X Meson aka η' and Kobayashi-Maskawa-'t Hooft Six-Quark Vertex

—— $U(1)_A$ *Anomaly and Generalized Nambu-Jona-Lasinio Model* ——

Teiji KUNIHIRO

Department of Physics, Kyoto University, Kyoto 606-8502, Japan

In 1970, Kobayashi and Maskawa concluded that an effective six-quark vertex with a determinantal form is necessary in the chiral effective models to account for the large mass of X meson, which is now called η'. The determinantal interaction has an $SU(3)_L \otimes SU(3)_R$ symmetry but not $U(3)_L \otimes U(3)_R$, and, hence accounts for the explicit breaking of $U(1)_A$ symmetry in quantum chromodynamics (QCD); the vertex was later derived by 't Hooft as an instanton-induced quark interaction. The vertex, which may be called the Kobayashi-Maskawa-'t Hooft (KMT) term, is widely used in quantitative analyses of hadron physics and QCD phase transitions at finite temperature and density. An account is made for the KMT term with recent extensive applications. Described are also personal experiences with Professor Maskawa and Professor Kobayashi, including an encounter with Professor Maskawa when the author first presented his work on the KMT term.

Subject Index: 230, 231, 232

§1. Introduction

The pseudoscalar meson $\eta'(958)$ was called X in the past for some time. It was difficult to understand its large mass within the chiral $SU(3)_L \otimes SU(3)_R$ with an explicit symmetry breaking term $-\epsilon_0 S_0 - \epsilon_8 S_8$, which may have been identified with the quark mass term with $S_a = \bar{q}\lambda_a q$; here, the isospin symmetry is assumed. Note that the existence of the quarks or *ur*-baryons was far from being established in those days. As early as 1970, Kobayashi and Maskawa indicated, in a paper[1] entitled "Chiral symmetry and eta-X mixing", that this is a serious problem, and concluded that there should exist a six-quark interaction with a determinantal form

$$\det_{i,j} \bar{q}_i(1 - \gamma_5)q_j + \text{h.c.,} \tag{1·1}$$

where h.c. stands for Hermite conjugate. Their analysis was based on the method of Gell-Mann-Oakes-Renner,[2] and a detailed account of the outcome of this vertex was reported in 1971 by Kobayashi, Kondo and Maskawa.[3] This vertex is contained in instanton-induced quark interaction derived by 't Hooft in 1976,[4] and is often called the 't Hooft vertex. However, one now sees that the appropriate name of this six-fermion determinantal vertex should be the Kobayashi-Maskawa or at least Kobayashi-Maskawa-'t Hooft term, which we adopt[5] and will be abbreviated as the KMT term or vertex.

The compatibility of the large mass of the η' with quantum chromodynamics (QCD) was formulated as the $U(1)_A$ problem by Weinberg[6] in 1975. The present understanding of the resolution of the problem is also described in the textbook by Weinberg[7] and also by Fujikawa and Suzuki,[8] for example. The basic ingredients

are the $U(1)_A$ anomaly for the divergence of the axial current in the flavor singlet and the instanton configuration[9] leading to the θ-vacuum.[10] The physical origin of the large mass of the η' is thus understood to be due to an explicit breaking of the $U(1)_A$ symmetry. It means that the low-energy effective theory of QCD should contain a vertex that explicitly breaks the $U(1)_A$ symmetry.

An interesting point of the work by Kobayashi and Maskawa is the fact that their proposal was based on the work by Nambu[11] who shared the Nobel prize with them in 2008. Noting that the original Nambu-Jona-Lasinio (NJL) model[11] only contains the four-fermion interaction and, hence, becomes inevitably $U(3)_L \otimes U(3)_R$ invariant even if one imposes $SU(3)_L \otimes SU(3)_R$ invariance to the model, Kobayashi and Maskawa[1] concluded that the six-fermion interaction of a determinantal form should be present in the chiral quark model as given by Nambu-Jona-Lasinio[11] for the η' to be described in the theory.[*]

The first serious and extensive analyses of such an extended NJL model with the six-fermion determinantal interaction were carried out around 1987 to 1988 by several people including the present author;[13]–[16] in these analyses, the nonperturbative vacuum is determined in the self-consistent mean-field theory, and the pseudoscalar and scalar mesons as collective excited states on top of the vacuum are also calculated as in the original work,[11] and, hence, the model parameters are determined explicitly. In those days, people became interested in the possible violation of the Okubo-Iizuka-Zweig rule in the baryon sector, which was prompted partly by the mysteriously large value of the π-N sigma term $\Sigma_{\pi N}$,[17] which may be related to the possible strangeness content of the nucleon in the scalar channel; see Ref. 18) for a recent status of the understanding of this subject. An interesting aspect of the KMT term is that it can give rise to a flavor mixing in the scalar as well as in the pseudo-scalar channels as in the η-η' system and, hence, may be an origin of a possible OZI rule violation in the baryon sector.[13],[15],[19] An extension of the model to include the vector and axial vector fields[16],[20]–[22] is reviewed in Refs. 23) and 24). The flavor mixing in the axial vector channel may be related to the "Spin Crisis".[25],[26]

In this article, we will describe the generalized NJL model with the KMT term and provide a brief review on how it works as an effective theory of QCD, particularly in describing the η-η' system, and the scalar meson dynamics together with other QCD phenomenology. We shall also provide a sketch on how the chiral quark models with the anomaly terms are utilized to explore the properties of the quark/hadronic matter at finite temperature and/or density. The first part of the following will be based on our previous review;[5] see Ref. 5) for the details of this part.[**] One can also refer to Refs. 23), 24) and 27) for a complementary account of the subjects and useful references. The author's personal experiences with Professor Maskawa and Professor Kobayashi will also be described, which include an encounter when the author made the first presentation of his work on the generalized Nambu-Jona-

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[*] The same model was later proposed independently by Mirelli and Schechter in 1976.[12]

[**] Reference 5) was cited in his Nobel lecture by Professor Y. Nambu, which was presented by Professor G. Jona-Lasinio.

Lasinio model incorporating the KMT term in 1987.

§2. The NJL model with the KMT term

2.1. *The model*

The effective model considered by Kobayashi and Maskawa[1] was a generalization of the Nambu-Jona-Lasinio (NJL) model[11] to the three-flavor case with the anomaly term incorporated in the form of the determinantal interaction:

$$\mathcal{L} = \bar{q}\,i\gamma \cdot \partial q + \sum_{a=0}^{8} \frac{g_S}{2}[(\bar{q}\,\lambda_a q)^2 + (\bar{q}\,i\lambda_a\gamma_5 q)^2] - \bar{q}\boldsymbol{m}q + g_D[\det \bar{q}_i(1 - \gamma_5)q_j + \text{h.c.}]$$

$$\equiv \mathcal{L}_0 + \mathcal{L}_S + \mathcal{L}_{\text{SB}} + \mathcal{L}_{\text{KMT}}, \tag{2.1}$$

where the quark field q_i has three colors ($N_c = 3$) and three flavors ($N_f = 3$), λ^a ($a = 0 - 8$) are the Gell-Mann matrices with $\lambda_0 = \sqrt{\frac{2}{3}}\boldsymbol{1}$. Although not explicitly written in their paper, this is exactly the model Lagrangian that Kobayashi and Maskawa considered in their 1970 paper. Therefore, we call the Lagrangian (2.1) the Kobayashi-Maskawa-Nambu-Jona-Lasinio (KM-NJL) model.

We emphasize that the model embodies three basic ingredients of QCD, i.e., the dynamical breaking of chiral symmetry (DBCS), $U(1)_A$ anomaly, and the explicit symmetry breaking due to the current quark masses. Extensive studies using this model showed[5] that the various empirical aspects of QCD are realized through the interplay among the three ingredients. It was emphasized in Ref. 5) that the constituent quark model and chiral symmetry are reconciled in a chiral quark model; the chiral quark model can account for most of the empirical facts on baryons as well as the low-lying mesons. Furthermore, such an effective model allows us to study the change in hadron properties in hot/dense medium in a self-consistent manner, as initiated by T. Hatsuda and the present author for the two-flavor case.[28] Before entering into the physical consequences, let us examine its symmetry properties.

2.2. *Symmetry properties*

To observe the transformation properties of each term in the KM-NJL model, it is convenient to introduce the 3×3 bosonic matrices by

$$\Phi_{ij} = \bar{q}_j(1 - \gamma_5)q_i = 2\bar{q}_{jR}q_{iL} = \bar{q}_j q_i + i\bar{q}_j i\gamma_5 q_i, \tag{2.2}$$

with $(\Phi^\dagger)_{ij} = \bar{q}_j(1 + \gamma_5)q_i = 2\bar{q}_{jL}q_{iR}$, where $q_{iL} \equiv 1/2 \cdot (1 - \gamma_5)\,q_i$ and $q_{iR} \equiv 1/2 \cdot (1 + \gamma_5)\,q_i$ are the left- and right-handed fields, respectively. We note that

$$\bar{q}_R\lambda_a q_L = \text{Tr}[\lambda_a\Phi]/2 \equiv \Phi_a \quad \text{and} \quad \bar{q}_L\lambda_a q_R = \text{Tr}[\lambda_a\Phi^\dagger]/2 \equiv \Phi_a^\dagger, \tag{2.3}$$

and accordingly, $\bar{q}\lambda_a q = (\Phi_a^\dagger + \Phi_a)/2$ and $\bar{q}i\gamma_5\lambda_a q = i(\Phi_a^\dagger - \Phi_a)/2$. Then, the Lagrangian is cast into a form reminiscent of the linear σ-model:

$$\mathcal{L}_S + \mathcal{L}_{\text{SB}} + \mathcal{L}_{\text{KMT}} = g_S\text{Tr}(\Phi^\dagger\Phi) - \frac{1}{2}\text{Tr}[\boldsymbol{m}(\Phi + \Phi^\dagger)] + g_D(\det\Phi + \text{h.c.}). \tag{2.4}$$

Now the chiral $SU(3)_L \otimes SU(3)_R$ transformation is defined by

$$q_{iL} \rightarrow [U(\boldsymbol{\theta}_L)]_{ij}q_{jL} \equiv L_{ij}q_{jL}, \quad q_{iR} \rightarrow [U(\theta_R)]_{ij}q_{jR} \equiv R_{ij}q_{jR}, \quad (2\cdot5)$$

with $U(\boldsymbol{\theta}) = \exp(i\sum_{a=1}^{8}\theta_a\lambda_a/2)$, the determinant of which is unity. Here, the repeated suffix implies a summation over it. Under the $SU(3)_L \otimes SU(3)_R$ transformation, the bosonic operators are transformed as

$$\Phi_{ij} \rightarrow L_{ik}\Phi_{kl}R_{lj}^{\dagger}, \quad \Phi_{ij}^{\dagger} \rightarrow R_{ik}\Phi_{kl}^{\dagger}L_{lj}^{\dagger}, \quad (2\cdot6)$$

which shows that they are representations $(3,\bar{3})$ and $(\bar{3},3)$ of $SU(3)_L \otimes SU(3)_R$, respectively. Thus, it is easily seen that $\mathcal{L}_S$ has an $SU(3)_L \otimes SU(3)_R$ invariance. A notable point is that $\mathcal{L}_S$ is also *automatically* invariant under the $U(1)_L \otimes U(1)_R = U(1)_V \otimes U(1)_A$ transformation defined by

$$q_L \rightarrow e^{i\alpha_0/2}q_L, \quad q_R \rightarrow e^{i\beta_0/2}q_R. \quad (2\cdot7)$$

In fact, under this transformation, the bosonic variables are transformed as

$$\Phi \rightarrow e^{i(\alpha_0-\beta_0)/2}\Phi, \quad \Phi^{\dagger} \rightarrow e^{-i(\alpha_0-\beta_0)/2}\Phi^{\dagger}. \quad (2\cdot8)$$

Conversely, it is impossible to construct a four-fermion vertex that is invariant under $SU(3)_L \otimes SU(3)_R$ but not $U(1)_A$. This point was emphasized by Kobayashi and Maskawa[1] and constitutes the basis of their proposal of the six-fermion vertex for accounting for the large mass of the X, i.e., the η'.

In fact, the determinantal terms $\det\Phi$ and $\det\Phi^{\dagger}$ are not invariant for the $U(1)_A$ transformation, although they are invariant under $SU(3)_L \otimes SU(3)_R$ because they are transformed as

$$\det\Phi \rightarrow e^{3i(\alpha_0-\beta_0)/2}\det\Phi, \quad \det\Phi^{\dagger} \rightarrow e^{-3i(\alpha_0-\beta_0)/2}\det\Phi^{\dagger}, \quad (2\cdot9)$$

which shows that $\mathcal{L}_{\mathrm{KMT}}$ is not invariant unless $\alpha_0 = \beta_0$, i.e., $U(1)_V$ transformation. Thus, one sees that the $\mathcal{L}_{\mathrm{KMT}}$ vertex takes care of the $U(1)_A$ anomaly. In short, (i) $\mathcal{L}_0$ and $\mathcal{L}_S$ are invariant under $U(3)_L \otimes U(3)_R$ transformation, while (ii) $\mathcal{L}_{\mathrm{KMT}}$ is invariant under $U(1)_V \otimes SU(3)_L \otimes SU(3)_R$ transformation but not invariant under $U(1)_A$ transformation.

$\mathcal{L}_{\mathrm{SB}}$ is the explicit $SU(3)_V$ breaking part with the current quark masses:

$$\mathcal{L}_{\mathrm{SB}} = -\bar{q}\boldsymbol{m}q = -\sum_{a=0,3,8}m_a S_a, \quad (2\cdot10)$$

where $m_0 = (m_u+m_d+m_s)/\sqrt{6}$, $m_3 = (m_u-m_d)/2$ and $m_8 = (m_u+m_d-2m_s)/2\sqrt{3}$ with $S_a = \bar{q}\lambda_a q$. If we assume the isospin symmetry, the mass term is reduced to $\mathcal{L}_{\mathrm{SB}} = -\epsilon_0 S_0 - \epsilon_8 S_8$ with $\epsilon_{0,8}$ being identified with $m_{0,8}$.

The fact that $\mathcal{L}_{\mathrm{KMT}}$ represents the $U(1)_A$ anomaly can be seen in the anomalous divergence of the flavor singlet axial current

$$\partial_\mu A_5^\mu = 2iN_f g_D(\det\Phi - \mathrm{h.c.}) + 2i\bar{q}\boldsymbol{m}\gamma_5 q, \quad (2\cdot11)$$

with $A_5^\mu = \bar{u}\gamma^\mu\gamma_5 u + \bar{d}\gamma^\mu\gamma_5 d + \bar{s}\gamma^\mu\gamma_5 s$. This equation is to be compared with the usual anomaly equation [29] written in terms of the topological charge density of the gluon field ,[4),7),8)]

$$\partial_\mu A_5^\mu = 2N_f \frac{g^2}{32\pi^2} F_{\mu\nu}^a \tilde{F}_a^{\mu\nu} + 2i\bar{q}\boldsymbol{m}\gamma_5 q. \tag{2.12}$$

Thus, one may say that the effect of the gluon operator $\frac{g^2}{32\pi^2} F_{\mu\nu}^a \tilde{F}_a^{\mu\nu}$ is simulated by the determinantal operator $i(\det\Phi - \text{h.c.}) = -2g_D \text{Im}(\det\Phi)$ in the quark sector. Note that the anomaly term $\mathcal{L}_{\text{KMT}}$ has a dimension 9. Away from the chiral limit, there arise other instanton-induced dimension-9 operators that break $U(1)_A$ symmetry but are proportional to the current quark masses.[30] The effects of such extra terms are discussed in Ref. 21).

2.3. *Dynamics of Kobayashi-Maskawa-Nambu-Jona-Lasinio Lagrangian*

So far we have discussed the symmetry properties of each term of the generalized Nambu-Jona-Lasinio model that Kobayashi and Maskawa proposed as the low-energy effective model. What is their dynamical role? It is easily verified that $\mathcal{L}_S$ can be decomposed into $\sim (\bar{u}\Gamma u)^2 + (\bar{d}\Gamma d)^2 + (\bar{s}\Gamma s)^2$ without the flavor mixing term, like $(\bar{u}\Gamma u)(\bar{d}\Gamma d)$ with $\Gamma = 1$ or γ_5, although there are terms like $(\bar{u}\Gamma d)(\bar{d}\Gamma u)$, and, hence does not cause a flavor mixing. On the other hand, the KMT term can cause a flavor mixing when the chiral symmetry is dynamically broken; indeed, it induces effective 4-fermion vertices such as $\langle\bar{d}d\rangle(\bar{u}u)(\bar{s}s)$ and $-\langle\bar{d}d\rangle(\bar{u}i\gamma_5 u)(\bar{s}i\gamma_5 s)$, where the former (latter) gives rise to a flavor mixing in the scalar (pseudo-scalar) channels. This flavor mixing in the pseudoscalar channel is found to be the origin of lifting of the η' mass to as high as 1 GeV.

The vacuum of this model Lagrangian is determined in the self-consistent mean field (SCMF) theory[13),14)] as is done for the usual NJL model,[11),31)] although the six-fermion interaction causes an additional complication in the analysis. The Lagrangian in the SCMF approximation reads

$$\mathcal{L}_{\text{MFA}} = \bar{q}(i\gamma\cdot\partial - \boldsymbol{M})q - g_S \text{Tr}(\phi^\dagger\phi) - 2g_D(\det\phi + \text{c.c.}). \tag{2.13}$$

Here, the four-fermion and six-fermion interactions are rewritten in the present approximation as

$$\bar{q}_i q_i \bar{q}_j q_j \to \langle\bar{q}_i q_i\rangle\bar{q}_j q_j + \langle\bar{q}_j q_j\rangle\bar{q}_i q_i - \langle\bar{q}_i q_i\rangle\langle\bar{q}_j q_j\rangle,$$

$$\bar{q}_i q_i \bar{q}_j q_j \bar{q}_k q_k \to \sum_{i,j,k;\ \text{cyclic}} \langle\bar{q}_i q_i\rangle\langle\bar{q}_j q_j\rangle\bar{q}_k q_k - 2\langle\bar{q}_i q_i\rangle\langle\bar{q}_j q_j\rangle\langle\bar{q}_k q_k\rangle. \tag{2.14}$$

The ϕ in (2.13) is a diagonal 3×3 c-number matrix defined in terms of the quark condensates; $\phi = \langle\Phi\rangle_0 \equiv \text{diag}(\langle\bar{u}u\rangle, \langle\bar{d}d\rangle, \langle\bar{s}s\rangle)$. The "constituent quark mass matrix" $\boldsymbol{M} = \text{diag}(M_u, M_d, M_s)$ is given in terms of the condensates,

$$M_u = m_u - 2g_S\alpha - 2g_D\beta\gamma,$$
$$M_d = m_d - 2g_S\beta - 2g_D\alpha\gamma,$$
$$M_s = m_s - 2g_S\gamma - 2g_D\alpha\beta, \tag{2.15}$$

with $(\alpha, \beta, \gamma) \equiv (\langle \bar{u}u \rangle, \langle \bar{d}d \rangle, \langle \bar{s}s \rangle)$. The respective quark condensate is, in turn, given with M_i,

$$\langle \bar{q}_i q_i \rangle = -2N_c \sum_{|\boldsymbol{p}| < \Lambda} M_i / \sqrt{M_i^2 + p^2}. \qquad (2\cdot16)$$

Here, the three momentum cutoff Λ is introduced. We call Eq. (2·16) together with Eq. (2·15) the gap equation because of its resemblance to the gap equation in the theory of superconductivity.[32] Note that the vacuum condensates and the constituent quark masses with different flavors are all coupled with each other owing to the KMT term. To determine these values in terms of the coupling constants, current quark masses, and the cutoff, one must solve this nonlinear coupled equation, (2·16) and (2·15). One should verify that the solution of this gap equation really gives the true vacuum state by evaluating the vacuum energy or the effective potential,

$$\mathcal{V}(\phi) = iN_c \text{Tr} \int \frac{d^4p}{(2\pi)^4} \ln \left(\frac{p \cdot \gamma - M}{p \cdot \gamma - m} \right) + g_S(\alpha^2 + \beta^2 + \gamma^2) + 4g_D \alpha\beta\gamma. \quad (2\cdot17)$$

Here, the first term represents the difference in the energy densities of the nonperturbative and perturbative Dirac seas and the second and third terms denote the repulsive interaction energy with which the double counting is avoided of the attractive interaction energy between quarks through the four-fermion and six-fermion interactions, respectively. The stationary condition, $\partial \mathcal{V}(\phi)/\partial \phi = 0$, is found to be equivalent to the gap equation, Eq. (2·16) with Eq. (2·15).

Once the vacuum is thus determined, one can discuss the meson states as q-$\bar{q}$ collective excited states on top of the vacuum. The residual interactions that develop the q-$\bar{q}$ collective excitations are the following effective four-quark interactions,

$$\begin{aligned}
\mathcal{L}_{\text{res}} = g_S &: \text{Tr}(\Phi^\dagger \Phi) : \\
&+ g_D : \left[\text{Tr}(\phi\Phi^2) - \text{Tr}(\phi\Phi)\text{Tr}\Phi - \frac{1}{2}\text{Tr}\Phi^2\text{Tr}\phi + \frac{1}{2}\text{Tr}\phi(\text{Tr}\Phi)^2 + \text{h.c.} \right] : \\
&+ g_D : (\det\Phi + \text{h.c.}) :,
\end{aligned} \qquad (2\cdot18)$$

where the normal ordering is taken with respect to the Fock vacuum of $\mathcal{L}_{\text{MFA}}$, and we have omitted the Fock terms.

The model can be utilized to describe the low-lying pseudoscalar mesons and the η' as well as the scalar mesons and other QCD phenomenology. As we shall see, the sign of the KMT coupling constant g_D is found to be negative to reproduce the mixing properties of the η-η'. This sign assignment is consistent with the identification of the KMT term as the instanton-induced vertex.[*] First, we shall show some details on how the KMT term can account for the η-η' system.

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[*] The sign of g_D is positive for the two-flavor case, because it is given by $\langle \bar{s}s \rangle \cdot g_D$.

§3. Flavor mixing of η and η' mesons

The relevant interaction in the η-η' channel is found to be

$$\mathcal{L}_{\mathrm{res}}^{\eta} = \frac{1}{2} \sum_{a,b=8,0} : \eta_a G_{ab}^P \eta_b :, \tag{3.1}$$

where $\eta_a \equiv \bar{q} i \gamma_5 \lambda_a q$, and G_{ab}^P denotes the coupling constant in the flavor basis,

$$\boldsymbol{G}^P = \begin{pmatrix} g_S + \frac{1}{3}(2\alpha + 2\beta - \gamma)g_D & -\frac{\sqrt{2}}{6}(2\gamma - \alpha - \beta)g_D \\ -\frac{\sqrt{2}}{6}(2\gamma - \alpha - \beta)g_D & g_S - \frac{2}{3}(\alpha + \beta + \gamma)g_D \end{pmatrix}. \tag{3.2}$$

The coupling among the modes η_0 and η_8 arises from both the $SU(3)_V$ breaking and the anomaly terms; here, we assume the isospin symmetry ($\alpha = \beta$). The effect of large m_s is primarily responsible for mixing the octet (η_8) and singlet (η_0) modes to make the physical η and η' mesons. One finds that the g_D contribution is positive in G_{88}^P but is negative in G_{00}^P because $g_D < 0$; accordingly,

$$G_{00}^P \equiv g_S - \frac{2}{3}(\alpha + \beta + \gamma)g_D < G_{88}^P \equiv g_S + \frac{1}{3}(2\alpha + 2\beta - \gamma)g_D. \tag{3.3}$$

This inequality implies that the mass of the singlet meson, $\eta_0 = (\bar{u} i \gamma_5 u + \bar{d} i \gamma_5 d + \bar{s} i \gamma_5 s)/\sqrt{3}$, is larger than the octet one, $\eta_8 = (\bar{u} i \gamma_5 u + \bar{d} i \gamma_5 d - 2\bar{s} i \gamma_5 s)/\sqrt{6}$, since the binding force is weaker in the singlet channel. Furthermore, noting that the g_D dependence is strongest in G_{00}^P, one can see that the mass of η_0 is most sensitive to the strength of the KMT term.

In the flavor basis, $(\bar{u} i \gamma_5 u, \bar{d} i \gamma_5 d, \bar{s} i \gamma_5 s)$, the coupling constant matrix $\boldsymbol{G}_{\eta\pi^0}$ for η mesons and π^0 reads

$$\boldsymbol{G}_{\eta\pi^0} = 2 \begin{pmatrix} g_S & -g_D\gamma & -g_D\beta \\ -g_D\gamma & g_S & -g_D\alpha \\ -g_D\beta & -g_D\alpha & g_S \end{pmatrix}. \tag{3.4}$$

The non-diagonal terms in $\boldsymbol{G}_{\eta\pi^0}$ are responsible for the flavor mixing and, hence, one sees that not only the strength of the anomaly term g_D but also the quark condensates (α, β and γ) affect the flavor mixing in the π^0 and η-η' system.

Now, the (un-normalized) propagator of the composite $\boldsymbol{\eta}$-system in the $SU(3)_V$ basis reads

$$\boldsymbol{D}(q^2) = -\boldsymbol{G}_P^{-1} \left(\frac{1}{1 + \boldsymbol{G}_P \boldsymbol{\Pi}^P(q^2)} \right), \tag{3.5}$$

where $(\boldsymbol{\Pi}^P)_{ab}$ is the polarization tensor. The mixing angle θ_η between the η and η' is obtained so that $\boldsymbol{D}^{-1}(q^2)$ is diagonalized as

$$\boldsymbol{T}(\theta_\eta)\boldsymbol{D}^{-1}(q^2)\boldsymbol{T}(\theta_\eta)^{-1} = \mathrm{diag}(D_\eta^{-1}(q^2), D_{\eta'}^{-1}(q^2)), \tag{3.6}$$

where the orthogonal matrix $\boldsymbol{T}(\theta_\eta)$ is given by

$$\boldsymbol{T}(\theta_\eta) = \begin{pmatrix} \cos\theta_\eta & -\sin\theta_\eta \\ \sin\theta_\eta & \cos\theta_\eta \end{pmatrix}. \tag{3.7}$$

Note that the mixing angle is inevitably energy-dependent in such a dynamical theory.

θ_η is determined through a competition between the anomaly (KMT term) and the explicit $SU(3)_V$-symmetry breaking ($m_s \gg m_{u,d}$); the former prefers pure $SU(3)_V$ states, i.e., small $|\theta_\eta|$ because of the large $u \leftrightarrow s$ and $d \leftrightarrow s$ transitions, while the latter prefers the mass eigenstates, i.e., large $|\theta_\eta|$. In fact,

- if the KMT term is absent ($g_D = 0$), $\boldsymbol{G}^P = g_S \cdot \boldsymbol{1}$ and the mass eigenstates are realized, leading to the ideal mixing $\theta_\eta = -54.75°$; $\eta = (\bar{u}i\gamma_5 u + \bar{d}i\gamma_5 d)/\sqrt{2}$ and $\eta' = \bar{s}i\gamma_5 s$. On the other hand,
- if $g_D \neq 0$ but $m_s = m_{u,d}$, $\theta_\eta = 0°$ and the flavor eigenstates are realized as $\eta = \eta_8 = (\bar{u}i\gamma_5 u + \bar{d}i\gamma_5 d - 2\bar{s}i\gamma_5 s)/\sqrt{6}$ and $\eta' = \eta_0 = (\bar{u}i\gamma_5 u + \bar{d}i\gamma_5 d + \bar{s}i\gamma_5 s)/\sqrt{3}$.

An explicit calculation[5),14),52)] for the general case gives the results for the mixing angle at the energy of η as $\theta_\eta(m_\eta^2) = -20.9°$, with $m_\eta = 486.5$ MeV and $m_{\eta'} = 957.5$ MeV (fitted).

The parameters of the KM-NJL model to be determined are as follows: The current quark masses $\boldsymbol{m} = \mathrm{diag}(m_u, m_d, m_s)$, the coupling constants g_S and g_D, and the momentum cutoff Λ characterizing the scale of the chiral symmetry breaking. For m_u and m_d, we assume the $SU(2)_V$ invariance and define $\hat{m} = (m_u + m_d)/2$. These parameters are determined[5),14),52)] so as to reproduce the four basic quantities

$$m_\pi = 138 \text{ MeV}, \ f_\pi = 93 \text{ MeV}, \ m_K = 495.7 \text{ MeV}, \text{ and } m_{\eta'} = 957.5 \text{ MeV}. \tag{3.8}$$

We have adopted 5.5 MeV as a value for $\hat{m}$ at 1 GeV scale in the following. Then, the resulting parameter set reads

$$\Lambda = 631.4 \text{ MeV}, \ \ g_S \Lambda^2 = 3.666, \ \ g_D \Lambda^5 = -9.288, \ \ m_s = 135.7 \text{ MeV}, \tag{3.9}$$

where we have used a three-momentum cutoff scheme.

§4.   The KMT term in scalar meson dynamics

As was mentioned earlier, the anomaly term of the determinantal form also gives rise to a flavor mixing in the scalar as well as in the pseudo-scalar channels. The scalar mesons may constitute a nonet as the low-lying pseudo-scalar mesons do, although some slight difference is surely present because of the absence of constraints, as given by the axial anomaly, in the scalar channel. Actually, the possible existence of the low-lying scalar mesons and their $SU(3)_V$ nonet scheme were quite controversial, although there were some pioneering works[33),34)] that emphasize the physical significance of the scalar meson, particularly the σ meson in QCD, which has (approximate) chiral symmetry as a fundamental property; see also Ref. 35). The situation has changed completely now since the two scalar mesons σ and κ have been established experimentally.[36)–40)]

The low-lying scalar mesons have attracted renewed interest since the 1990s when extensive analyses claimed the existence of the σ meson pole in the complex energy plane of the S-matrix for the π-π scattering in the $I = J = 0$ channel.[41)–44)] In these analyses, the significance of respecting chiral symmetry, unitarity, and crossing

symmetry was recognized and emphasized to reproduce the phase shifts in both the σ (s)- and ρ (t)-channels with a low-mass σ pole.[45] One of the most elaborate analyses[46] identifies the σ pole at $M_\sigma = 441 - i272$ MeV. The existence of such low-lying scalar mesons can be a puzzle in QCD.[47],[48] In the nonrelativistic constituent quark model,[49] the meson with the quantum number $J^{PC} = 0^{++}$ is in the $^3P_0$ state, which normally implies that the mass lies in the region from 1.2 to 1.6 GeV. Several mechanisms have been proposed to lower the mass with an amount as large as 600 – 800 MeV; see Ref. 47) for the issues concerning the low-lying scalar mesons. A first idea was a diquark-anti-diquark (or tetraquarks) structure proposed by Jaffe,[50] who showed that the color magnetic interaction between the diquark and the anti-diquark gives a sufficiently large attraction to decrease the masses of the scalar mesons to approximately 600 MeV. Another time-honored idea is attributable to Nambu,[11] in which the smallness of the mass is attributed to the possible collective nature of the scalar mesons as possessed by the pion. It is well known that the scalar meson appears as a consequence of the chiral symmetry and its dynamical breaking as the pion does, and the mass of the sigma satisfies the Nambu relation,[51] $m_\sigma = 2M_f$, with M_f being the dynamically generated fermion (quark) mass, which should be valid within any Nambu-Jona-Lasinio type model. If we put $M_f = 300$ MeV, m_σ becomes 600 MeV, in fairly good agreement with the experiment. It is shown that this feature essentially persists even when the $U(1)_A$ anomaly term is incorporated although there arises a small but sizable flavor mixing between the $\sigma \sim (\bar{u}u + \bar{d}d)/\sqrt{2}$ and $f_0 \sim \bar{s}s$;[14],[52] see also some subsequent works.[53]–[55] The wave functions of scalar mesons should also have components of meson-resonance states as these states are seen through the π-π or π-K scattering.

Although the reality should be that the wave functions of these mesons are linear combinations of these components, the most popular idea is the tetraquarks.[50],[56]–[58] In this scheme, the $SU(3)$ nonet structure is composed of the quark content as follows; $\sigma = [ud][\bar{u}\bar{d}]$, $\kappa^0 = [su][\bar{u}\bar{d}]$, $\kappa^- = [sd][\bar{u}\bar{d}]$, $f_0 = ([su][\bar{s}\bar{u}] + [sd][\bar{s}\bar{d}])/\sqrt{2}$, $a_0^+ = [su][\bar{s}\bar{d}]$, and other members, $\kappa^{+,-}$, $a_0^{0,-}$, are constructed in a similar manner.

The merit of the tetraquark scheme lies in the fact[50] that it can naturally identify the nonet scheme in the scalar mesons of the masses of less than 1 GeV and also explain the multiplet scheme that has an inverted form of the vector-meson nonet, i.e., the ρ, ω, K^*, and ϕ. However, there are at least two problems to be clarified to establish this scheme:[59] (i) If the flavor mixing is ideal as given above, the $f_0 \to 2\pi$ coupling vanishes in contrast to that in the experiment, and the $a_0 \to \eta\pi$ coupling is too large to be consistent with the experimental data. (ii) As is mentioned above, there should be a mixing between the tetraquark and $q\bar{q}$ states more or less to make the physical states, which may possibly imply the existence of the scalar mesons mainly composed of $q\bar{q}$ with a small mixture of the tetraquark states. Although there is some work on these problems,[60] several people[59],[61] have recently shown that the KMT term in the scalar meson dynamics can nicely resolve these problems. By making a Fierz transformation, one can see that the KMT term contains a tetraquark-$q\bar{q}$ coupling,

$$\mathcal{L}_{4q-2q} = G_{4q-2q}\mathrm{Tr}(\tilde{S}S), \tag{4·1}$$

where $\tilde{S}_{ij} = [\bar{d}]_i[d]_j$ and $S_{ij} = \bar{q}_j q_i$ with $[d]_{i\alpha} = \epsilon_{ijk}\epsilon_{\alpha\beta\gamma}\bar{q}_c^{j\beta}\gamma_5 q^{k\gamma}$ being the spin-0 diquark operator. The Latin and Greek indices denote flavor and color state, respectively, and q_c is the charge conjugate of the quark field. This form of the vertex was first considered in Ref. 56) in the context of the scalar meson mixing.[*] Although the coupling constant G_{4q-2q} is in principle given by the KMT coupling g_D, the phenomenological analysis indicates[56] that $|G_{4q-2q}| \simeq 0.6$ GeV^2. See Refs. 59) and 61) for the details of the roles of the KMT vertex in the tetraquark-$q\bar{q}$ mixing and the resulting phenomenology for the scalar meson dynamics.

§5. Other phenomenology with the KMT term

It has been shown that the KM-NJL model well describes the vacuum properties related to chiral symmetry and its spontaneous breaking including their flavor dependence.[5],[13],[14] We have seen that the model gives a systematic description of the low-energy hadrons in the pseudo-scalar and scalar channels. The model can be a good starting point even for the octet and decuplet *baryons*;[5],[66],[**] one can use the vertex to analyze the possible violation of the OZI rule in the baryon sector. A fundamental reason for such successes of the KM-NJL model lies in the fact that the model can be regarded as a field theoretic version of the constituent quark model under the identification of the constituent quark masses with those generated dynamically by the chiral symmetry breaking: The new ingredients of the KM-NJL model beyond the conventional constituent quark model are (i) it gives a self-consistent description of the *vacuum* and the excited states (*hadrons*), and (ii) the model properly takes into account the *collective* nature of the vacuum and the mesons. These points are emphasized in Ref. 5). The recent development of the phenomenology based on the KMT term may be seen in Ref. 71) and the references cited therein.

§6. Application to finite temperature and density systems

The role of the $U(1)_A$ anomaly at finite temperature T and/or baryon density ρ_B or the chemical potential μ is a big issue, and there are many studies on this problem; see, for example, Refs. 72)–74). As an effective model embodying the $U(1)_A$ anomaly, the KM-NJL model is also applied to the finite temperature and density. Here, we will pick some topics on these subjects.

6.1. *Phase diagram*

In Ref. 75), the quark condensates and meson excitations in the hadronic phase at finite temperature T are investigated by the present author in the KM-NJL model for the first time; see also Refs. 13) and 76). It was shown that the order of the chiral transition is of *crossover* against an expectation that the cubic term owing to the KMT term would lead to a first-order phase transition. This is because the explicit

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[*] We remark that the vertex in the form of Eq. (4·1) was also considered in the context of the color superconductivity in dense quark matter;[62]–[65] see §6.

[**] See also Refs. 67)–69), in which the role of the instanton-induced interaction is examined for the existence or nonexistence of the H-dibaryon.[70]

symmetry terms owing to the current quark masses, particularly that of the strange quark, are so large that the order of the phase transition becomes smooth. This is the result within the mean-field approximation. Thus, it would be intriguing to apply the functional renormalization group to explore the chiral phase transition with the KMT term and the explicit breaking with the current quark masses.

The mean field theory at finite T and μ goes much the same way as that at zero temperature discussed in a previous section: The vacuum expectation value $\langle O \rangle$ is replaced by the statistical average $\langle\langle O \rangle\rangle$. Then, the quark condensates as the variational parameters are now T- and μ-dependent; $\langle\langle \bar{u}u \rangle\rangle \equiv \tilde{\alpha}$, $\langle\langle \bar{d}d \rangle\rangle \equiv \tilde{\beta}$, $\langle\langle \bar{s}s \rangle\rangle \equiv \tilde{\gamma}$. Thus, the Hamiltonian to be used in this approximation has the same form as that at zero temperature, with the quark mass matrix $\boldsymbol{M} = \mathrm{diag}(M_u, M_d, M_s)$ given by Eq. (2·15) but now being T- and μ- dependent: $M_u = m_u - 2g_S\tilde{\alpha} - 2g_D\tilde{\beta}\tilde{\gamma}$, $M_d = m_d - 2g_S\tilde{\beta} - 2g_D\tilde{\alpha}\tilde{\gamma}$, and $M_s = m_s - 2g_S\tilde{\gamma} - 2g_D\tilde{\alpha}\tilde{\beta}$. We note again that the contribution of the anomaly term to the constituent quark masses is dependent on the condensates of other flavors. It indicates that a change, say, in $\langle\langle \bar{u}u \rangle\rangle$ causes a change in M_s and accordingly in $\langle\langle \bar{s}s \rangle\rangle$, and *vice versa*. Thus, the properties of the strange quark can change even in the matter composed of the u and d quarks, i.e., nuclear matter.

The thermodynamical potential in the mean-field approximation can be readily calculated with

$$K_{\mathrm{MFA}} = H_{\mathrm{MFA}} - \sum_{i=u,d,s} \mu_i N_i. \tag{6·1}$$

The result is

$$\Omega_{\mathrm{MFA}}(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}) = \mathcal{V}(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}) \cdot V - 2N_c T \sum_{i=u,d,s,|p|<\Lambda} [\ln\{1 + \exp(-e_{i\boldsymbol{p}}^{(-)}/T)\}$$
$$+ \ln\{1 + \exp(-e_{i\boldsymbol{p}}^{(+)}/T)\}], \tag{6·2}$$

where $e_{i\boldsymbol{p}}^{(\pm)} = E_{i\boldsymbol{p}} \pm \mu_i$ with $E_{i\boldsymbol{p}} = \sqrt{M_i^2 + \boldsymbol{p}^2}$ and V being the volume of the system, and

$$\mathcal{V}(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}) = -2N_c \sum_{i=u,d,s} \int^\Lambda \frac{d\boldsymbol{p}}{(2\pi)^3} E_{i\boldsymbol{p}} + \{g_S(\tilde{\alpha}^2 + \tilde{\beta}^2 + \tilde{\gamma}^2) + 4g_D\,\tilde{\alpha}\tilde{\beta}\tilde{\gamma}\} \tag{6·3}$$

is the vacuum energy term, which has the same form as the effective potential at $T = 0$, although the condensates are now temperature-dependent. We have neglected the constant term $\Omega(0, 0, 0)$, which is irrelevant for the following argument. The equilibrium state can be determined as the point where the thermodynamical potential takes the minimum with $\tilde{\alpha}$, $\tilde{\beta}$, and $\tilde{\gamma}$ as the variational parameters:

$$\frac{\partial \Omega_{\mathrm{MFA}}}{\partial Q_i} = 0, \qquad (Q_i = \tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}) \tag{6·4}$$

which ensures that the condensates assumed are the statistical averages calculated with the corresponding mean-field Hamiltonian.

A numerical calculation[75),76)] shows that the thermodynamical potential $\Omega(\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma})$ as a function of the condensates has only one minimum for all the temperatures with vanishing chemical potentials. This implies that the phase transition described in this model is a smooth one, or a crossover.[75)] However, the transition turns out to be a first-order one at finite density with low temperatures ($T \leq 29$ MeV $\equiv T_{c1}$) like a liquid-gas phase transition: At $T = 0$, a chirally restored phase with a high density ($\rho_B \sim 6\rho_0$, $\rho_0 = .17\,\mathrm{fm}^{-3}$) coexists with a chirally broken phase with a small density. As T increases, the difference in the densities of the coexisting phase becomes smaller and smaller; then, at $T = T_{c1}$, the phase transition ceases to be a first-order one. For $T > T_{c1}$, the phase transition is a smooth one, and the pressure increases monotonically as the density is increased.[75),76)]

6.2. *The mesonic excited states; effective restoration of $U(1)_A$ symmetry*

Now let us proceed to the examination of the meson states as collective excitations in the system.

The collective excitations are generated primarily by the four-fermion interactions; the effective coupling constants G_α in the various channels are tabulated in Table 3.1 in Ref. 5). For the η-η' channel, the coupling matrix is given in Eq. (3·2) or Eq. (3·4). Note, however, that the condensates appearing there are now T- and μ_i-dependent.[75)] Furthermore, since the condensates are multiplied by g_D there, the net effects of the anomaly as manifested in the mixing properties of the η and η' mesons should become smaller when T is increased. This means that the chiral restoration effectively causes a partial restoration of the $U(1)_A$ symmetry.[75)]

The information of collective excitations is all contained in the corresponding retarded Green's functions or the response functions given by

$$R_{\alpha\beta}(\omega, \boldsymbol{q}) = -i \int \frac{d^4x}{(2\pi)^4} e^{-iq\cdot x}\theta(t)\langle\langle[O_{K\alpha}(t,\boldsymbol{x}), O_{K\beta}(0,\boldsymbol{0})]_-\rangle\rangle, \qquad (6\cdot5)$$

where

$$O_{K\alpha}(t,\boldsymbol{x}) = \bar{q}_K(t,\boldsymbol{x})\Gamma_\alpha q_K(t,\boldsymbol{x}) - \langle\langle\bar{q}_K(t,\boldsymbol{x})\Gamma_\alpha q_K(t,\boldsymbol{x})\rangle\rangle, \qquad (6\cdot6)$$

with $q_K(t,\boldsymbol{x}) = \exp(-iKt)q(0,\boldsymbol{x})\exp(iKt)$ being the real-time operator. Here, $\Gamma_\alpha(\Gamma_\beta)$ denotes a product of Dirac and Gell-Mann matrices that specifies the quantum numbers of the collective modes; for example, $\Gamma_\alpha = \Gamma_\beta = i\gamma_5\lambda_{4\pm i5}$ for kaons $K^\pm$. For the sigma mesons and η and η' mesons, the response functions become matrices owing to the octet-singlet couplings. The poles of the response function (or the determinant of the response functions for sigma and η mesons) give the dispersion relations $\omega_\alpha = \omega_\alpha(\boldsymbol{q})$ of the mode. To evaluate the response function, one may use the *imaginary-time* formalism.

It was shown[75)] that the η and η' mesons change their nature owing to both the temperature dependence of the quark condensates and the possible decrease in the KMT coupling constant g_D with T. The coupling constant g_D of the KMT term may be dependent on temperature and baryon chemical potential because the instanton density is dependent on them.[73),74)] When such a possible temperature dependence is considered, the mixing angle θ_η can also be T dependent, and θ_η increases in

the absolute value and the mixing between the η and η' approaches the ideal one. Although the η_0 component in the physical η' decreases as T is increased, the η' mass decreases gradually with increasing T, because the η_0 tends to acquire the nature of the ninth Nambu-Goldstone boson of the $SU(3)_L \otimes SU(3)_R \otimes U(1)_A$ symmetry and decreases its mass rapidly. This tendency is also observed with an explicit use of the instanton-induced interaction.[73),77)] This is an effective "restoration" of $U(1)_A$ anomaly at finite temperature as seen in the η-η' spectrum, which was first suggested by Pisarski and Wilczek[72)] using a linear σ model with a determinant term in the chiral limit. Such an anomalous decrease in the η' mass might have been observed in the relativistic heavy ion collisions at RHIC.[78)]

6.3. *Possible temperature and density dependence of the KMT coupling g_D*

The temperature dependence of g_D can be deduced by utilizing the lattice data, as was done by Fukushima, Ohnishi, and Ohta.[79)] The finite density case is examined in Ref. 80). The possibility of the effective restoration of the chiral $U(1)_A$ anomaly in finite nuclei is discussed in Ref. 81) where a possibility to create bound states of the $\eta'(958)$ meson in nuclei is examined. Further developments in this direction may be found in Refs. 82) and 83). The density dependence of the coupling constant g_D has also been considered[84)] for exploring whether the QCD critical point suggested in some effective models[85)] can actually be absent as is shown in some lattice simulation.[86)]

6.4. *Incorporation of color superconductivity; the $U(1)_A$ anomaly versus vector interaction*

At extremely high density with moderate temperature, various forms of color superconductivity may occur; see Refs. 27) and 87) for the recent reviews. In the three-flavor case in the chiral limit, the most symmetric pairing can be realized, which is called the color-flavor locked (CFL) phase.[62)] On the basis of the pattern of the symmetry breaking in the CFL phase, Schafer and Wilczek[88)] suggested a hadron-quark continuity, i.e., the transition from hadron to quark matter can be smooth. As was stated before in the context of the tetraquark structure of the scalar mesons, a Fierz transformation of the KMT term gives a tetraquark-$q\bar{q}$ coupling that breaks the $U(1)_A$ symmetry.[62)–65)] It was speculated[65)] that the existence of such a vertex would give rise to another QCD critical point in the low-temperature region and can be essential to realize the hadron-quark continuity.[88)] Here, we remark that the possible existence of multiple critical points in QCD phase diagram was first shown in Ref. 89) where the vector interaction plays the essential role; see also Ref. 90). The combined effects of the KMT term as well as the vector interaction on the QCD phase diagram have been recently examined in Ref. 91), together with the charge neutrality and the beta equilibrium constraints.

§7. Concluding remarks

In the present article, we have described the significance of the work by Kobayashi and Maskawa in 1970,[1)] which introduced the determinantal six-quark interaction

to account for the large mass of X meson, which is now called the η', although the determinantal term is often called the 't Hooft vertex because it can be derived from the instanton-induced interaction. Then, we proposed to call the determinantal six-quark interaction the Kobayashi-Maskawa-'t Hooft (KMT) term. We have also indicated that the effective Lagrangian suggested in the Kobayashi-Maskawa paper in 1970 is actually the generalized Nambu-Jona-Lasinio model with the KMT term, which has now been widely used both in QCD phenomenology and in the extensive study of the condensed matter physics of QCD at finite temperature and density. Some focus was put on the recent active studies on the scalar meson dynamics with diquark correlations, which, in turn, can give rise to color superconductivity in high-density quark matter.

Finally, I wish to tell my personal experiences with Professor Maskawa and Professor Kobayashi, particularly that related to their work.[1] I gave a talk on my work on the generalized NJL model with the KMT term at a workshop held at the Research Institute for Fundamental Physics (RIFP)*) from November 4 to 6, 1987. My talk[13] entitled "An Effective Theory of QCD — SU(3)-Nambu-Jona-Lasinio Model Incorporating the Anomaly Term —" consisted of a part that corresponds to §§2 and 3 in the present article and a sketch on the application of the model to finite temperature and density,[75],[76] together with a discussion on the vector mesons. The proceedings of the meeting were published in a Japanese journal called Soryushiron Kenkyu[13] in July 1988. I became aware of the work by Kobayashi and Maskawa[1] and the subsequent work by Kobayashi-Kondo-Maskawa[3] after the domestic meeting, and therefore had not cited their papers in the proceedings.[13] Although I do not remember exactly, my collaborator, Tetsuo Hatsuda, might possibly have told me of their papers; although he was involved in other projects[92] at KEK as a postdoc there, he kindly helped me by my request after the meeting at RIFP to rapidly finalize the paper,[14] which was submitted at the very end of 1987. I remember that in a certain meeting held at RIFP in July or August, 1988, Professor Maskawa sat next to me and, to my surprise, talked to me, and we chatted (in Japanese, of course), roughly as follows:

Maskawa: Are you the author who did the analysis of the determinantal interaction that is reported in the latest Soken?**)

T. K.: Yes, I am.

Maskawa: Some years ago, we made an analysis of the η' and concluded that there must be a six-fermion determinantal interaction for describing the η'.

T. K.: I know of your papers.

This brief chat with Professor Maskawa was a significant event and of great encouragement to me, since I had been feeling that my works were not fully appreciated, although I had a strong confidence in my work, particularly in that presented in the meeting at RIFP, as is described in the Introduction of the proceedings.[13] Professor Maskawa happened to be the director of YITP when I earned a position there in 2000. Unfortunately, I failed to ask him whether he remembered the event described

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*) The former and original name of the Yukawa Institute for Theoretical Physics (YITP).
**) An abbreviation of Soryushiron Kenkyu in Japanese.

above.

Professor Kobayashi was the supervisor of our exercise class on electromagnetism when I was a student at Kyoto University. He was still young and an assistant professor then. Tetsuo Matsui, a former classmate of mine and now at the University of Tokyo, was brave enough to ask Professor Kobayashi to tutor our group who planned to read the textbook on quantum mechanics of Dirac. He kindly accepted our request. That was from 1972 to 1973, which means that although he might have been busy developing the Kobayashi-Maskawa theory on the CP violation,[93] he was kind enough to take the time to supervise us in reading a textbook on quantum mechanics. I was fortunate that Professor Kobayashi was also in charge of the seminar on elementary particle physics when I entered the graduate school of Kyoto University. Moreover, he chose the paper by Nambu-Jona-Lasinio[11] as one of the papers that we were to read and report in the course. I now appreciate how much my career is owed to these two great physicists. It is a great honor and pleasure for me to contribute to this special issue to celebrate the Nobel Prize awarded to Professor Maskawa and Professor Kobayashi by writing an article on the subject through which I crossed paths with them.

Acknowledgements

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