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What Are the Gauge Bosons Made of?

Hidezumi TERAZAWA, Keiichi AKAMA* and Yūichi CHIKASHIGE

Institute for Nuclear Study, University of Tokyo, Tanashi, Tokyo 188
**Saitama Medical College, Kawakado, Moroyama, Iruma-Gun Saitama 350-04*

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The recent work of Saito and Shigemoto for deriving the Weinberg-Salam model from leptonic system is extended to a more realistic model including quarks. The photon and the weak-vector bosons are considered composites of lepton-antilepton or quark-antiquark pair while the color-octet gluons are considered those of quark-antiquark pair. The Weinberg angle is determined to be $\sin^2\theta_W=3/8$ for fractionally charged quarks, which coincides with the prediction of Georgi and Glashow in their unified gauge model. The gluon coupling constant is also determined to be 8/3 times the fine structure constant.

What "elementary" particles are further made of is always an important subject in particle physics. In 1961, by adopting the nonlinear fermion interaction of the Heisenberg type, Nambu and Jona-Lasinio[1] made a dynamical model of elementary particles based on an analogy with superconductivity. Succesively, Bjorken[2] showed that the photon can be considered a "collective excitation" of fermion-antifermion pair. Eguchi and Sugawara[3] found a set of equations which describes the collective motion of fermions and which is equivalent to the Higgs Lagrangian. Kikkawa[4] further made this approach to the collective motion very transparent by using the functional integration method.

On the other hand, an attractive picture for interactions of elementary particles has been given by the unified gauge theory of Weinberg and Salam[5] for the weak and electromagnetic interactions of leptons and quarks and by the asymptotically free gauge theory of Gross, Wilczek and Politzer[6] for the strong interaction of quarks. In addition to the familiar photon, the weak-vector bosons and the color-octet massless gluons form a set of elementary gauge fields inherent to these theories. Recently, two of the present authors (K. A. and H. T.)[7] have suggested the possibility that all of these gauge bosons are composite states of fermion subquarks which are the building-blocks of the ordinary quarks.

Very lately, Saito and Shigemoto,[8] starting with a Lagrangian of self-interacting leptons, have constructed an effective Lagrangian of the Weinberg-Salam type and proposed that the photon, the Higgs boson and the weak-vector bosons are all composites of lepton-antilepton pair. They have fixed, among other things, the Weinberg angle to be $\sin^2\theta_W=1/4$. Their model is, however, certainly incomplete since it does not include quarks. The purpose of this paper is to extend their model to a more realistic one including quarks. In our picture, the photon and the weak-vector bosons are considered composite states of lepton-antilepton or quark-antiquark pair while the color-octet gluons are considered those of quark-antiquark pair. As a result, the Weinberg angle is determined to be $\sin^2\theta_W=3/8$ for fractionally charged quarks, which coincides with the prediction of Georgi and Glashow[9] in their unified gauge model of all elementary-particle forces. A new feature in our model is that the gluon coupling constant is also determined to be 8/3 times the fine structure constant. In this short paper, we shall show only the essence of our model and skip all the detailed procedure, which will be presented elsewhere.

Our model consists of the leptons l_L and l_R and the quarks q_L, u_R and d_R. They belong to representations of $(1,2,1)$, $(1,1,1)$, $(3,2,1)$, $(3,1,1)$ and $(3,1,1)$, respectively, of the $SU(3)\times SU(2)\times U(1)$ group where $SU(3)$ is the color symmetry of quarks and $SU(2)\times U(1)$ is the symmetry of Weinberg and Salam. The left- and right-handed fields are defined by $\psi_L=(1/2)(1-\gamma_5)\psi$ and $\psi_R=1/2\times(1+\gamma_5)\psi$. We start with the Lagrangian

$$L=\bar{l}_L i\gamma\partial l_L+\bar{l}_R i\gamma\partial l_R+\bar{q}_L i\gamma\partial q_L+\bar{u}_R i\gamma\partial u_R+\bar{d}_R i\gamma\partial d_R + (\text{interaction terms}), \tag{1}$$

where the interaction terms contains all the possible quartic forms of lepton and quark fields that are invariant under the *global* $SU(3)\times SU(2)\times U(1)$ symmetry. Introducing the auxiliary fields[10),4)] and expanding them in terms of scalar and vector fields, we obtain the Lagrangian

$$\begin{aligned} L'=&\bar{l}_L i\gamma^\mu(\partial_\mu+iU_\mu+i\boldsymbol{\tau}\boldsymbol{U}_\mu)l_L+\bar{l}_R i\gamma^\mu(\partial_\mu+iV_\mu)l_R\\ &+\bar{q}_L i\gamma^\mu(\partial_\mu+iu_\mu+i\boldsymbol{\tau}\boldsymbol{u}_\mu+i\lambda^a u_\mu{}^a)q_L\\ &+\bar{u}_R i\gamma^\mu(\partial_\mu+iw_\mu+i\lambda^a w_\mu{}^a)u_R+\bar{d}_R i\gamma^\mu(\partial_\mu+iv_\mu+i\lambda^a v_\mu{}^a)d_R+\cdots, \end{aligned} \tag{2}$$

which is equivalent to the original one (1). In Eq. (2), we have kept only some vector auxiliary fields U, $\boldsymbol{U}$, V, u, $\boldsymbol{u}$, u^a, w, w^a, v and v^a, which transforms as $(1,1)$, $(1,3)$, $(1,1)$, $(1,1)$, $(1,3)$, $(8,1)$, $(1,1)$, $(8,1)$, $(1,1)$ and $(8,1)$ respectively, omitting all other terms that are irrelevant in this paper. The τ^i $(i=1,2,3)$ and λ^a $(a=1,\cdots,8)$ are the 2×2 Pauli matrices for $SU(2)$ and 3×3 Gell-Mann's matrices for $SU(3)$. The effective Lagrangian for these vector fields can be calculated up to the logarithmically divergent terms to be

$$\begin{aligned} L_{\text{eff}}=&-c[N_1U_{\mu\nu}U^{\mu\nu}+N_2\boldsymbol{U}_{\mu\nu}\boldsymbol{U}^{\mu\nu}+V_{\mu\nu}V^{\mu\nu}\\ &+N_1n_1u_{\mu\nu}u^{\mu\nu}+N_2n_1\boldsymbol{u}_{\mu\nu}\boldsymbol{u}^{\mu\nu}+N_1n_2u^a_{\mu\nu}u^{a\mu\nu}\\ &+n_1w_{\mu\nu}w^{\mu\nu}+n_2w^a_{\mu\nu}w^{a\mu\nu}+n_1v_{\mu\nu}v^{\mu\nu}+n_2v^a_{\mu\nu}v^{a\mu\nu}]+\cdots, \end{aligned} \tag{3}$$

where $N_1=\text{tr}\ I^2=2$ and $N_2\delta_{ij}=\text{tr}\ \tau^i\tau^j=2\delta_{ij}$ for $SU(2)$, $n_1=\text{tr}\ I^2=3$ and $n_2\delta_{ab}=\text{tr}\ \lambda^a\lambda^b=2\delta_{ab}$ for $SU(3)$, $c=(1/96\pi^2)\ln\Lambda^2$ with the cutoff parameter Λ^2 and $U_{\mu\nu}$, etc., are the usual (covariant) field strengths.

In order that the Lagrangian L' contains both the Weinberg-Salam gauge theory for the weak and electromagnetic interactions of leptons and quarks and the asymptotically free gauge theory for the strong interaction of quarks, the vector fields must satisfy the relations

$$u_\mu{}^a = w_\mu{}^a = v_\mu{}^a\,,\quad \boldsymbol{U}_\mu = \boldsymbol{u}_\mu \text{ and}$$
$$U_\mu : V_\mu : u_\mu : w_\mu : v_\mu = Y_{l_L} : Y_{l_R} : Y_{q_L} : Y_{u_R} : Y_{d_R}\,, \tag{4}$$

where Y's are the weak hypercharges of leptons and quarks related with the charges and weak isospins by $Q = I_3 + (1/2)Y$. Together with these relations, the Lagrangian L' and the effective Lagrangian L_{eff} lead to a desired gauge theory described by the Lagrangian

$$\begin{aligned} L'' = & -\tfrac{1}{4}\boldsymbol{A}_{\mu\nu}\boldsymbol{A}^{\mu\nu} - \tfrac{1}{4}B_{\mu\nu}B^{\mu\nu} - \tfrac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} \\ & + \bar{l}_L i\gamma^\mu(\partial_\mu + ig'\tfrac{1}{2}Y_{l_L}B_\mu + ig\tfrac{1}{2}\boldsymbol{\tau}\boldsymbol{A}_\mu) l_L + \bar{l}_R i\gamma^\mu(\partial_\mu + ig'\tfrac{1}{2}Y_{l_R}) l_R \\ & + \bar{q}_L i\gamma^\mu(\partial_\mu + iq'\tfrac{1}{2}Y_{q_L}B_\mu + ig'\tfrac{1}{2}\boldsymbol{\tau}\boldsymbol{A}_\mu + if\tfrac{1}{2}\lambda^a G_\mu{}^a) q_L \\ & + \bar{u}_R i\gamma^\mu(\partial_\mu + ig'\tfrac{1}{2}Y_{u_R}B_\mu + if\tfrac{1}{2}\lambda^a G_\mu{}^a) u_R + \bar{d}_R i\gamma^\mu(\partial_\mu + ig'\tfrac{1}{2}Y_{d_R}B_\mu + if\tfrac{1}{2}\lambda^a G_\mu{}^a) d_R \\ & + \cdots\,, \end{aligned} \tag{5}$$

if the coupling constants g, g' and f for the gauge bosons $\boldsymbol{A}_\mu$ and B_μ of Weinberg and Salam and for the color-octet gauge gluons $G_\mu{}^a$ obey the relation

$$\begin{aligned} g^2 : (g')^2 : f^2 = & [N_2(1+n_1)]^{-1} : [N_1 Y_{l_L}^2 + Y_{l_R}^2 + n_1(N_1 Y_{q_L}^2 + Y_{u_R}^2 + Y_{d_R}^2)]^{-1} \\ & : [n_2(N_1+2)]^{-1}\,. \end{aligned} \tag{6}$$

From this relation, we finally reach the main results:

$$\begin{aligned} \sin^2\theta_W = \frac{(g')^2}{g^2 + (g')^2} & = \frac{N_2(1+n_1)}{N_2(1+n_1) + N_1 Y_{l_L}^2 + Y_{l_R}^2 + n_1(N_1 Y_{q_L}^2 + Y_{u_R}^2 + Y_{d_R}^2)} \\ & = \frac{\operatorname{tr} I_3{}^2}{\operatorname{tr} Q^2} \end{aligned} \tag{7}$$

and

$$\frac{f^2}{g^2} = \frac{N_2(1+n_1)}{n_2(N_1+2)} = \frac{\text{the number of isodoublets}}{\text{the number of color-triplets}} = 1\,. \tag{8}$$

Since $Y_{l_L} = -1$, $Y_{l_R} = -2$, $Y_{q_L} = 1/3$, $Y_{u_R} = 4/3$ and $Y_{d_R} = -2/3$ for fractionally charged quarks, the relation (7) gives $\sin^2\theta_W = 3/8$, which coincides with the prediction of Georgi and Glashow[9] in their unified gauge theory. The relation (8) together with $g^2 = e^2/\sin^2\theta_W$[5] fixes the gluon coupling constant to be $f^2/4\pi = \alpha/\sin^2\theta_W = (8/3)\alpha$ where α is the fine structure constant.

In the light of the general argument by Georgi, Quinn and Weinberg,[11] these relations suggest that our model must possess a symmetry larger than $SU(3) \times SU(2) \times U(1)$. In fact, our model contains more gauge bosons than are needed for the local $SU(3)\times SU(2)\times U(1)$ symmetry. How to build an appropriate hierarchy of interactions[11] by making use of the generated scalars which we have omitted in this paper will be discussed in our forthcoming paper.

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