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# On Subsidiary Condition in Quantum Theory of Non-Abelian Gauge Fields

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It is shown that gauge noncovariance character of the condition usually imposed on the divergence of a non-Abelian gauge field is inherited by $S$-matrix elements. The method for proving gauge invariance of quantum electrodynamics is generalized to non-Abelian local gauge transformation groups. That is, the transformation group represented in the Heisenberg picture is translated into a group operating on operators and state vectors in the interaction picture, and then invariance of the Tomonaga-Schwinger equation under the reduced group is examined. A new criterion is also proposed to discuss whether the self-interaction of a non-Abelian gauge field can generate a non-vanishing self-energy.

## § 1. Introduction and summary

Recently the domain of application of non-Abelian gauge theory[1] has been widened to a large extent by introducing the Higgs mechanism,[2)~5)] and the theory has made considerable progress in its technical side.[6),7)] However, two very primitive questions immanent in quantum theory of non-Abelian gauge fields have not been solved, and are deliberately glossed over in some technical approaches. The first question is whether the theory is really invariant under non-Abelian local gauge transformation group. Its origin is in the fact that the divergence of a gauge field is never a covariant expression. Provided the expression is regarded covariant extraordinarily, the gauge function becomes a functional of the gauge field. Moreover even in such a standpoint it is very difficult to insist that the theory is covariant under the group, as was discussed by one of the authors in the radiation gauge formalism.[8] Mathematically the covariant derivative of an arbitrary function, which is technically regarded as transforming as a general field, may be considered as an arbitrary function, but physically it is very hard for us to accept such a view. The second problem is whether the self-interaction of a non-Abelian gauge field can generate a non-vanishing self-energy. There is no definite argument on this problem yet in our opinion. This is because it is very likely that invariance of gauge theories is not guaranteed by their invariance under non-Abelian local gauge transformation groups *in the mathematical sense*, and because in such systems space-time delta functions with vanishing arguments which are not dimensionless appear in quantum field theory.[9] Here we should make notice that there are some attempts to introduce a mass term without violating local gauge invariance.[10),11)] But in this article we are not discussing such a possibility.

There are several kinds of approaches to formulate $S$-matrix theory of non-Abelian gauge fields.[12)~16)] It seems rather questionable, however, whether the arguments on covariance of $S$-matrix elements under non-Abelian local gauge transformation groups presented in some articles are enough, and their arguments are not so transparent as in quantum electrodynamics. The main purpose of the present article is to examine the first question explained above through the most elementary method used in quantum electrodynamics.[17),18)] That is, we shall translate a non-Abelian local gauge transformation group represented in the Heisenberg picture into a group operating on operators and state vectors in the interaction picture through the subsidiary condition in the latter picture, and then examine covariance of the Tomonaga-Schwinger equation under the reduced group. Our result shows that the Tomonaga-Schwinger equation is not invariant under the group in contrast to quantum electrodynamics. Therefore, gauge noncovariance character of the condition usually imposed on the divergence of a non-Abelian gauge field is inherited by $S$-matrix elements. In the next section we shall settle operator relations in the Heisenberg picture for an isolated non-Abelian gauge field and a corresponding set of free vector fields. The operator Lagrange multiplier formalism[19),8)] is taken to describe the gauge fixing term in the lagrangian density operator. Some people are considering formalisms in which the operator Lagrange multiplier is regarded as an independent field operator.[16),20)] However, we shall not take such a standpoint as will be explained at the end of § 2. The argument in the Fermi gauge proceeds in a way almost parallel to the classical work by Tomonaga[17)] and Schwinger.[18)] The hamiltonian operator in the interaction picture is fixed in § 3.1, and then the subsidiary condition in the Heisenberg picture is translated into the one in the interaction picture (§ 3.2). In § 4.1 the reduced group in the interaction picture is determined and covariance of the Tomonaga-Schwinger equation under the group is examined. The hamiltonian operator in the interaction picture has a term which depends on a time-like unit normal vector. But it will be shown in § 4.2 that the normal dependent term is completely cancelled out in $S$-matrix *if the subsidiary condition is put aside*, as in the case of the pseudovector coupling of pion with nucleon.[21)] The Landau gauge formalism is controversial.[20)] The trouble is in generalization of canonical commutation relations of a *free* vector field to a relativistic form. In § 2.2 we shall take the standpoint that the exact correspondence between canonical commutation relations in the Heisenberg picture and the ones in the interaction picture (the ones for the free vector fields) must be taken into account seriously. Then the present problem can be discussed through both the Fermi and the Landau gauge formalisms in a unified way. In the interaction picture the characteristic features of these formalisms appear only in the operator relations for field operators. In other words the Tomonaga-Schwinger equation and the subsidiary condition have the same forms in these formalisms, as will be seen in § 3. Sometimes in the Landau gauge formalism canonical commutation relations of free vector fields are not respected.

Then we are considering Heisenberg operators which cannot be connected with operators in the interaction picture. It is very hard to get a physical image to such an extraordinary situation.

The second problem explained at the beginning is briefly discussed in § 5. The problem is presented as a question whether we can quantize an apparently massless non-Abelian gauge field as a massive vector field, which has *three* independent space-time components, starting with a gauge *invariant* energy-momentum tensor density operator. There the noncovariant condition on the divergence of a gauge field can be renounced.

## § 2. Operator relations in the Heisenberg picture

2.1 In order to discuss the Landau and the Fermi gauge formalisms of quantum theory of a non-Abelian gauge field $\phi_{k\alpha}(x)$ inclusively, we start with the lagrangian density operator

$$\mathcal{L}(x) = -\tfrac{1}{4}G^{kl}G_{kl} - G.(\partial^k\phi_k - \tfrac{1}{2}\alpha G). \qquad (2\cdot1)$$

This is a slight modification of $\mathcal{L}(x)$ given by Schwinger in the Landau gauge formalism.[19),8)] Operator $G_{kl} = \partial_{[k}\phi_{l]} - if\,'t\phi_k'.\phi_l$ is the covariant derivative of the gauge field, and operator $G$ is defined by the Euler equations

$$\nabla_l.G^{kl} = \partial^k G \quad \text{and} \quad \partial_k\phi^k = \alpha G \qquad (2\cdot2\text{a and b})$$

with $\nabla_k = \partial_k - if\,'t\phi_k'$. Parameter $\alpha$ is 0 and 1 for the Landau and the Fermi gauge, respectively. Hermitian matrix $t^\alpha$ is defined as $(t^\alpha)_{\beta\gamma} = -c_{\alpha\beta\gamma}$ through completely anti-symmetric structure constants $c_{\alpha\beta\gamma}$, and symbol '$tA$' means $t^\alpha A_\alpha$. Internal symmetry indices will be omitted for economy of space, unless any confusion is expected. Moreover we have introduced notations $A_{[k}B_{l]} = A_kB_l - A_lB_k$ and $A.B = (AB + BA)/2$, where the matrix element with respect to internal symmetry indices should be taken before the symmetrization procedure. Our metric in Minkowski space is such that $x^0 = -x_0 = t$ and $x^a = x_a$. Letters $a, b, c$, etc., are used for space indices, and indices $k, l, m$, etc., run from 0 to 3. Finally, volume and surface elements are expressed as $d^4x = -dx^k d\sigma_k$ and $d\sigma_k = n_k d\sigma$, respectively, where $n_k$ ($n^0 > 0$) is a time-like unit normal vector to the surface element.

Non-vanishing canonical commutation relations are

$$[\phi^a{}_\alpha(x), G^{0b}{}_\beta(y)] = -i\delta_{\alpha\beta}g^{ab}\delta(\boldsymbol{x}-\boldsymbol{y}) \qquad (2\cdot3\text{a})$$

and

$$[\phi^0{}_\alpha(x), G_\beta(y)] = -i\delta_{\alpha\beta}\delta(\boldsymbol{x}-\boldsymbol{y}), \qquad (2\cdot3\text{b})$$

and all the other commutators among $\phi_k$, $G^{kl}$ and $G$ vanish, especially $[\phi_{k\alpha}(x), \phi_{l\beta}(y)] = 0$ for $x^0 = y^0$. It should be noticed that operator relations (2·2) are consistent with these commutation relations, and that we have $[G_\alpha(x), \partial_0 G_\beta(y)] = 0$ and $[\partial_0 G_\alpha(x), \partial_0 G_\beta(y)] = -f\,'t\partial_0 G'_{\alpha\beta}\delta(\boldsymbol{x}-\boldsymbol{y})$. Local gauge transformation

$\Omega' = \exp(iG_{\delta\lambda})\Omega\exp(-iG_{\delta\lambda})$ is generated by $G_{\delta\lambda} = -\int d\sigma_k j_{\delta\lambda}{}^k$ with

$$j_{\delta\lambda}{}^k = -f^{-1}\{G.(\nabla^k\delta\lambda) - \delta\lambda(\nabla_l.G^{kl}) + \partial_l(\delta\lambda G^{kl})\}. \tag{2·4}$$

Hence we have $\delta\phi^k = f^{-1}\nabla^k\delta\lambda$, $\delta G^{kl} = i\,{}^t\delta\lambda' G^{kl}$ and $\delta G = i\,{}^t\delta\lambda' G$. Operator $\mathcal{L}(x)$ is not invariant under the local gauge transformation unless $\delta\lambda$ is an *operator* satisfying $\nabla_k\partial^k\delta\lambda = 0$. In an earlier article[8] we discussed the symmetry character in the radiation gauge formalism using operator gauge functions $\delta\lambda$. Even in that case, however, the symmetry was proved to be violated in the Heisenberg picture. This time we do not impose the invariance of $\mathcal{L}(x)$, and $\delta\lambda$ is a *numerical* gauge function. Then our system is a self-interacting linear field, and we have

$$\partial_k j_{\delta\lambda}{}^k = -f^{-1}G.(\nabla^k.\partial_k\delta\lambda). \tag{2·5}$$

Operator $G_{\delta\lambda}$ satisfies the relation $-i[G_{\delta\lambda'}, G_{\delta\lambda''}] = G_{\delta\lambda}$ with $\delta\lambda = -i(\delta\lambda'{}^t\delta\lambda'')$, and gauge invariance of physical states can be translated into a subsidiary condition on state vectors

$$G^{(+)}(x)\Phi = 0 \quad \text{for } all \text{ space-time points}, \tag{2·6}$$ *)

as was proposed by Schwinger.[19] For operator $G$ an equation $\nabla_k.\partial^k G = 0$ comes out from Eq. (2·2a), but *we do not regard it as a field operator independent from $\phi^k$ and $G^{kl}$*. Finally displacement operators in Minkowski space are $P_l = \int d\sigma_k \Theta^k{}_l$ with

$$\Theta^k{}_l = -G^{km}.G_{lm} + (\nabla_m.G^{km}).\phi_l + \phi^k.\partial_l G$$
$$+ g^k{}_l\{\tfrac{1}{4}G^{mn}G_{mn} - (\phi^m.\partial_m G + \tfrac{1}{2}\alpha G^2)\}. \tag{2·7}$$

2.2. Before constructing our interaction picture, we should set up relativistic commutation relations for free field $b_{k\alpha}(x)$, which corresponds to the field $\phi_{k\alpha}(x)$ described in § 2.1. Lagrangian density operator $\mathcal{L}^f(x)$, energy-momentum tensor density operator $\Theta^{f,k}{}_l$, which will be used in displacement operators in Minkowski space $P^f{}_l = \int d\sigma_k \Theta^{f,k}{}_l$, the Euler equations and non-vanishing canonical commutation relations among operators $b_k$, $F^{kl} = \partial^{[k}b^{l]}$ and $F$, which is defined by Eqs. (2·10a and b), are given as follows:

$$\mathcal{L}^f(x) = -\tfrac{1}{4}F^{kl}F_{kl} - F.(\partial^k b_k - \tfrac{1}{2}\alpha F), \tag{2·8}$$

$$\Theta^{f,k}{}_l(x) = -F^{km}.F_{lm} + \partial_m F^{km}.b_l + b^k.\partial_l F$$
$$+ g^k{}_l\{\tfrac{1}{4}F^{mn}F_{mn} - (b^m.\partial_m F + \tfrac{1}{2}\alpha F^2)\}, \tag{2·9}$$

$$\partial_l F^{kl} = \partial^k F \quad \text{and} \quad \partial_k b^k = \alpha F, \tag{2·10a and b}$$

$$[b^a{}_\alpha(x), F^{0b}{}_\beta(y)] = -i\delta_{\alpha\beta}g^{ab}\delta(\boldsymbol{x}-\boldsymbol{y}) \tag{2·11a}$$

and

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*) The positive (negative) frequency part of any operator $\chi(x)$ can be defined as $\chi^{(\pm)}(x) = (2\pi i)^{-1}\int_C \chi(x \mp \tau n)d\tau/\tau$, where the contour $C$ is extended from $-\infty$ to $+\infty$, deformed below the singularity at $\tau = 0$.[22]

$$[b^0{}_\alpha(x), F_\beta(y)] = -i\delta_{\alpha\beta}\delta(\boldsymbol{x}-\boldsymbol{y}). \tag{2·11b}$$

Equations (2·10a and b) are equivalent to

$$\Box b_k = (\alpha-1)\partial_k F \quad \text{and} \quad \Box F = 0. \tag{2·12a and b}$$

The initial value problem for Eq. (2·12b) can be solved to give

$$F(x) = -\int d\sigma^l(y)\{D(x-y)\partial_l F(y) + \partial_l D(x-y)F(y)\} \tag{2·13}$$

in terms of function $D(x)$, which is defined by $\Box D(x)=0$, $D(-x)=-D(x)$ and $\partial D(\boldsymbol{x},0)=-\delta(\boldsymbol{x})$ with $\partial = n^k\partial_k$. Then equal-time commutation relations of $F$ with $F$, $b_k$ and $F^{kl}$ can be generalized to relativistic forms

$$[F_\alpha(x), F_\beta(y)] = 0, \tag{2·14}$$

$$[F_\alpha(x), b_{k\beta}(y)] = i\delta_{\alpha\beta}\partial_k D(x-y) \tag{2·15a}$$

and

$$[F_\alpha(x), F^{kl}{}_\beta(y)] = 0. \tag{2·15b}$$

Relativistic commutation relations of $b_k$ with $b_l$ and $F^{kl}$ can be expressed as

$$[b_{k\alpha}(x), b_{l\beta}(y)] = i\delta_{\alpha\beta}\{g_{kl}D(x-y) - a\mathscr{D}_{kl}(x-y)\} \tag{2·16a}$$

and

$$[F_{lm\alpha}(x), b_{k\beta}(y)] = i\delta_{\alpha\beta}g_{k[m}\partial_{l]}D(x-y) \tag{2·16b}$$

with $a=1-\alpha$. It is easy to check consistency of Eq. (2·16) with operator relations (2·10). Function $\mathscr{D}_{kl}(x)$ is defined by equations

$$\Box\mathscr{D}_{kl}(x) = \partial_k\partial_l D(x), \qquad \mathscr{D}_{kl}(x) = -\mathscr{D}_{lk}(-x),$$
$$\mathscr{D}_{kl}(\boldsymbol{x},0) = 0 \quad \text{and} \quad \partial^k\mathscr{D}_{kl}(x) = \partial_l D(x), \tag{2·17}$$

and has properties $\partial^a\mathscr{D}_{a0}(\boldsymbol{x},0) = \partial\mathscr{D}_{kl}(\boldsymbol{x},0) = 0$. The third condition of Eq. (2·17) is necessary to reproduce canonical commutation relation $[b_{k\alpha}(x), b_{l\beta}(y)]=0$ for $x^0=y^0$. In order to settle the interaction picture which is connected with the Heisenberg picture on some space-like surface, exact correspondence of canonical commutation relations in the two picture is essential. It is already known[23] that Eqs. (2·17) can be satisfied by function

$$\mathscr{D}_{kl}(x) = \partial_k\partial_l E(x) \tag{2·18a}$$

with

$$E(x) = -i(2\pi)^{-3}\int d^4p\,\varepsilon(p^0)\delta'(p^2)\exp(ipx), \tag{2·18b}$$

where first the $p^0$-integration should be performed and the meaning of $\delta'(p^2)$ is

$$\delta'(p^2) = \lim_{\mu^2\to 0}(\partial/\partial\mu^2)\delta(p^2+\mu^2). \tag{2·19}$$

When we consider free field $b_k$ in the Heisenberg picture, we should impose a subsidiary condition

$$F^{(+)}(x)\Phi=0 \quad \text{for } all \text{ space-time points} \tag{2·20}$$

on physical states as in § 2.1. Then Eq. (2·12a) means

$$\Box\langle\Phi|b_k|\Phi'\rangle=0\,. \tag{2·21}$$

Condition (2·20), however, will be modified in § 3.2, where $b_k$ is regarded as an operator in the interaction picture. The reason why we do not regard operator $F$ (and then operator $G$) as an independent field is threefold: First commutation relation (2·14) can be hardly regarded as the one for a free field satisfying the Klein-Gordon equation. Secondly, it is very dangerous to read a particle interpretation off only through wave equations without referring to the structure of lagrangian density operator.[24] (If we add the kinetic energy part for $F$ to $\mathcal{L}^f(x)$ when $\alpha>0$, field $F$ should be represented as an operator in a Hilbert space with negative metric.) Thirdly, the zeroth component of Eq. (2·10a) is a constraint. That is, $\partial^0 F$ is completely determined by $F^{0\alpha}$. Our standpoint is to consider $F$ as an operator defined by Eqs. (2·10).

For the free field we have only a global symmetry and current $\hat{j}^k{}_\alpha=-i({}^tb_l{}'.F^{kl}+{}^tb^{k\prime}.F)_\alpha$ is conserved. For the first term $s^k{}_\alpha=-i({}^tb_l{}'.F^{kl})_\alpha$ we have $\partial_k s^k{}_\alpha=i({}^tb_l{}'.\partial^l F)_\alpha$ and $\hat{j}^k.b_k=s^k.b_k$.

## § 3. Interaction picture

3.1. Along with the line explained in an earlier article,[25] we shall settle an interaction picture in which we impose relation $b_k(s)=\phi_k(s)$ at all points $s$ on a standard space-like surface $\tau_s$. (In the following $\tau_x$ means a space-like surface passing through point $x$, and we shall consider only parallel displacement of space-like flat surfaces $\tau_x$, for simplicity.) Then we have

$$b_k(x)=V(\tau_x,\tau_s)\phi_k(x)V(\tau_x,\tau_s)^{-1} \tag{3·1}$$

with $V(\tau_x,\tau_s)=U_f(\tau_x,\tau_s)U(\tau_x,\tau_s)^{-1}$, where $U_f(\tau_x,\tau_s)=\exp(i(\tau_x-\tau_s)P^f)$ with $\tau_x=-n^kx_k$ and $P^f=-n^kP^f{}_k$ and $U(\tau_x,\tau_s)=\exp(i(\tau_x-\tau_s)P)$ with $P=-n^kP_k$. Operator $V(\tau_x,\tau_s)$ satisfies equations

$$i\frac{\partial}{\partial\tau_x}V(\tau_x,\tau_s)=H(\tau_x)V(\tau_x,\tau_s) \quad\text{and}\quad V(\tau_s,\tau_s)=1 \tag{3·2}$$

with hamiltonian $H(\tau_x)=U_f(\tau_x,\tau_s)(P-P^f)U_f(\tau_x,\tau_s)^{-1}$. Defining state vectors by $\Psi(\tau_x)=V(\tau_x,\tau_s)\Psi(\tau_s)$, we get the Tomonaga-Schwinger equation

$$i\frac{d}{d\tau_x}\Psi(\tau_x)=H(\tau_x)\Psi(\tau_x)\,. \tag{3·3}$$

Then we identify $\Psi(\tau_s)$ with state vectors $\Phi$ in the Heisenberg picture. It is easy to derive relations

$$F_{kl}(x)=V(\tau_x,\tau_s)G_{kl}(x)V(\tau_x,\tau_s)^{-1}$$
$$+if(\text{`}tb_k\text{'}.b_l)(x)+n_{[k}B_{l]}(x), \tag{3·4a}$$
$$\partial_k b^k(x)=V(\tau_x,\tau_s)\partial_k\phi^k(x)V(\tau_x,\tau_s)^{-1}+(nB)(x) \tag{3·4b}$$

and

$$\partial_k F(x)=V(\tau_x,\tau_s)\partial_k G(x)V(\tau_x,\tau_s)^{-1}$$
$$+if\{\text{`}tb^{l}\text{'}.(F_{kl}-if\text{`}tb_k\text{'}.b_l-n_{[k}B_{l]}-\partial_l b_k+n_l B_k)\}(x)+\partial^l(n_{[k}B_{l]})(x)$$
$$+if\{\text{`}tb_k\text{'}.(\alpha F-(nB))\}(x)+in^l[H(\tau_x),(F_{kl}-n_{[k}B_{l]})(x)], \tag{3·4c}$$

where $B_k(x)=i[H(\tau_x),b_k(x)]$ and $(nB)=n_kB^k$. Equation (3·4c) is attained by using Eqs. (2·10) and (2·2a), and can be used even if $\alpha=0$. When $\alpha\neq0$ we can get simple relations

$$\alpha F(x)=\alpha V(\tau_x,\tau_s)G(x)V(\tau_x,\tau_s)^{-1}+(nB)(x) \tag{3·5a}$$

and

$$\alpha\partial_k F(x)=\alpha V(\tau_x,\tau_s)\partial_k G(x)V(\tau_x,\tau_s)^{-1}+\partial_k(nB)(x)$$
$$+in_k[H(\tau_x),(\alpha F-(nB))(x)] \tag{3·5b}$$

with the help of Eqs. (2·10b) and (2·2b). However, in order to proceed inclusively with our discussion as far as possible, we shall not use Eqs. (3·5b) to fix operators $B_k(x)$ and $H(\tau_x)$.

To determine operators $B_k(x)$ and $H(\tau_x)$ it is enough for us to fix them on $\tau_s$. Operator $H(\tau_s)$ can be expressed as $H(\tau_s)=H^{(1)}(\tau_s)+H^{(2)}(\tau_s)+H^{(3)}(\tau_s)$ with

$$H^{(1)}(\tau_s)=\int_{\tau_s}d\sigma(n_kn^l+\tfrac{1}{4}g_k{}^l)\{-2F^{km}.(n_{[l}B_{m]}+if\text{`}tb_l\text{'}.b_m)$$
$$+n^{[k}B^{m]}.(n_{[l}B_{m]}+2if\text{`}tb_l\text{'}.b_m)-f^2(\text{`}tb^{k}\text{'}.b^m).(\text{`}tb_l\text{'}.b_m)\},$$
$$H^{(2)}(\tau_s)=\int_{\tau_s}d\sigma(2n_kn^l+g_k{}^l)\{if(F^{km}-if\text{`}tb^{k}\text{'}.b^m-n^{[k}B^{m]}-\partial^m b^k+n^mB^k).(\text{`}tb_l\text{'}.b_m)$$
$$+b_l.(in_m[H(\tau_s),(F^{km}-n^{[k}B^{m]})(s)]+\partial_m(n^{[k}B^{m]}))\}$$

and

$$H^{(3)}(\tau_s)=\alpha\int_{\tau_s}d\sigma(\alpha F-\tfrac{1}{2}(nB)).(nB), \tag{3·6}$$

in terms of operators in the interaction picture. Here it should be noticed that the $\alpha$-dependent term in Eq. (3·4c) does not appear in $H^{(2)}(\tau_s)$ since $(2n_kn^l+g_k{}^l)(b_l.\text{`}tb^{k}\text{'}).(\alpha F-(nB))=0$. Comparing Eq. (2·3a) with Eq. (2·11a) on $\tau_s$, we can presume that $B_l(s)$ commutes with $b_k(s)$, and then we can derive an equation for $B_k(s)$

$$i\int d\sigma(s')b_{l\alpha}(s').\{n_m[F^{lm}{}_\alpha(s'),B_{k\beta}(s)]$$

$$+(g_m{}^l+n_m n^l)[\partial B^m{}_\alpha(s'),b_{k\beta}(s)]\}$$
$$=(1-\varepsilon)n_k(nB)_\beta(s)-2if(`t(nb)'.b_k)_\beta(s) \qquad (3\cdot7)$$

through the definition of $B_{k\alpha}(s)$, where $(nb)=n^k b_k$ and $\varepsilon=1$ and 0 for $\alpha\neq0$ and $\alpha=0$, respectively. It is not difficult to see Eq. (3·7) has a solution

$$B_k=if(`t(nb)'.b_k), \qquad (3\cdot8)$$

which has an important property $(nB)=0$. Therefore, the $\varepsilon$-term in Eq. (3·7) is effectively zero. The term $H^{(3)}(\tau_s)$ depends on $\alpha$, and Eq. (3·5a) has been used to get the form (3·6). However, neither this term survive. Thus $B_k(s)$ and $H(\tau_s)$ do not depend explicitly on $\alpha$. It is easy to check that solution (3·8) correctly connects equations of motion and canonical commutation relations in the Heisenberg and the interaction pictures with each other.

Substituting solution (3·8) into Eq. (3·6), we can express $H(\tau_s)$ in the form $H(\tau_s)=X(\tau_s)+[H(\tau_s),Y(\tau_s)]$ with $Y(\tau)=i\int_\tau d\sigma_l(b_k.F^{kl})$ and $X(\tau)=if\int_\tau d\sigma\cdot\{A_1+3A_2-if(3/2)(\frac{1}{2}A_3+A_4)\}$, where $A_1=(`tb_k'.b_l).F^{kl}$, $A_2=(`t(nb)'.b_l).(n_k\cdot F^{kl})$, $A_3=(`tb_k'.b_l).(`tb^{k}{}'.b^l)$ and $A_4=(`t(nb)'.b_k).(`t(nb)'.b^k)$. Equation (3·6) can be solved by assuming a form $H(\tau)=if\int_\tau d\sigma\{a_1A_1+a_2A_2+if(a_3A_3+a_4A_4)\}$, because we have commutation relations $[Y(\tau_s),A_1(s)]=-3(A_1+2A_2)(s)$, $[Y(\tau_s),A_2(s)]=0$, $[Y(\tau_s),A_3(s)]=-4(A_3+A_4)(s)$ and $[Y(\tau_s),A_4(s)]=-2A_4(s)$. The solution is $a_1=-1/2$, $a_2=0$, $a_3=1/4$ and $a_4=1/2$, and then

$$H(\tau)=-\tfrac{1}{2}if\int_\tau d\sigma\{F^{kl}.(`tb_k'.b_l)-if(n_kn^l+\tfrac{1}{2}g_k{}^l)(`tb_l'.b_m).(`tb^{k}{}'.b^m)\} \qquad (3\cdot9)$$

for any space-like surface $\tau$. After having fixed the hamiltonian operator, we can get a relation between $F(x)$ and $G(x)$, although for the case with $\alpha=0$ $F(x)$ and $G(x)$ themselves are not explicitly determined through the Euler equations. First by Eq. (3·4c) we have

$$\partial_kF(x)=V(\tau_x,\tau_s)\partial_kG(x)V(\tau_x,\tau_s)^{-1}-ifn_kn^l(`tb^{m}{}'.F_{lm}). \qquad (3\cdot10)$$

and then $\partial^t{}_kF(x)=V(\tau_x,\tau_s)\partial^t{}_kG(x)V(\tau_x,\tau_s)^{-1}$, where $\partial^t{}_k=(g_k{}^l+n_kn^l)\partial_l$. This relation suggests us a more fundamental relation

$$F(x)=V(\tau_x,\tau_s)G(x)V(\tau_x,\tau_s)^{-1} \qquad (3\cdot11)$$

and it is easy to see that Eq. (3·11) correctly reproduces Eq. (3·10). When $\alpha\neq0$, the last relation is simply a result of property $(nB)=0$.

Finally it should be emphasized that operators $B_k(x)$ and $H(\tau_x)$ do not depend on parameter $\alpha$. This is due to the third condition in Eq. (2·17), especially due to commutability of $b_0(x)$ with $b_a(y)$ on a space-like surface.

3.2. First the subsidiary condition in the Heisenberg picture must be translated into the one on state vectors in the interaction picture. When point $x$ is on surface $\tau_x$, Eq. (2·6) simply means $F^{(+)}(x)\Psi(\tau_x)=0$. This is, however, not enough since condition (2·6) also requires $\partial_kG^{(+)}(x)\Phi=0$. This requirement can be expressed as

$$\Omega^{(+)}(x,\tau)\Psi(\tau)=0 \quad \text{for } \textit{all} \text{ space-time points } x, \tag{3·12}$$

where $\Omega(x,\tau)=\Omega_1(x)+\Omega_2(x,\tau)$ with

$$\Omega_1(x)=F(x) \quad \text{and} \quad \Omega_2(x,\tau)=-f\int_\tau d\sigma^k(y)D(x-y)s_k(y). \tag{3·13}$$

Operator $s_k$ was defined in § 2.2. The argument to get generalized form (3·12), where point $x$ is generally not on surface $\tau$, is completely parallel to the one in quantum electrodynamics.[18] The key point is Eq. (2·13). It is not so difficult to check consistency among condition (3·12) for *all* space-time points

$$[\Omega^{(+)}(x,\tau),\Omega^{(+)}(y,\tau)]\Psi(\tau)=0 \tag{3·14}$$

for any pair of space-time points $x$ and $y$ and consistency between Eqs. (3·12) and (3·3)

$$\left\{i\frac{d}{d\tau}\Omega^{(+)}(x,\tau)-[H(\tau),\Omega^{(+)}(x,\tau)]\right\}\Psi(\tau)=0. \tag{3·15}$$

Moreover property

$$\partial_k\partial^k\Omega(x,\tau)=0 \tag{3·16}$$

will also be used later. Here it should be noticed that operand $\Psi(\tau)$ is necessary in Eqs. (3·15) and (3·16) in contrast to the case of quantum electrodynamics. It is because we have

$$[\Omega(x,\tau)_\alpha,\Omega(y,\tau)_\beta]=-fV(\tau,\tau_s)\int_\tau d\sigma(z)D(x-z)\text{`}t\partial G(z)\text{'}_{\alpha\beta}V(\tau,\tau_s)^{-1} \tag{3·17}$$

and

$$i\frac{d}{d\tau}\Omega(x,\tau)|_{x\Lambda\tau}=[H(\tau),\Omega(x,\tau)]$$
$$-fV(\tau,\tau_s)\int_\tau d\sigma(y)D(x-y)(\text{`}t\phi^{k}\text{'}.\partial_k G)(y)V(\tau,\tau_s)^{-1}. \tag{3·18a}$$

In Eq. (3·18a) $x\Lambda\tau$ means that point $x$ is not on surface $\tau$. When point $x$ is on surface $\tau$ the left-hand side must be supplemented with another term

$$\frac{d}{d\tau}\Omega_1(x)-f\int_\tau d\sigma^k(y)\partial D(x-y)s_k(y)=V(\tau,\tau_s)\partial G(x)V(\tau,\tau_s)^{-1}, \tag{3·18b}$$

since operator $d/d\tau$ must be operated also on $x$ and then $d/d\tau=\partial$.

Finally we make a notice that the first term of $H(\tau)$ can be expressed as $(1/2)f\int_\tau d\sigma\hat{j}^k.b_k$ in terms of the conserving current $\hat{j}^k$ but the operator $s_k$ in Eq. (3·13) cannot be replaced by $\hat{j}_k$.

## § 4. Local gauge transformation in interaction picture and Matthews' theorem

4.1. First we consider the physical meaning of subsidiary condition (3·12) in the severest form: Any observable quantities defined on surface $\tau$, $\chi(\tau)$, should satisfy relations

$$[\chi(\tau), \Omega(x,\tau)]=0 \quad \text{and} \quad [\chi(\tau), \partial_k\Omega(x,\tau)]=0\,. \tag{4·1}$$

(Owing to property (3·16) we need not consider higher derivatives of $\Omega(x,\tau)$.) Equation (4·1) can be unified into the form

$$[\chi(\tau), \Gamma(\tau)]=0 \tag{4·2}$$

by introducing a generating operator

$$\Gamma(\tau)=f^{-1}\int_\tau d\sigma^k(x)\{\partial_k\lambda_\alpha(x)\Omega(x,\tau)_\alpha-\lambda_\alpha(x)\partial_k\Omega(x,\tau)_\alpha\} \tag{4·3}$$

with an arbitrary numerical function $\lambda_\alpha(x)$. In Eq. (4·3) operator $\partial_k\Omega(x,\tau)$ should be understood as a sum of two terms

$$\partial_k\Omega_1(x)=\partial_k F(x) \quad \text{and} \quad \partial_k\Omega_2(x,\tau)=-n_k(ns)(x)\,. \tag{4·4}$$

Now any observable quantities are functional of operators $b_k$, $F^{kl}$ and $F$. Therefore, it is interesting to examine transformation laws $\chi'(x)=\exp(i\Gamma(\tau_x))\chi(x)\exp(-i\Gamma\cdot(\tau_x))$ of these operators. The result is

$$i[\Gamma(\tau_x), b^k(x)]=f^{-1}\{\partial^k\lambda-if(g^{kl}+n^kn^l)`tb_l'\lambda\}(x), \tag{4·5a}$$

$$i[\Gamma(\tau_x), F^{kl}(x)]=i`t\lambda'F^{kl}(x)$$
$$-i\{`tb^{[l}{'}\partial^{k]}+n^{[k}(`tb^{l]}{'}\lambda-`t(nb)'\partial^{l]}\lambda)\}(x) \tag{4·5b}$$

and

$$i[\Gamma(\tau_x), F(x)]=0\,. \tag{4·5c}$$

It is interesting to see that operator $\widehat{G}^{kl}=F^{kl}-if`tb^{k}{'}.b^l-n^{[k}B^{l]}$ has a very simple property

$$i[\Gamma(\tau_x), \widehat{G}^{kl}(x)]=i`t\lambda'\widehat{G}^{kl}(x)\,. \tag{4·6}$$

The asymmetry with respect to the time-like and the space-like components of $b^k$ in Eq. (4·5a) and the too simple property of Eq. (4·5c) come from the fact that in $\Omega_2(x,\tau)$ operator $s^k$ appears instead of conserving current $\hat{j}^k$. In other words, provided we consider a modified operator $\hat{\Gamma}(\tau)$. which is obtained from $\Gamma(\tau)$ by the substitution of $s^k$ with $\hat{j}^k$, we have commutation relations

$$i[\hat{\Gamma}(\tau_x), b^k(x)]=f^{-1}(\nabla^k\lambda)(x),$$

$$i[\hat{\Gamma}(\tau_x), F^{kl}(x)]=i[\Gamma(\tau_x), F^{kl}(x)],$$

$$i[(\hat{\Gamma}(\tau_x), F(x)]=i(`t\lambda'F)(x)$$

and

$$i[\hat{\Gamma}(\tau_x), \widehat{G}^{kl}(x)]=i(`t\lambda'\widehat{G}^{kl})(x)\,. \tag{4·7}$$

Now, after the usual attitude in quantum electrodynamics, we shall alter condition (4·2) into invariance of matrix elements of observables. That is, we divide our transformation rule into two parts;

$$\chi'(x)=\exp(i\Gamma_1(\tau_x))\chi(x)\exp(-i\Gamma_1(\tau_x))$$

and

$$\Psi'(\tau_x)=\exp(-iG(\tau_x))\Psi(\tau_x), \tag{4·8}$$

where

$$G(\tau)=\Gamma_2(\tau)+\frac{1}{2}i[\Gamma_1(\tau),\Gamma_2(\tau)]=\Gamma_2(\tau)+\frac{i}{2}f^{-1}\int_\tau d\sigma^k(x)(\partial^l\lambda`tF_{kl}'\lambda)(x) \tag{4·9}$$

and

$$\Gamma_i(\tau)=f^{-1}\int_\tau d\sigma^k(x)\{\partial_k\lambda_\alpha(x)\Omega_i(x,\tau)_\alpha-\lambda_\alpha(x)\partial_k\Omega_i(x,\tau)_\alpha\}.\ (i=1,2) \tag{4·10}$$

In Eq. (4·10) $\Omega_1(x,\tau)$ should be understood as $\Omega_1(x)$. Transformation law (4·8) means

$$b^{k\prime}=b^k+f^{-1}\partial^k\lambda,\ F'=F \text{ and } F^{kl\prime}=F^{kl}. \tag{4·11}$$

In order that wave equations and canonical commutation relations are invariant under this transformation, we must restrict gauge function $\lambda$ by condition

$$\partial^k\partial_k\lambda=0. \tag{4·12}$$

Finally we should examine invariance of Eq. (3·3) under transformation (4·8). Unfortunately, this equation is proved not to be invariant. The transformed equation is

$$i\frac{d}{d\tau}\Psi(\tau)=\left\{-ie^{iG(\tau)}\frac{d}{d\tau}e^{-iG(\tau)}+e^{i\Gamma(\tau)}H(\tau)e^{-i\Gamma(\tau)}\right\}\Psi(\tau), \tag{4·13}$$

and violation is already observed in the first order of $\lambda(x)$:

$$-\frac{d}{d\tau}G(\tau)+i[\Gamma(\tau),H(\tau)]=\int_\tau d\sigma(x)\{-\lambda\partial^k s_k-if((ns).`t(nb)'\lambda)$$
$$+if^2(\tfrac{1}{2}g^{kl}+n^kn^l)(b_m.(`tb_l'.(`t(`tb^{m}'.b_k)'\lambda)))\}(x). \tag{4·14}$$

4.2. The Tomonaga-Schwinger equation (3·3) can be integrated to give $S$-matrix

$$S=T\exp\left[-i\int d^4x\mathcal{H}(x)\right] \tag{4·15}$$

with $\mathcal{H}(x)=\mathcal{H}_1(x)+\mathcal{H}_2(x)+A(x;n)$, where

$$\mathcal{H}_1(x)=-if\partial^kb^l.(`tb_k'.b_l),\qquad \mathcal{H}_2(x)=\tfrac{1}{4}(if)^2(`tb^k'.b^l)(`tb_k'.b_l)$$

and

$$A(x;\ n)=\tfrac{1}{2}(if)^2(`t(nb)'.b^k)(`t(nb)'.b_k). \tag{4·16}$$

The structure of our hamiltonian is very similar to the pseudovector coupling of pion with nucleon, and $n$-dependent term $A(x;\, n)$ can be eliminated by introducing the $T^*$-product notation

$$S = T^* \exp\left[-i\int d^4x(\mathcal{H}_1(x)+\mathcal{H}_2(x))\right]. \tag{4·17}$$

In other words Matthews' theorem does hold for our system. For instance, we have $T(\mathcal{H}_i(x),\, \mathcal{H}_j(y)) = \frac{1}{2}\{\mathcal{H}_i(x),\, \mathcal{H}_j(y)\} + \varepsilon(x-y)/2[\mathcal{H}_i(x),\, \mathcal{H}_j(y)]$, and $\varepsilon(x-y)[\partial_k b_{l\alpha}(x),\, b_{m\beta}(y)] = \partial^x{}_k\{\varepsilon(x-y)[b_{l\alpha}(x),\, b_{m\beta}(y)]\}$ and $\varepsilon(x-y)[\partial_k b_{l\alpha}(x),\, \partial_m b_{n\beta}(y)] = \partial^x{}_k\partial^y{}_m\{\varepsilon(x-y)[b_{l\alpha}(x),\, b_{m\beta}(y)]\} - 2ig^0{}_k g^0{}_m g_{ln}\delta_{\alpha\beta}\delta^4(x-y)$. Hence we have $T(\mathcal{H}_1(x),\mathcal{H}_2(y)) = T^*(\mathcal{H}_1(x),\, \mathcal{H}_2(y))$ and $T(\mathcal{H}_1(x),\, \mathcal{H}_1(y)) = T^*(\mathcal{H}_1(x),\, \mathcal{H}_1(y)) - 2i\delta^4(x-y)A(x;\, n)$. The factor $-2i$ should be multiplied by another factor $-i/2$ coming from the expansion of Eq. (4·15), and the first order contribution from $A(x;\, n)$ is cancelled out. Such an analysis can be extended to all orders.

The absence of vector $n$ in Eq. (4·17), however, does not immediately mean that $S$-matrix *elements* are $n$-independent. This is because subsidiary condition (3·12), which depends on vector $n$, is not taken into account in integrating Eq. (3·3) to get Eq. (4·15). The most fundamental way to deal with the subsidiary condition is to solve Eq. (3·3) after shifting to the radiation gauge. For instance, in the Fermi gauge formalism it is enough for us to decompose the gauge field operator into $b_k = -n_k\partial\Lambda + \partial^t{}_k\Lambda' + \mathcal{B}_k$ and to eliminate variables $\Lambda$ and $\Lambda'(\mathcal{B}_k)$ from Eq. (3·3) (from Eq. (3·12)) through a unitary transformation. (Operators $\Lambda$, $\Lambda'$ and $\mathcal{B}_k$ are defined by properties $n^k\mathcal{B}_k = \partial^k\mathcal{B}_k = \Box\mathcal{B}_k = 0$ and $\Box\Lambda = \Box\Lambda' = 0$). This transformation is not so simple in the present case as in quantum electrodynamics, since operator $s_k$ is a function of the gauge field itself and is not conserved although we have $[(ns)_\alpha(x),\, (ns)_\beta(y)] = -\delta(x-y)\,{}^\backprime t(ns)'_{\alpha\beta}$ when $x^0 = y^0$. (The radiation gauge formalism was already discussed in the most compact form by starting with the Heisenberg picture described in the gauge.[26),8)]) When $S$-matrix elements in the radiation gauge are translated back into the Landau or the Fermi gauge description, it is expected that the ghost in the path-integral formalism[14)] will come in.

## § 5. Is it possible to build up a massive quanta formalism of apparently massless non-Abelian gauge fields?

It is a generally accepted opinion that non-Abelian gauge fields cannot be massive since there is no parameter with dimension of length in the gauge invariant lagrangian density. However, this argument is not enough. First the self-interaction of a non-Abelian gauge field is non-linear, and then the interaction itself and higher order effects essentially involve delta functions coming from the order of operators, which is not dimensionless. (See the argument on quantum field theory of non-linear realization of a group on its sub-group.[9)]) Secondly, it is very questionable that quantum theory of non-Abelian gauge field is really invariant, as was discussed in the preceding section and in an earlier article.[8] Thus, as one of reliable criterions, we may ask whether we can quantize an apparently massless non-Abelian gauge field as a massive vector field which has three independent space-time components starting with gauge invariant energy-momentum tensor density operator. The classical constraint $\nabla_a G^{0a}=0$ diminishes the number of independent components of vector field by one, and then we can thraw additional condition $\partial_k\phi^k=0$, which is an origin of the trouble in discussing covariance of non-Abelian gauge field theory. Here we should make notice that we shall evade the canonical quantization method and use the method based on characteristics of displacement operators in Minkowski space.[27),8)] This is because we are discussing the possibility itself of the quantization of the kind explained above.

Along the line described above, first we consider the most primitive energy-momentum tensor density operator

$$T^k{}_l(x)=\{-G^{km}.G_{lm}+\tfrac{1}{2}\beta g^k{}_l G^{mn}G_{mn}\}(x), \tag{5·1}$$

where parameter $\beta$ should be determined through condition $i\partial_0\phi_a=[P_0, \phi_a]$. This operator is defined uniquely by the conditions that $T^k{}_l$ is a second-rank Lorentz tensor in the sense of classical field theory, and that it is invariant under our local gauge transformation group in the mathematical sense. Moreover the operator must be hermitian, and involve covariant derivative $G^{kl}$ at most in bilinear forms. Displacement operators $P_k=-\int d\boldsymbol{x}T^0{}_k$ can be expressed as

$$P_a=\int d\boldsymbol{x}\{(\nabla_b.G^{0b}).\phi_a+G^{0b}.\partial_a\phi_b+if\tfrac{1}{4}[[G^{0b}{}_\alpha,\phi_{a\beta}],\text{‘}t\phi_b\text{’}_{\alpha\beta}]\}$$

and

$$P_0=\int d\boldsymbol{x}\{(1-\tfrac{1}{2}\beta)((\nabla_b.G^{0b}).\phi_0+G^{0b}.\partial_0\phi_b+if\tfrac{1}{4}[[G^{0b}{}_\alpha,\phi_{0\beta}],\text{‘}t\phi_b\text{’}_{\alpha\beta}])-\tfrac{1}{4}\beta(G^{ab})^2\}. \tag{5·2}$$

Requirements $i\partial_a\phi_b=[P_a,\phi_b]$ and $i\partial_a G^{0b}=[P_a, G^{0b}]$ give pairs of two commutation relations

$$[\phi_{b\alpha}(x), G^{0a}{}_\beta(y)]=-i\delta_{\alpha\beta}g_b{}^a\delta(\boldsymbol{x}-\boldsymbol{y}), \tag{5·3a}$$

$$\left[\phi_{b\alpha}(x),\ \int d\boldsymbol{y}((\nabla_a.G^{0a}).\phi_c)(y)\right]=0 \tag{5·3b}$$

$$[G^{0b}{}_\alpha(x), G^{0a}{}_\beta(y)]=0 \tag{5·4a}$$

and

$$\left[G^{0b}{}_\alpha(x),\ \int d\boldsymbol{y}((\nabla_a.G^{0a}).\phi_c)(y)\right]=0, \tag{5·4b}$$

respectively. Equations (5·3b) and (5·4b) mean that operator $\int d\boldsymbol{y}((\nabla_a G^{0a}).\phi_c)$ is a $c$-number since we are considering the set of operators $\phi_a$ and $G^{0a}$ to be complete.

Moreover this fact implies that relation

$$\nabla_a G^{0a} = 0 \tag{5·5}$$

should hold as an operator relation. This is because operator $\phi_{c\beta}$ in the above expression is arbitrary and we have no $c$-number with an internal symmetry index. (In addition to this, it is easy to see $\partial_k(\nabla_a G^{0a})=0$.) Now it is self-evident that operator relation (5·5) is inconsistent with Eq. (5·3a) and then expression (5·1) does not fit in with our problem. This kind of trouble does not occur in the radiation gauge formalism.[8] In deriving Eqs. (5·3) and (5·4), equal-time commutation relation $[\phi_{a\alpha}(x), \phi_{b\beta}(y)]=0$ is enough. Provided we regard $\phi_0$ as an independent component, $\phi_0$ should commute also with $\phi_k$ and $G^{0a}$ and then we have $\partial_a\phi_0=0$. It is interesting to examine conditions which must be satisfied by the Lorentz covariant quantum field theories of this kind,

$$[T^{00}(x), T^{00}(y)] = i\partial_a\delta(\boldsymbol{x}-\boldsymbol{y})(T^{0a}(x)+T^{0a}(y)) \tag{5·6}$$

and

$$[T^0{}_a(x), T^0{}_b(y)] = i\{\partial_a\delta(\boldsymbol{x}-\boldsymbol{y})T^0{}_b(x)+\partial_b\delta(\boldsymbol{x}-\boldsymbol{y})T^0{}_a(y)\}+\tau_{ab}(x,y), \tag{5·7}$$

where $\tau_{ab}$ should have properties $\tau_{ab}(x,y)=-\tau_{ba}(y,x)$ and $\int d\boldsymbol{x}\tau_{ab}(x,y)=\int d\boldsymbol{x}\cdot(x_a\tau_{bc}(x,y)-x_b\tau_{ac}(x,y))=0$. In the present case Eq. (5·6) does hold and we have $\tau_{ab}(x,y)=i\delta(\boldsymbol{x}-\boldsymbol{y})(\nabla_c.G^{0c}).G_{ab}$. Therefore, Eq. (5·5) must be an operator relation in order that $\tau_{ab}$ should have the required properties.

One may imagine that the trouble explained above can be resolved by modifying the expression of $T^{0k}$ into $\widehat{T}^0{}_k = T^0{}_k + (\nabla_a.G^{0a}).\phi_k$ at the expense of covariance of $\widehat{T}^0{}_k$ with respect to our group, and by introducing a subsidiary condition $(\nabla_a.G^{0a})^{(+)}\Phi=0$. However, even this alternative does not give a consistent massive quanta formalism. In order to realize the relation $i\partial_0\phi_a=[\widehat{P}_0, \phi_a]$, we must assume $[\phi_0(x), \phi_a(y)]=0$ and then we have $i\partial_a\phi_0(x)=[\widehat{P}_a, \phi_0(x)]=\int d\boldsymbol{y}[\phi_0(x), G^{0b}{}_\alpha(y)].\partial_a\phi_{b\alpha}(y)$. It would be impossible that the last integral reproduces the left-hand side. Finally, this time we have no trouble about Eq. (5·7) since $\tau_{ab}=0$, but we cannot have satisfactory result for relation (5·6). Operator $\Theta^l{}_k$ given in Eq. (2·7) is a modification of $\widehat{T}^l{}_k$ which gives a self-consistent scheme and will be unique.

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**Note added:**

After completion of this work, Mr. K. Shizuya called our attention to the article written by T, Goto and R. Utiyama [Prog. Theor. Phys. Suppl. Nos. 37 & 38 (1966), 323], where the interaction picture for the non-Abelian gauge field theory was studied in the Fermi gauge. The discussion presented here is more exhaustive and critical than the earlier study.