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Meson-Baryon and Baryon-Antibaryon Ratios in Two Way Quark-Cascade Model

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Particle ratios at large p_T, including baryons and antibaryons, in energetic hadron collisions are studied in the two way quark-cascade model[1] based on simple quark counting. Model predictions are compared with the data for the ratios of the cross sections $p+p$ (or W)$\rightarrow h+X$, where h can be any of $(p, \bar{p}, \pi^+, \pi^-)$.

On the basis of the basic notions of hadron structure and interaction, two way quark-cascade model was proposed[1] by one (S.M.) of the present authors in order to explain the large p_T particle composition data of the Chicago-Princeton group.[2] The model has also been applied to various kinds of scattering processes including on-shell photon projectile.[3] The analyses lend strong support to the quark model of hadrons and to the applicability of naive quark counting rule[4] for large p_T production of hadrons. In all the previous studies, however, only the meson emission has been treated. In looking at the reported data, protons or antiprotons are also produced considerably, so that it is very interesting and urgent to include baryon emission in the idea of two way quark-cascade.

The purpose of this paper is to present a possible way of treating the baryon emission and try to explain the observed particle ratios of baryons to antibaryons and baryons to mesons in terms of the two way quark-cascade model. We shall compare our results with the data taken by the Chicago-Princeton Collaboration at FNAL[2] and by the British-Scandinavian Collaboration at ISR.[5]

In the approach of the two way quark-cascade model, large angle scattering goes through incoherent hard scattering between the two quarks initially present inside the colliding particles. Then the scattered quark starts cascade before being emitted as an energetic hadron. As a dominant process of first order approximation, we assume "two way cascade" for the scattered quark; a *soft* cascade with soft hadron emission allowing the produced quark to carry away most of the energy of the parent quark and a *hard* cascade of hard hadron emission with almost all energy of the parent quark taken away by the emitted hadron leaving the wee quark only to be absorbed by the scattering remnants. We postulate that the cascade process starts to proceed with soft cascades within the interaction region and terminates once a hard cascade develops. The essential point is that the soft cascade steps determine the quark distribution functions ("quark function") within the hadronic matter. Then the observed distribution of various hadrons with large

p_T will be proportional to the corresponding quark function through one hard cascade step.*)

One way to handle various hadron emissions including baryon and antibaryon emissions will be the following. In the first place one divides the emitted hadrons into three sectors corresponding to baryon-sector, antibaryon-sector and meson-sector and tries to determine the relative probabilities of producing baryons, anti-baryons and mesons at finite x_T *without particle identification* for $SU(3)$ quantum number. Then within each sector the relative emission probability of hadrons with various $SU(3)$ quantum numbers will be obtained by multiplying the corresponding relative weights with respect to $SU(3)$ quantum number within the sector. Similar methods of treating meson and baryon emissions were employed by Bjorken and Farrar[6] and other people.[7] Their approaches, however, are more dependent on statistical quark counting and the underlying dynamics is quite different from ours. Further, their concern was not on large p_T production.

For hadrons observed at 90° in the center-of-mass system, we have $p_{\parallel}=0$. Therefore, the relevant variable here is

$$x_T=\frac{2p_T}{\sqrt{s}}$$

or

$$y_T=\frac{1}{2}\ln\frac{\sqrt{p_T{}^2+m^2}+p_T}{\sqrt{p_T{}^2+m^2}-p_T},$$

where m is the mass of the observed hadron and s is the total energy squared in the C.M. system. For finite x_T and large s we have

$$e^{y_T}\simeq\frac{\sqrt{s}}{m}x_T$$

and

$$dy_T=\frac{dx_T}{x_T}.$$

Now we shall consider the problem of determining the relative probabilities of producing hard particles with various baryon numbers corresponding to baryon, antibaryon and meson without particle identification for $SU(3)$ quantum number. For this purpose the following picture[8] will be sufficient. Imagine that the "quarks" are $SU(3)$ singlets, carrying only baryon number, $+1/3$ for quarks and $-1/3$ for antiquarks. Now we shall suppose that the scattered quark starts the following cascades in two ways (i.e., soft and hard) (see Fig. 1): q emits a meson M and goes on as a q, or emits a baryon B and goes on as a $\bar{q}\bar{q}$ state. On the other hand, we shall assume that the $\bar{q}\bar{q}$ state can emit a meson M and goes on

*) For the details we refer the reader to the original paper (Ref. 1)).

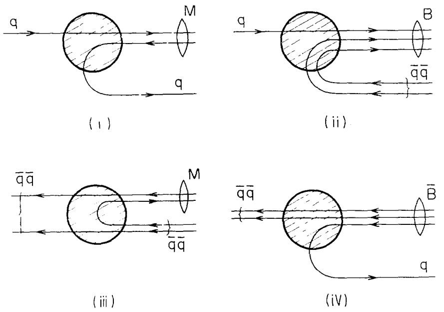

Fig. 1. Cascade scheme. The initial quark q (i) emits a meson and goes on as a q, or (ii) emits a baryon and goes on as a $\bar{q}\bar{q}$ state, while a $\bar{q}\bar{q}$ state (iii) emits a meson and goes on as a $\bar{q}\bar{q}$, or (iv) emits an antibaryon and goes on as a q. Furthermore the above four cascades are assumed to proceed in two ways, i.e. soft or hard.

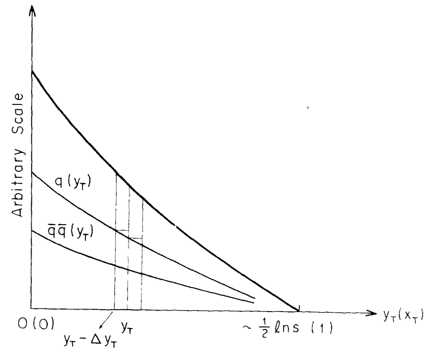

Fig. 2. Quark distribution along transverse rapidity y_T. The distribution is assumed to be sharply bound by the over-all p_T cut-off.

as $\bar{q}\bar{q}$, or emit an antibaryon $\overline{B}$ and goes on as a q.*)

On the basis of the above picture it is straightforward to construct equations for determining the corresponding quark functions. The procedure is essentially the same as in the case with only meson emissions.[1] Let us introduce the quark functions in y_T space as

$$Q(y_T) = \begin{pmatrix} q(y_T) \\ \bar{q}\bar{q}(y_T) \end{pmatrix},$$

where $Q(y_T)$ generally has s dependence as well. The quark distributions will be given by what is illustrated in Fig. 2. Dividing the y_T space into discrete cells and taking into account the quark number conservation as a consequence of soft and hard emissions, the following equations are suggested at a given cell according to Ref. 1).

$$q(y_T - \varDelta y_T) - q(y_T) = g(y_T)\varDelta y_T q(y_T) - h\varDelta y_T q(y_T)$$
$$- w(B)_q \varDelta y_T q(y_T) + w(\overline{B})_{\bar{q}\bar{q}} \varDelta y_T \bar{q}\bar{q}(y_T),$$

*) In this paper we have adopted a closed system for quark cascade, where we take the base of the quark function to be $\begin{pmatrix} q \\ \bar{q}\bar{q} \end{pmatrix}$. However, there are other various possibilities. For example, it is possible to take a base of $\begin{pmatrix} q \\ \bar{q} \end{pmatrix}$ and to assume that each single quark or antiquark develops cascade independently. But if we accept that the overall p_T cut-off function g and the strength of the hard vertex h are common for q and $\bar{q}$ we come to see that choosing $\begin{pmatrix} q \\ \bar{q} \end{pmatrix}$ as the base is not able to explain the date, especially the steep rise at smaller x_T of the $p/\bar{p}$ ratio whatever parameter values we choose. Anyway the dynamics of large p_T phenomena is not yet understood completely and it is an open question what scheme we should choose.

$$\bar{q}\bar{q}(y_T - \Delta y_T) - \bar{q}\bar{q}(y_T) = g(y_T)\Delta y_T \bar{q}\bar{q}(y_T) - h\Delta y_T \bar{q}\bar{q}(y_T)$$
$$- w(\bar{B})_{\bar{q}\bar{q}}\Delta y_T \bar{q}\bar{q}(y_T) + w(B)_q \Delta y_T q(y_T), \tag{1}$$

where the first term on the right-hand side represents the effect of the overall p_T cut-off which is supplied by some unknown interaction mechanism. This term will give rise to the well-known p_T cut-off of observed hadrons. The second term represents the total loss due to the hard emission. The quantities $g(y_T)$ and $h(y_T)$ are taken to be common for q and $\bar{q}\bar{q}$.*) The third and fourth terms represent the $q(\bar{q}\bar{q})$ changing into $\bar{q}\bar{q}(q)$ through the soft emission of baryon (antibaryon) respectively. We also remark that the contributions due to the soft emission of mesons from q and $\bar{q}\bar{q}$ cancel in the difference, the left-hand side of Eq. (1). Now we denote the strengths of the emission vertices corresponding to $w(M)_q$, $w(M)_{\bar{q}\bar{q}}$, $w(B)_q$ and $w(\bar{B})_{\bar{q}\bar{q}}$, as

$$w(M)_q = \alpha\,, \quad (q \to Mq) \qquad w(M)_{\bar{q}\bar{q}} = \gamma\,, \quad (\bar{q}\bar{q} \to M\bar{q}\bar{q})$$
$$w(B)_q = \beta\,, \quad (q \to B\bar{q}\bar{q}) \qquad w(\bar{B})_{\bar{q}\bar{q}} = \delta\,. \quad (\bar{q}\bar{q} \to \bar{B}q)$$

All quantities which have been introduced in the above argument are generally dependent on s as well as on y_T. Then, by taking the limit of $\Delta y_T \to 0$ in Eq. (1), the differential equation for the quark function is given in a simple form by

$$\frac{dQ(y_T)}{dy_T} = -SQ(y_T)\,, \tag{2}$$

where

$$S = (g-h)I + T\,,$$
$$T = \begin{pmatrix} -\beta & \delta \\ \beta & -\delta \end{pmatrix}.$$

It is straightforward to solve the above equation (2). The eigenvalues λ_i and the corresponding eigenvectors v_i of T are

$$\lambda_1 = 0\,, \quad \lambda_2 = -\beta - \delta\,,$$
$$v_1 = \begin{pmatrix} \delta \\ \beta \end{pmatrix}, \quad v_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

The corresponding eigenvalues μ_i of S are then given by

$$\mu_i = \lambda_i + g - h\,.$$

In the following α, β, γ and δ will be assumed to be constant.**) Then the solution satisfying Eq. (2) will be given by

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*) This will give a universal p_T cut-off for hard hadron emission which seems to be supported by the present data.

**) The minimum requirement for the following treatment is actually that β/δ be constant in y_T.

$$Q(y_T) = G(y_T)\sum_{i=1}^{2} a_i \exp\left(\int_{y_T}^{y_{T\max}} \lambda_i dy_T\right) v_i = G(y_T)\sum_{i=1}^{2} a_i x_T^{-\lambda_i} v_i\,, \tag{3}$$

where

$$G(y_T) = \exp\left(\int_{y_T}^{y_{T\max}} (g-h)\, dy_T\right)$$

and we have used the relation $\exp(y_T - y_{T\max}) \simeq x_T$ for large s. The a_i's are determined as the expansion coefficients of a given initial vector $Q(y_{T\max})$ in terms of the eigenvectors v_i. Once we obtain the quark functions, it is immediate to write down the relative probabilities for producing mesons, baryons and antibaryons. These are given by

$$\begin{aligned} M(y_T) &= Ah[\alpha q(y_T) + \gamma \overline{q}\overline{q}(y_T)], \\ B(y_T) &= Ah\beta q(y_T), \\ \overline{B}(y_T) &= Ah\delta \overline{q}\overline{q}(y_T), \end{aligned} \tag{4}$$

where A is an appropriate normalization factor.

The next problem we have to consider is what is the relative probabilities of producing hadrons with various $SU(3)$ quantum number. This is not so difficult according to the picture we have assumed to employ here. For example, the probability of producing a hard meson M_i at y_T will be given by

$$M_i(y_T) = M(y_T)\frac{P_{M_i}(y_T)}{\sum_j P_{M_j}(y_T)} = M(y_T)\left\{\sum_j \frac{M_j(y_T)}{M_i(y_T)}\right\}^{-1}, \tag{5}$$

where $M(y_T)$ is already given in the above, while the ratios $M_j(y_T)/M_i(y_T)$ of meson emission were discussed in Ref. 1). As to baryon emission, only protons are observed experimentally at large p_T. Other octet baryons are of course expected to be being produced. However, in the present experimental set up those are supposed to contribute to proton counting due to their decay. The net baryons we observe far away from the colliding section will be dominantly protons and neutrons. Furthermore, in our two way quark-cascade scheme we expect to have roughly the same emission probabilities for proton and neutron over almost the entire range of x_T as is suggested from $SU(2)$ symmetry, except that possibly around $x_T=1$ some deviation will be predicted. For our present purpose and within our accuracy of comparing our results with the data, it is enough to assume

$$\begin{aligned} p(y_T) &\cong n(y_T) \cong \tfrac{1}{2}B(y_T), \\ \overline{p}(y_T) &\cong \overline{n}(y_T) \cong \tfrac{1}{2}\overline{B}(y_T). \end{aligned} \tag{6}$$

Now we shall proceed to compare our results with the data. The Chicago-Princeton experiment was done for the nuclear targets Be, Ti and W. Since the case of W target was studied in Ref. 1), we shall focus our attention on the data corresponding to that. Let us first consider the proton-antiproton ratio $p/\overline{p}$. For scattering of proton beam on W target we have only quarks in the colliding particles. Therefore, the initial vector $Q(y_{T\max})$ is given by

$$Q(y_{T\max}) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

which can be expanded in terms of the two eigenvectors as

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\beta+\delta}\begin{pmatrix} \delta \\ \beta \end{pmatrix} + \frac{\beta}{\beta+\delta}\begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

Thus, from Eq. (3) the quark function at a given x_T will be

$$Q(x_T) = G(x_T)\left[\frac{1}{\beta+\delta}\begin{pmatrix} \delta \\ \beta \end{pmatrix} + \frac{\beta}{\beta+\delta}\begin{pmatrix} 1 \\ -1 \end{pmatrix} x_T^{\beta+\delta}\right]. \tag{7}$$

From Eqs. (4), (6) and (7) we obtain

$$p/\bar{p} \cong B/\bar{B} = \frac{\delta + \beta x_T^{\beta+\delta}}{\delta - \delta x_T^{\beta+\delta}}.$$

As to the meson-baryon ratio, let us note that both of the q and $\bar{q}\bar{q}$ channels contribute to meson production. We consider, for example, pion-proton ratio p/π^+. From Eq. (5) we have

$$\pi^+(y_T) = Ah[\alpha q(y_T) + \gamma \bar{q}\bar{q}(y_T)] \frac{P_{\pi^+}(y_T)}{\sum_j P_{M_j}(y_T)}.$$

In the summation over M_i we shall include

$$\sum_i M_i = \pi^+ + \pi^0 + \pi^- + K^+ + K^0 + \bar{K}^0 + K^-.$$

The meson emissions included above can be treated using the results of Ref. 1). For example, the emission probability of neutral pions can be obtained from the $SU(2)$ symmetry as*)

$$\pi^0 = \tfrac{1}{2}(\pi^+ + \pi^-).$$

As a consequence, we obtain

$$\frac{\pi^+(y_T)}{\sum_i M_i(y_T)} = \frac{\mathfrak{p}(y_T)}{2\mathfrak{p}(y_T) + 2\mathfrak{n}(y_T) + \lambda(y_T)}$$

in terms of the $SU(3)$ quark functions $\mathfrak{p}(y_T)$, $\mathfrak{n}(y_T)$, $\lambda(y_T)$. As to the explicit expressions for these functions, refer to the original reference 1). For the proton beam on the tungsten target we have

$$\frac{\pi^+(x_T)}{\sum_i M_i(x_T)} = \frac{10 + 5x_T^{2/3} + 3x_T^{10/9}}{50 + 10x_T^{2/3}}. \tag{8}$$

*) The η meson emission is not considered here, since we do not want to introduce additional parameters. There is some experimental evidence that η mesons are produced at large p_T with about half the emission probability of π^0. We have checked that the inclusion of η with this probability does not change significantly our results in the present paper.

From Eqs. (4) $\sim$ (8) we therefore get

$$\frac{p}{\pi^+}(x_T)=\frac{1}{2}\frac{\delta+\beta x_T^{\beta+\delta}}{((\delta\alpha/\beta)+\gamma)+(\alpha-\gamma)x_T^{\beta+\delta}}\cdot\frac{50+10x_T{}^{2/3}}{10+5x_T{}^{2/3}+3x_T{}^{10/9}}\,.$$

Similarly, we can also obtain the $\bar{p}/\pi^-$ ratio as

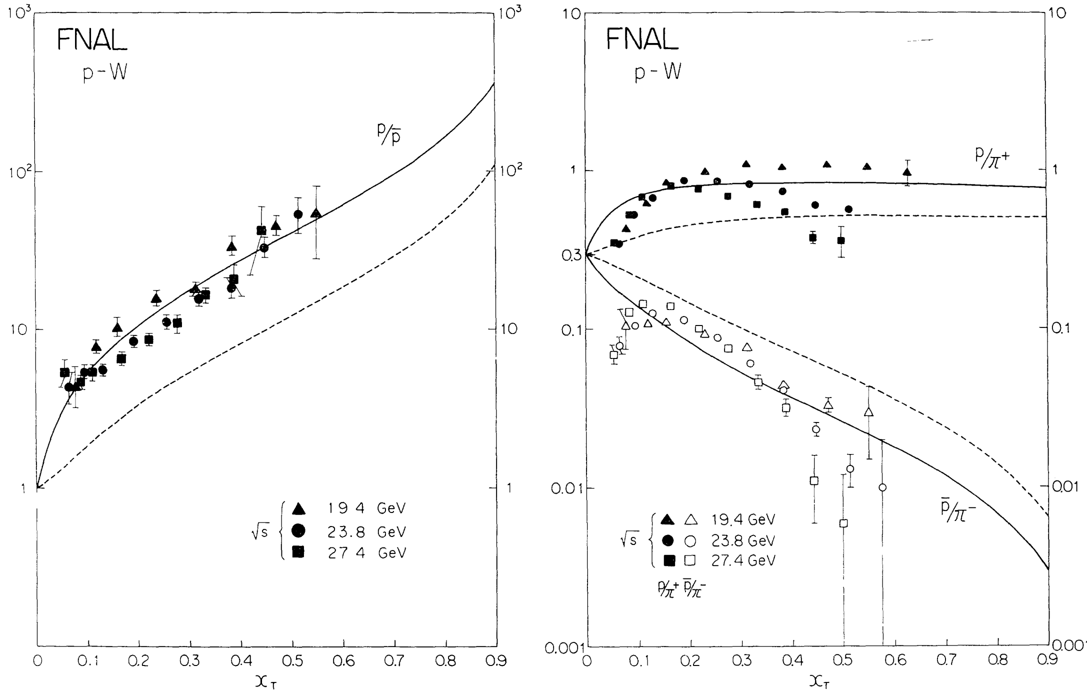
$$\frac{\bar{p}}{\pi^-}(x_T)=\frac{1}{2}\frac{\delta-\delta x_T^{\beta+\delta}}{((\delta\alpha/\beta)+\gamma)+(\alpha-\gamma)x_T^{\beta+\delta}}\cdot\frac{50+10x_T{}^{2/3}}{10+5x_T{}^{2/3}-3x_T{}^{10/9}}\,.$$

Note that the unknowns such as A, g and h are factored out and cancelled in these particle ratios.

In comparing our results with the experimental data we have found that the following choice of the four parameters α, β, γ and δ is good enough to fit nicely the Chicago-Princeton data.

$$\alpha=2.153\,,\quad \beta=1\,,\quad \gamma=0.153\,,\quad \delta=0.025\,.$$

Our comparison is shown in Fig. 3. From the fact that γ and δ are very small compared to α and β, we conclude that the hard hadron emission in the $\bar{q}q$ channel is substantially suppressed. The parameter α measures the emission probability



(a) (b)

Fig. 3. Plot of the particle ratios (a) $p/\bar{p}$ and (b) p/π^+, $\bar{p}/\pi^-$ in comparison with the Chicago-Princeton p-W experiment. Best fits to the data are shown by the solid lines, whereas the curves for p-W scattering with the values of the ISR fits as presented in Fig. 4 are drawn by the dotted lines.

per unit y_T that one "quark" emit any kind of meson. It means that, for example, if "q" is a proton quark $\mathfrak{p}$, α will be given as the following sum:

$$\alpha \approx w(\pi^+) + w(\pi^0) + w(K^+) + \cdots,$$

where $w(\pi^+)$ represents the emission probability corresponding to the vertex $\mathfrak{p} \to \mathfrak{n} + \pi^+$ and so on. If we use the values given in Ref. 1) for these w's, we find that our choice of $\alpha = 2.153$ is reasonable in magnitude.

If one takes into account the great simplicity of our analysis the agreement with the data shown in Fig. 3 is remarkably good, especially for $p/\bar{p}$. Some deviations at low x_T for the p/π^+, $\bar{p}/\pi^-$ ratios are to be explained by the effect of "pion accumulation" which is argued in detail in Ref. 1). The reason for the observed deviation at finite x_T in the p/π^+ at higher energies is not clear to us. However, quite probably it may be due to a possible effect of proton emission from the scattering center without going through our two way quark-cascade, or it may also be due to possible s dependence of the parameters.

In Fig. 4 we also present our fits to the ISR data for comparison. The data are limited to small x_T, and so here we are not going to draw any definite conclusion from these results. The values of the four parameters are chosen as

$$\alpha = 4, \quad \beta = 1.2, \quad \gamma = 0.4, \quad \delta = 0.08$$

for the ISR case. This suggests that we may not be able to neglect the s dependence of the relevant parameters in two way quark-cascade.

For comparison the curves with the parameter values for the ISR case are also shown as dotted lines in Fig. 3.

In conclusion we have seen that the two way quark-cascade model can explain quantitatively the various observed particle ratios. Some of the discrepancies with the data may be understood by refining the two way quark-cascade idea. However, we are not sure yet whether it is meaningful to proceed to next order approximation by modifying our two way quark-cascade model or in any other way as long as we base our argument on the naive quark counting rule. On the other hand, we are convinced

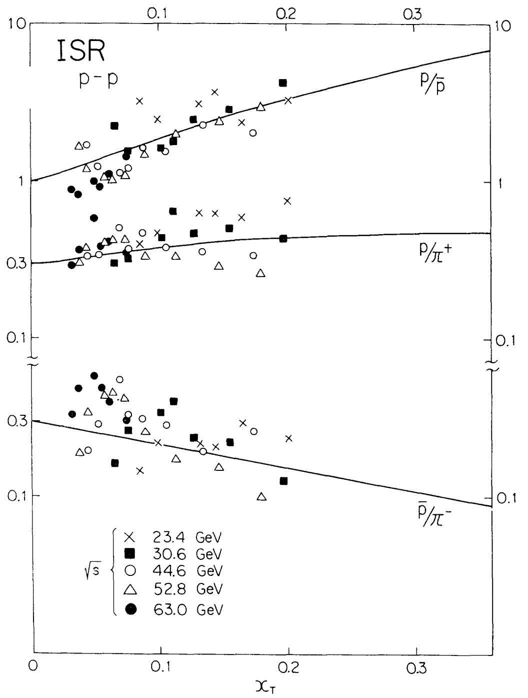


Fig. 4. Plot of the particle ratios $p/\bar{p}$, p/π^+ and $\bar{p}/\pi^-$ in comparison with the ISR data.

that in the first approximation the simple idea of quark additivity based on constituent quark model is in operation in high energy scattering of inclusive reactions.

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