

An Uncertainty Relation of Space-Time

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We propose an uncertainty relation of space-time. This relation is characterized by $G\hbar T \lesssim \delta V$, where T and δV denote a characteristic time scale and a spatial volume, respectively. Using this uncertainty relation, we give qualitative estimations for the entropies of a black hole and our universe. We obtain qualitative agreements with the known results. The holographic principle of 't Hooft and Susskind is reproduced. We also discuss cosmology and give a relation to the cosmic holographic principle of Fischler and Susskind. However, as for the maximal entropy of a system with an energy E , we obtain the formula $\sqrt{EV/G\hbar^2}$, with V denoting the volume of the system, which is distinct from the Bekenstein entropy formula ER/\hbar with R denoting the length scale of the system.

§1. Introduction

Despite various noteworthy attempts, the cosmological constant problem remains as one of the major mysteries in physics.¹⁾ If we believe the locality of our world, the fundamental degrees of freedom of a system should be the degrees of freedom at a sufficiently small scale, because, understanding these, we can make a prediction at any scale larger than this small scale. The natural small scale associated with gravity is the Planck length. The fundamental degrees of freedom of gravity are believed to be somehow associated with this Planck length. Our general knowledge of quantum field theory tells us that the vacuum quantum fluctuations of these degrees of freedom induce a cosmological constant on the huge order (Planck length)⁻⁴, since a cosmological constant can exist in general relativity, which is well established as the theory of gravity for macroscopic phenomena.

On the other hand, turning to the matter sector, QCD is well established as a quantum field theory of $SU(3)$ non-abelian gauge fields (and quark fields). QCD is an asymptotically free field theory, and non-abelian gauge fields are weakly interacting at a sufficiently small scale. On such a small scale, the non-abelian gauge field behaves like a classical field, and we may safely assume that the fundamental degrees of freedom of QCD are the (classical) non-abelian gauge field. However, in quantum gravity, we have a conflict between the locality and the weakness of the interactions. Although the metric tensor field describes macroscopic phenomena quite well, the interaction becomes stronger on a smaller scale. If we impose the locality of our world, the fundamental degrees of freedom of gravity should have quantum mechanical properties as their basic properties. The quantum mechanical properties may be incorporated by an uncertainty relation of space-time.

In this paper, we propose a space-time uncertainty relation and question whether

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the fundamental scale associated with gravity is really the Planck length. Our proposal looks quite unusual at first sight, but it turns out to be consistent with known results. In §2, we explain the motivation for our proposal. Based on the space-time uncertainty relation, we find that the density of the degrees of freedom vanishes in an infinitely stretching space-time. This might be a nice property to solve the cosmological constant problem. In §3, we evaluate the entropy in a spatial region as a function of the volume and the energy of the region and compare it with the previously obtained results. In §4, we discuss thermodynamics. In §5, we evaluate the entropy and the energy density originating from the space-time itself in our universe. In §6, we summarize our results and give some discussion. In the Appendix, we give the derivation of the entropy formula used in §3. Throughout this paper, we use a unit system in which $c = 1$.

§2. A space-time uncertainty relation

The fundamental degrees of freedom of general relativity are the metric tensor field, which is associated with each point in the (3+1)-dimensional space-time. It is implicitly assumed that we can construct a coordinate system and measure the values of the metric tensor field. This process seems to have no difficulties in classical mechanics. We could arrange “clocks” in an appropriate way and measure the metric field by interchanging light or some particles among them. As one arranges more “clocks” in a spatial region, the measurement becomes finer. One might worry about the influence from the masses of the “clocks” themselves when the density of the “clocks” becomes large. But it would not contradict the principles of classical mechanics to make a “clock” with an arbitrarily small mass, though it would become an unlimitedly difficult engineering problem.

However, in quantum mechanics, one notices a serious difficulty in the above process.¹³⁾ As the mass of each “clock” becomes smaller, the coordinate system decays in a shorter time. To see this, let us consider the construction of a coordinate system for a time interval T and with a spatial fineness δx in a Minkowski space-time. Since a “clock” must be localized in a region with the scale δx , the “clock” inevitably has a momentum of the order $p \sim \hbar/\delta x$, obtained from the uncertainty relation of quantum mechanics. Thus the “clocks” move with a finite velocity of order $v \sim \hbar/m\delta x$, where m denotes the masses of the “clocks”. This implies that the coordinate system will be destroyed by the quantum effect in a finite period $\delta x/v \sim m(\delta x)^2/\hbar$. This period must be larger than the time interval T of the coordinate. Hence we obtain

$$T \lesssim m(\delta x)^2/\hbar. \quad (2.1)$$

This gives a lower bound for the “clock” mass m for given T and δx .

From (2.1), we need “clocks” with a larger mass to construct a finer coordinate system. However we also have a maximum value of a “clock” mass, because no “clock” should become a black hole. To measure the gravitational field, it should be possible to interchange light or particles among “clocks”. Thus the Schwarzschild

radius should not exceed the uncertainty δx of each “clock”:

$$Gm \lesssim \delta x. \quad (2.2)$$

The “clock” mass can be chosen arbitrary if it satisfies (2.1) and (2.2). Thus the condition for the existence of such a “clock” mass is

$$G\hbar T \lesssim (\delta x)^3. \quad (2.3)$$

Since we cannot construct an appropriate coordinate system unless the inequality (2.3) is satisfied, it would be natural to propose that the inequality (2.3) is the property of a space-time itself with T and δx denoting the time scale and the spatial uncertainty of the space-time, respectively. Regarding $(\delta x)^3$ as the volume associated with each “clock”, (2.3) would be rewritten in the form

$$G\hbar T \lesssim \delta V, \quad (2.4)$$

where δV denotes a volume uncertainty.

A flaw in the above derivation is that we neglected the dynamical effect caused by the gravitational field generated by the “clocks”. One easily finds from (2.1) that a coordinate system is destroyed in a much shorter time by the gravitational effects among the “clocks” themselves than by the quantum effects. To justify neglecting this effect, we should consider the case in which the distances among “clocks” are much larger than the fineness δx . In this case, the fineness δx represents the uncertainty of the coordinate associated to each “clock”.

An unusual point of the inequality (2.3) is that the minimal spatial length δx has a positive correlation with the time interval T . Although it seems as if the left-hand side of (2.3) can be made arbitrarily small, we cannot detect an arbitrarily small spatial length scale. To show this, let us suppose a test particle is injected into a system to detect a structure with a length scale δx of the system by a collision experiment. Since the speed of the particle is smaller than the speed of light, the collision experiment needs a time longer than δx . Thus, from (2.3), we have

$$G\hbar\delta x < G\hbar T \lesssim (\delta x)^3, \quad (2.5)$$

which dictates that the minimal length detectable is indeed the Planck length $\sqrt{G\hbar}$.

When we take the time scale T very large, the inequality (2.3) tells us quite an unusual thing for a space-time. If the time scale T diverges, the fineness δx also diverges. This means that a space-time with an infinite characteristic time scale has a vanishing density of degrees of freedom. We will use this property in the discussion of the cosmological constant problem in §5.

From (2.4), it would be natural to propose that a fundamental degree of freedom of quantum gravity is associated with a finite spatial volume determined by

$$\delta V = c_{bt} G\hbar T, \quad (2.6)$$

where c_{bt} is a numerical constant. We discuss the value of this constant in §5, but for the time being we simply leave it undetermined. We call each such fundamental space-time region a “space-time bit” in this paper.

§3. The number of bits in a region with a given energy

In this section, we discuss the maximal number of the space-time bits in a spatial region with a total volume V and a total energy E , and give a formula for the entropy associated with the region.

Suppose a spatial region is composed of bits labeled by an integer $i = 1, \dots, N$. We assume the total spatial volume V is partitioned by the bits

$$V = \sum_{i=1}^N \delta V_i. \quad (3.1)$$

We now use the uncertainty inequality for energy, $\delta E \geq \hbar/T$. We assume also that the total energy is partitioned by the bits. Then we have

$$E = \sum_{i=1}^N \delta E_i \gtrsim \sum_{i=1}^N \frac{\hbar}{T_i} \sim \sum_{i=1}^N \frac{G\hbar^2}{\delta V_i}. \quad (3.2)$$

In the last relation, we have used (2.6). Under the constraint (3.1), the quantity $\sum_{i=1}^N \frac{G\hbar^2}{\delta V_i}$ in (3.2) takes its minimum value when the volume of each bit, δV_i , takes the same value V/N . Thus we obtain an inequality

$$E \gtrsim \frac{G\hbar^2 N^2}{V}, \quad (3.3)$$

or

$$N \lesssim \sqrt{\frac{EV}{G\hbar^2}}. \quad (3.4)$$

There is no reason to believe that the maximum value of the number of the bits in a region agrees with the entropy associated with it. However, in the Appendix, we give discussion that illustrates that the right-hand side of (3.4) is in fact approximately proportional to the entropy:*)

$$S(E, V) \sim c_0 \sqrt{\frac{EV}{G\hbar^2}}, \quad (3.5)$$

where c_0 is a proportionality factor that is a constant in the leading approximation.

The right-hand side of (3.4) becomes $\sqrt{ER^3/G\hbar^2}$ for a system with a characteristic scale R . Thus Eq. (3.4) has an extra factor $\sqrt{R/EG}$, compared with the so-called Bekenstein entropy bound ER/\hbar .^{2), 3)} In the case that the Schwarzschild bound $GE < R$ is satisfied, the bound (3.4) is larger than the Bekenstein entropy bound. Thus (3.4) is distinct from the Bekenstein entropy bound but does not contradict it. In the case $GE > R$, the entropy formula (3.5) gives a value smaller than the Bekenstein bound. This plays an important role in explaining the entropy of our universe in §5.

*) The entropy in this paper is dimensionless.

Substituting (3.4) with the Schwarzschild bound $GE < R$, we obtain

$$N \lesssim \frac{R^2}{G\hbar}. \quad (3.6)$$

This is the basic inequality of the holographic principle of 't Hooft and Susskind⁴⁾ that the world can be described by the degrees of freedom on a two-dimensional surface. In the case of a black hole, the Schwarzschild bound is saturated. Regarding R^2 as the area of the black hole, we obtain qualitative agreement with the Bekenstein-Hawking entropy formula.^{5), 3)}

§4. Thermodynamics

In this section we discuss thermodynamics based on the entropy formula (3.5). Using the first law of the thermodynamics $dE = TdS - PdV$, we find

$$T = \left(\frac{\partial S}{\partial E} \right)^{-1} = \frac{2}{c_0} \sqrt{\frac{G\hbar^2 E}{V}}, \quad (4.1)$$

and

$$P = T \left(\frac{\partial S}{\partial V} \right) = \frac{E}{V}. \quad (4.2)$$

If we assume that the energy distribution is uniform, from (4.2) we obtain

$$P = \rho, \quad (4.3)$$

where ρ denotes the energy density. If we regard R as $V^{1/3}$, we instead obtain $P = E/3V = \rho/3$ from the Bekenstein entropy formula $S \sim ER$. This is the same relation between the energy density and the pressure as that of radiation, while we obtain a distinct relation (4.3). The relation (4.3) describes the most incompressible fluid that is consistent with special relativity. In this fluid, "sound" propagates at the velocity of light. This light speed propagation might be understood as that of a graviton. Equation (4.3) is distinct from the relation $P = -\rho = -\Lambda$, also, which we would have if the cosmological constant term were interpreted as an energy-momentum tensor.

The pressure (4.3) should also play important roles in black hole physics and cosmology.

§5. Cosmological implications

In this section, we discuss the entropy of our universe and then discuss the cosmological constant problem.

The total entropy of our universe would be accurately estimated by the entropy produced at the Planckian time $t_P = l_P = \sqrt{G\hbar}$ from the big bang. Substituting $T = t_P$ into (2.6), the volume δV_P of a bit at the Planckian time is

$$\delta V_P \sim G\hbar t_P = (l_P)^3. \quad (5.1)$$

Thus the total number of the bits in the universe at the Planckian time is estimated as

$$N_P \sim \frac{R_P^3}{\delta V_P} \sim \left(\frac{R_P}{l_P} \right)^3, \quad (5.2)$$

where R_P denotes the size of the observable part of our universe at the Planckian time. Thus, using (3.5) and $E \sim N_P \hbar / t_P$, we obtain the entropy of our universe as

$$S_U \sim \sqrt{\frac{N_P V}{t_P G \hbar}} \sim N_P \sim \left(\frac{R_P}{l_P} \right)^3 \sim 10^{90}. \quad (5.3)$$

Here we have used $R_P \sim 10^{30} l_P$ obtained from the Friedman-Robertson-Walker (FRW) cosmological model with the epoch of matter-radiation density equality $t_{\text{eq}} \sim 10^{11}$ sec. The value (5.3) is very near the entropy obtained from the 2.7 K cosmic microwave background.

This value (5.3) of the entropy of our universe has been recently discussed from the point of view of the cosmic holographic principle,⁶⁾⁻⁸⁾ Fischler and Susskind⁶⁾ have shown that the cosmic holographic principle gives a bound on the expansion rate and that it can be translated into a bound on the equation of state. The bound is saturated by the most incompressible perfect fluid, which we discussed in §4. We can obtain the same bound by imposing the second law of thermodynamics, namely that the entropy of our universe should not decrease with time. To show this, let us assume $a(t) \sim t^p$, where $a(t)$ is the scale factor of the FRW metric. Then, $\delta V(t) \sim t$ from (2.6) and $V(t) \sim t^{3p}$. Thus, by a similar argument as that used in deriving (5.3), we find the behavior of entropy to be described by

$$S(t) \sim N(t) \sim t^{3p-1}. \quad (5.4)$$

Thus, for the second law to be satisfied, we must have

$$p \geq \frac{1}{3}, \quad (5.5)$$

which can be translated into the bound $\gamma \leq 1$ for the equation of state $P = \gamma \rho$. Relations between the generalized second law and the cosmic holographic principle are discussed in Ref. 8).

As discussed in §2, the notion of a space-time bit seems to seriously contradict the classical notion of the (3+1)-dimensional space-time, if the range of time is infinite. This suggests that the space-time volume “operator” $\int d^4x \sqrt{-g}$ does not exist in “quantum gravity”. Since the cosmological constant is the coupling constant associated with this “operator”, the suggestion indicates that we cannot introduce the cosmological constant into “quantum gravity”. However, our universe does not have an infinite range of time, so we expect there to be a term analogous to the cosmological constant. We can estimate this as follows. We choose the age of the universe T_U as the characteristic time scale. Then, from (2.6), the volume of a space-time bit of our universe is

$$\delta V_U \sim c_{bt} G \hbar T_U. \quad (5.6)$$

Each bit will have an energy \hbar/T_U , from the uncertainty relation. Thus the energy density of our universe originating from the space-time itself is obtained as

$$\rho_{st} \sim \frac{\hbar}{T_U \delta V_U} \sim \frac{1}{c_{bt} G T_U^2}. \tag{5.7}$$

This is on the same order as the critical density $\rho_c = 3H^2/8\pi G$, where H is the Hubble constant, and so might be large enough to change significantly the evolution scenario of our universe.

We now give an argument to estimate the numerical constant c_{bt} in (2.6). Let us consider an FRW cosmological model with a vanishing spatial curvature, and, as a matter, consider a perfect fluid with the thermodynamic property given in §4. The conservation law of the stress-energy tensor, $\dot{\rho} + 3(\rho + P)\dot{a}/a = 0$, determines the density as

$$\rho = \frac{C}{a^6}, \tag{5.8}$$

where C is a numerical constant. Substituting (5.8) into the evolution equation $3\dot{a}^2/a^2 = 8\pi G\rho$, we obtain $a^3 = \sqrt{24\pi G C}t$, and hence

$$\rho = \frac{1}{24\pi G t^2}. \tag{5.9}$$

Since general relativity gives a perfect description, at least for a macroscopic object, it would be natural to demand that the result (5.9) agree with (5.7) for $T_U = t$. Moreover, since $a(t) \sim t^{1/3}$, this evolution is an adiabatic process, as can be seen from (5.4), and is consistent with the conservation of the stress-energy tensor used in the derivation of (5.8). Thus we obtain

$$c_{bt} \sim 24\pi. \tag{5.10}$$

An important comment is that the perfect fluid with the thermodynamic properties in §4 does not fully represent the properties of space-time bits. As an example in the FRW model of a flat spatial curvature, let us consider the case in which there exists dust. Since the energy density of dust behaves as $1/a^3$ and decreases more slowly than (5.8), the evolution of the universe is dominated by dust after a sufficiently long time. Then the scale factor behaves as $a \sim t^{2/3}$ in this regime, and, substituting this into (5.8), we obtain $\rho \sim 1/t^4$. This contradicts (5.7), which was derived simply from the uncertainty relation of energy and time. We propose that the thermodynamic properties change somehow in the regime and (5.7) is the correct answer. Substituting (5.10) into (5.7), the ratio of the energy density originating from the space-time itself to the critical density becomes

$$\Omega_{st} = \frac{\rho_{st}}{\rho_c} \sim \frac{1}{9(HT_U)^2}. \tag{5.11}$$

Comparing this with the observational data, since $HT_U \sim 1$, we see that a good portion of the total energy density of our universe originates from the space-time itself.

Since the time scale of our universe is very large, it would be interesting to estimate the length scale of a space-time bit. We obtain

$$(c_{bt}G\hbar T_U)^{1/3} \sim 10^{-14} \text{ m} \sim (10 \text{ MeV})^{-1}, \quad (5.12)$$

where we have taken $T_U = 10^{10}$ year. Although this energy scale is much lower than the Planck energy and is within the range of high energy experiments, it is not clear whether or how this length scale can be observed in a high energy experiment or an astrophysical observation.

§6. Summary and discussion

In this paper, we have proposed an uncertainty relation for space-time and have estimated some quantities in quantum gravity. An interesting point is that, although the proposed uncertainty relation is very different from the usually expected relations such as $\delta t \delta x \gtrsim l_P^2$ or $\delta x \gtrsim l_P$, we have obtained results qualitatively consistent with the known results, and have not found any serious deviations from them. The purpose of this paper is merely to show the surprise of the consistencies, but not to give a concrete way to calculate quantities in quantum gravity.

Of course we hope that the uncertainty relation we have proposed turns out to be an intermediate notion which catches an essential property of quantum gravity. The most peculiar point is that our space-time uncertainty relation is not consistent with the cosmological constant term. Presently the most promising approach to quantum gravity is string theory. The microscopic derivation of the entropy formula for a BPS black hole from D-brane dynamics is impressive.⁹⁾ The uncertainty relations in string theory are of the form $\delta t \delta x \gtrsim l_s^2$ and $\delta x \gtrsim l_{\min}$,¹⁰⁾ which work evidently as an ultraviolet cutoff. The reduction of the degrees of freedom in the infrared limit is realized through a duality which interchanges small scale and large scale dynamics.^{11), 10)} Thus there is the possibility for an uncertainty relation similar to ours to be derived from string theory. We hope this as well as our space-time uncertainty relation will be helpful in constructing a new theory describing quantum gravity.

Finally, in lattice approaches to quantum gravity, an uncertainty relation of the kind $\delta x, \delta t \gtrsim l_{\min}$ seems to be assumed implicitly.¹²⁾ Our uncertainty relation may provide a new direction for such approaches.

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Appendix

In this appendix, we evaluate a quantity which is similar to entropy in statistical mechanics to illustrate that the right-hand side of (3·4) can be regarded as a quantity proportional to the entropy of space-time. What we consider here is just an example. It is not a proof, because we do not have any reliable microscopic theory of quantum gravity.

Let us consider a space-time with a total number of bits N , a total volume V and a total energy E . The quantity we evaluate is the phase volume of energies and volumes with the constraint (2·4):

$$\begin{aligned} \Omega(N, V, E) &= \prod_{i=1}^N \left(\int_0^\infty \int_0^\infty \frac{dV_i dE_i}{G\hbar^2} \theta(V_i E_i - G\hbar^2) \right) \\ &\quad \times \delta \left(\sum_{i=1}^N V_i - V \right) \delta \left(\sum_{i=1}^N E_i - E \right), \end{aligned} \tag{A·1}$$

where $\theta(y)$ denotes the step function: $\theta(y) = 1$ for $y \geq 0$ and otherwise vanishing. The Laplace transform of (A·1) is evaluated as

$$\begin{aligned} \Omega(N, \alpha, \beta) &= \int_0^\infty \int_0^\infty \frac{dV dE}{G\hbar^2} e^{-\alpha V - \beta E} \Omega(N, V, E) \\ &= \prod_{i=1}^N \int_0^\infty \int_0^\infty \frac{dV_i dE_i}{G\hbar^2} e^{-\alpha V_i - \beta E_i} \theta(V_i E_i - G\hbar^2) \\ &= \left[\int_0^\infty \frac{dV}{G\hbar^2 \beta} e^{-G\hbar^2 \beta / V - \alpha V} \right]^N \\ &= \left[\frac{2}{\sqrt{G\hbar^2 \alpha \beta}} K_1(2\sqrt{G\hbar^2 \alpha \beta}) \right]^N \\ &= \left[\frac{\sqrt{\pi}}{(G\hbar^2 \alpha \beta)^{3/4}} \exp(-2\sqrt{G\hbar^2 \alpha \beta}) h(2\sqrt{G\hbar^2 \alpha \beta}) \right]^N, \end{aligned} \tag{A·2}$$

where $K_1(z)$ is a Bessel function of imaginary argument, and $h(z)$ is a function with the following asymptotic series for large z :

$$h(z) = \sum_{n=0}^{\infty} \frac{\Gamma(3/2 + n)}{n! \Gamma(3/2 - n) (2z)^n}. \tag{A·3}$$

The function $h(z)$ can be approximated as 1 if z is sufficiently large. In the calculation below, we neglect $h(z)$. The consistency of this simplification is checked later.

The quantity $\Omega(N, V, E)$ is obtained by the inverse Laplace transform of (A·2):

$$\Omega(N, V, E) = \int_{-i\infty}^{i\infty} \int_{-i\infty}^{i\infty} d\alpha d\beta \Omega(N, \alpha, \beta) \exp(\alpha V + \beta E). \tag{A·4}$$

The integration can be approximated by the saddle point method. The saddle point is determined by

$$S(N, \alpha, \beta) \equiv N \left(\frac{1}{2} \ln(\pi) - \frac{3}{4} \ln(G\hbar^2 \alpha \beta) - 2\sqrt{G\hbar^2 \alpha \beta} \right) + \alpha V + \beta E,$$

$$\frac{\partial S(N, \alpha, \beta)}{\partial \alpha} = \frac{\partial S(N, \alpha, \beta)}{\partial \beta} = 0. \quad (\text{A}\cdot 5)$$

The solution is

$$\alpha_0 = \frac{3N}{4V(1 - \sqrt{G\hbar^2 N^2 / VE})},$$

$$\beta_0 = \frac{3N}{4E(1 - \sqrt{G\hbar^2 N^2 / VE})}. \quad (\text{A}\cdot 6)$$

Substituting this back, we obtain

$$\ln(\Omega(N, V, E)) \sim S(N, \alpha_0, \beta_0) = N_0 f\left(\frac{N}{N_0}\right), \quad (\text{A}\cdot 7)$$

$$f(z) = \left(\frac{1}{2} \ln(\pi) - \frac{3}{2} \ln\left(\frac{3}{4}\right) + \frac{3}{2} \right) z - \frac{3}{2} z \ln\left(\frac{z}{1-z}\right),$$

$$N_0 = \sqrt{\frac{EV}{G\hbar^2}}. \quad (\text{A}\cdot 8)$$

For given E and V , the maximum value of (A·7) with respect to N will give its total entropy. The function $f(z)$ takes its maximum value $f(z_0) \sim 1.3$ at $z_0 \sim 0.5$. The error introduced by neglecting $h(z)$ is just $z_0 \ln(h(z_0)) \sim 0.1$, and hence is of next higher order. Thus we obtain

$$S(V, E) \sim 1.4 \sqrt{\frac{EV}{G\hbar^2}}. \quad (\text{A}\cdot 9)$$

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