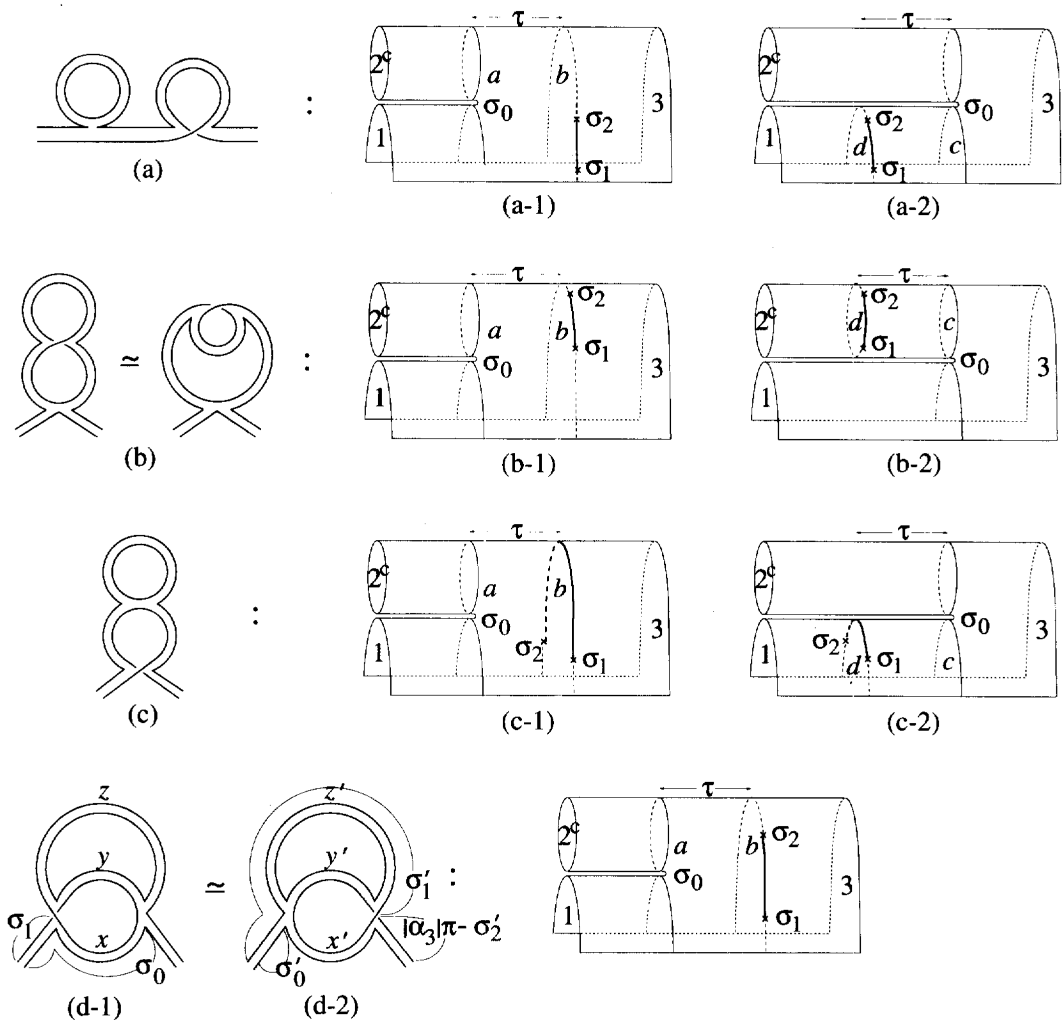


Fig. 23. Four configurations for (T16) terms. For brevity, σ_0, σ_1 and σ_2 on the ρ planes, denote $\sigma_0^{(1)}$, $\sigma_1^{(b)}$ and $\sigma_2^{(b)}$ for (a-1), (b-1), (c-1) and (d) diagrams, and $\sigma_0^{(3)}$, $\sigma_1^{(d)}$ and $\sigma_2^{(d)}$ for (a-2), (b-2) and (c-2) diagrams, respectively.

states have opposite signs ($|\Psi\rangle_{13} = -|\Psi\rangle_{31}$), and hence the (a-1) and (a-2) terms cancel each other.

Next consider the type (b) configuration. The (b-1) diagram corresponds to $\langle U_\Omega|\,\langle V_\propto|\,|R^{\rm o}\rangle$ and (b-2) to $\langle U_\Omega|\,\langle V_\infty|\,|R^{\rm c}\rangle$. The former (b-1) is just the (a-1) diagram in the region with $\sigma_0^{(1)} \le \sigma_1^{(b)} \le \sigma_2^{(b)} \le \sigma_0^{(1)} + 2\alpha_{2^c}\pi$. In this region, the presence of string 1 plays no important role. If we forget string 1, then the vertex $\langle U_\Omega|$ reduces to $\langle U|$, and in fact the diagrams (b-1) and (b-2) are the same as those we encountered in I for proving the cancellation between $\langle U|\,\langle V_\propto|\,|R^{\rm o}\rangle$ and $\langle U|\,\langle V_\infty|\,|R^{\rm c}\rangle$, that is (T10) in Eq. (3·22). Therefore, the same calculation as I proves the cancellation of (b-1) and (b-2) terms here when the coupling relation

$$x_\infty = 4\pi i x_\propto \tag{4·68}$$

is satisfied. One may wonder about discrepancy of the relative weights of the coefficients, $-2x_u x_\propto : 2x_u x_\infty$ in Eq. (3·22), and the present one $2x_\Omega x_\propto : -x_\Omega x_\infty$ in Eq. (3·29). However, from the second term $\langle U_\Omega(1,3,\check{a}^{\rm c})|\ \langle V_\infty(b^{\rm c},2^{\rm c})|\ |R^{\rm c}(a^{\rm c},b^{\rm c})\rangle$ in the latter, the specific diagram (b-2) with a definite set of string lengths appears twice since the term with 3 and 1 exchanged in $\langle U_\Omega(1,3,\check{a}^{\rm c})|$ also give the same diagram. So the actual relative weight for the present case is also $-2x_\propto : 2x_\infty = -x_\propto : x_\infty$, thus leading to the same coupling relation (4·68).

Next is the type (c) configuration, which is realized in two ways of gluing (c-1) and (c-2) at $\tau = 0$ in Fig. 23. These are nothing but (a-1) and (a-2) terms with moduli parameters in different regions, respectively, and thus can be described by the same equations as (4·66) and (4·67): with the LPP vertices for the glued configurations

$$\begin{aligned}\langle\tilde{v}(132^{\rm c};\sigma_0^{(1)},\sigma_1^{(b)},\sigma_2^{(b)})| &\equiv \langle u_\Omega(1a2^{\rm c};\sigma_0^{(1)})|\ \langle v_\propto(b3;\sigma_1^{(b)},\sigma_2^{(b)})|\ |R^{\rm o}(ab)\rangle\,,\\ \langle\tilde{v}(132^{\rm c};\sigma_0^{(3)},\sigma_1^{(d)},\sigma_2^{(d)})| &\equiv \langle u_\Omega(3c2^{\rm c};\sigma_0^{(3)})|\ \langle v_\propto(d1;\sigma_1^{(d)},\sigma_2^{(d)})|\ |R^{\rm o}(cd)\rangle\,,\end{aligned} \tag{4·69}$$

(c-1) and (c-2) appear in the forms

$$(\text{c-1}) = \int d\sigma_0^{(1)} d\sigma_1^{(b)} d\sigma_2^{(b)}\ \langle\tilde{v}(132^{\rm c};\sigma_0^{(1)},\sigma_1^{(b)},\sigma_2^{(b)})|\, b_{\sigma_1^{(b)}} b_{\sigma_2^{(b)}} b_{\sigma_0^{(1)}} (b_0^- \mathcal{P})^{(2^{\rm c})}, \tag{4·70}$$

$$(\text{c-2}) = -\int d\sigma_0^{(3)} d\sigma_1^{(d)} d\sigma_2^{(d)}\ \langle\tilde{v}(132^{\rm c};\sigma_0^{(3)},\sigma_1^{(d)},\sigma_2^{(d)})|\, b_{\sigma_1^{(d)}} b_{\sigma_2^{(d)}} b_{\sigma_0^{(3)}} (b_0^- \mathcal{P})^{(2^{\rm c})}, \tag{4·71}$$

where the common external states $|\Phi\rangle_{2^{\rm c}}\, |\Psi\rangle_{31}$ are omitted. The minus sign of (c-2) has come from $|\Psi\rangle_{13} = -\,|\Psi\rangle_{31}$. Hence we have only to compare the anti-ghost factors $b_{\sigma_1^{(b)}} b_{\sigma_2^{(b)}} b_{\sigma_0^{(1)}}$ and $b_{\sigma_1^{(d)}} b_{\sigma_2^{(d)}} b_{\sigma_0^{(3)}}$. Note that, since strings 3 and d carry negative string lengths, $\sigma_1^{(d)}$, $\sigma_2^{(d)}$ and $\sigma_0^{(3)}$ represent distances on the ρ plane measured from the opposite edge of the open string. Then the condition for these two diagrams (c-1) and (c-2) to reduce to a common glued configuration at $\tau = 0$ is that the positions of those interaction points coincide:

$$\sigma_1^{(b)} = \alpha_1\pi - \sigma_2^{(d)}, \quad |\alpha_3|\,\pi - \sigma_2^{(b)} = \sigma_1^{(d)}, \quad \sigma_1^{(b)} + \sigma_2^{(b)} - \sigma_0^{(1)} = |\alpha_3|\,\pi - \sigma_0^{(3)}. \tag{4·72}$$

The left-hand side of the third condition means that the interaction point $\sigma_0^{(1)}$ appears at the place $\sigma_1^{(b)} + \sigma_2^{(b)} - \sigma_0^{(1)}$ when crossing the cross cap cut $[\sigma_1^{(b)},\ \sigma_2^{(b)}]$ on the (c-1) diagram. As is clear from the diagrams, however, the configurations of (c-1) and (c-2) at $\tau = 0$ are not quite equal to each other as they stand, since the whole region of the closed string $2^{\rm c}$ in the (c-1) case is wrapped by the cross cap cut. Therefore the glued vertices coincide with each other only when the twist operator $\Omega^{(2^{\rm c})}$ acts on the (c-2) glued vertex:

$$\left\langle\tilde{v}(1,3,2^{\rm c};\sigma_0^{(1)},\sigma_1^{(b)},\sigma_2^{(b)})\right| = \left\langle\tilde{v}(1,3,2^{\rm c};\sigma_0^{(3)},\sigma_1^{(d)},\sigma_2^{(d)})\right|\,\Omega^{(2^{\rm c})}. \tag{4·73}$$

Similarly to the previous (T15) [type (b)] case, we can see from Eq. (4·72) the relation

$$b_{\sigma_1^{(b)}} b_{\sigma_2^{(b)}} b_{\sigma_0^{(1)}} = \Omega^{(2^c)^{-1}} (-b_{\sigma_2^{(d)}})(-b_{\sigma_1^{(d)}}) b_{\sigma_0^{(3)}} \Omega^{(2^c)} = -\Omega^{(2^c)^{-1}} b_{\sigma_1^{(d)}} b_{\sigma_2^{(d)}} b_{\sigma_0^{(3)}} \Omega^{(2^c)} \tag{4·74}$$

in the presence of $(b_0^- \mathcal{P})^{(2^c)}$, and hence we obtain, noting also that $b_0^{-(2^c)}$ is odd under $\Omega^{(2^c)}$, the relation

$$\begin{aligned} &\left\langle \tilde{v}(1,3,2^c;\sigma_0^{(1)},\sigma_1^{(b)},\sigma_2^{(b)}) \right| b_{\sigma_1^{(b)}} b_{\sigma_2^{(b)}} b_{\sigma_0^{(1)}} b_0^{-(2^c)} \\ &= -\left\langle \tilde{v}(1,3,2^c;\sigma_0^{(3)},\sigma_1^{(d)},\sigma_2^{(d)}) \right| b_{\sigma_1^{(d)}} b_{\sigma_2^{(d)}} b_{\sigma_0^{(3)}} \Omega^{(2^c)} b_0^{-(2^c)} \\ &= +\left\langle \tilde{v}(1,3,2^c;\sigma_0^{(3)},\sigma_1^{(d)},\sigma_2^{(d)}) \right| b_{\sigma_1^{(d)}} b_{\sigma_2^{(d)}} b_{\sigma_0^{(3)}} b_0^{-(2^c)} \Omega^{(2^c)}. \end{aligned} \tag{4·75}$$

This implies that the (c-1) and (c-2) terms cancel each other.

Finally consider the type (d) configuration, which corresponds to the (a-1) term with the moduli parameters in the region

$$\begin{aligned} R: &\quad \sigma_1^{(b)} \le \sigma_0^{(1)} \le \sigma_2^{(b)}, \\ R': &\quad |\alpha_3|\,\pi - \sigma_2^{(b)} \le \sigma_0^{(1)} \le |\alpha_3|\,\pi - \sigma_1^{(b)}. \end{aligned} \tag{4·76}$$

This case is more similar to the (T15) type (b) case. Cancellation occurs between the configurations in these two regions R and R'. These two configurations can be seen to give the same pattern of gluing; indeed, if we redraw the (d-2) diagram in the region R' by exchanging the two handles y' and z', then it can be recognized as possessing the same pattern as the (d-1) diagram in the region R. It is also the same as before that the orientation of the closed string 2^c is reversed in this exchanging process of y' and z'. Therefore, we find the conditions for these two to give a common configuration:

$$\begin{aligned} \text{the left leg}: &\quad \sigma_1 = \sigma_0', \\ \text{the right leg}: &\quad \alpha_1\pi - \sigma_0 = |\alpha_3|\,\pi - \sigma_2', \\ \text{length of } y \text{ and } z': &\quad \sigma_2 - \sigma_0 = \sigma_1' - \sigma_0'. \end{aligned} \tag{4·77}$$

Here we have omitted the superscripts (b) and (1) and added primes to denote the parameters in the R' region to distinguish them from those in R. The glued vertices with these corresponding parameter sets coincide with each other when the twist operator $\Omega^{(2^c)}$ acts:

$$\langle \tilde{v}(1,3,2^c;\sigma_0,\sigma_1,\sigma_2)| = \langle \tilde{v}(1,3,2^c;\sigma_0',\sigma_1',\sigma_2')|\,\Omega^{(2^c)}. \tag{4·78}$$

Noting the presence of the twist operation, we have the correspondence of the anti-ghost factors,

$$b_{\sigma_1} b_{\sigma_2} b_{\sigma_0} = \Omega^{(2^c)^{-1}} (b_{\sigma_0'})(b_{\sigma_1'})(b_{\sigma_2'}) \Omega^{(2^c)} = +\Omega^{(2^c)^{-1}} b_{\sigma_1'} b_{\sigma_2'} b_{\sigma_0'} \Omega^{(2^c)}, \tag{4·79}$$

which is again valid in the presence of the $(b_0^- \mathcal{P})^{(2^c)}$ factor. We thus find

$$\langle \tilde{v}(1,3,2^c;\sigma_0,\sigma_1,\sigma_2)|\, b_{\sigma_1} b_{\sigma_2} b_{\sigma_0} b_0^{-(2^c)}$$

$$= \langle \tilde{v}(1,3,2^{c};\sigma'_0,\sigma'_1,\sigma'_2)|\, b_{\sigma'_1} b_{\sigma'_2} b_{\sigma'_0} \Omega^{(2^c)} b_0^{-(2^c)}$$
$$= -\langle \tilde{v}(1,3,2^{c};\sigma'_0,\sigma'_1,\sigma'_2)|\, b_{\sigma'_1} b_{\sigma'_2} b_{\sigma'_0} b_0^{-(2^c)} \Omega^{(2^c)} \,, \qquad (4\cdot 80)$$

implying that (d) type configurations also cancel between R and R' parameter regions. This finishes the proof for (T16) case.

§5. Summary

We have presented the full SFT action (1·1) for the unoriented open-closed mixed system and determined the BRS/gauge transformation laws for open and closed string fields. We have shown that the action (1·1) indeed satisfies the BRS invariance at the 'tree' level; namely, all the terms (T1) $\sim$ (T16) vanish provided that the coupling constants satisfy the relations (4·2) $\sim$ (4·6). Also, for the other remaining terms (L1) $\sim$ (L5) we have identified which one loop diagrams they are expected to cancel.

The tasks to show that those loop diagrams are indeed anomalous and the terms (L1) $\sim$ (L5) really cancel them, are left to a forthcoming paper. Because of this, two of the coupling constants, say x_u and $x_\infty = nx_u^2$, are still left as free parameters at this stage. We will show that these are indeed determined by the requirements of anomaly cancellations in that paper. In particular, this determines that the gauge group $SO(n)$ must be $SO(2^{13})$ in this bosonic unoriented theory case.[3),4)]

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Appendix A
—— *BRS and Gauge Transformations* ——

We here summarize the general rule for obtaining the BRS and gauge transformation laws from the action with a precise treatment of the statistics of the fields.[34)-37)] The BRS invariance of the action implies what is called a BV master equation[38)] and automatically implies the gauge invariance of the action.

A.1. *Notation and differentiation rules*

We introduce the notation Φ_I and Φ^I, denoting the open and closed field unifiedly:

$$\left.\begin{matrix}|\Phi\rangle \\ |\Psi\rangle\end{matrix}\right\} \longleftrightarrow \Phi_I\,, \qquad \left.\begin{matrix}\langle\Phi| \\ \langle\Psi|\end{matrix}\right\} \longleftrightarrow \Phi^I\,. \qquad (\text{A}\cdot 1)$$

As a convention, we take the $SL(2;C)$ ket vacuum $|0\rangle$ to be Grassmann *even*, and so the ket Fock vacuum $|1\rangle$ must be Grassmann *odd*. Then, the $SL(2;C)$ bra vacuum $\langle 0|$ must be Grassmann *odd*, and the bra Fock vacuum must be Grassmann *even*, as enforced by the relation

$$\langle 0| c_{-1}c_0c_1 |0\rangle = \langle 1| c_0 |1\rangle = 1. \tag{A·2}$$

Taking this into account we have the following Grassmann even-odd property: we cite here the statistics indices also for the quantities appearing below.

$$\begin{array}{lcl} \Phi_I,\ \dfrac{\delta}{\delta\Phi_I} & : & 1\ \text{(odd) always odd independently of open or closed}, \\ \Phi^I,\ \dfrac{\delta}{\delta\Phi^I} & : & I \equiv \begin{cases} 0\ \text{(even)} & \text{if } \Phi_I \text{ is open}, \\ 1\ \text{(odd)} & \text{if } \Phi_I \text{ is closed}, \end{cases} \\ R_{IJ},\ R^{IJ} & : & I+1 = J+1 \quad \text{since no open-closed transition}, \\ \delta^I{}_J,\ \delta_I{}^J & : & 0\ \text{(even)}. \end{array} \tag{A·3}$$

We next introduce a metric R_{IJ} and R^{IJ} for lowering and raising the indices, which are the same as the reflector

$$\left.\begin{array}{rcl} {}_1\langle\Phi| &=& \langle R^{c}(1,2)|\,|\Phi\rangle_2 \\ {}_1\langle\Psi| &=& \langle R^{o}(1,2)|\,|\Psi\rangle_2 \end{array}\right\} \longleftrightarrow \quad \Phi^I = R^{IJ}\Phi_J,$$

$$\left.\begin{array}{rcl} |\Phi\rangle_1 &=& {}_2\langle\Phi|\,|R^{c}(2,1)\rangle \\ |\Psi\rangle_1 &=& {}_2\langle\Psi|\,|R^{o}(2,1)\rangle \end{array}\right\} \longleftrightarrow \quad \Phi_I = \Phi^J R_{JI}, \tag{A·4}$$

where the (pair of upper and lower) repeated indices imply contractions (or summations). Note that R_{IJ} and R^{IJ} have only open-open and closed-closed diagonal components. We have the following property of the reflector (or metric):

$$\begin{array}{rcl} R_{IJ} = (-)^{I+1}R_{JI} & : & \begin{cases} \text{anti-symmetric for open}, \\ \text{symmetric for closed}, \end{cases} \\ R^{IJ} = R^{JI} & : & \text{symmetric}. \end{array} \tag{A·5}$$

This satisfies

$$R^{IJ}R_{JK} = \delta^I{}_K = (-)^{I+1}\delta_K{}^I. \tag{A·6}$$

Care should be taken with regard to the order of the indices of the Kronecker deltas $\delta_K{}^I$ and $\delta^I{}_K$, in particular, for the *open string* case, for which $(-)^{I+1}$ is negative. The Kronecker deltas $\delta_K{}^I$ and $\delta^I{}_K$ are defined by the following property:

$$\begin{array}{l} \delta_I{}^J\Phi_J = \Phi_I, \quad \text{but} \quad \delta^J{}_I\Phi_J = (-)^{I+1}\Phi_I, \\ \Phi^J\delta_J{}^I = \Phi^I, \quad \text{but} \quad \Phi^J\delta^I{}_J = (-)^{I+1}\Phi^I. \end{array} \tag{A·7}$$

To make it easy to translate into the bra-ket notation, we always use the convention:

$$\begin{array}{lcl} \dfrac{\delta}{\delta\Phi_I} \sim \dfrac{\delta}{\delta\,|\Phi\rangle_1} & : & \text{differentiation from } \textit{right}, \\ \dfrac{\delta}{\delta\Phi^I} \sim \dfrac{\delta}{\delta\,{}_1\langle\Phi|} & : & \text{differentiation from } \textit{left}. \end{array} \tag{A·8}$$

We have the following rule:

$$\frac{\delta\Phi_I}{\delta\Phi_J} = \delta_I{}^J, \qquad \frac{\delta\Phi^I}{\delta\Phi_J} = R^{IJ}\,,$$
$$\frac{\delta\Phi^I}{\delta\Phi^J} = \delta_J{}^I, \qquad \frac{\delta\Phi_I}{\delta\Phi^J} = R_{JI}\,. \tag{A·9}$$

It should be noted that, as seen from $\delta\Phi_I/\delta\Phi_J \propto \delta_I{}^J$ and $(\delta/\delta\Phi^J)\Phi^I \propto \delta_J{}^I$, the derivatives $\delta/\delta\Phi$ have the same Grassmann even-odd properties as Φ in the denominator: that is, $\delta/\delta\Phi_I$ is always odd and $\delta/\delta\Phi^I$ is I (even for open and odd for closed).

Note that if we use the notation

$$\frac{\delta F}{\delta\Phi_I} \equiv F^I, \qquad \frac{\delta F}{\delta\Phi^I} \equiv F_I, \tag{A·10}$$

then, for any (Grassmann even) derivation δ, we have

$$\delta F = F^I\delta\Phi_I, \qquad \delta F = \delta\Phi^J F_J. \tag{A·11}$$

For (Grassmann odd) anti-derivation $\boldsymbol{\delta}_A$, we can convert $\boldsymbol{\delta}_A$ into the usual derivation $\lambda\boldsymbol{\delta}_A$ by multiplying a Grassmann odd constant λ, and then we have

$$\lambda\boldsymbol{\delta}_A F = F^I\lambda\boldsymbol{\delta}_A\Phi_I = \lambda(-)^{(F+1)}F^I\boldsymbol{\delta}_A\Phi_I, \qquad \lambda\boldsymbol{\delta}_A F = \lambda\boldsymbol{\delta}_A\Phi^J F_J, \tag{A·12}$$

from which follows

$$\boldsymbol{\delta}_A F = (-)^{(F+1)}F^I\boldsymbol{\delta}_A\Phi_I, \qquad \boldsymbol{\delta}_A F = \boldsymbol{\delta}_A\Phi^J F_J. \tag{A·13}$$

From this Eq. (A·11), we can also derive an identity which gives a relation between F_I and F^I. From Eq. (A·4),

$$\delta\Phi^J = R^{JI}\delta\Phi_I, \qquad \delta\Phi_I = \delta\Phi^J R_{JI}. \tag{A·14}$$

Substituting this into

$$F^I\delta\Phi_I = \delta\Phi^J F_J, \tag{A·15}$$

we have, noting $\left|F^I\right| = F+1$ and $|F_J| = F+J$,

$$\begin{aligned}
F^I\delta\Phi_I &= \delta\Phi^J F_J = R^{JI}\delta\Phi_I F_J \\
&= (-)^{F+J}R^{JI}F_J\delta\Phi_I = (-)^{F+I}R^{IJ}F_J\delta\Phi_I \\
\Rightarrow&\quad F^I = (-)^{F+I}R^{IJ}F_J\,, \\
\delta\Phi^J F_J &= F^I\delta\Phi_I = F^I\delta\Phi^J R_{JI} \\
&= (-)^{J(F+1)}\delta\Phi^J F^I R_{JI} = (-)^{I(F+1)}(-)^{I+1}\delta\Phi^J F^I R_{IJ} \\
\Rightarrow&\quad F_J = (-)^{IF+1}F^I R_{IJ}\,.
\end{aligned} \tag{A·16}$$

Let us introduce generic notation:

$$F_{I_1\cdots I_n}^{J_1\cdots J_m} \equiv \frac{\delta^{n+m}F}{\delta\Phi^{I_1}\cdots\delta\Phi^{I_n}\delta\Phi_{J_1}\cdots\cdots\delta\Phi_{J_m}} \equiv \frac{\overrightarrow{\delta}^n}{\delta\Phi^{I_1}\cdots\delta\Phi^{I_n}}F\frac{\overleftarrow{\delta}^m}{\delta\Phi_{J_1}\cdots\cdots\delta\Phi_{J_m}}. \tag{A·17}$$

Then, since the left and right derivatives commute, we clearly have

$$F_I^J = \frac{\overrightarrow{\delta}}{\delta\Phi^I} F \frac{\overleftarrow{\delta}}{\delta\Phi_J} = \frac{\delta}{\delta\Phi^I} F^J = \frac{\delta}{\delta\Phi_J} F_I\,. \tag{A·18}$$

A.2. *BRS transformation*

Define the "BRS" transformation from the action S by

$$\delta_{\mathrm{B}}\Phi_I \equiv \frac{\delta S}{\delta\Phi^I} \equiv S_I, \tag{A·19}$$

which actually stands for

$$\delta_{\mathrm{B}}\Phi_I = \begin{cases} \boldsymbol{\delta}_{\mathrm{B}}\Phi_I & \text{for open } (I=0)\,, \\ \boldsymbol{\delta}_{\mathrm{B}} b_0^- \Phi_I & \text{for closed } (I=1)\,, \end{cases} \tag{A·20}$$

where $\boldsymbol{\delta}_{\mathrm{B}}$ is the true BRS transformation. This BRS transformation $\boldsymbol{\delta}_{\mathrm{B}}$ is an *anti-derivation*, so that it obeys the rule (A·13). Using that rule, we have

$$\boldsymbol{\delta}_{\mathrm{B}} S = -S^I \boldsymbol{\delta}_{\mathrm{B}}\Phi_I. \tag{A·21}$$

We note that S^I for $I=1$ (closed case) always has a b_0^- factor on the extreme right, and so we can multiply it by $c_0^- b_0^-$ from the right, since $b_0^- c_0^- b_0^- = b_0^-$. So, inserting $(c_0^-)^I (b_0^-)^I$ which is $c_0^- b_0^-$ for $I=1$ and 1 for $I=0$, we obtain

$$\begin{aligned} \boldsymbol{\delta}_{\mathrm{B}} S &= -S^I (c_0^-)^I (b_0^-)^I \boldsymbol{\delta}_{\mathrm{B}}\Phi_I = -S^I(-c_0^-)^I(\boldsymbol{\delta}_{\mathrm{B}}(b_0^-)^I\Phi_I) \\ &= -S^I(-c_0^-)^I \delta_{\mathrm{B}}\Phi_I = -S^I(-c_0^-)^I S_I. \end{aligned} \tag{A·22}$$

So, if the action is BRS invariant, we have an identity, usually called BV master equation:

$$S^I(-c_0^-)^I S_I = 0. \tag{A·23}$$

If the BV master equation is satisfied, the nilpotency of the BRS transformation automatically follows as

$$\begin{aligned} (\boldsymbol{\delta}_{\mathrm{B}})^2 (b_0^-)^I \Phi_I &= \boldsymbol{\delta}_{\mathrm{B}} \delta_{\mathrm{B}} \Phi_I = \boldsymbol{\delta}_{\mathrm{B}} S_I = (-)^{S_I+1} \frac{\delta S_I}{\delta\Phi_J} \boldsymbol{\delta}_{\mathrm{B}}\Phi_J \\ &= (-)^{I+1} \frac{\delta S_I}{\delta\Phi_J} (c_0^-)^J (b_0^-)^J \boldsymbol{\delta}_{\mathrm{B}}\Phi_J = (-)^{I+1}\frac{\delta S_I}{\delta\Phi_J}(-c_0^-)^J(\boldsymbol{\delta}_{\mathrm{B}}(b_0^-)^J\Phi_J) \\ &= (-)^{I+1}\frac{\delta S_I}{\delta\Phi_J}(-c_0^-)^J\delta_{\mathrm{B}}\Phi_J = (-)^{I+1} S_I^J (-c_0^-)^J S_J \\ &= (-)^{I+1}\frac{1}{2}\frac{\delta}{\delta\Phi^I}\left(S^J(-c_0^-)^J S_J\right) = 0. \end{aligned} \tag{A·24}$$

A.3. *Gauge invariance*

The gauge transformation is defined by

$$\delta(\Lambda)(b_0^-)^I\Phi_I \equiv (-)^{I+J}\frac{\delta(\delta_{\mathrm{B}}\Phi_I)}{\delta\Phi_J}\Lambda_J = (-)^{I+J} S_I^J \Lambda_J. \tag{A·25}$$

Then, if the BV master equation is satisfied, the gauge invariance of the action also follows automatically:

$$\begin{aligned}\delta(\Lambda)S &= S^I\delta(\Lambda)\Phi_I = S^I(c_0^-)^I(b_0^-)^I\delta(\Lambda)\Phi_I = S^I\Big((c_0^-)^I\delta(\Lambda)(b_0^-)^I\Phi_I\Big)\\ &= (-)^J S^I(-c_0^-)^I S_I^J\Lambda_J = (-)^J\frac{1}{2}\frac{\delta}{\delta\Phi^J}\Big(S^I(-c_0^-)^I S_I\Big)\Lambda_J = 0. \qquad (\text{A}\cdot 26)\end{aligned}$$

It should hold that

$$S^I(-c_0^-)^I S_I = S_I\cdot S^I(-c_0^-)^I\,, \qquad (\text{A}\cdot 27)$$

since $|S_I| = I$ and $\left|S^I(-c_0^-)^I\right| = I+1 = 0$. We can confirm this directly by the lowering and raising index identities (A·16) which now read

$$S^I = (-)^I R^{IJ}S_J, \qquad S_I = (-)^1 S^J R_{JI}. \qquad (\text{A}\cdot 28)$$

Using these, we see

$$\begin{aligned}&S^I(-c_0^-)^I S_I\\ &= (-)^{I+1}R^{IJ}S_J(-c_0^-)^I S^K R_{KI} = (-)^{I+1}R^{IJ}S_J\cdot S^K(+c_0^-)^I R_{KI}\\ &= (-)^{I+1}R^{IJ}S_J\cdot(-)^{IK}S^K(-c_0^-)^K R_{KI} \qquad \leftarrow (+c_0^-)^I R_{KI} = (-c_0^-)^K R_{KI}\\ &= R^{IJ}R_{KI}S_J\cdot S^K(-c_0^-)^K \qquad \leftarrow \left|S_J S^K(-c_0^-)^K\right| = 1+J+K = 1+2I = 1\\ &= (-)^{I+1}R^{JI}R_{IK}S_J\cdot S^K(-c_0^-)^K \qquad \leftarrow R^{IJ} = R^{JI}, \quad R_{KI} = (-)^{I+1}R_{IK}\\ &= \delta_K{}^J S_J\cdot S^K(-c_0^-)^K = S_K\cdot S^K(-c_0^-)^K\,. \qquad (\text{A}\cdot 29)\end{aligned}$$

Essentially the same procedures prove the identities used above:

$$\begin{aligned}S_J^I(-c_0^-)^I S_I &= \frac{1}{2}\frac{\delta}{\delta\Phi^J}\Big(S^I(-c_0^-)^I S_I\Big)\,,\\ S^I(-c_0^-)^I S_I^J &= \frac{1}{2}\frac{\delta}{\delta\Phi_J}\Big(S^I(-c_0^-)^I S_I\Big)\,. \qquad (\text{A}\cdot 30)\end{aligned}$$

A.4. *Anomaly*

If the integration measure is not BRS invariant, then the BV master equation gets a contribution from it and is modified into

$$S^I(-c_0^-)^I S_I = \hbar\Big(\frac{\delta}{\delta\Phi^I}(-c_0^-)^I\frac{\delta}{\delta\Phi_I}\Big)S = \hbar S^{IJ}(-c_0^-)^J R_{JI}. \qquad (\text{A}\cdot 31)$$

But this expression is too formal, and it needs a suitable regularization to properly define the RHS. We shall discuss this point in a forthcoming paper.

Appendix B
—— *GGRT* ——

The GGRT formulas have been proved by LPP[23] and the present authors (AKT).[24] However, these proofs are restricted to the simplest situation in which only open strings exist. To apply the formulas in this mixed system of open and closed strings, we need some generalizations of the original GGRT, which we shall give in this appendix.

In our SFT, there appear seven LPP vertices, which can be classified into the following three different classes, depending on the type of CFT which is referred to in the definition of the vertex:

I. tree level open-type vertices $\langle v_{\rm I}|$ (Grassmann odd)

$$\begin{aligned} &\text{3-pt:} \quad \langle v_3^{\rm o}(1,2,3)|\,, \quad \langle u(1,2^{\rm c})|\,, \\ &\text{4-pt:} \quad \langle v_4^{\rm o}(1,2,3,4;\sigma_0)|\,, \quad \langle v_\infty(1^{\rm c},2^{\rm c};\sigma_0)|\,, \quad \langle u_\Omega(1,2,3^{\rm c};\sigma_0)|\,, \end{aligned} \tag{B·1}$$

II. tree level closed-type vertex $\langle v_{\rm II}^{\rm c}|$ (Grassmann even)

$$\langle v_3^{\rm c}(1^{\rm c},2^{\rm c},3^{\rm c})|\,, \tag{B·2}$$

III. 1-loop level open-type vertex $\langle v_{\rm L}|$ (Grassmann even)

$$\langle v_\propto(1,2;\sigma_1,\sigma_2)|\,. \tag{B·3}$$

Here the tree and 1-loop level vertices refer to the CFT on sphere and torus, respectively. The closed-type vertex (which is now uniquely $\langle v_3^{\rm c}(1^{\rm c},2^{\rm c},3^{\rm c})|$) refers to a pair of CFT's corresponding to the holomorphic and anti-holomorphic degrees of freedom, separately, while the open-type one refers to a single CFT on a complex z plane. (This explains why $\langle v_\infty(1^{\rm c},2^{\rm c};\sigma_0)|$, for instance, is classified into the open-type vertex, although it is the vertex of purely closed strings.)

Because of this, the closed-type vertex is always given by a tensor product of a pair of 'open-type' vertices representing the holomorphic and anti-holomorphic parts: in the present case, the closed 3-string vertex takes the form

$$\langle v_3^{\rm c}(1^{\rm c},2^{\rm c},3^{\rm c})| = \langle \bar{v}_3(\bar{1},\bar{2},\bar{3})| \otimes \langle v_3(1,2,3)|\,. \tag{B·4}$$

The reflectors $\langle R^{\rm o}(1,2)|$ and $\langle R^{\rm c}(1^{\rm c},2^{\rm c})|$ are, of course, 2-point vertices, and similarly can be classified into the open-type and closed-type vertices, respectively. Indeed the latter closed reflector, and its ket counterpart also, are given in the following tensor product forms:

$$\begin{aligned} \langle R^{\rm c}(1^{\rm c},2^{\rm c})| &= \langle \bar{R}(\bar{1},\bar{2})| \otimes \langle R(1,2)|\,, \\ |R^{\rm c}(1^{\rm c},2^{\rm c})\rangle &= |R(1,2)\rangle \otimes |\bar{R}(\bar{1},\bar{2})\rangle\,. \end{aligned} \tag{B·5}$$

This can be confirmed by inspecting the explicit expressions (2·11) and (2·12) in I: precisely speaking, Eq. (B·5) holds if the exponent $E_{12}^{\rm c}$ in (2·12) of I is replaced by

$$E_{12}^{\rm c} = \sum_{n\geq 1}(-)^{n+1}\left(\frac{1}{n}\alpha_n^{\mu(1)}\alpha_{n\mu}^{(2)} + c_n{}^{(1)}b_n{}^{(2)} - b_n{}^{(1)}c_n{}^{(2)}\right) + \text{a.h.}, \tag{B·6}$$

where the alternating sign factor $(-)^n$ has been added. Although the reflectors with and without this sign factor are equivalent in the presence of the closed string projection operator $\mathcal{P}$, it is necessary for the equality (B·5) itself to hold in the absence of $\mathcal{P}$.

The 1-loop level vertex is defined by the CFT on the torus:

$$\langle v_L(\{\Phi_i\};\tau)| \prod_i |\mathcal{O}_{\Phi_i}\rangle_{\Phi_i} = \Big\langle \prod_i \hat{h}_{\Phi_i}[\mathcal{O}_{\Phi_i}] \Big\rangle_{\text{torus } \tau} \equiv -\mathrm{Tr}\left[(-1)^{N_{\mathrm{FP}}} q^{2L_0} \prod_i \hat{h}_{\Phi_i}[\mathcal{O}_{\Phi_i}] \right], \tag{B·7}$$

where $q = e^{i\pi\tau}$ and $(-1)^{N_{\mathrm{FP}}}$ is the FP ghost number defined by

$$N_{\mathrm{FP}} = c_0 b_0 + \sum_{n\geq 1}(c_{-n}b_n - b_{-n}c_n), \tag{B·8}$$

which counts the ghost number from the Fock vacuum.

As was shown in LPP and AKT, we have the following tree level GGRT which holds for any two tree level open-type vertices $\langle v_{\mathrm{I}}(\{B_j\}, D)|$ and $\langle v_{\mathrm{I}}(C, \{A_i\})|$:

$$\langle v_{\mathrm{I}}(\{B_j\}, D)|\,\langle v_{\mathrm{I}}(C, \{A_i\})|\,|R^{\mathrm{o}}(D, C)\rangle = \langle v_{\mathrm{I}}(\{B_j\}, \{A_i\})|\,. \tag{B·9}$$

The resultant LPP vertex $\langle v_{\mathrm{I}}(\{B_j\}, \{A_i\})|$ for the glued configuration also becomes a tree level open-type vertex.

We need another type of gluing already at the tree level, the gluing of a tree level open-type vertex $\langle v_{\mathrm{I}}(\{B_j\}, D^{\mathrm{c}})|$ containing at least one closed string D^{c} and a tree level closed-type vertex $\langle v^{\mathrm{c}}_{\mathrm{II}}(C^{\mathrm{c}}, \{A^{\mathrm{c}}_i\})|$, by contraction using $|R^{\mathrm{c}}(D^{\mathrm{c}}, C^{\mathrm{c}})\rangle$. However, applying the above GGRT twice, we can show that the same form of GGRT holds also for this case:

$$\langle v_{\mathrm{I}}(\{B_j\}, D^{\mathrm{c}})|\,\langle v^{\mathrm{c}}_{\mathrm{II}}(C^{\mathrm{c}}, \{A^{\mathrm{c}}_i\})|\,|R^{\mathrm{c}}(D^{\mathrm{c}}, C^{\mathrm{c}})\rangle = \langle v_{\mathrm{I}}(\{B_j\}, \{A^{\mathrm{c}}_i\})|\,. \tag{B·10}$$

Indeed, separating the various closed string quantities into holomorphic and anti-holomorphic parts and writing $D^{\mathrm{c}} = (\bar{D}, D)$, etc., we have

$$\begin{aligned}
&\langle v_{\mathrm{I}}(\{B_j\}, D^{\mathrm{c}})|\,\langle v^{\mathrm{c}}_{\mathrm{II}}(C^{\mathrm{c}}, \{A^{\mathrm{c}}_i\})|\,|R^{\mathrm{c}}(D^{\mathrm{c}}, C^{\mathrm{c}})\rangle \\
&\quad = \langle v_{\mathrm{I}}(\{B_j\}, \bar{D}, D)|\,\langle \bar{v}(\bar{C}, \{\bar{A}_i\})|\,\langle v(C, \{A_i\})|\,|R(D, C)\rangle\,|\bar{R}(\bar{D}, \bar{C})\rangle \\
&\quad = \langle v_{\mathrm{I}}(\{B_j\}, \bar{D}, D)|\,\langle v(C, \{A_i\})|\,|R(D, C)\rangle \cdot \langle \bar{v}(\bar{C}, \{\bar{A}_i\})|\,|\bar{R}(\bar{D}, \bar{C})\rangle \\
&\quad = \langle \tilde{v}_{\mathrm{I}}(\{B_j\}, \bar{D}, \{A_i\})|\,\langle \bar{v}(\bar{C}, \{\bar{A}_i\})|\,|\bar{R}(\bar{D}, \bar{C})\rangle \\
&\quad = \langle v_{\mathrm{I}}(\{B_j\}, \{\bar{A}_i\}, \{A_i\})| = \langle v_{\mathrm{I}}(\{B_j\}, \{A^{\mathrm{c}}_i\})|\,.
\end{aligned} \tag{B·11}$$

Next consider the gluing of two tree level open-type vertices each containing a closed string by contraction using $|R^{\mathrm{c}}\rangle$:

$$\begin{aligned}
&\langle v_{\mathrm{I}}(D^{\mathrm{c}}, \{B_j\})|\,\langle v_{\mathrm{I}}(C^{\mathrm{c}}, \{A_i\})|\,|R^{\mathrm{c}}(D^{\mathrm{c}}, C^{\mathrm{c}})\rangle \\
&\quad = \langle v_{\mathrm{I}}(\bar{D}, D, \{B_j\})|\,\langle v_{\mathrm{I}}(\bar{C}, C, \{A_i\})|\,|R(D, C)\rangle\,|\bar{R}(\bar{D}, \bar{C})\rangle\,.
\end{aligned} \tag{B·12}$$

But this has exactly the same form as the 1-loop level GGRT proved in AKT:

$$\langle v_{\mathrm{I}}(D, F, \{B_j\})|\,\langle v_{\mathrm{I}}(C, \{A_k\}, E)|\,|R^{\mathrm{o}}(D, C)\rangle\,|R^{\mathrm{o}}(E, F)\rangle = \langle v_{\mathrm{L}}(\{B_j\}, \{A_k\}; \tau)|\,. \tag{B·13}$$

Therefore, we immediately obtain

$$\langle v_{\rm I}(D^{\rm c},\{B_j\})|\,\langle v_{\rm I}(C^{\rm c},\{A_i\})|\,|R^{\rm c}(D^{\rm c},C^{\rm c})\rangle = -\,\langle v_{\rm L}(\{B_j\},\{A_i\})|\;, \qquad ({\rm B}\cdot 14)$$

with $\langle v_{\rm L}(\{B_j\},\{A_i\})|$ being the 1-loop level LPP vertex resulting from this gluing. (Note, however, that there is actually a sign ambiguity here on the right-hand side, since the exchange of the two vertices $\langle v_{\rm I}|$ on the left-hand side gives rise to a sign change, in contrast to the case of tree level GGRT formula (B·9).)

Finally, consider the gluing of a 1-loop level vertex and a tree level open-type vertex by contraction using $|R^{\rm o}\rangle$:

$$\langle v_{\rm L}(D,\{B_j\})|\,\langle v_{\rm I}(C,\{A_i\})|\,|R^{\rm o}(D,C)\rangle = \langle v_{\rm L}(\{B_j\},\{A_i\})|\;. \qquad ({\rm B}\cdot 15)$$

This can also be proved by using the tree level GGRT. To do this, we first note that the loop level vertex $\langle v_{\rm L}(\{\Phi_i\})|$ can generally be reduced to a tree level vertex in the following form:

$$\langle v_{\rm L}(\{\Phi_i\})| = \langle v_{\rm I}(F,\{\Phi_i\},E)|\,|R^{\rm o}(E,F)\rangle\;. \qquad ({\rm B}\cdot 16)$$

This is clear since if we cut the loop of the 1-loop diagram corresponding to the vertex $\langle v_{\rm L}(\{\Phi_i\})|$, then both sides of the cutting line correspond to the intermediate (open) strings E and F, and the diagram becomes a tree level vertex $\langle v_{\rm I}(F,\{\Phi_i\},E)|$ before contraction by $|R^{\rm o}(E,F)\rangle$. Then Eq. (B·15) is proved by the tree level GGRT as follows:

$$\begin{aligned}
&\langle v_{\rm L}(\{B_j\}),D|\,\langle v_{\rm I}(C,\{A_i\})|\,|R^{\rm o}(D,C)\rangle \\
&\quad= \langle v_{\rm I}(F,\{B_j\},D,E)|\,|R^{\rm o}(E,F)\rangle\,\langle v_{\rm I}(C,\{A_i\})|\,|R^{\rm o}(D,C)\rangle \\
&\quad= \langle v_{\rm I}(F,\{B_j\},D,E)|\,\langle v_{\rm I}(C,\{A_i\})|\,|R^{\rm o}(D,C)\rangle\,|R^{\rm o}(E,F)\rangle \\
&\quad= \langle v_{\rm I}(F,\{B_j\},\{A_i\},E)|\,|R^{\rm o}(E,F)\rangle = \langle v_{\rm L}(\{B_j\},\{A_i\})|\,. \qquad ({\rm B}\cdot 17)
\end{aligned}$$

In summary, we have shown that we can apply the naive GGRT formula to all the cases we have discussed in the text.

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