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Lorentz Anomaly in Semi-Light-Cone Gauge Superstrings

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We study the Lorentz invariance of $D = 4$ and 6 superstrings in the double-spinor formalism, which are equivalent to the $D = 4$ and 6 superstrings in the pure-spinor formalism in the sense of the BRST cohomology. We first re-examine how the conformal and Lorentz anomalies appear in the $D = 4$ and 6 Green-Schwarz superstrings in the semi-light-cone gauge in the framework of BRST quantization. We construct a set of BRST invariant Lorentz generators and show that they do not form a closed algebra, even cohomologically. We then turn to the construction of Lorentz generators in the $D = 4$ and 6 double-spinor superstrings, and show that the Lorentz invariance is again anomalous. We also discuss the relation between the anomaly-free Lorentz generators in the lower-dimensional pure-spinor formalisms and that obtained in this paper.

§1. Introduction

Recently, it has been recognized that the covariant quantization of superstrings using pure spinors[1)] can be naturally understood in terms of a Green-Schwarz-like superstring with twice as many fermionic degrees of freedom, the double-spinor (DS) formalism.[2)] The superstring in the DS formalism possesses an additional local symmetry, and is classically gauge equivalent to the ordinary Green-Schwarz (GS) superstring. Imposing the semi-light-cone gauge condition on one half of the fermionic variables, Aisaka and Kazama completed a Dirac/BRST quantization of the $D = 10$ DS superstring, finding that the resulting system is cohomologically equivalent to the PS superstring.[*)] In this way, they uncovered the "origin" of the formalism, and, in particular, they derived the previously mysterious seventeen first-class constraints[4)] assumed to clarify the relation between GS and PS superstrings.

In a previous paper, Ref. 5), we applied this idea to lower-dimensional ($D = 4$ and 6) cases.[6),7)] The primary motivation of that work was to understand how the concept of the critical dimension emerges in the PS formalism. We have shown that, starting from similar Lagrangians, $D = 4$ and $D = 6$ DS superstrings can be BRST quantized to yield free CFTs similar to the semi-light-cone gauge GS superstrings, along with additional conjugate pair systems and extra constraints. The BRST charges again reduce to those of the lower-dimensional PS superstrings

[*)] See Ref. 3) for a different formulation which also relates GS and PS superstrings.

through similarity transformations.

Thus, the DS superstrings "interpolate" between the GS and PS superstrings, but this raises some questions. The GS superstring theories have a Lorentz anomaly in lower dimensional cases, while the PS superstring theories have anomaly-free Lorentz generators.[6),7)] Where does this difference come from? Then, as a related question, what do "quantum mechanically consistent $D = 4$ and 6 superstrings" describe?

The DS superstrings are closely related to the GS superstrings in the semi-light-cone gauge.[2)] The presence or absence of Lorentz and conformal anomalies for the $D = 10$ semi-light-cone gauge GS superstring was a subject of great debate in the late 1980s and early 1990s. In Ref. 8), it was revealed that, contrary to the prevailing belief at that time,[9)] the $D = 10$ GS superstring in the semi-light-cone gauge has a non-vanishing conformal anomaly. Later, it was shown that this conformal anomaly is canceled by introducing a certain local counterterm, and the Lorentz algebras become closed with a suitable modification of the Lorentz generators.[10),11)] This local counterterm can be viewed as a coupling to a certain dilaton background. More recently, the Lorentz invariance of the $D = 10$ GS superstring in the semi-light-cone gauge has been re-examined and proved using the BRST method.[4)]

In this paper, we first examine the conformal and Lorentz anomalies of the $D = 4$ and 6 GS superstrings in the semi-light-cone gauge. We BRST quantize these lower-dimensional GS superstrings in a manner similar to that for the DS superstring in Ref. 5). The key step in this procedure is the modification of the quantum constraints, and we argue that it effectively changes the background from a flat space-time to a linear-dilaton-like one. We then construct a set of BRST invariant Lorentz generators and show that they are not closed, as expected.

Next, we turn to an examination of the Lorentz invariance of the $D = 4$ and 6 DS superstrings studied in Ref. 5). We present a complete set of BRST-invariant Lorentz generators in both cases. We then show that they form the correct Lorentz algebra, except for the commutators between the "$i-$" generators, which, again, are not BRST exact. Finally, we investigate the relation between these charges and the anomaly-free Lorentz generators in the $D = 4$ PS formalism described in Refs. 6) and 7).

The organization of this paper is as follows. In §2, we study the conformal and Lorentz anomalies of the $D = 4$ and 6 GS superstrings in the semi-light-cone gauge using the BRST method. We derive the BRST-invariant Lorentz generators of the semi-light-cone gauge DS superstring and compute their algebras in §3. In the final section, we discuss the difference between the anomaly-free Lorentz generators of Refs. 6) and 7) and those obtained in this paper.

§2. The Lorentz invariance of lower-dimensional GS superstrings in the semi-light-cone gauge

2.1. *The $D = 4$ GS superstring in the semi-light-cone gauge*

The Lagrangian of the $D = 4$ Green-Schwarz (GS) superstring is an obvious generalization of the $D = 10$ GS Lagrangian,[2)] with an appropriate spinor structure in four dimensions:

$$\mathcal{L} = \mathcal{L}_K + \mathcal{L}_{WZ}, \tag{2·1a}$$

$$\mathcal{L}_K = -\frac{1}{2}\sqrt{-g}g^{ab}\Pi_a^\mu \Pi_{\mu b}, \tag{2·1b}$$

$$\mathcal{L}_{WZ} = \epsilon^{ab}\Pi_a^\mu (W_{\mu b} - \hat{W}_{\mu b}) - \epsilon^{ab} W_a^\mu \hat{W}_{\mu b} \tag{2·1c}$$

with

$$\Pi_a^\mu = \partial_a X^\mu - \sum_{A=1}^{2} W_a^{A\mu}, \tag{2·2}$$

$$W_a^{A\mu} = i\theta^A \sigma^\mu \partial_a \bar{\theta}^A - i\partial_a \theta^A \sigma^\mu \bar{\theta}^A. \tag{2·3}$$

Here, we employ the notation used in Ref. 5): $\mu, \nu = 0, 1, 2, 3$ are the flat space-time indices with the metric $\eta^{\mu\nu} = \mathrm{diag}[+1, -1, -1, -1]$, $a, b = 0, 1$ are the worldsheet indices, σ^μ are the two-by-two hermitian off-diagonal blocks of the gamma matrices in the chiral representation, θ^A are complex Weyl spinors, with $A = 1, 2$ labeling the left and right degrees of freedom after the semi-light-cone gauge fixing. We also adopt the notation $W_a^\mu = W_a^{A=1,\mu}$, $\hat{W}_a^\mu = W_a^{A=2,\mu}$, etc.

The fermionic constraints are simply

$$D_\alpha^A = k_\alpha^A - i(k^\mu + \eta_A(\Pi_1^\mu + W_1^{\mu\bar{A}}))(\sigma_\mu \bar{\theta}^A)_\alpha \approx 0, \tag{2·4a}$$

$$\bar{D}_{\dot{\alpha}}^A = \bar{k}_{\dot{\alpha}}^A - i(k^\mu + \eta_A(\Pi_1^\mu + W_1^{\mu\bar{A}}))(\theta^A \sigma_\mu)_{\dot{\alpha}} \approx 0, \tag{2·4b}$$

where $\bar{A} = 1(2)$ if $A = 2(1)$. Parameterizing the worldsheet metric as

$$g_{ab} = \begin{pmatrix} -N^2 + \gamma(N^1)^2 & \gamma N^1 \\ \gamma N^1 & \gamma \end{pmatrix}, \tag{2·5}$$

in the ADM form, we obtain the Hamiltonian

$$\mathcal{H} = \frac{N}{\sqrt{\gamma}} T_0 + N^1 T_1 + \dot{\theta}^{A\alpha} D_\alpha^A + \dot{\bar{\theta}}^{A\dot{\alpha}} \bar{D}_{\dot{\alpha}}^A, \tag{2·6}$$

where

$$T_+ = \frac{1}{2}(T_0 + T_1) = \frac{1}{4}\Pi^\mu \Pi_\mu, \tag{2·7a}$$

$$T_- = \frac{1}{2}(T_0 - T_1) = \frac{1}{4}\hat{\Pi}^\mu \hat{\Pi}_\mu, \tag{2·7b}$$

$$\Pi^\mu = k^\mu + X'^\mu - 2W_1^\mu, \tag{2·8a}$$
$$\hat{\Pi}^\mu = k^\mu - X'^\mu + 2\hat{W}_1^\mu. \tag{2·8b}$$

In fact, all the above formulas can be derived from the corresponding ones in the $D = 4$ DS formalism[5)] by setting all the variables with tildes to zero. Assuming the Poisson brackets

$$\{X^\mu(\sigma), k^\nu(\sigma')\}_P = \eta^{\mu\nu}\delta(\sigma - \sigma'), \tag{2·9a}$$
$$\{\theta^{A\alpha}(\sigma), k^B_\beta(\sigma')\}_P = -\delta^{AB}\delta^\alpha_\beta\delta(\sigma - \sigma'), \tag{2·9b}$$
$$\{\bar{\theta}^{A\dot{\alpha}}(\sigma), \bar{k}^B_{\dot{\beta}}(\sigma')\}_P = -\delta^{AB}\delta^{\dot{\alpha}}_{\dot{\beta}}\delta(\sigma - \sigma'), \tag{2·9c}$$

we find that two of the four fermionic constraints are first class, generating the kappa symmetry, and the other two are second class. Imposing the semi-light-cone gauge condition

$$\theta^2 \approx \bar{\theta}^{\dot{2}} \approx 0, \tag{2·10}$$

the kappa symmetry is fixed, and all the fermionic constraints become second class. Then, the only first-class constraints are the left and right Virasoro constraints generated by (2·7).

The Dirac bracket can be computed straightforwardly, and the result is identical to the DS superstring given in Ref. 5), with the variables with tildes replaced by variables without tildes, and T and $\Pi^+ (\equiv \Pi^0 + \Pi^3)$ replaced by variables appropriate for the GS superstring. In this case, unlike in the case of the DS superstrings, the Dirac brackets among X^μ and k^ν remain canonical; the only necessary modifications are the familiar rescalings

$$S \equiv \sqrt{2\Pi^+}\theta^1, \qquad \bar{S} \equiv \sqrt{2\Pi^+}\bar{\theta}^{\dot{1}}, \tag{2·11}$$

which satisfy the relations

$$\{S(\sigma), \bar{S}(\sigma')\}_D = i\delta(\sigma - \sigma'), \tag{2·12a}$$
$$\{X^\mu(\sigma), S(\sigma')\}_D = 0, \tag{2·12b}$$
$$\{X^\mu(\sigma), \bar{S}(\sigma')\}_D = 0. \tag{2·12c}$$

We now turn to the quantization of the $D = 4$ GS superstring. As in Ref. 5), we replace the Dirac brackets obtained above with appropriate OPEs. With some rescalings, the left constraint, $T_0 + T_1$, becomes the energy-momentum tensor $T_{\text{matter}}(z)$ composed of free fields:

$$T(z) = \frac{1}{2}\partial X^\mu \partial X_\mu - \frac{1}{2}(S\partial\bar{S} - \partial S\bar{S}). \tag{2·13}$$

The OPEs for the basic holomorphic fields are

$$X^\mu(z)X^\nu(w) \sim \eta^{\mu\nu}\log(z - w), \tag{2·14a}$$
$$S(z)\bar{S}(w) \sim \frac{1}{z - w}. \tag{2·14b}$$

Because the central charge of $T(z)$ is 5, the ghost contribution -26 cannot be cancelled in four dimensions. To compensate for the shortage, we modify the energy-momentum tensor $T(z)$ similarly to that in Ref. 5), as

$$T(z) \to \check{T}(z) = \frac{1}{2}\partial X^\mu \partial X_\mu - \frac{1}{2}(S\partial\bar{S} - \partial S\bar{S}) + \frac{7}{8}\partial^2 \log \partial X^+, \qquad (2{\cdot}15)$$

with $\eta^{+-} = 2$, $\eta^{ij} = -\delta^{ij}$. In general, a family of energy-momentum tensors

$$T_{X^+X^-}(z) = \frac{1}{2}\partial X^+ \partial X^- + \xi\ \partial^2 \log \partial X^+ \qquad (2{\cdot}16)$$

with a parameter ξ has central charge

$$c(\xi) = 1 + 24\xi \qquad (2{\cdot}17)$$

if $X^+(z)X^-(w) \sim +2\log(z-w)$. Therefore, the logarithm term correctly shifts the central charge to 26. Using this modified energy-momentum tensor, we can construct a standard nilpotent BRST charge:

$$Q_{\mathrm{GS}} = \oint \frac{dz}{2\pi i}\left(c\check{T} + bc\partial c\right). \qquad (2{\cdot}18)$$

Note that although this modification of the energy-momentum tensor may seem ad hoc, it *is* required even in the $D = 10$ GS superstring in the semi-light-cone gauge. Indeed, a one-loop analysis reveals the existence of a conformal anomaly of $c = -12$, including the bc ghosts, which can only be canceled with a special dilaton coupling introduced as a local counterterm.[10),11)] This causes a change of the energy-momentum tensor as in (2·15), though with a coefficient of 1/2 instead of 7/8. The inclusion of the counterterm also results in a modification of the spacetime Lorentz transformation rules, which have been shown to have no anomaly.[10),11),4)] Similarly, we can add a local counterterm to the $D = 4$ GS action so that the total conformal anomaly vanishes, and this gives rise to a change of the energy-momentum tensor (2·15). The question is whether, with that counterterm, the rigid Lorentz symmetry is preserved in the theory. Below we examine this point.

A Lorentz generator for the GS superstrings in the semi-light-cone gauge basically consists of a Noether current and, if it does not preserve the semi-light-cone gauge condition (2·10), an additional, compensating kappa-symmetry current. In addition, we need some extra terms for the BRST invariance of the generators. For the $D = 4$ case, we find

$$N^{ij} = \frac{1}{4}\left(-X^i\partial X^j + X^j \partial X^i + i\epsilon^{ij}S\bar{S}\right), \qquad (2{\cdot}19\mathrm{a})$$

$$N^{+-} = \frac{1}{4}\left(-X^+\partial X^- + X^- \partial X^+\right), \qquad (2{\cdot}19\mathrm{b})$$

$$N^{i+} = \frac{1}{4}\left(-X^i\partial X^+ + X^+ \partial X^i\right), \qquad (2{\cdot}19\mathrm{c})$$

$$N^{i-} = \frac{1}{4}\left(-X^i\partial X^- + X^- \partial X^i + 2i\epsilon^{ij}\frac{\partial X^j}{\partial X^+}S\bar{S} - \frac{7}{2}\frac{\partial^2 X^i}{\partial X^+}\right), \qquad (2{\cdot}19\mathrm{d})$$

where $i, j = 1, 2$ and $\epsilon^{12} = -\epsilon^{21} = 1$, $\epsilon^{11} = \epsilon^{22} = 0$. The third term in N^{i-} (2·19d) comes from the compensating kappa transformation, and the fourth term is required for the BRST invariance.*) Lorentz generators constructed from these currents all commute with Q_{GS}. Defining the charges as

$$M^{\mu\nu} = \oint \frac{dz}{2\pi i} N^{\mu\nu}(z), \tag{2·20}$$

it can be verified that they form the $D = 4$ Lorentz algebra, except for $[M^{1-},\ M^{2-}]$, which is given by

$$[M^{1-},\ M^{2-}] = \oint \frac{dz}{2\pi i}\left(i\frac{S\bar{S}}{(\partial X^+)^2}\left(\frac{1}{2}\partial X^\mu \partial X_\mu - \frac{7}{8}\frac{\partial^3 X^+}{\partial X^+} + \frac{7}{4}\frac{(\partial^2 X^+)^2}{(\partial X^+)^2}\right) - \frac{3}{4}\frac{\partial X^1 \partial^2 X^2 - \partial^2 X^1 \partial X^2}{(\partial X^+)^2}\right). \tag{2·21}$$

Unlike the $D = 10$ GS superstring analyzed in Ref. 4), the right-hand side cannot be BRST-exact. This can be proven as follows. Suppose that the terms proportional to $S\bar{S}$ in (2·21) could be written as a commutator of Q_{GS} and some BRST "parent." Then, since $\check{T}$ does not have such a term, the parent itself must contain $S\bar{S}$. It is not difficult to show that the only possible choice is $\frac{bS\bar{S}}{(\partial X^+)^2}$ multiplied by some constant. However, we have

$$\left[Q_{\mathrm{GS}},\ \oint \frac{dz}{2\pi i} i\frac{bS\bar{S}}{(\partial X^+)^2}(z)\right] = \oint \frac{dz}{2\pi i}\left(i\frac{S\bar{S}}{(\partial X^+)^2}\left(\frac{1}{2}\partial X^\mu \partial X_\mu - \frac{1}{8}\frac{\partial^3 X^+}{\partial X^+} - \frac{7}{8}\frac{(\partial^2 X^+)^2}{(\partial X^+)^2}\right) + \frac{3}{4} i \frac{S\partial^2\bar{S} + \partial^2 S\bar{S}}{(\partial X^+)^2}\right), \tag{2·22}$$

which is inconsistent. Thus, we have shown that (2·21) does not vanish, even cohomologically, and therefore the Lorentz invariance is broken. This is a natural result, because we know that the Lorentz algebra is not closed in the light-cone quantization, and this should be independent of the gauge choice.

2.2. *The $D = 6$ GS superstring in the semi-light-cone gauge*

The BRST quantization of the $D = 6$ GS superstring in the semi-light-cone gauge is completely analogous, and therefore we give only a brief summary. Again, the Dirac brackets for the $D = 6$ GS superstring are derived from the $D = 6$ DS superstring[5)] by similar replacements. The matter energy-momentum tensor is given by

$$T(z) = \frac{1}{2}\partial X^\mu \partial X^\mu - \frac{1}{2} S^I_a \partial S^a_I. \tag{2·23}$$

*) An analogous term is also needed for the $D = 10$ GS superstring. In this case, one must add $+\frac{\partial^2 X^i}{\partial X^+}$ to N^{i-} in Eq. (3.6) of Ref. 4).

The relevant OPEs are

$$X^\mu(z)X^\nu(w) \sim \eta^{\mu\nu}\log(z-w), \tag{2·24a}$$

$$S_I^a(z)S_J^b(w) \sim -\frac{\epsilon_{IJ}\epsilon^{ab}}{z-w}. \tag{2·24b}$$

Again, we modify the energy-momentum tensor to

$$\check{T}(z) = \frac{1}{2}\partial X^\mu \partial X^\mu - \frac{1}{2}S_a^I \partial S_I^a + \frac{3}{4}\partial^2 \log \partial X^+, \tag{2·25}$$

so that the BRST charge

$$Q_{\rm GS} = \oint \frac{dz}{2\pi i}\left(c\check{T} + bc\partial c\right) \tag{2·26}$$

becomes nilpotent. The BRST-invariant Lorentz generators are found to be

$$N^{ij} = \frac{1}{4}\left(-X^i\partial X^j + X^j \partial X^i + \frac{i}{2}(S^I\gamma^{ij}S_I)\right), \tag{2·27a}$$

$$N^{+-} = \frac{1}{4}\left(-X^+\partial X^- + X^-\partial X^+\right), \tag{2·27b}$$

$$N^{i+} = \frac{1}{4}\left(-X^i\partial X^+ + X^+\partial X^i\right), \tag{2·27c}$$

$$N^{i-} = \frac{1}{4}\left(-X^i\partial X^- + X^-\partial X^i + i\frac{\partial X^j}{\partial X^+}(S^I\gamma^{ij}S_I) - 3\frac{\partial^2 X^i}{\partial X^+}\right). \tag{2·27d}$$

It can be verified that they form the correct $D=6$ Lorentz algebra, except that

$$\begin{aligned}
&[M^{i-},\ M^{j-}] \\
&= \oint \frac{dw}{2\pi i}\left(\frac{i}{2}\frac{(S^I\gamma^{ij}S_I)}{(\partial X^+)^2}\left(\frac{1}{2}\partial X^\mu \partial X_\mu - \frac{1}{8}S_b^J\partial S_J^b - \frac{3}{4}\frac{\partial^3 X^+}{\partial X^+} + \frac{(\partial^2 X^+)^2}{(\partial X^+)^2}\right)\right. \\
&\qquad -\frac{1}{2}\frac{\partial X^i \partial^2 X^j - \partial^2 X^i \partial X^j}{(\partial X^+)^2} + \frac{i}{4}\frac{(S^I\gamma^{ij}\partial^2 S_I)}{(\partial X^+)^2} \\
&\qquad \left. + \frac{i}{8}\frac{(S^I\gamma^{ij}\partial S_J)}{(\partial X^+)^2}S_b^J S_I^b\right).
\end{aligned} \tag{2·28}$$

Again, the right-hand side is not BRST-exact: As in the $D=4$ case, the S-bilinear terms can only arise from a product of $c\check{T}$ and something proportional to $\frac{S^I\gamma^{ij}S_I}{(\partial X^+)^2}$, but we have

$$\begin{aligned}
&\left[Q_{\rm GS},\ \oint \frac{dz}{2\pi i} i\frac{b(S^I\gamma^{ij}S_I)}{2(\partial X^+)^2}(z)\right] \\
&= \oint \frac{dw}{2\pi i}\left(\frac{i}{2}\frac{(S^I\gamma^{ij}S_I)}{(\partial X^+)^2}\left(\frac{1}{2}\partial X^\mu \partial X_\mu - \frac{1}{2}S_b^J\partial S_J^b + \frac{7}{4}\frac{\partial^3 X^+}{\partial X^+} - \frac{3}{4}\frac{(\partial^2 X^+)^2}{(\partial X^+)^2}\right)\right. \\
&\qquad \left. + \frac{3}{4}i\frac{(S^I\gamma^{ij}\partial^2 S_I)}{(\partial X^+)^2}\right),
\end{aligned} \tag{2·29}$$

which does not coincide with (2·28).

§3. The Lorentz invariance of the lower-dimensional DS superstrings

3.1. *The $D=4$ DS superstring*

We now focus on the issue of the Lorentz invariance of the lower-dimensional DS superstrings studied in Ref. 5). We first briefly review the relevant results in the $D=4$ case. The Lagrangian of the $D=4$ DS superstring is

$$\mathcal{L} = \mathcal{L}_K + \mathcal{L}_{WZ}, \tag{3·1a}$$

$$\mathcal{L}_K = -\frac{1}{2}\sqrt{-g}g^{ab}\Pi_a^\mu \Pi_{\mu b}, \tag{3·1b}$$

$$\mathcal{L}_{WZ} = \epsilon^{ab}\Pi_a^\mu(W_{\mu b} - \hat{W}_{\mu b}) - \epsilon^{ab}W_a^\mu \hat{W}_{\mu b}, \tag{3·1c}$$

with

$$\Pi_a^\mu = \partial_a X^\mu - \sum_{A=1}^{2} i\partial_a(\theta^A\sigma^\mu\tilde{\bar{\theta}}^A - \tilde{\theta}^A\sigma^\mu\bar{\theta}^A) - \sum_{A=1}^{2} W_a^{A\mu} \tag{3·2}$$

and

$$W_a^{A\mu} = i\Theta^A\sigma^\mu\partial_a\bar{\Theta}^A - i\partial_a\Theta^A\sigma^\mu\bar{\Theta}^A, \tag{3·3a}$$

$$\Theta^A = \tilde{\theta}^A - \theta^A, \qquad \bar{\Theta}^A = \tilde{\bar{\theta}}^A - \bar{\theta}^A. \tag{3·3b}$$

Here, $\tilde{\theta}^A$ and $\tilde{\bar{\theta}}^A$ are the spinors newly added to the GS superstring, and if they are set to zero, the Lagrangian reduces to that of the GS superstring. Following Ref. 2), we impose the semi-light-cone gauge condition only on the spinors with tildes and compute the Dirac bracket. Then, we obtain a new set of canonical variables with respect to the Dirac bracket, in terms of which the remaining holomorphic first-class constraints read as follows:[5)]

$$D_1 = d_1 - i\sqrt{2\pi^+}\bar{S}, \tag{3·4a}$$

$$D_2 = d_2 - i\sqrt{\frac{2}{\pi^+}}\pi\bar{S} - \frac{2}{\pi^+}S\bar{S}\partial\bar{\theta}^{\dot{2}}, \tag{3·4b}$$

$$\bar{D}_{\dot{1}} = \bar{d}_{\dot{1}} + i\sqrt{2\pi^+}S, \tag{3·4c}$$

$$\bar{D}_{\dot{2}} = \bar{d}_{\dot{2}} + i\sqrt{\frac{2}{\pi^+}}\bar{\pi}S + \frac{2}{\pi^+}S\bar{S}\partial\theta^2, \tag{3·4d}$$

$$\begin{aligned}\mathcal{T} = &-\frac{1}{2}\frac{\pi^\mu\pi_\mu}{\pi^+} - \frac{1}{2}\frac{S\partial\bar{S}}{\pi^+} + \frac{1}{2}\frac{\partial S\bar{S}}{\pi^+} + i\sqrt{\frac{2}{\pi^+}}(S\partial\bar{\theta}^{\dot{1}} + \partial\theta^1\bar{S}) \\ &+i\sqrt{\frac{2}{(\pi^+)^3}}\left(\bar{\pi}S\partial\bar{\theta}^{\dot{2}} + \pi\partial\theta^2\bar{S}\right) + 4\frac{S\bar{S}\partial\theta^2\partial\bar{\theta}^{\dot{2}}}{(\pi^+)^2},\end{aligned} \tag{3·4e}$$

where

$$d_\alpha = p_\alpha - i\partial X^\mu(\sigma_\mu\bar{\theta})_\alpha - \frac{1}{2}\left((\theta\sigma^\mu\partial\bar{\theta}) - (\partial\theta\sigma^\mu\bar{\theta})\right)(\sigma_\mu\bar{\theta})_\alpha, \tag{3·5a}$$

$$\bar{d}_{\dot{\alpha}} = \bar{p}_{\dot{\alpha}} - i\partial X^\mu(\theta\sigma_\mu)_{\dot{\alpha}} - \frac{1}{2}\left((\theta\sigma^\mu\partial\bar{\theta}) - (\partial\theta\sigma^\mu\bar{\theta})\right)(\theta\sigma_\mu)_{\dot{\alpha}}, \tag{3·5b}$$

$$\pi^\mu = i\partial X^\mu + \theta\sigma^\mu\partial\bar{\theta} - \partial\theta\sigma^\mu\bar{\theta}. \tag{3·5c}$$

Here, the symbol ∂ represents $\frac{\partial}{\partial z}$. The quantities $\pi^{\pm} = \pi^0 \pm \pi^3$, $\pi = \pi^1 + i\pi^2$ and $\bar{\pi} = \pi^1 - i\pi^2$ are also introduced.

The relevant OPEs among the basic fields are all free:

$$X^\mu(z)X^\nu(w) \sim \eta^{\mu\nu} \log(z-w), \tag{3·6a}$$

$$p_\alpha(z)\theta^\beta(w) \sim \frac{\delta_\alpha^\beta}{z-w}, \tag{3·6b}$$

$$\bar{p}_{\dot{\alpha}}(z)\bar{\theta}^{\dot{\beta}}(w) \sim \frac{\delta_{\dot{\alpha}}^{\dot{\beta}}}{z-w}, \tag{3·6c}$$

$$S(z)\bar{S}(w) \sim \frac{1}{z-w}. \tag{3·6d}$$

Again, the algebras of the constraints (3·4) are not closed, due to the presence of multiple contractions in the OPEs, and this prevents us from constructing a nilpotent BRST charge. To remedy this, as in Ref. 2), we modify the constraints as

$$D_1 \rightarrow \check{D}_1 \equiv D_1, \tag{3·7a}$$

$$\bar{D}_{\dot{1}} \rightarrow \check{\bar{D}}_{\dot{1}} \equiv \bar{D}_{\dot{1}}, \tag{3·7b}$$

$$D_2 \rightarrow \check{D}_2 \equiv D_2 - \frac{\partial^2\bar{\theta}^{\dot{2}}}{\pi^+} + \frac{1}{2}\frac{\partial\pi^+\partial\bar{\theta}^{\dot{2}}}{(\pi^+)^2}, \tag{3·7c}$$

$$\bar{D}_{\dot{2}} \rightarrow \check{\bar{D}}_{\dot{2}} \equiv \bar{D}_{\dot{2}} - \frac{\partial^2\theta^2}{\pi^+} + \frac{1}{2}\frac{\partial\pi^+\partial\theta^2}{(\pi^+)^2}, \tag{3·7d}$$

$$\mathcal{T} \rightarrow \check{\mathcal{T}} \equiv \mathcal{T} + \frac{\partial\theta^2\partial^2\bar{\theta}^{\dot{2}}}{(\pi^+)^2} - \frac{\partial^2\theta^2\partial\bar{\theta}^{\dot{2}}}{(\pi^+)^2} - \frac{1}{8}\frac{\partial^2\log\pi^+}{\pi^+}. \tag{3·7e}$$

The additional terms above can be viewed as arising from the normal-ordering ambiguities of the constraints, and the precise values of the coefficients have been determined so that the algebras are closed. One can verify that these modified constraints have the OPE

$$\check{D}_2(z)\check{\bar{D}}_{\dot{2}}(w) \sim \frac{4\check{\mathcal{T}}(w)}{z-w}, \tag{3·8}$$

without higher singularities, and is regular otherwise. In this way, we obtain a set of first-class constraints which can be used to construct a nilpotent BRST charge in a conventional manner as

$$\tilde{Q} = \oint \frac{dz}{2\pi i}\left(\lambda^\alpha\check{D}_\alpha + \bar{\lambda}^{\dot{\alpha}}\check{\bar{D}}_{\dot{\alpha}} + c\check{\mathcal{T}} - 4\lambda^2\bar{\lambda}^{\dot{2}}b\right). \tag{3·9}$$

Here, b and c are the usual fermionic ghosts, satisfying

$$b(z)c(w) \sim \frac{1}{z-w}, \tag{3·10}$$

while λ^α and $\bar{\lambda}^{\dot{\alpha}}$ are *unconstrained* bosonic spinor ghosts, a part of which is identified as the pure spinor ghosts after the similarity transformations described in the next section.[*)]

Let us now consider the Lorentz generators. All of them but N^{i-} are obtained by adding generators constructed from p, θ, λ and ω, the conjugate of λ, with

$$\lambda^{\alpha}(z)\omega_{\beta}(w) \sim \frac{\delta^{\alpha}_{\beta}}{z-w}, \tag{3·11a}$$

$$\bar{\lambda}^{\dot{\alpha}}(z)\bar{\omega}_{\dot{\beta}}(w) \sim \frac{\delta^{\dot{\alpha}}_{\dot{\beta}}}{z-w}, \tag{3·11b}$$

to those of the GS superstring in the semi-light-cone gauge,

$$\begin{aligned} N^{ij} &= \frac{1}{4}\Big(-X^i\partial X^j + X^j\partial X^i + i\epsilon^{ij}S\bar{S} \\ &\qquad + i\epsilon^{ij}(\theta\sigma_3 p + \bar{p}\sigma_3\bar{\theta} - \lambda\sigma_3\omega + \bar{\omega}\sigma_3\bar{\lambda})\Big), \end{aligned} \tag{3·12a}$$

$$\begin{aligned} N^{+-} &= \frac{1}{4}\Big(-X^+\partial X^- + X^-\partial X^+ \\ &\qquad + 2(\theta\sigma_3 p - \bar{p}\sigma_3\bar{\theta} - \lambda\sigma_3\omega - \bar{\omega}\sigma_3\bar{\lambda}) + 4bc\Big), \end{aligned} \tag{3·12b}$$

$$\begin{aligned} N^{i+} &= \frac{1}{4}\Big(-X^i\partial X^+ + X^+\partial X^i \\ &\qquad + 2(s_i\theta^2 p_1 + \bar{s}_i\bar{\theta}^{\dot{2}}\bar{p}_{\dot{1}} - s_i\lambda^2\omega_1 - \bar{s}_i\bar{\lambda}^{\dot{2}}\bar{\omega}_{\dot{1}})\Big), \end{aligned} \tag{3·12c}$$

where

$$s_i = \begin{cases} 1 & (i=1) \\ i & (i=2) \end{cases} \quad \text{and} \quad \bar{s}_i = \begin{cases} 1 & (i=1) \\ -i & (i=2) \end{cases}. \tag{3·13}$$

The generator N^{+-} also contains a contribution from the bc-ghost. This is because that these ghost fields are not Lorentz scalars, which can be seen from the form of the BRST charge (3·9). On the other hand, N^{i-} involves extra terms coming from the compensating κ symmetry, and also other terms for the BRST invariance. The result is

$$\begin{aligned} N^{i-} = \frac{1}{4}\Bigg(& -X^i\partial X^- + X^-\partial X^i + 2(\bar{s}_i\theta^1 p_2 + s_i\bar{\theta}^{\dot{1}}\bar{p}_{\dot{2}} - \bar{s}_i\lambda^1\omega_2 - s_i\bar{\lambda}^{\dot{1}}\bar{\omega}_{\dot{2}}) \\ & + \frac{4\pi^i bc}{\pi^+} + 2i\epsilon^{ij}\frac{\pi^j S\bar{S}}{\pi^+} + 4\sqrt{2}i\frac{bc(\bar{s}_i S\partial\bar{\theta}^{\dot{2}} + s_i\partial\theta^2\bar{S})}{(\pi^+)^{\frac{3}{2}}} - \frac{3}{2}\frac{\partial\pi^i}{\pi^+} \\ & - 2\sqrt{2}i\frac{\bar{s}_i\partial S\partial\bar{\theta}^{\dot{2}} + s_i\partial\theta^2\partial\bar{S}}{(\pi^+)^{\frac{3}{2}}} - \sqrt{2}i\frac{\bar{s}_i S\partial\bar{\theta}^{\dot{2}} + s_i\partial\theta^2\bar{S}}{(\pi^+)^{\frac{5}{2}}}\partial\pi^+ \\ & + 12i\epsilon^{ij}\frac{\pi^j\partial\theta^2\partial\bar{\theta}^{\dot{2}}}{(\pi^+)^2} + 6\frac{\bar{s}_i\partial\theta^1\partial\bar{\theta}^{\dot{2}} - s_i\partial\theta^2\partial\bar{\theta}^{\dot{1}}}{\pi^+} \\ & + 4\sqrt{2}i\frac{b(\bar{s}_i S\partial\bar{\lambda}^{\dot{2}} - s_i\partial\lambda^2\bar{S})}{(\pi^+)^{\frac{1}{2}}}\Bigg). \end{aligned} \tag{3·14}$$

[*)] Note that the $D=4$ pure spinor condition implies $\lambda^{\alpha}=0$ or $\bar{\lambda}^{\dot{\alpha}}=0$, treating them as independent quantities (rather than complex conjugates), as usual in the PS formalism.

With the exception of $[M^{i-}, M^{j-}]$, these generators form the correct $D = 4$ Lorentz algebra:

$$[M^{\mu\nu},\ M^{\rho\sigma}] = -\frac{1}{2}(\eta^{\nu\rho}M^{\mu\sigma} - \eta^{\mu\rho}M^{\nu\sigma} - \eta^{\nu\sigma}M^{\mu\rho} + \eta^{\mu\sigma}M^{\nu\rho}), \tag{3·15}$$

$$M^{\mu\nu} \equiv \oint \frac{dz}{2\pi i} N^{\mu\nu}(z). \tag{3·16}$$

The commutator $[M^{i-},\ M^{j-}]$ is given by

$$\begin{aligned}
&[M^{i-},\ M^{j-}] \\
&= \oint \frac{dz}{2\pi i}\Bigg[\frac{1}{2} i\epsilon^{ij}\frac{\pi^+\pi^- - \pi\bar{\pi}}{(\pi^+)^2}S\bar{S} + \frac{3}{4}\frac{-\pi^i\partial\pi^j + \pi^j\partial\pi^i}{(\pi^+)^2} \\
&\quad +\sqrt{2}\epsilon^{ij}bc\left(\frac{\bar{\pi}S\partial\bar{\theta}^{\dot{2}} - \pi\partial\theta^2\bar{S}}{(\pi^+)^{5/2}} + \frac{S\partial\bar{\theta}^{\dot{1}} - \partial\theta^1\bar{S}}{(\pi^+)^{3/2}}\right) \\
&\quad -i\epsilon^{ij}\partial\left(\frac{bc}{\pi^+}\right)\frac{S\bar{S}}{\pi^+} + 4i\epsilon^{ij}\left(-2\frac{\partial(bc)}{(\pi^+)^3} + \frac{bc\partial\pi^+}{(\pi^+)^4}\right)\partial\theta^2\partial\bar{\theta}^{\dot{2}} \\
&\quad +\sqrt{2}\epsilon^{ij}\Bigg(\frac{3}{2}\frac{\partial\bar{\pi}S\partial\bar{\theta}^{\dot{2}} - \partial\pi\partial\theta^2\bar{S}}{(\pi^+)^{5/2}} - \frac{1}{2}\frac{\bar{\pi}\partial S\partial\bar{\theta}^{\dot{2}} - \pi\partial\theta^2\partial\bar{S}}{(\pi^+)^{5/2}} \\
&\qquad -\frac{7}{4}\frac{(\bar{\pi}S\partial\bar{\theta}^{\dot{2}} - \pi\partial\theta^2\bar{S})\partial\pi^+}{(\pi^+)^{7/2}} - \frac{1}{2}\frac{\partial S\partial\bar{\theta}^{\dot{1}} - \partial\theta^1\partial\bar{S}}{(\pi^+)^{3/2}} - \frac{1}{4}\frac{(S\partial\bar{\theta}^{\dot{1}} - \partial\theta^1\bar{S})\partial\pi^+}{(\pi^+)^{5/2}}\Bigg) \\
&\quad +3i\epsilon^{ij}\left(\frac{(\pi^+\pi^- - 2\pi\bar{\pi})\partial\theta^2\partial\bar{\theta}^{\dot{2}}}{(\pi^+)^2} - \frac{\bar{\pi}\partial\theta^1\partial\bar{\theta}^{\dot{2}} + \pi\partial\theta^2\partial\bar{\theta}^{\dot{1}}}{(\pi^+)^2} - \frac{\partial\theta^1\partial\bar{\theta}^{\dot{1}}}{\pi^+}\right) \\
&\quad -i\epsilon^{ij}\left(\frac{\partial^2\partial\theta^2\partial^2\partial\bar{\theta}^{\dot{2}}}{(\pi^+)^3} + \frac{\partial\pi^+\partial(\partial\theta^2\partial\bar{\theta}^{\dot{2}})}{(\pi^+)^4} - 4\frac{(\partial\pi^+)^2\partial\theta^2\partial\bar{\theta}^{\dot{2}}}{(\pi^+)^5}\right) \\
&\quad +\sqrt{2}\epsilon^{ij}\left(\frac{b(\bar{\pi}S\bar{\lambda}^{\dot{2}} + \pi\lambda^2\bar{S})}{(\pi^+)^{3/2}} + \frac{b(S\bar{\lambda}^{\dot{1}} + \lambda^1\bar{S})}{\sqrt{\pi^+}}\right) \\
&\quad +2i\epsilon^{ij}\left(2\partial b\frac{\partial\bar{\theta}^{\dot{2}}\lambda^2 - \partial\theta^2\bar{\lambda}^{\dot{2}}}{(\pi^+)^2} + b\frac{\partial^2\bar{\theta}^{\dot{2}}\lambda^2 - \partial^2\theta^2\bar{\lambda}^{\dot{2}}}{(\pi^+)^2} - b\frac{\partial\bar{\theta}^{\dot{2}}\lambda^2 - \partial\theta^2\bar{\lambda}^{\dot{2}}}{(\pi^+)^3}\partial\pi^+\right) \\
&\quad +2i\epsilon^{ij}\frac{-\partial(S\partial\bar{\theta}^{\dot{2}})\partial\theta^2\bar{S} + \partial(\partial\theta^2\bar{S})S\partial\bar{\theta}^{\dot{2}}}{(\pi^+)^3} \\
&\quad +4i\epsilon^{ij}\frac{bS\bar{S}(\lambda^2\partial\bar{\theta}^{\dot{2}} + \partial\theta^2\bar{\lambda}^{\dot{2}})}{(\pi^+)^2}\Bigg].
\end{aligned} \tag{3·17}$$

We can show that the right-hand side cannot be written in a BRST exact form as follows. First, suppose that all the terms in (3·17) could be written in the form

$$\left[\tilde{Q},\ \oint\frac{dz}{2\pi i}(\text{parent})\right] \tag{3·18}$$

for some (parent). Then, note that (parent) cannot contain ω_α or $\bar{\omega}_{\dot{\alpha}}$, because (3·17) contains neither ω_α and $\bar{\omega}_{\dot{\alpha}}$ nor p_α and $\bar{p}_{\dot{\alpha}}$, which necessarily follows from the contraction with $\lambda^\alpha d_\alpha + \bar{\lambda}^{\dot{\alpha}}\bar{d}_{\dot{\alpha}}$. In analogy to the previous section, let us focus on terms that do not contain any of λ, $\bar{\lambda}$, $\partial\theta$ and $\partial\bar{\theta}$:

$$[M^{i-},\ M^{j-}] = \oint \frac{dz}{2\pi i}\left(\epsilon^{ij}\frac{iS\bar{S}}{\pi^+}\left(\frac{1}{2}\frac{\pi^\mu\pi_\mu}{\pi^+} - \partial\left(\frac{bc}{\pi^+}\right)\right) + \frac{3}{4}\frac{-\pi^i\partial\pi^j + \pi^j\partial\pi^i}{(\pi^+)^2}\right) + \mathcal{O}(\partial\theta) + \mathcal{O}(\lambda). \tag{3·19}$$

This contribution could only arise from contraction with $c\check{\mathcal{T}}$, and thus (parent) must contain b. Taking into account the π^+ dependence of (3·19), the $S\bar{S}$ terms can only arise from the OPE between terms of $c\mathcal{T}$ ($\equiv c\mathcal{T}_0$), that are independent of both $\partial\theta$ and $\partial\bar{\theta}$, and $\epsilon^{ij}\frac{S\bar{S}}{\pi^+}$. However, we find

$$\left[\oint \frac{dz}{2\pi i}c\mathcal{T}_0,\ \oint\frac{dw}{2\pi i}\left(-i\epsilon^{ij}\frac{bS\bar{S}}{\pi^+}\right)\right] = \oint\frac{dz}{2\pi i}\left(\epsilon^{ij}\frac{iS\bar{S}}{\pi^+}\left(\frac{1}{2}\frac{\pi^\mu\pi_\mu}{\pi^+} - \partial\left(\frac{bc}{\pi^+}\right)\right)\right. - \frac{3}{8}\frac{\partial^2\pi^+}{(\pi^+)^2} + \frac{15}{8}\frac{(\partial\pi^+)^2}{(\pi^+)^3} \left. - \frac{3}{4}i\frac{\partial^2 S\bar{S} + S\partial^2\bar{S}}{(\pi^+)^2}\right), \tag{3·20}$$

which is inconsistent with (3·19). Therefore, the commutator (3·17) is not BRST-exact. Thus we have shown that the $D = 4$ DS superstring has only partial Lorentz invariance, like the $D = 4$ GS superstring in the light-cone or semi-light-cone gauge.

3.2. *The $D = 6$ DS superstring*

The Lagrangian of the $D = 6$ DS superstrings is similarly given by

$$\mathcal{L}_K = -\frac{1}{2}\sqrt{-g}g^{mn}\mathit{\Pi}^\mu_m\mathit{\Pi}_{\mu n}, \tag{3·21a}$$

$$\mathcal{L}_{WZ} = \epsilon^{mn}\mathit{\Pi}^\mu_m(W_{\mu n} - \hat{W}_{\mu n}) - \epsilon^{mn}W^\mu_m\hat{W}_{\mu n}, \tag{3·21b}$$

where

$$\mathit{\Pi}^\mu_m = \partial_m X^\mu - \sum_{A=1}^{2} i\partial_m(\theta^{IA}C\gamma^\mu\tilde{\theta}^A_I) - \sum_{A=1}^{2} W^{A\mu}_m, \tag{3·22}$$

$$W^{A\mu}_m = i(\mathit{\Theta}^{IA}C\gamma^\mu\partial_m\mathit{\Theta}^A_I), \tag{3·23}$$

$$\mathit{\Theta}^A_I = \tilde{\theta}^A_I - \theta^A_I. \tag{3·24}$$

Here we use the same convention as in Ref. 5), except that, for later convenience, we put a bar on the lower component in the light-cone decomposition of a $SU(2)$ Majorana-Weyl (MW) spinor:

$$\theta^\alpha_I = \begin{pmatrix}\theta^a_I \\ \bar{\theta}^{\dot{a}}_I\end{pmatrix}, \qquad (a, \dot{a} = 1, 2) \tag{3·25}$$

where a and $\dot{a}$ are the spinor indices of the transverse rotation $SO(4) \sim SU(2) \times SU(2)$. The $SU(2)$ MW condition is given by

$$(\theta_I^a)^* = \epsilon^{IJ}\theta_J^b\epsilon_{ba} \equiv \theta_a^I, \tag{3·26a}$$

$$(\bar{\theta}_I^{\dot{a}})^* = \epsilon^{IJ}\bar{\theta}_J^{\dot{b}}\epsilon_{\dot{b}\dot{a}} \equiv \bar{\theta}_{\dot{a}}^I. \tag{3·26b}$$

After some field redefinitions, we find that the constraint generators are classically given by

$$D_a^I = d_a^I + \sqrt{2\pi^+}S_a^I, \tag{3·27a}$$

$$\bar{D}_{\dot{a}}^I = \bar{d}_{\dot{a}}^I + \sqrt{\frac{2}{\pi^+}}\pi^i(S^I\bar{\gamma}^i)_{\dot{a}} + \frac{2}{\pi^+}S_b^I S_J^b \partial\bar{\theta}_{\dot{a}}^J, \tag{3·27b}$$

$$\begin{aligned}\mathcal{T} = &-\frac{1}{2}\frac{\pi^\mu\pi_\mu}{\pi^+} - \frac{1}{2}\frac{S_a^J\partial S_J^a}{\pi^+} - \sqrt{\frac{2}{\pi^+}}\partial\theta_a^J S_J^a \\ &-\sqrt{\frac{2}{\pi^+}}\frac{\pi^i(\partial\bar{\theta}^J\gamma^i S_J)}{\pi^+} + 2\frac{\partial\bar{\theta}_{\dot{a}}^I\partial\bar{\theta}_J^{\dot{a}}S_a^J S_I^a}{(\pi^+)^2},\end{aligned} \tag{3·27c}$$

where the super-covariant currents d_α^I and π^μ are defined by

$$d_\alpha^I = p_\alpha^I + i\partial X^\mu(C\gamma_\mu\theta^I)_\alpha + \frac{1}{2}(\theta^J C\gamma^\mu\partial\theta_J)(C\gamma_\mu\theta^I)_\alpha, \tag{3·28a}$$

$$\pi^\mu = i\partial X^\mu + (\theta^I C\gamma^\mu\partial\theta_I). \tag{3·28b}$$

The redefined fields are free and satisfy the relations

$$X^\mu(z)X^\nu(w) \sim \eta^{\mu\nu}\log(z-w), \tag{3·29a}$$

$$p_\alpha^I(z)\theta_J^\beta(w) \sim \frac{\delta_J^I\delta_\alpha^\beta}{z-w}, \tag{3·29b}$$

$$S_I^a(z)S_J^b(w) \sim -\frac{\epsilon_{IJ}\epsilon^{ab}}{z-w}. \tag{3·29c}$$

Including quantum corrections, we define $\check{D}_a^I$, $\check{\bar{D}}_{\dot{a}}^I$ and $\check{\mathcal{T}}$ as

$$\check{D}_a^I = D_a^I, \tag{3·30a}$$

$$\check{\bar{D}}_{\dot{a}}^I = \bar{D}_{\dot{a}}^I - 2\frac{\partial^2\bar{\theta}_{\dot{a}}^I}{\pi^+} + \frac{\partial\pi^+\partial\bar{\theta}_{\dot{a}}^I}{(\pi^+)^2} + \frac{8}{3}\frac{\partial\bar{\theta}_{\dot{b}}^I\partial\bar{\theta}_J^{\dot{b}}\partial\bar{\theta}_{\dot{a}}^J}{(\pi^+)^2}, \tag{3·30b}$$

$$\check{\mathcal{T}} = \mathcal{T} - \frac{1}{4}\frac{\partial^2\log\pi^+}{\pi^+} - 2\frac{\partial^2\bar{\theta}_{\dot{b}}^J\partial\bar{\theta}_J^{\dot{b}}}{(\pi^+)^2} + \frac{8}{3}\frac{\partial\bar{\theta}_{\dot{a}}^I\partial\bar{\theta}_J^{\dot{a}}\partial\bar{\theta}_{\dot{b}}^J\partial\bar{\theta}_I^{\dot{b}}}{(\pi^+)^3}, \tag{3·30c}$$

then they satisfy

$$\check{\bar{D}}_{\dot{a}}^I(z)\check{\bar{D}}_{\dot{b}}^J(w) \sim -\frac{4\epsilon^{IJ}\epsilon_{\dot{a}\dot{b}}\check{\mathcal{T}}(w)}{z-w}, \tag{3·31a}$$

$$[\text{all other combinations}] \sim 0. \tag{3·31b}$$

The BRST charge can be straightforwardly constructed from this constraint algebra as

$$\tilde{Q} = \oint \frac{dz}{2\pi i} \left(\lambda_I^\alpha \check{D}_\alpha^I + c\check{\mathcal{T}} - 2\bar{\lambda}_{\dot{a}}^I \bar{\lambda}_I^{\dot{a}} b \right), \tag{3·32}$$

with the unconstrained bosonic ghost pair λ_I^α and ω_α^I and the fermionic ghost pair b and c, with

$$c(z)b(w) \sim \frac{1}{z-w}, \tag{3·33a}$$

$$\lambda_I^\alpha(z)\omega_\beta^J(w) \sim \frac{\delta_\beta^\alpha \delta_I^J}{z-w}. \tag{3·33b}$$

The BRST charge given in (3·32) is exactly nilpotent.

Using the light-cone decomposition, it is convenient to use the rewritten forms

$$\pi^+ = i\partial X^+ + 2\bar{\theta}_{\dot{a}}^I \partial\bar{\theta}_I^{\dot{a}}, \tag{3·34a}$$

$$\pi^- = i\partial X^- + 2\theta_a^I \partial\theta_I^a, \tag{3·34b}$$

$$\pi^i = i\partial X^i + (\partial\bar{\theta}^I \gamma^i \theta_I) - (\bar{\theta}^I \gamma^i \partial\theta_I), \tag{3·34c}$$

$$d_I^a = p_I^a - i\partial X^+ \theta_I^a - i\partial X^i (\bar{\gamma}^i \theta^I)^a + \partial\bar{\theta}_J^{\dot{b}} \bar{\theta}_{\dot{b}}^J \theta_I^a - \bar{\theta}_I^{\dot{b}} \partial\bar{\theta}_{\dot{b}}^J \theta_J^a + \bar{\theta}_I^{\dot{b}} \bar{\theta}_{\dot{b}}^J \partial\theta_J^a, \tag{3·34d}$$

$$\bar{d}_I^{\dot{a}} = \bar{p}_I^{\dot{a}} - i\partial X^- \bar{\theta}_I^{\dot{a}} - i\partial X^i (\gamma^i \theta^I)^{\dot{a}} + \partial\theta_J^b \theta_b^J \bar{\theta}_I^{\dot{a}} - \theta_I^b \partial\theta_b^J \bar{\theta}_J^{\dot{a}} + \theta_I^b \theta_b^J \partial\bar{\theta}_J^{\dot{a}}, \tag{3·34e}$$

where we have used the notation

$$\gamma_i = i\sigma_i \quad (i=1,2,3), \quad \gamma_4 = 1_2, \tag{3·35a}$$

$$\bar{\gamma}_i = -i\sigma_i \quad (i=1,2,3), \quad \bar{\gamma}_4 = 1_2, \tag{3·35b}$$

which are 2×2 blocks of the gamma matrices defined in Ref. 5).*) Their standard index positions are $(\gamma_i)^{\dot{a}}{}_b$ and $(\bar{\gamma}_i)^a{}_{\dot{b}}$. Using these 2×2 matrices, we also define

$$(\gamma_{ij})^a{}_b \equiv -\frac{i}{2}(\bar{\gamma}_i\gamma_j - \bar{\gamma}_j\gamma_i)^a{}_b, \tag{3·36a}$$

$$(\bar{\gamma}_{ij})^{\dot{a}}{}_{\dot{b}} \equiv -\frac{i}{2}(\gamma_i\bar{\gamma}_j - \gamma_j\bar{\gamma}_i)^{\dot{a}}{}_{\dot{b}}, \tag{3·36b}$$

$$(\gamma_{ijk})^{\dot{a}}{}_b \equiv +\frac{1}{6}(\gamma_i\bar{\gamma}_j\gamma_k - \gamma_i\bar{\gamma}_k\gamma_j + \gamma_j\bar{\gamma}_k\gamma_i - \gamma_j\bar{\gamma}_i\gamma_k + \gamma_k\bar{\gamma}_i\gamma_j - \gamma_k\bar{\gamma}_j\gamma_i)^{\dot{a}}{}_b. \tag{3·36c}$$

The Lorentz generators, except for N^{i-}, can be easily obtained as

$$N^{ij} = -\frac{1}{4}X^i\partial X^j + \frac{1}{4}X^j\partial X^i + \frac{i}{4}(\theta^I\gamma^{ij}p_I) + \frac{i}{4}(\bar{\theta}^I\bar{\gamma}^{ij}\bar{p}_I) - \frac{i}{4}(\lambda^I\gamma^{ij}\omega_I) - \frac{i}{4}(\bar{\lambda}^I\bar{\gamma}^{ij}\bar{\omega}_I) + \frac{i}{8}(S^I\gamma^{ij}S_I), \tag{3·37a}$$

$$N^{+-} = -\frac{1}{4}X^+\partial X^- + \frac{1}{4}X^-\partial X^+ + \frac{1}{2}\theta_a^I p_I^a - \frac{1}{2}\bar{\theta}_{\dot{a}}^I \bar{p}_I^{\dot{a}} - \frac{1}{2}\lambda_a^I \omega_I^a + \frac{1}{2}\bar{\lambda}_{\dot{a}}^I \bar{\omega}_I^{\dot{a}} + bc, \tag{3·37b}$$

$$N^{i+} = -\frac{1}{4}X^i\partial X^+ + \frac{1}{4}X^+\partial X^i + \frac{1}{2}(\bar{\theta}^I\gamma^i p_I) - \frac{1}{2}(\bar{\lambda}^I\gamma^i\omega_I). \tag{3·37c}$$

*) These are denoted by $\tilde{\gamma}_i$ and $\tilde{\bar{\gamma}}_i$ in Ref. 5).

The remaining generator N^{i-} is given by

$$\begin{aligned}N^{i-} = & -\frac{1}{4}X^i\partial X^- + \frac{1}{4}X^-\partial X^i + \frac{1}{2}(\theta^I\bar{\gamma}^i\bar{p}_I) - \frac{1}{2}(\lambda^I\bar{\gamma}^i\bar{\omega}_I) \\ & +\frac{\pi^i bc}{\pi^+} + \frac{1}{4}\frac{i\pi^j(S^I\gamma^{ij}S_I)}{\pi^+} - \frac{1}{4}\frac{\partial\pi^i}{\pi^+} - \sqrt{2}\frac{bc(\partial\bar{\theta}^I\gamma^i S_I)}{(\pi^+)^{3/2}} \\ & -\frac{\sqrt{2}}{3}\frac{(\partial\bar{\theta}^I\gamma^i S_J)S^J_a S^a_I}{(\pi^+)^{3/2}} + \frac{1}{\sqrt{2}}\frac{\partial\pi^+(\partial\bar{\theta}^I\gamma^i S_I)}{(\pi^+)^{5/2}} - \frac{i\pi^j(\partial\bar{\theta}^I\bar{\gamma}^{ij}\partial\bar{\theta}_I)}{(\pi^+)^2} \\ & -\frac{(\partial\bar{\theta}^I\gamma^i\partial\theta_I)}{\pi^+} - \frac{8\sqrt{2}}{3}\frac{(\partial\bar{\theta}^I\gamma^i S_J)\partial\bar{\theta}^J_{\dot{a}}\partial\bar{\theta}^{\dot{a}}_I}{(\pi^+)^{5/2}} + \sqrt{2}\frac{b(\bar{\lambda}^I\gamma^i S_I)}{(\pi^+)^{1/2}}. \end{aligned} \tag{3·37d}$$

The integrated generators

$$M^{\mu\nu} = \oint\frac{dz}{2\pi i}N^{\mu\nu}(z) \tag{3·38}$$

are BRST invariant, satisfying $[\tilde{Q}, M^{\mu\nu}] = 0$, and form the Lorentz algebra, except for

$$\begin{aligned}&[M^{i-}, M^{j-}] \\ &= \oint\frac{dz}{2\pi i}\Bigg(-\frac{1}{2}\left(\delta^{ik}\delta^{jl} - \frac{1}{2}\epsilon^{ijkl}\right)\frac{\pi^k\partial\pi^l - \pi^l\partial\pi^k}{(\pi^+)^2} \\ &+\frac{i}{2}\frac{(S^I\gamma^{ij}S_I)}{\pi^+}\left(\frac{1}{2}\frac{\pi^\mu\pi_\mu}{\pi^+} + \frac{1}{8}\frac{S^J_a\partial S^a_J}{\pi^+} + \frac{1}{4}\frac{\partial^2\pi^+}{(\pi^+)^2} - \partial\left(\frac{bc}{\pi^+}\right)\right) \\ &-\frac{i}{4}\frac{(S^I\gamma^{ij}\partial^2 S_I)}{(\pi^+)^2} - \frac{i}{8}\frac{(S^I\gamma^{ij}\partial S_J)S^J_a S^a_I}{(\pi^+)^2} \\ &+\sqrt{2}b\left(\frac{\pi^i(\bar{\lambda}^I\gamma^j S_I)}{(\pi^+)^{3/2}} - \frac{\pi^j(\bar{\lambda}^I\gamma^i S_I)}{(\pi^+)^{3/2}} + \frac{\pi^k(\bar{\lambda}^I\gamma^{ijk}S_I)}{(\pi^+)^{3/2}} + \frac{i(\lambda^I\gamma^{ij}S_I)}{(\pi^+)^{1/2}}\right) \\ &-\sqrt{2}bc\left(\frac{\pi^i(\partial\bar{\theta}^I\gamma^j S_I)}{(\pi^+)^{5/2}} - \frac{\pi^j(\partial\bar{\theta}^I\gamma^i S_I)}{(\pi^+)^{5/2}} + \frac{\pi^k(\partial\bar{\theta}^I\gamma^{ijk}S_I)}{(\pi^+)^{5/2}} + \frac{i(\partial\theta^I\gamma^{ij}S_I)}{(\pi^+)^{3/2}}\right) \\ &-\frac{1}{\sqrt{2}}\Bigg(3\frac{\partial\pi^i(\partial\bar{\theta}^I\gamma^j S_I)}{(\pi^+)^{5/2}} - 3\frac{\partial\pi^j(\partial\bar{\theta}^I\gamma^i S_I)}{(\pi^+)^{5/2}} - 4\frac{\partial\pi^+\pi^i(\partial\bar{\theta}\gamma^j S_I)}{(\pi^+)^{7/2}} \\ &\qquad +4\frac{\partial\pi^+\pi^j(\partial\bar{\theta}\gamma^i S_I)}{(\pi^+)^{7/2}} - \frac{\partial\pi^+\pi^k(\partial\bar{\theta}^I\gamma^{ijk}S_I)}{(\pi^+)^{7/2}} - \frac{i\partial\pi^+(\partial\theta^I\gamma^{ij}S_I)}{(\pi^+)^{5/2}}\Bigg) \\ &-2b\frac{i\partial\pi^+(\bar{\lambda}^I\bar{\gamma}^{ij}\partial\bar{\theta}_I)}{(\pi^+)^3} + \frac{b}{3}\Bigg(8\frac{(\bar{\lambda}^I\gamma^i S_I)(\partial\bar{\theta}^J\gamma^j S_J)}{(\pi^+)^2} - 8\frac{(\bar{\lambda}^I\gamma^j S_I)(\partial\bar{\theta}^J\gamma^i S_J)}{(\pi^+)^2} \\ &-4\frac{(\bar{\lambda}^I\gamma^i S_J)(\partial\bar{\theta}^J\gamma^j S_I)}{(\pi^+)^2} + 4\frac{(\bar{\lambda}^I\gamma^j S_J)(\partial\bar{\theta}^J\gamma^i S_I)}{(\pi^+)^2} - 12\frac{i(\partial\bar{\lambda}^I\bar{\gamma}^{ij}\partial\bar{\theta}_I)}{(\pi^+)^2} + 6\frac{i\partial\pi^+(\bar{\lambda}^I\bar{\gamma}^{ij}\partial\bar{\theta}_I)}{(\pi^+)^3} \\ &-4\frac{i(\bar{\lambda}^I\bar{\gamma}^{ij}\partial\bar{\theta}_J)S^J_a S^a_I}{(\pi^+)^2}\Bigg) - \frac{\sqrt{2}}{3}\Bigg(\frac{\pi^i(\partial\bar{\theta}^I\gamma^j S_J)S^J_a S^a_I}{(\pi^+)^{5/2}} - \frac{\pi^j(\partial\bar{\theta}^I\gamma^i S_J)S^J_a S^a_I}{(\pi^+)^{5/2}}\end{aligned}$$

$$
\left. + \frac{\pi^k(\partial\bar{\theta}^I\gamma^{ijk}S_J)S_a^J S_I^a}{(\pi^+)^{5/2}} + \frac{i(\partial\theta^I\gamma^{ij}S_J)S_a^J S_I^a}{(\pi^+)^{3/2}} \right)
$$

$$
\begin{aligned}
&-\frac{i(\partial\bar{\theta}^I\bar{\gamma}^{ij}\partial\bar{\theta}_I)}{(\pi^+)^2}\left(\pi^- + \frac{S_b^J\partial S_J^b}{\pi^+} + \frac{1}{2}\frac{\partial^2\log\pi^+}{\pi^+} - 4\frac{b\partial c}{\pi^+}\right) \\
&+2\frac{i\pi^i\pi^k(\partial\bar{\theta}^I\bar{\gamma}^{kj}\partial\bar{\theta}_I)}{(\pi^+)^3} + 2\frac{i\pi^k\pi^j(\partial\bar{\theta}^I\bar{\gamma}^{ik}\partial\bar{\theta}_I)}{(\pi^+)^3} - 2\frac{\pi^i(\partial\bar{\theta}^I\gamma^j\partial\theta_I)}{(\pi^+)^2} + 2\frac{\pi^j(\partial\bar{\theta}^I\gamma^i\partial\theta_I)}{(\pi^+)^2} \\
&-2\frac{\pi^k(\partial\bar{\theta}^I\gamma^{ijk}\partial\theta_I)}{(\pi^+)^2} - \frac{i(\partial\theta^I\gamma^{ij}\partial\theta_I)}{\pi^+} - \frac{(\partial\bar{\theta}^I\gamma^i S_I)\partial(\partial\bar{\theta}^J\gamma^j S_J)}{(\pi^+)^3} + \frac{(\partial\bar{\theta}^I\gamma^j S_I)\partial(\partial\bar{\theta}^J\gamma^i S_J)}{(\pi^+)^3} \\
&-\frac{(\partial\bar{\theta}^I\gamma^i S_J)\partial(\partial\bar{\theta}^J\gamma^j S_I)}{(\pi^+)^3} + \frac{(\partial\bar{\theta}^I\gamma^j S_J)\partial(\partial\bar{\theta}^J\gamma^i S_I)}{(\pi^+)^3} \\
&+2\frac{i(\partial\bar{\theta}^I\bar{\gamma}^{ij}\partial^2\bar{\theta}_J)S_a^J S_I^a}{(\pi^+)^3} - \frac{32b}{3}\frac{i(\bar{\lambda}^I\bar{\gamma}^{ij}\partial\bar{\theta}_J)\partial\bar{\theta}_{\dot{a}}^J\partial\bar{\theta}_I^{\dot{a}}}{(\pi^+)^3} + \frac{i}{3}\frac{(S^I\gamma^{ij}S_I)S_a^J S_K^a\partial\bar{\theta}_{\dot{a}}^K\partial\bar{\theta}_J^{\dot{a}}}{(\pi^+)^3} \\
&-\frac{8\sqrt{2}}{3}\left(2\frac{\pi^i(\partial\bar{\theta}^I\gamma^j S_J)\partial\bar{\theta}_{\dot{a}}^J\partial\bar{\theta}_I^{\dot{a}}}{(\pi^+)^{7/2}} - 2\frac{\pi^j(\partial\bar{\theta}^I\gamma^i S_J)\partial\bar{\theta}_{\dot{a}}^J\partial\bar{\theta}_I^{\dot{a}}}{(\pi^+)^{7/2}} + \frac{\pi^k(\partial\bar{\theta}^I\gamma^{ijk}S_J)\partial\bar{\theta}_{\dot{a}}^J\partial\bar{\theta}_I^{\dot{a}}}{(\pi^+)^{7/2}}\right) \\
&+2\sqrt{2}\left(\frac{(\partial\bar{\theta}^I\gamma^i S_J)(\partial\bar{\theta}_I\gamma^j\partial\theta^J)}{(\pi^+)^{5/2}} - \frac{(\partial\bar{\theta}^I\gamma^j S_J)(\partial\bar{\theta}_I\gamma^i\partial\theta^J)}{(\pi^+)^{5/2}} - 2\frac{i(\partial\theta^I\gamma^{ij}S_J)\partial\bar{\theta}_{\dot{a}}^J\partial\bar{\theta}_I^{\dot{a}}}{(\pi^+)^{5/2}}\right) \\
&\left.+\frac{i(\partial\bar{\theta}^I\bar{\gamma}^{ij}\partial\bar{\theta}_I)\partial\bar{\theta}_{\dot{a}}^J\partial^2\bar{\theta}_J^{\dot{a}}}{(\pi^+)^4} - 2\frac{i(\partial\bar{\theta}^I\bar{\gamma}^{ij}\partial^2\bar{\theta}_J)\partial\bar{\theta}_{\dot{a}}^J\partial\bar{\theta}_I^{\dot{a}}}{(\pi^+)^4} + \frac{8}{3}\frac{i(\partial\bar{\theta}^I\bar{\gamma}^{ij}\partial\bar{\theta}_I)\partial\bar{\theta}_{\dot{a}}^J\partial\bar{\theta}_K^{\dot{a}}S_a^K S_J^a}{(\pi^+)^4}\right).
\end{aligned}
\tag{3·39}
$$

In particular, we have

$$
\begin{aligned}
&[M^{i-}, M^{j-}] \\
&\quad = \oint\frac{dz}{2\pi i}\Bigg(-\frac{1}{2}\left(\delta^{ik}\delta^{jl} - \frac{1}{2}\epsilon^{ijkl}\right)\frac{\pi^k\partial\pi^l - \pi^l\partial\pi^k}{(\pi^+)^2} \\
&\qquad\qquad + \frac{i}{2}\frac{(S^I\gamma^{ij}S_I)}{\pi^+}\left(\frac{1}{2}\frac{\pi^\mu\pi_\mu}{\pi^+} + \frac{1}{8}\frac{S_a^J\partial S_J^a}{\pi^+} + \frac{1}{4}\frac{\partial^2\pi^+}{(\pi^+)^2} - \partial\left(\frac{bc}{\pi^+}\right)\right) \\
&\qquad\qquad - \frac{i}{4}\frac{(S^I\gamma^{ij}\partial^2 S_I)}{(\pi^+)^2} - \frac{i}{8}\frac{(S^I\gamma^{ij}\partial S_J)S_a^J S_I^a}{(\pi^+)^2}\Bigg) + \mathcal{O}(\partial\theta) + \mathcal{O}(\lambda).
\end{aligned}
\tag{3·40}
$$

On the other hand, we find

$$
\begin{aligned}
&\left\{\oint\frac{dz}{2\pi i}c\mathcal{T}_0, \oint\frac{dz}{2\pi i}\left(-\frac{i}{2}\frac{b(S^I\gamma^{ij}S_I)}{\pi^+}\right)\right\} \\
&\quad = \oint\frac{dz}{2\pi i}\Bigg(\frac{i}{2}\frac{(S^I\gamma^{ij}S_I)}{\pi^+}\left(\frac{1}{2}\frac{\pi^\mu\pi_\mu}{\pi^+} + \frac{1}{2}\frac{S_a^J\partial S_J^a}{\pi^+} - \frac{1}{4}\frac{\partial^2\pi^+}{(\pi^+)^2} + \frac{7}{4}\frac{(\partial\pi^+)^2}{(\pi^+)^3} - \partial\left(\frac{bc}{\pi^+}\right)\right) \\
&\qquad\qquad - \frac{3}{4}\frac{i(S^I\gamma^{ij}\partial^2 S_I)}{(\pi^+)^2}\Bigg),
\end{aligned}
\tag{3·41}
$$

where, as above, $\mathcal{T}_0$ is the $\partial\theta$-independent part of $\mathcal{T}$. Then, repeating the same argument as in the previous section, we find that the commutator is not BRST exact.

§4. Conclusions and discussion

In this paper, we have shown that the $D = 4$ and 6 double-spinor (DS) superstrings do not possess the full Lorentz symmetry, as in the light-cone and semi-light-cone gauge quantizations of lower-dimensional Green-Schwarz superstrings.

We have emphasized that the modification of the energy-momentum tensor is a common procedure employed to preserve quantum conformal invariance in the semi-light-cone gauge quantization, even in the critical case. One can rewrite the logarithmic term of the energy-momentum tensor (2·15) or, more generally, (2·16) in the usual linear-dilaton form by bosonization. Owing to the relation

$$\partial X^+(z)X^-(w) \sim \frac{2}{z-w}, \tag{4·1}$$

we can identify them as a $\beta\gamma$-system. Therefore, we define

$$\partial X^+(z) = \gamma(z) \;=\; e^{\phi-\chi}(x), \tag{4·2a}$$

$$X^-(z) = 2\beta(z) \;=\; 2\partial\chi e^{-\phi-\chi}(x), \tag{4·2b}$$

where $\gamma(z)\beta(w) \sim \frac{1}{z-w}$, $\phi(z)\phi(w) \sim -\log(z-w)$ and $\chi(z)\chi(w) \sim +\log(z-w)$. Plugging these into (2·16), we obtain

$$T_{X^+X^-}(z) = -\frac{1}{2}(\partial\phi)^2 + \left(\frac{1}{2}+\xi\right)\partial^2\phi + \frac{1}{2}(\partial\chi)^2 + \left(\frac{1}{2}-\xi\right)\partial^2\chi, \tag{4·3}$$

where $\xi = \frac{7}{8}$ $(D = 4)$, $\frac{3}{4}$ $(D = 6)$ and $\frac{1}{2}$ $(D = 10)$. Therefore, the modification of the energy-momentum tensor can be regarded as a change of the background from flat to linear-dilaton, although the dilaton is only linear with respect to the special bosonized coordinates. This way of viewing the modification is consistent with that in recent works on the relation between the lower-dimensional PS and non-critical superstrings.[12)] It is also interesting that the χ field becomes a normal scalar in the critical ($D = 10$) case. However, the meaning of this observation is yet unclear.

We showed in Ref. 5) that the physical spectra of the $D = 4$ and $D = 6$ DS superstrings coincide with those of the pure-spinor (PS) formalisms in the same numbers of dimensions. Let us now compare the Lorentz generators given in Refs. 6) and 7) and ours obtained in the DS formalism. In four dimensions, the necessary similarity transformations relating the BRST charges of the two $D = 4$ theories are[5)]

$$X = -\frac{1}{4}\oint\frac{dz}{2\pi i}\frac{c\check{\bar{D}}_{\dot{2}}}{\tilde{\lambda}^2}, \tag{4·4}$$

$$Y = -\frac{1}{2}\oint\frac{dz}{2\pi i}S\bar{S}\log\pi^+, \tag{4·5}$$

$$Z = \oint\frac{dz}{2\pi i}\left(\frac{i}{\sqrt{2}}\bar{d}_{\dot{1}}\bar{S} + \frac{\partial\theta^2\partial\bar{\theta}^{\dot{2}}}{\pi^+}\right). \tag{4·6}$$

Then, the BRST charge $\tilde{Q}$ is transformed to

$$(e^Z e^Y e^X)\tilde{Q}(e^Z e^Y e^X)^{-1} = Q + \delta_b + \delta, \tag{4·7}$$

$$Q = \oint \frac{dz}{2\pi i}\lambda^\alpha d_\alpha, \tag{4·8}$$

$$\delta_b = -4\oint \frac{dz}{2\pi i}\lambda^2\bar{\lambda}^{\dot{2}}b, \tag{4·9}$$

$$\delta = \sqrt{2}i\oint \frac{dz}{2\pi i}\bar{\lambda}^{\dot{1}}S, \tag{4·10}$$

where δ_b and δ anti-commute with Q and have trivial cohomologies of the BRST quartets $(b, c; \bar{\lambda}^{\dot{2}}, \bar{\omega}_{\dot{2}})$ and $(S, \bar{S}; \bar{\lambda}^{\dot{1}}, \bar{\omega}_{\dot{1}})$ (where $\bar{\omega}_{\dot{\alpha}}$ is the field conjugate to $\bar{\lambda}^{\dot{\alpha}}$). One can alternatively decouple λ^α instead of $\bar{\lambda}^{\dot{\alpha}}$. Taking the quotients with respect to the Hilbert space of these BRST trivial fields leaves precisely the $D = 4$ PS Hilbert space with the BRST charge Q proposed in Refs. 6) and 7).

The $D = 4$ PS superstring has an anomaly-free set of level-1 Lorentz currents. If they are similarity-transformed back to the DS theory by using the above X, Y and Z, they do not coincide with the Lorentz generators we considered in the previous section. This is obvious, because the Lorentz generators in the PS formalism do not act on the BRST-quartet fields decoupled through the similarity transformations. This can also be verified by an explicit calculation. Thus, we conclude that, although the generators of the PS formalism realize a representation of the $D = 4$ Lorentz group on the PS fields, they are not directly related to the symmetries of the DS Lagrangian. A similar statement holds in the $D = 6$ case.

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