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Effective α-α t-Matrix Interaction at Medium Energies

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The α-α transition operator-effective interaction, $t_{\alpha\alpha}(E,\vec{r})$ has been evaluated using phenomenological α-α optical potentials at several bombarding energies, E_α from 77 to 200 MeV. The results are obtained by solving the Schrödinger equation with appropriate optical potentials for various partial waves and then clubbing these with their respective optical potentials and initial state plane waves. The change of the α-α optical potential with or without hard core has been shown to influence the effective α-α t-matrix interactions drastically. The behaviour of the calculated $t_{\alpha\alpha}(E,\vec{r})$ effective interaction indicates towards a solution of the large anomalies found in the $(\alpha,2\alpha)$ reaction analyses.

§1. Introduction

Quasifree proton knockout, $(p,2p)$ reaction has been proved to be a powerful tool for the investigation of proton hole states in light and medium mass nuclei.[1)–7)] Similar method of knockout has been applied for the study of α-clustering in light and medium mass nuclei through quasifree α-knockout in $(p,p\alpha)$ and $(\alpha,2\alpha)$ reactions.[8)–13)] The zero range distorted wave impulse approximation (ZR-DWIA) has been the main reaction model for the analysis of these reactions. The extracted absolute α-clustering spectroscopic factors from the $(p,p\alpha)$ reactions have been found to be in reasonable agreement with the nuclear structure calculations.[10),11),14)] In the case of $(\alpha,2\alpha)$ reactions however, there arise orders of magnitude anomalies in the extracted α-spectroscopic factors.[12),13),15),16)] For the $(\alpha,2\alpha)$ reactions even the shapes of the energy sharing spectra have large mismatch between the ZR-DWIA predictions and the corresponding data (especially for spectra which have pronounced structure, such as for $\ell \neq 0$ knockout).[15),16)] So far it has remained a puzzle that the ZR-DWIA formalism which seems to work nicely for the $(p,2p)$ and $(p,p\alpha)$ reactions fails miserably in its predictions for the $(\alpha,2\alpha)$ and other knockout reactions involving the alpha particle beams.[17)–19)] However, at 200 MeV in the case of ^{9}Be$(\alpha,2\alpha)^5$He and ^{12}C$(\alpha,2\alpha)^8$Be reactions[20),21)] there were hardly any discrepancies observed between the ZR-DWIA predictions and the data. No explanation has been forthcoming to understand this sharp energy dependence[20),21)] as well large anomalies found in the analyses of data up to 140 MeV.[10),12),13)]

While the inputs to the ZR-DWIA calculations are generally well defined it is worth verifying some of the basic simple minded approximations made within the ZR-DWIA formalism. One of the basic approximations which has been explicitly made

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use of is the factorization approximation.[10),22)] Here it has been clearly assumed that the α-α t-matrix interaction operator called the effective α-α interaction (which is responsible for the transfer of large energy and/or momentum) is sufficiently short ranged so that the optical distortions do not change significantly over this range. An empirical test of this factorization approximation for the $(\alpha, 2\alpha)$ reactions was obtained from an entity derived from the $(\alpha, 2\alpha)$ data as a function of $\theta_{\alpha\alpha}$. Indeed, as expected, this entity had been found to be following the proper trend over three orders of magnitude variations of $\frac{d\sigma}{d\Omega}\,|_{\alpha\alpha}$.[12)] A further check has also been obtained through the appearance of the 19.8 MeV α-α resonance in the $(\alpha, 2\alpha)$ data.[8),9)]

Theoretically it has been argued by Jackson and Berggren[3)]and Jacob and Maris[1)] that the factorization arises not only due to the short range nature of the t-matrix effective interaction but also due to the constancy of the optical distortion factors over the significant range of the t-matrix effective interaction. For example in the extreme case of no optical distortions or plane waves the factorization would be exact even when the t-matrix effective interaction is of sufficiently large range. However, the ratio of the plane wave to the distorted wave cross sections ($\frac{\sigma_{PW}}{\sigma_{DW}}$, Table II of Ref. 10)) is more than 3-orders of magnitude at medium energies and the plane wave impulse approximation, PWIA analyses give much less anomalous results than the ZR-DWIA analyses. This is an indication that somehow ZR-DWIA is overestimating the distortion effects. Thus actually the results include much less distortion effects in the data than what had been incorporated through the ZR-DWIA.[16)] Extending this argument further one may conclude that the observed empirical factorization[8),9),12)] results from lesser optical distortions than from the short range nature of the t-matrix effective interaction. Therefore necessitating a thorough examination of the zero-range approximation in the ZR-DWIA through a study of the nature of the t-matrix effective interaction.

One of the important findings of the study of $(p, 2p)$ reactions using zero range-DWIA and finite range-DWIA formalisms by Kudo and Miyazaki,[23)] Ikebata[5)] and Cooper and Maxwell[24),25)] has been the demonstration of large differences in both the shapes as well as magnitudes of the zero range and finite range results. However, they could do the finite range DWIA $(p, 2p)$ calculations because one of the essential ingredients, i.e the p-p t-matrix effective interaction was available from the works of Love and Franey.[26),27)] For the $(\alpha, 2\alpha)$ reaction however, the α-α t-matrix effective interaction, $t_{\alpha\alpha}(E, \vec{r})$ is an essential ingredient for the finite range-DWIA calculations. These α-α t-matrix effective interactions are not available in the literature and through this paper we report our calculations on these α-α t-matrix effective interactions. Our study of the α-α t-matrix effective interaction will indicate whether the factorization is a result of the short range nature of the t-matrix effective interaction or it is because of the weaker distortions. A short ranged α-α t-matrix effective interaction will indicate the validity of the zero range-DWIA while a longer ranged one will suggest using a finite range-DWIA analysis which will correspond to weak optical distortions.

The main purpose of the present work is to find out the extent to which the α-α t-matrix effective interaction, $t_{\alpha\alpha}(E, \vec{r})$ differs from $\delta(r)$ in the region where the $(\alpha, 2\alpha)$ reactions show large anomalies. In §2 the formalism is presented for the evaluation of $t_{\alpha\alpha}(E, \vec{r})$. In §3 the nature of the realistic α-α optical potentials is discussed. The calculations, results and discussions are included in §4 and finally the conclusions are presented in §5.

§2. Formalism for α-α transition operator $t_{\alpha\alpha}(E, \vec{r})$

In terms of the Møller wave operators, $\Omega^{\pm}$ and scattering potential, V the effective interaction operators $T^{\pm}$ are defined[28] as

$$T^{\pm} = V\Omega^{\pm}.$$

Here $\Omega^{\pm}$ are such that they transform the plane wave states, Φ into scattering states, $\Psi^{\pm}$.

For α-α scattering the potential, $V(\vec{r})$ is central, $V(r)$ (which may be energy and ℓ dependent) and the solution of the corresponding Schrödinger equation in terms of partial waves is,

$$\Psi^{\pm}_{\alpha\alpha}(\vec{r}) = \sum_{\ell=0,2,4,..} i^{\ell}(2\ell+1)\frac{u_{\ell}(kr)}{kr}e^{i\sigma_{\ell}}P_{\ell}(\hat{r}). \tag{2·1}$$

Only even partial waves, ℓ $(= 0, 2, 4, ..)$ contribute to the $\Psi^{\pm}_{\alpha\alpha}(\vec{r})$ because of the symmetric nature of the α-α scattering state. Here $\vec{k}$, r, and $\hat{r}$ are the wave vector of the relative α-α motion, α-α separation and the angle between $\vec{k}$ and $\vec{r}$ respectively. The radial wave function, $u_{\ell}(kr)$ is the solution of the radial Schrödinger equation and σ_{ℓ} is the Coulomb phase shift. The α-α t-matrix effective interaction, $t^{+}_{\alpha\alpha}(E, \vec{r})$ will now take the form as

$$t^{+}_{\alpha\alpha}(E, \vec{r}) = e^{-ikz}V(\vec{r})\Psi^{+}_{\alpha\alpha}(\vec{r}) \tag{2·2}$$

$$\equiv \sum_{L=0,1,2,3...} t_{L}(E, r)P_{L}(\hat{r}). \tag{2·3}$$

Here it is to be noted that all the multipoles, L-values (= 0, 1, 2, 3,...), even as well as odd contribute in the expansion of Eq. (2·3) of α-α t-matrix effective interaction. This arises because the partial wave expansion of plane wave, e^{-ikz} of Eq. (2·2) will contain all partial waves, even as well as odd, which combining with the even partial waves of $\Psi^{\pm}_{\alpha\alpha}(\vec{r})$ will lead to a combination of even and odd multipoles in the expansion of the t-matrix effective interaction. The assumption of even L-values in the expansion of the t-matrix effective interaction by Sharma and Jain[29] is not correct. Now the t-matrix is given by

$$t^{+}_{\alpha\alpha}(E, \vec{r}) = e^{-ikz}\sum_{\ell=0,2,4,..} V_{\ell}(r)i^{\ell}(2\ell+1)\frac{u_{\ell}(kr)}{kr}e^{i\sigma_{\ell}}P_{\ell}(\hat{r}). \tag{2·4}$$

The $t_{L}(E, r)$ of Eq. (2·3) (which are used to calculate the differential cross section for elastic scattering) can be evaluated by expanding e^{-ikz} in terms of partial waves.

Then

$$t_L(E,r) = \frac{2L+1}{2}\int_{-1}^{+1} P_L^*(\cos\theta)t^+_{\alpha\alpha}(E,\vec{r})d(\cos\theta)$$

$$= \frac{2L+1}{2}\sum_{\ell,m} V_\ell(r)i^\ell(2\ell+1)\frac{u_\ell(kr)}{kr}\jmath_m(kr)(-i)^m(2m+1)$$

$$e^{i\sigma_\ell}\int_{-1}^{+1} P_L^*(\cos\theta)P_\ell(\cos\theta)P_m(\cos\theta)d(\cos\theta). \qquad (2\cdot5)$$

Here $\jmath_m(kr)$ is the spherical Bessel function. It can be easily seen that while ℓ in Eq. (2·1) takes on even values due to the symmetry of the α-α wave function, the L in Eq. (2·3) will have both even as well as odd values due to the asymmetry of e^{-ikz}. The scattering amplitude for elastic scattering at angle $\Theta_{\text{C.M.}}$ is given by

$$f(\Theta_{\text{C.M.}}) = -\frac{\mu}{2\pi\hbar^2}T_{fi}(\vec{k}_f,\vec{k}_i),$$

where μ is the reduced mass and

$$T_{fi}(\vec{k}_f,\vec{k}_i) = \langle \Phi_f(\vec{k}_f) \mid t^+_{\alpha\alpha}(E,\vec{r}) \mid \Phi_i(\vec{k}_i)\rangle, \qquad (2\cdot6)$$

with the Φ representing the plane wave states. Now

$$f(\Theta_{\text{C.M.}}) = -\frac{\mu}{2\pi\hbar^2}\int e^{-i\vec{k}_f.\vec{r}} \sum_{L=0,1,2,3...} t_L(E,r)P_L(\hat{r})e^{i\vec{k}_i.\vec{r}}\vec{dr}.$$

The elastic angular distribution, $\sigma(\Theta_{\text{C.M.}})$ was obtained using either the conventional phase shifts from the α-α optical potentials or the α-α t-matrix effective interaction, $t_{\alpha\alpha}(E,\vec{r})$ of Eqs. (2·3) to (2·6) evaluated from the same realistic α-α optical potentials.

§3. α-α optical potentials

For the evaluation of the α-α t-matrix effective interaction, $t_{\alpha\alpha}(E,\vec{r})$ one of the main input requirements is the realistic α-α interaction between the two alphas. The second requirement is the symmetry of the system which in α-α case, requires the α-α relative wave function to be symmetric in the exchange of the two alphas. The realistic interaction between the two alphas can be obtained by fitting the α-α elastic scattering phase shift data by phenomenological optical potential (which represents our realistic interaction). It is well known that optical model potential obtained by this procedure is not unique. These optical model potentials have continuous as well as discrete ambiguities in their various parameters.

Theoretically, however, one can obtain the optical potentials microscopically using various prescriptions, models and assumptions. A special characteristic of nuclear scattering involving two complex particles, such as the two alphas, is that the Pauli

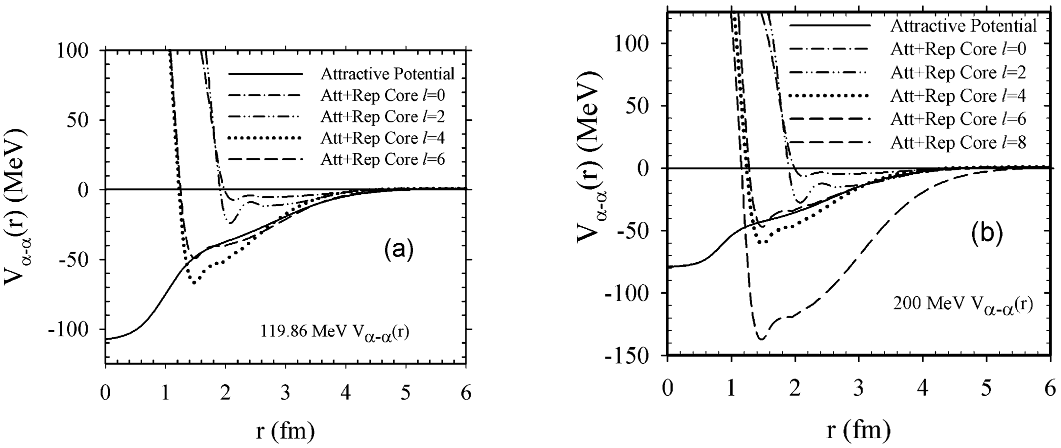

Fig. 1. Typical α-α real optical potentials $V_{\alpha-\alpha}(r)$ ℓ independent fully attractive as well as ℓ dependent with a repulsive core (for $\ell = 0, 2, 4, 6, 8$ and 10) (a) for $E_\alpha = 119.86$ MeV and (b) for $E_\alpha = 200$ MeV.

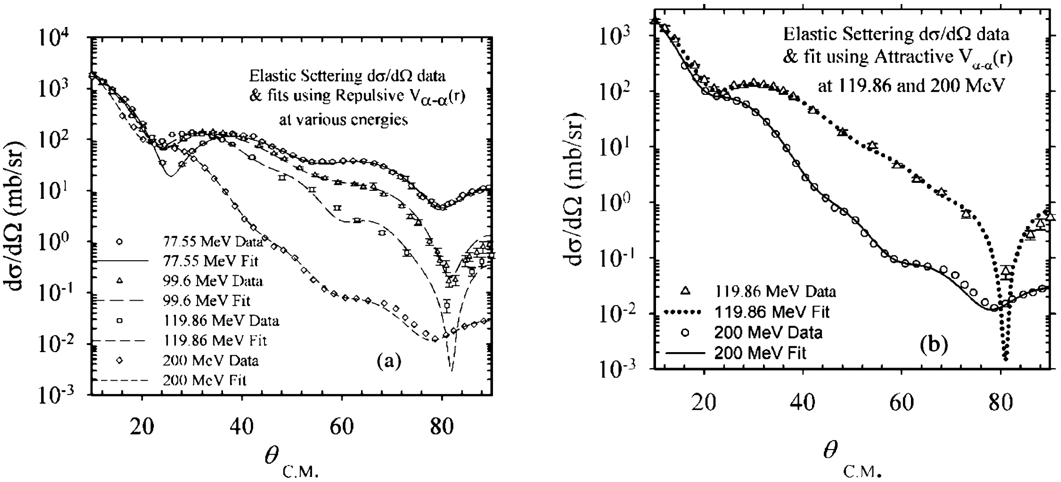

Fig. 2. Comparison of the calculated α-α elastic scattering angular distributions with the data (a) using ℓ dependent α-α optical potentials with repulsive core at $E_\alpha = 77.5$, 99.6, 119.86 and 200 MeV, and (b) using ℓ independent fully attractive α-α optical potentials at $E_\alpha = 119.86$ MeV and 200 MeV.

exclusion principle forbids the formation of compound cluster states with certain intercluster quantum numbers. For the excluded compound states the phase equivalent two-body potential should have a repulsive core arising from a nonlocal term. The source of this is the Pauli exclusion principle in the resonating group method, (RGM) of Wildermuth et al.[30] and Aoki and Horiuchi[31] or in the orthogonality condition model, (OCM) of Saito, Okai, Tanaka, Tamagaki and others.[32)–34] These equivalent local optical potentials with very short range hard cores are energy independent and angular momentum dependent.[31)–34] However these local potentials do not contain

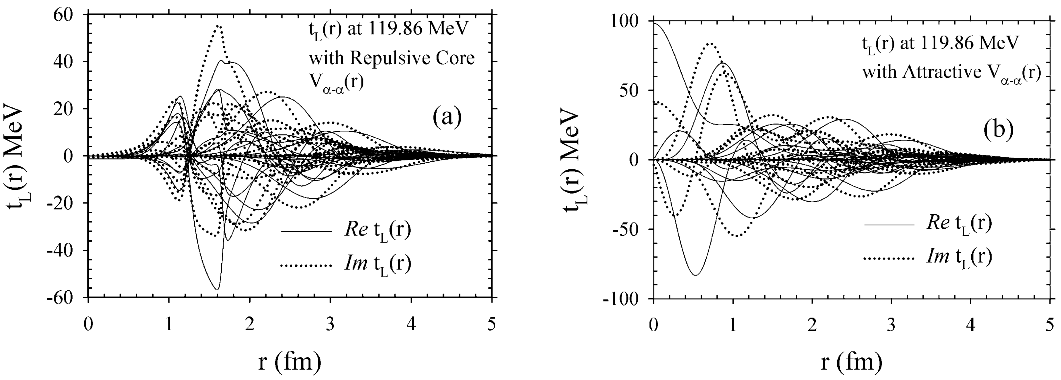

Fig. 3. Effective α-α t-matrix interaction, $t_L(r)$ vs r at 119.86 MeV for many L-values, (a) using $V_{\ell,\alpha\text{-}\alpha}(r)$ with repulsive core with a longer range attraction, and (b) using an all through attractive $V_{\alpha\text{-}\alpha}(r)$.

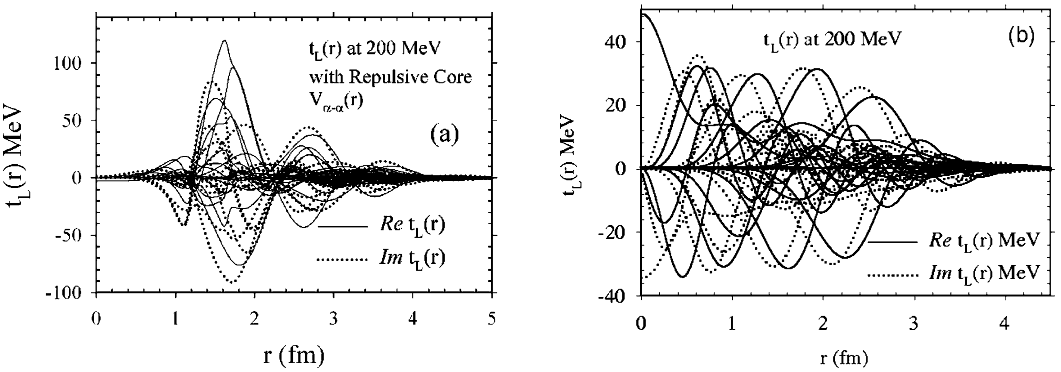

Fig. 4. Effective α-α t-matrix interaction, $t_L(r)$ vs r at 200 MeV for many L-values, (a) using $V_{\ell,\alpha\text{-}\alpha}(r)$ with repulsive core with a longer range attraction, and (b) using an all through attractive $V_{\alpha\text{-}\alpha}(r)$.

such a strong and "long range" repulsive force as that of the phenomenological α-α potentials of Darriulat et al.[35] On the other hand, it has been advocated by Neudatchin et al.[36] that one should use energy dependent deep attractive potentials containing forbidden bound states. It had been shown that a choice of the real part as a sum of two attractive Woods-Saxon potentials,[37),38)] provide excellent fits to the α-α scattering data over a wide range of energies. Thus these two types of α-α potentials widely differ in their nature in terms of their shape (see Figs. 1(a) and (b)), their energy and orbital angular momentum dependence but they all reproduce the elastic scattering angular distributions very well (see Figs. 2(a) and (b)).

There have been a lot of discussions[31),33),34),36),39)–41)] on the merits and demerits of both types of these optical potentials citing their dynamical versus static Pauli correlation nature. Without digressing much from the main point of our discussion we emphasize that so far for other applications there has been no way to choose one type of the optical potential to be more suitable in comparison to the other from the analysis of the elastic scattering data alone, as witnessed in Figs. 1 and 2. However it is expected, from the definition of the t-matrix effective interaction, Eq. (2·2) (being the product of a plane wave, $V_{\alpha\alpha}(r)$ and $\Psi^{\pm}_{\alpha\alpha}(\vec{r})$), that these $t_{\alpha\alpha}$-may be drastically different from their respective realistic interactions. The use of these different t-matrix effective interactions in a finite range-plane wave impulse approximation, FR-PWIA analysis of knockout reactions will not differentiate between them due to the applicability of the exact factorization. However in the distorted wave case the FR-DWIA analyses are expected[5),23)–25)] to differ significantly from the ZR-DWIA analyses. In fact, different FR-DWIA results were obtained[23),24)] for the $(p, 2p)$ reactions by the use of different t-matrices.[26),27)] We have therefore calculated the t-matrix effective interactions using the two types of realistic α-α interactions, (1) Attractive but with a repulsive core and (2) all through Attractive. The available real (ℓ -dependent) α-α potentials[35),37),38)] (1) having repulsive core with a longer range attraction and the other (2) bearing an all through attractive nature are seen for $E_\alpha = 119.86$ MeV and 200 MeV in Figs. 1(a) and 1(b) respectively. Both of these real optical potentials along with their mild imaginary counterparts[35),37)] provide angular distributions at 119.86 and 200 MeV which reproduces the α-α scattering data rather well in Fig. 2.[35),37)] It is to be remarked that the angular distributions are quite similar from 77.55 MeV to 200 MeV, the attractive ℓ-independent optical potentials for 120 MeV and 200 MeV are quite different from the ℓ- dependent optical potentials with repulsive core. As the fully attractive $V_{\alpha\alpha}(r)$ are not available in the literature for 77.55 MeV and 99.6 MeV here we compare the $t_{\alpha\alpha}(r)$ results for two the types of potentials for large number of L-values for $E_\alpha = 119.86$ MeV and 200 MeV in Figs. 3 and 4. It is seen that qualitatively the repulsive core results for $t_{\alpha\alpha}(r)$ are pushed out to larger r's in comparison to the $t_{\alpha\alpha}(r)$ results corresponding to the all through attractive $V_{\alpha\alpha}(r)$. The peak of the repulsive core results is seen to be shifted to $r \sim 1.7$ fm while the results for the all through attractive $V_{\alpha\alpha}(r)$ peak at $r = 0$.

§4. Calculation of $t_{\alpha\alpha,L}(E, \vec{r})$, results and discussion

Evaluation of the α-α t-matrix effective interaction, $t_{\alpha\alpha}(E, \vec{r})$ is done at four α-laboratory energies E_α, 77.55, 99.60, 119.86 and 200 MeV where the $(\alpha, 2\alpha)$ reaction results were existing on various target nuclei.[12),15),20),21)] For the evaluation of $t_{\alpha\alpha,L}(E, r)$ the Schrödinger equation is solved with initial state boundary condition for given α-α centre of mass energy and optical potential. The various radial wave functions $u_\ell(kr)$ for different even partial waves were obtained. These $u_\ell(kr)$ are multiplied with respective optical potentials, $V_\ell(r)$ the spherical Bessel functions, $\jmath_m(kr)$ and various constants for every L of $t_{\alpha\alpha,L}(E, r)$ in Eq. (2·5), the product of the three $P_\ell(\cos\theta)$ integrated over θ and then summing over ℓ and m gives the $t_{\alpha\alpha,L}(E, r)$. From Eq. (2·5) it is clear that for any L-value of $t_L(E, r)$ contributions from various ℓ-values, the realistic interaction $V_\ell(r)$ and $\frac{u_\ell(kr)}{kr}$ will contribute. Moreover while ℓ-values of the partial waves in the wave function $\Psi^+(\vec{r})$ are restricted to be only

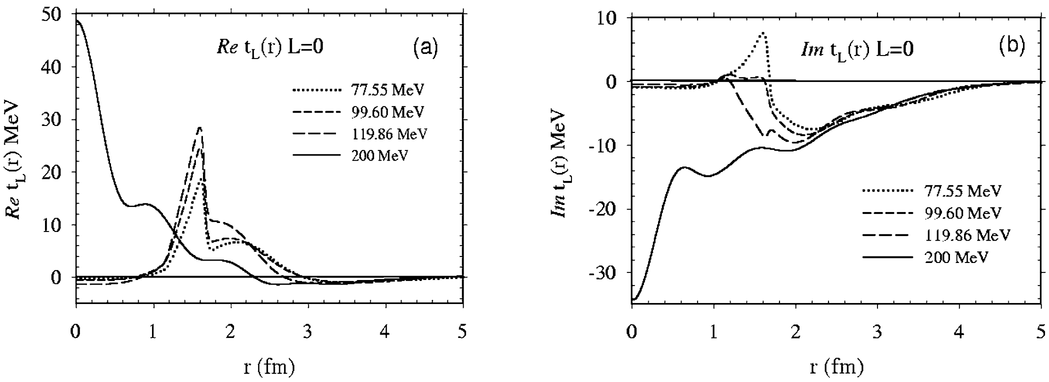

Fig. 5. (a) Real part of the α-α t-matrix effective interaction, $t_{\alpha\alpha,L}(E,r)$ for $L=0$ using $V_{\ell,\alpha\text{-}\alpha}(r)$ with a repulsive core at $E_\alpha = 77.55$, 99.6 and 119.86 MeV and using an all through attractive $V_{\alpha\text{-}\alpha}(r)$ at $E_\alpha = 200$ MeV. (b) Same as for Fig. 5(a) except that it is for the imaginary part of the α-α t-matrix effective interaction, $t_{\alpha\alpha,L}(E,r)$ for $L=0$.

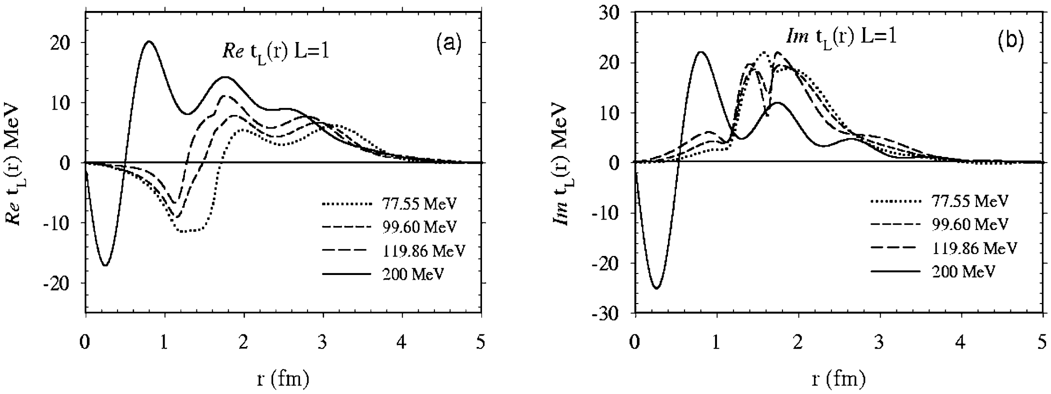

Fig. 6. Same as for Fig. 5 but for $L=1$.

even values, the multipole L-values of $t_L(E,r)$ can take even as well as odd values. This arises when in Eq. (2·3) both even and odd partial waves of the partial wave expansion of e^{-ikz} couple with the even partial waves of $V(r)\Psi^+_{\alpha\alpha}(\vec{r})$.

The $t_L(r)$ results are plotted as a function of $r(r = r_{\alpha\alpha})$ in Figs. 5(a) and (b) to Figs. 8(a) and (b) for the chosen energies and the multipole L-values from 0, 1, 2 and 3 respectively. In Figs. 5(a) to 8(a) only real values of $t_L(r)$ are plotted while in Figs. 5(b) to 8(b) only imaginary values of the $t_L(r)$ are plotted. A comparison of the even and odd L-value contributions to the α-α t-matrix effective interaction in Figs. 5(b) to 8(b) gives the feeling that for lower energy cases and except for $L=0$ monopole case most of the $\Im m t_L(r)$ are largely positive.

It is seen in Figs. 5 to 8 that when all through attractive potentials are employed the $t_L(r)$ results are strikingly different from the $t_L(r)$ results of $V_{\alpha\text{-}\alpha}(r)$ with a

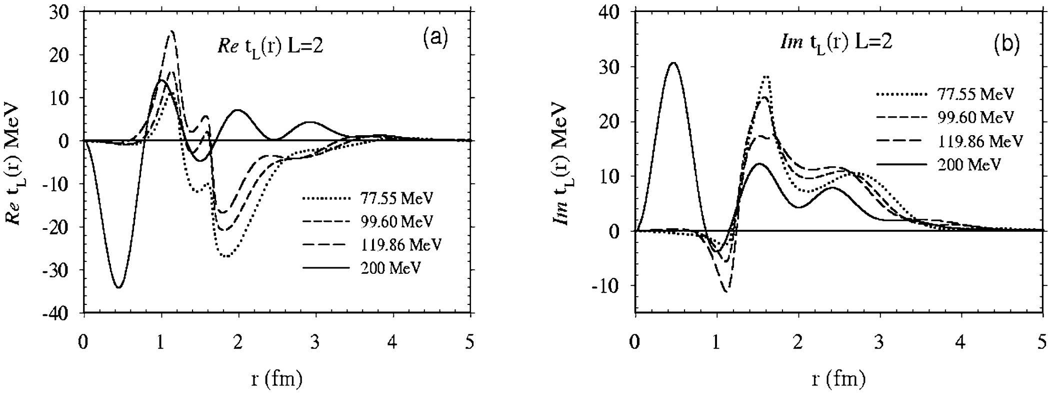

Fig. 7. Same as for Fig. 5 but for $L = 2$.

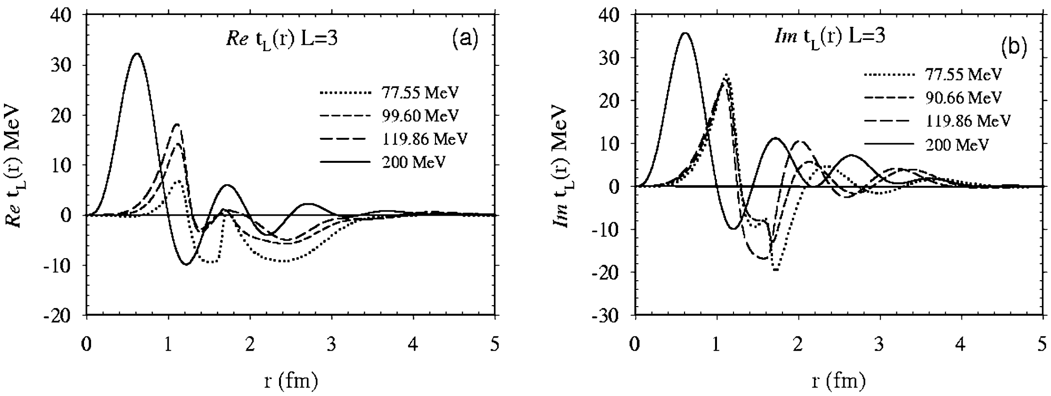

Fig. 8. Same as for Fig. 5 but for $L = 3$.

shorter range repulsive core. Moreover as seen in Figs. 5(a) and (b) the results of $t_0(r)$ (both real as well as imaginary components) with attractive $V_{\alpha\text{-}\alpha}(r)$ have a very prominent peak at $r = 0$. This is only to be expected from an all through attractive potential (as at 200 MeV and 120 MeV) because the $\ell = 0$, s-wave in $\Psi^+_{\alpha\alpha}(\vec{r})$ does not have to overcome the centrifugal barrier resulting in both $V_0(r)$ and $\frac{u_0(kr)}{kr}$ peaking at $r \sim 0$. In contrast to this, all the lower energy $L = 0$ results using repulsive core $V_{\alpha\text{-}\alpha}(r)$ as seen in Fig. 5 here have almost negligible contributions close to $r = 0$. This again is to be expected as the repulsive core in the $V_0(r)$ optical potentials will lead to negligible radial functions, $\frac{u_0(kr)}{kr}$ at short α-α separation in the repulsive core region. It is to be emphasized that in the α-α elastic scattering around $\Theta_{\text{C.M.}} \sim 90°$, the largest contribution comes from the $\ell = 0$ partial wave (although the collective contribution from larger partial waves is significant). It is to be recalled that the ZR-DWIA analyses of the symmetric coplanar data containing the zero recoil momentum uses the $\Theta_{\text{C.M.}} \sim 90°$ free α-α elastic scattering cross sections as input. As these 90° results are affected strongly by the $L = 0$ $t_0(r)$, the wild difference between the $(\alpha, 2\alpha)$ results at 200 MeV and at lower energies may be associated to the different finite range nature of the $t_L(r)$ at these energies arising from the change of the nature of $V_{\alpha\text{-}\alpha}(r)$.

Thus on the basis of the present knowledge of the $t_{\alpha\alpha}(r)$ it is but natural that the FR-DWIA results for $(\alpha, 2\alpha)$ reactions on any target nucleus where $V_{\alpha\text{-}\alpha}(r)$ is all through attractive (at 200 MeV) are expected to be different where $V_{\alpha\text{-}\alpha}(r)$ has short range repulsion (as at lower energies, 77.55, 99.60 and 119.86 MeV).

It has indeed been found by us in very preliminary FR-DWIA calculations that the α-α vertex is much more important in explaining the $(\alpha, 2\alpha)$ reaction anomalies[12),20),21)] than the α-residual nucleus vertex. As the FR-DWIA calculations we have performed are very cumbersome, time-consuming and require huge resources in terms of computer hardware, so far we could only achieve very preliminary results vindicating only our initiative in this venture.

Invariably the ZR-DWIA analyses provide α-clustering probabilities that are energy dependent. The higher the energy one uses the lesser the anomaly one gets in the ZR-DWIA analyses of the $(\alpha, 2\alpha)$ reactions. Qualitatively this result can also be seen to arise when one compares the lower energy (77.55, 99.60 and 119.86 MeV) $t_L(r)$ results amongst themselves. It is seen in Figs. 5 to 8 that for repulsive core results themselves there is a gradual shift of the $t_L(r)$ vs r curves towards lower values of r as the energy increases. This qualitatively amounts to improved applicability of the ZR-approximation with increasing incident energy.

Due to the simple structure of the $\ell = 0$ (here $\ell = 0$ is meant to be the orbital angular momentum of the bound α-residual nucleus state of the target nucleus) $(\alpha, 2\alpha)$ knockout spectra one may not be able to envisage the influence of the finite range form of the t-matrix α-α effective interaction on the shape of these spectra.

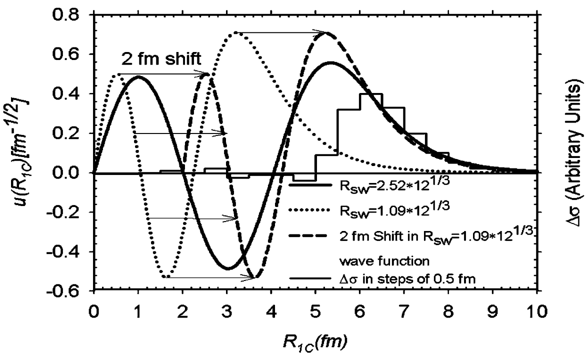

Fig. 9. 3S α-^{12}C g.s. intercluster bound wave function for ^{16}O using different Woods-Saxon wells of various radii ($R_{SW} = r_0 \times 12^{1/3}$). Continuous line (_____) with $r_0 = 2.52$ fm. Dotted line $(\cdots\cdots)$ using $r_0 = 1.09$ fm. Dashed line (_ _ _) same as with $r_0 = 1.09$ fm but shifted by 2 fm. Histogram of $\Delta\sigma$ (change in calculated ^{16}O$(\alpha, 2\alpha)^{12}$C cross section) for 0.5 fm change in cutoff in ZR-DWIA using $r_0 = 2.52$ fm.

However, for the $\ell = 1$ $(\alpha, 2\alpha)$ knockout spectra which have characteristic pronounced structures, the ^{7}Li$(\alpha, 2\alpha)^3$H reaction data[12] at lower energies have witnessed lesser influence of optical distortions[16] (dip at small recoil momenta is very pronounced in the 77 MeV data as compared to the 119.86 MeV data) contrary to the zero range DWIA predictions. This indicates that the zero range assumption for the t-matrix α-α effective interaction amounts to larger distortion in the DWIA analyses of the ^{7}Li$(\alpha, 2\alpha)^3$H data suggesting a proper finite range analysis to resolve the anomalies.

In Fig. 9 various bound state wavefunctions are shown which had been used earlier to analyse the α-knockout from ^{16}O-nucleus.[12] Using a realistic bound state well radius of $r_0 \times 12^{\frac{1}{3}}$, with $r_0 = 1.09$ fm yields an unrealistic α-spectroscopic factor of about two orders of magnitude too large[10] in comparison to the expectations based on the shell model estimates. In order to circumvent such large spectroscopic factors in general, Wang et al.[12] and Chant et al.[13] increased the radius parameter (α-residual nucleus vertex) r_0 from 1.09 to 2.52 fm and got the α-spectroscopic factors which were consistent with data of $(p, p\alpha)$ and $(\alpha, 2\alpha)$ reactions as well as with the shell model theory. It is also seen in Fig. 9 (histogram, r.h.s. scale) that for the ^{16}O$(\alpha, 2\alpha)^{12}$C g.s. reaction at 140 MeV the zero recoil momentum ($\vec{K}_{^{12}\mathrm{C}} \sim 0$) contributions to the $(\alpha, 2\alpha)$ cross section are localized beyond $R_{1\mathrm{C}}$ $\sim$5 fm. It is interesting to note that the bound state intercluster wave function, $u_0(R_{1\mathrm{C}})$ is also nicely overlapping with the corresponding $u_0(R_{1\mathrm{C}})$ for $r_0 = 1.09$ fm but which is shifted by ≈ 2 fm in the region where the reaction, $\Delta\sigma$ is localized. At this stage it is only to be visualized that this shift is akin to the shift one observes in Figs. 3 and 4 for the $t_L(r)$ vs r for repulsive and attractive realistic interactions $V_{\alpha\alpha}(r)$ respectively. It is to be conceived (see Fig. 10), that if $t_{\alpha\alpha}(r)$ is shorter ranged, as in the case of fully attractive $V_{\alpha\alpha}(r)$, then for the knockout to occur the incident α- has to enter and then emerge from the strong absorption region where the α- is bound with the target, thus validating the large distortion effects of the ZR-DWIA. On the contrary when the $t_{\alpha\alpha}(r)$ is longer ranged, as is the case with $V_{\alpha\alpha}(r)$ with repulsive core then for the knockout to occur the incident α-can knockout the bound α-cluster even without entering the strong absorption region. In this case the cross section is expected to be comparatively large due to the reduced attenuation in the incoming and emerging α-particles as witnessed in the lower energy $(\alpha, 2\alpha)$ results.

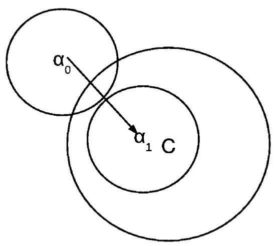

Fig. 10. Schematics of incident α_0 knocking out the bound α_1 from inside the nucleus due to α-α repulsion.

Another point worth attention that has emerged from the present study is that almost twice as many multipole L-values are required with the α-α effective interaction to fit the elastic scattering data than the corresponding ℓ in the realistic optical potential interaction. This is the result of a certain partial wave, say $L_{\max}$, of $\Psi^+_{\alpha\alpha}(\vec{r})$, lying in the optical potential range, coupling their angular momentum with a similar partial wave of the plane wave expansion, e^{-ikz} lying in the same range. Naturally the angular momentum coupling will permit double the angular momentum values in the same radial zone.

§5. Conclusions

From the present study of the evaluation the effective α-α t-matrix interaction at different energies few points have emerged very clearly. In the ZR-DWIA the t-matrix α-α effective interaction is assumed to be a δ-function while this has been calculated here and is shown to be fairly long ranged. This, as discussed in the Introduction, indicates that the factorization obtained in the $(\alpha, 2\alpha)$ reactions is not a result of the zero range α-α vertex. With the finite longer range α-α vertex found in the present work the factorization should be a result of weak optical distortions only. The results obtained in the past using the ZR-DWIA analyses indicating the strong optical distorions is in fact the result of not accounting the finite range effects of $t_{\alpha\alpha}(\vec{r})$ as worked out in this report. The indications of this happening had been seen and reported earlier[16),42)] proving thereby that the zero range approximation for the t-matrix α-α effective interaction is a very crude approximation.

One of the notable points that has come out here, contrary to the previous assumption,[29)] is that while there are only even ℓ-values in the realistic interaction arising out from the symmetric nature of the colliding system the α-α t-matrix effective interaction contains both even as well as odd L-values. The symmetric character of the wave function is reflected in the strong correlations between various $t_{\alpha\alpha,L}(r)$. Another point concerns the differences between the effective interactions obtained from fully attractive and from repulsive core realistic interactions. While the $t_0(r)$ for the fully attractive $V_{\alpha\alpha}(r)$ is peaked at $r = 0$ the $t_0(r)$ for $V_{\alpha\alpha}(r)$ with a repulsive core is peaked away from the $r = 0$, the shift is about the size of the repulsive core. For other L-values all the $t_L(r)$ whether from attractive or from repulsive core $V_{\alpha\alpha}(r)$ are peaked away from $r = 0$. In general the $t_L(r)$ corresponding to the repulsive core $V_{\alpha\alpha}(r)$ are peaked more outside in comparison to the ones obtained from fully attractive $V_{\alpha\alpha}(r)$. In terms of the energy dependence it is found that the lower energy $t_L(r)$ are pushed out more in comparison to the ones for higher energies. Another significant finding of the present study is that the maximum L-values for $t_L(r)$ that contributes significantly to the elastic scattering angular distribution is almost double the maximum ℓ-values that contribute from the realistic interaction. Use of the evaluated $t_L(r)$ in very preliminary FR-DWIA calculations indicates that the anomalies found in various $(\alpha, 2\alpha)$ reactions may find a reasonable solution in terms of the finite range effects and the energy dependence of the effective α-α t-matrix interaction at the α-α knockout vertex.

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