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Equation of Motion for Leptons and Quarks in a Dynamical Subquark Model*)

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A program to obtain an equation of motion for leptons and quarks in a dynamical subquark model is discussed. The inverse problem, how to find the subquark interactions, given the equation of motion for a lepton or quark, is investigated. As an example, a nonlinear and nonlocal dynamical model of subquark interactions is found to reproduce the Dirac equations for a free lepton or quark. Possible relevance of supercurrent interactions, nonlinear interactions and a mass-dimensional parameter for subquark dynamics is suggested.

As lepton-quark physics stands now with six leptons and five flavors and three colors of quarks, it does not seem too early to consider seriously the possibility that leptons and quarks are further made of more fundamental particles, the subquarks. In fact, many subquark models of leptons and quarks have already been proposed and discussed.[1] The proposed models can be classified into the following two categories: 1) the one in which flavor and color quantum numbers of a lepton or quark are attributed to those of subquarks which build the lepton or quark although each one of subquarks has either flavor or color quantum number,[2] and 2) the other in which flavor and color quantum numbers appear only at the lepton and quark level due to certain combinations of subquarks but have no definite meaning at the subquark level.[3] As far as the ways of considering gauge bosons (and Higgs scalars) also as composites of subquarks and the ways of achieving universality of interactions at the lepton and quark level are concerned, subquark models in the former category seem to be more natural than those in the latter.

In either case, however, there are many fundamental problems left to be solved in these subquark models. They include 1) the lepton and quark mass spectra, 2) the quark mixings, 3) the lepton and quark magnetic moments, 4) possible leptons and quarks of spin 3/2 and 5) exotic states of subquarks. In 1), one should explain all the mass spectra of leptons and quarks as one could do all the level spectra of the hydrogen atom in quantum mechanics. In 2), one should explain why and how much the Cabibbo-like mixings of quarks happen in the weak interaction. In 3), one should explain why leptons and quarks have the magnetic

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moments so close to the Dirac ones in spite of their possible substructure.[4] In 4) and 5), one should explain why leptons and quark of spin 3/2 or exotic states of subquarks which may exist in some subquark models do not exist at all or have much higher masses than the ordinary ones.

All these problems are closely linked to a single problem, the unknown subquark dynamics. In this paper, a serious consideration of the subquark dynamics will be made. In particular, a general program to obtain an equation of motion for leptons and quarks in a dynamical subquark model will be discussed. Furthermore, the inverse problem, how to find the form of subquark interactions, given the equation of motion for a lepton or quark, will be investigated. As an example, a nonlinear and nonlocal dynamical model of subquark interactions is found to reproduce the Dirac equation for a free lepton or quark.

Among the subquark models in the category 1), one of the most universal ones is our spinor subquark model[5] in which leptons and quarks are made of three spinor subquarks w^i $(i=1,2)$, h^n $(n=1,2,3,\cdots,N)$ and C^a $(a=0,1,2,3)$ as

$$f^{ina}=(w^i h^n C^a), \tag{1}$$

where f^{1n0}, f^{2n0}, f^{1na} and f^{2na} for $a=1,2,3$ are neutrinos (ν^n), charged leptons (l^n), up-quarks (u^{na}) and down-quarks (d^{na}), respectively. The left-handed wakems $w_L{}^1$ and $w_L{}^2$ form a doublet of the Glashow-Weinberg-Salam $SU(2)$, the right-handed ones are singlets, the chroms C^0 and C^a for $a=1,2,3$ form a singlet and a triplet of color $SU(3)$ and the hakams $h_1, h_2, h_3, \cdots, h_N$ may form an N-plet of an unknown horizontal (or generation) symmetry.

Our program to obtain a unified equation of motion for leptons and quarks can be expressed in a word as follows: given the subquark dynamics written in terms of subquarks (w^i, h^n and C^a), find an effective equation of motion for leptons and quarks (f^{ina}). An important phenomenological requirement in such a program is that the resultant equation of motion should express both the Glashow-Weinberg-Salam gauge theory of $SU(2)\times U(1)$ for electroweak interactions of leptons and quarks and quantum chromodynamics, the Yang-Mills gauge theory of color $SU(3)$, for strong interaction of quarks. A way to satisfy this requirement most naturally is to consider gauge bosons, the photon γ, the weak bosons $W^{\pm}$ and Z and the gluons G^A $(A=1\sim8)$ as well as the Higgs scalar ϕ as composite states of subquark-antisubquark pair. Such a dynamical subquark model of gauge bosons has been discussed in detail in Refs. 1) and 5). Notice, however, that what has been shown there is that, strictly speaking, the resultant "pregauge theory"[6] is an effective gauge theory for subquark interactions but not for lepton and quark interactions. Whether or not it also becomes an effective gauge theory for lepton and quark interactions strongly depends on how subquarks are bound to become a lepton or quark.

To be more precise, let us suppose the subquark dynamics is described by the following Lagrangian:

$$
\begin{aligned}
L' = & \overline{w^i} i\gamma\partial w^i + \overline{h^n} i\gamma\partial h^n + \overline{C^a} i\gamma\partial C^a \\
& + (\overline{w}_L\gamma_\mu Y_{w_L} w_L + \overline{w}_R\gamma_\mu Y_{w_R} w_R + \overline{h}\gamma_\mu Y_h h + \overline{C}\gamma_\mu Y_C C) B^\mu - \frac{1}{2} f_1 (B_\mu)^2 \\
& + (\overline{w}_L \gamma_\mu \boldsymbol{\tau} w_L) \boldsymbol{A}^\mu - \frac{1}{2} f_2 (\boldsymbol{A}_\mu)^2 \\
& + (\overline{C}\gamma_\mu \lambda^A C) G^{A\mu} - \frac{1}{2} f_3 (G_\mu{}^A)^2 \\
& + (-a_1 \overline{w_R{}^G} w_L{}^{1c} + a_2 \overline{w}_L w_R{}^2)\phi + \text{h.c.} - f_4 \phi^\dagger \phi \\
& + f_5 (\overline{f^{ina}} F^{ina} + \text{h.c.} - \overline{f^{ina}} f^{ina}) ,
\end{aligned} \tag{2}
$$

where $\gamma\partial = \gamma^\mu \partial_\mu$; f_i $(i=1\sim5)$ and a_i $(i=1, 2)$ are constants; Y's, $\boldsymbol{\tau}$ and λ^A are the weak hypercharges, the Pauli $SU(2)$ matrices and the Gell-Mann $SU(3)$ matrices; c and G denote the charge-conjugate and G-parity-conjugate states; and B_μ, $\boldsymbol{A}_\mu$, $G_\mu{}^A$, ϕ and f^{ina} are the auxiliary fields for the gauge fields of the $U(1)$, $SU(2)$ and $SU(3)$ symmetries, for the Higgs scalar and for the leptons and quarks, respectively. In this Lagrangian model, the whole secret on how subquarks are bound to become a lepton or quark is contained in an unknown function

$$
F^{ina} \equiv F^{ina}(w^i, \overline{w^i}, \partial_\mu w^i, \partial_\mu \overline{w^i}, h^n, \overline{h^n}, \partial_\mu h^n, \partial_\mu \overline{h^n}, C^a, \overline{C^a}, \partial_\mu C^a, \partial_\mu \overline{C^a}) . \tag{3}
$$

It has been shown in Refs. 1) and 5) that this form of Lagrangian (2) in the absence of the last term is effectively equivalent to the Lagrangian which combines the Glashow-Weinberg-Salam gauge theory for electroweak interactions of subquarks and quantum chromodynamics for strong interaction of chroms.

In order to proceed along our program, define the effective Lagrangian by integrating the subquark fields out as

$$
\exp\left[i \int d^4x L_{\text{eff}}\right] = \int dw^i d\overline{w^i} dh^n d\overline{h^n} dC^a d\overline{C^a} \exp\left[i \int d^4x L'\right]. \tag{4}
$$

Then, the final goal of the program is to find that the effective Lagrangian L_{eff} as a function of B_μ, $\boldsymbol{A}_\mu$, $G_\mu{}^A$, ϕ and f^{ina} is, at least approximately, equal to the one which combines the Glashow-Weinberg-Salam gauge theory for electroweak interactions of leptons and quarks and quantum chromodynamics for strong interaction of quarks. Actually, however, it is not easy to perform the path-integration over the subquark fields explicitly to produce the desired lepton and quark dynamics even if an appropriate form of the function F^{ina} is given. It should be emphasized here that such an attempt by assuming the form of

$$
F^{ina} = P w^i h^n C^a , \tag{5}
$$

where P is the projection operator spin for 1/2 states, has been under way by

Akama[7] with some success of explaining the Dirac moment of a lepton or quark.

The purpose of the present work is not to make such a dynamical calculation by specifying a particular form of the subquark dynamics, which seems to be ambiguous at this stage, but to obtain some general idea about what the subquark dynamics looks like. To this end, let us make the problem completely reversed. Namely, given the known lepton and quark dynamics, find the unknown subquark dynamics. To solve this inverse problem is desperately difficult in the actual presence of lepton and quark interactions. Even in the absence of the interactions, it is still non-trivial. We have found only a satisfactory (but not necessary) solution to this most simplified inverse problem, which we shall explain in what follows.

The most simplified inverse problem is to find an appropriate form of subquark interactions which reproduces the Dirac equation of motion for a free lepton or quark. For simplicity, let us take the simplest subquark model in which a lepton or quark f is made of a spinor subquark w and a complex scalar subquark C with the equal mass M. Let L and L' be an unknown Lagrangian for the subquark dynamics, which is a function of w, $\bar{w}$, C and $C^\dagger$, and another unknown Lagrangian, which is a function of w, $\bar{w}$, C, $C^\dagger$, f and $\bar{f}$. The Lagrangian L' is effectively equivalent to the Lagrangian L in a sense that

$$\int df d\bar{f} \exp\left[i\int d^4x L'\right] = \text{const} \times \exp\left[i\int d^4x L\right]. \tag{6}$$

Let us also introduce an effective Lagrangian for the lepton or quark dynamics L_{eff} as

$$\exp\left[i\int d^4x L_{\text{eff}}\right] = \text{const} \times \int dw d\bar{w} dC dC^\dagger \exp\left[i\int d^4x L'\right]. \tag{7}$$

We can then reduce the most simplified inverse problem to the following mathematical problem: Find an appropriate form of L as a function of w, $\bar{w}$, C and $C^\dagger$ such that L_{eff} simply becomes

$$L_{\text{eff}} = \bar{f}(i\gamma\partial - m)f\,, \tag{8}$$

where m is the lepton or quark mass.

The simplest mathematical solution to this problem is given by

$$L' = \bar{w}(i\gamma\partial - M)w + \partial_\mu C^\dagger \partial^\mu C - M^2 C^\dagger C + \bar{f}(i\gamma\partial - m)f$$

and

$$L = \bar{w}(i\gamma\partial - M)w + \partial_\mu C^\dagger \partial^\mu C - M^2 C^\dagger C\,. \tag{9}$$

This trivial solution is physically meaningless since the subquarks are free and unable to produce a lepton or quark as a composite. It is, however, yet instructive, suggesting how to find a nontrivial solution to the problem.

Suppose that L' is written in the form of

$$L' = \bar{w}'(i\gamma\partial - M)w' + \partial_\mu C'^\dagger \partial^\mu C' - M^2 C'^\dagger C' + \bar{f}(i\gamma\partial - m)f \tag{10}$$

and that the transformation of w and C into w' and C' satisfies

$$\frac{\partial(w', \bar{w}', C', C'^\dagger)}{\partial(w, \bar{w}, C, C^\dagger)} = \text{const.} \tag{11}$$

It is then obvious that such L' gives the answer (8) to L_{eff}. The problem, therefore, can be reduced to how to find an appropriate transformation which satisfies the condition (11) and which gives a physically nontrivial and meaningful form of L.

A satisfactory example of such transformations is given by

$$w' = w + \frac{1}{\mu^2}\gamma^\mu f \partial_\mu C^\dagger, \quad \bar{w}' = \bar{w} + \frac{1}{\mu^2}\partial_\mu C \bar{f}\gamma^\mu,$$
$$C' = C - \frac{1}{\mu^2} i\bar{w}f, \qquad C'^\dagger = C^\dagger + \frac{1}{\mu^2} i\bar{f}w, \tag{12}$$

where μ is a certain mass-dimensional parameter. This would become nothing but a supersymmetric transformation of Wess and Zumino[8] if f were replaced by an infinitesimal Majorana-spinor parameter. It is easy to check that this finite transformation satisfies the condition (11) as

$$\frac{\partial(w', \bar{w}', C', C'^\dagger)}{\partial(w, \bar{w}, C, C^\dagger)} = \exp \operatorname{tr} \ln\left(1 - \frac{1}{\mu^2}\begin{bmatrix} & & & \gamma\partial f \\ & & \bar{f}\gamma\overleftarrow{\partial} & \\ & if & & \\ i\bar{f} & & & \end{bmatrix}\right) = 1. \tag{13}$$

Furthermore, this example is nontrivial since

$$L' = \bar{w}(i\gamma\partial - M)w + \partial_\mu C^\dagger \partial^\mu C - M^2 C^\dagger C + \bar{f}Af + \bar{f}a + \bar{a}f, \tag{14}$$

where

$$A = i\gamma\partial - m + \frac{1}{\mu^4}[(\gamma\partial C)(i\gamma\partial - M)(\gamma\partial C^\dagger) + (\overleftarrow{\partial}_\mu + \overrightarrow{\partial}_\mu)w\bar{w}(\overleftarrow{\partial}^\mu + \overrightarrow{\partial}^\mu) - M^2 w\bar{w}],$$

$$a = \frac{1}{\mu^2}[(\gamma\partial C)(i\gamma\partial - M) - i(\Box + M^2)C]w$$

and

$$\bar{a} = \frac{1}{\mu^2}\bar{w}[(-i\gamma\overleftarrow{\partial} - M)(\gamma\partial C^\dagger) + i(\Box + M^2)C^\dagger]. \tag{15}$$

By performing path-integration formally in (6) with (14), the Lagrangian for subquark dynamics can be written as

$$\int d^4xL = \int d^4x[\bar{w}(i\gamma\partial - M)w + \partial_\mu C^\dagger \partial^\mu C - M^2 C^\dagger C]$$

$$-i \text{ tr} \ln A - \int d^4x \text{ tr } \bar{a}\frac{1}{A}a \,, \qquad (16)$$

which is highly nonlinear and nonlocal.

Several remarks are in order, concerning this nontrivial solution to the most simplified inverse problem: 1) The solution is satisfactory but by no means necessary. 2) The lepton or quark couples with the divergence of subquark supercurrent.[9] 3) The subquark dynamics involves an infinite series of multiple interactions among subquarks. 4) In finding an explicit form of subquark interactions the cutoff momentum must be introduced since all the involved loop integrals quartically diverge. However, all the integrals can be considered to be finite if the mass-dimensional parameter μ is taken to be as large as the cutoff momentum, i.e.,

$$\delta^4(x-x) = \mu^4. \qquad (17)$$

Although we have no intention of insisting on the Lagrangian (16), we have gained from this investigation some insight into real subquark dynamics, if any: 1) The supercurrent interaction between subquarks and a lepton or quark may be relevant. 2) The subquark dynamics may be highly nonlinear. Otherwise, the real equation of motion for a lepton or quark would be highly nonlinear. It would look as if it were linear simply because the energy involved is much lower than a certain mass scale. A suggestive example of such possibilities is given by the model Lagrangian for a lepton or quark

$$\int d^4xL = -i \text{ tr} \ln\left(1 + \frac{1}{\mu^4}\bar{f}i\gamma\partial f\right) - \int d^4x m\bar{f}f$$

$$= \int d^4x\left[\bar{f}(i\gamma\partial - m)f - \frac{1}{2\mu^4}(\bar{f}i\gamma\partial f)^2 + \cdots\right], \qquad (18)$$

where the relation (17) has been assumed. In any case, such possible nonlinearity of subquark dynamics is nothing that we should be afraid of. Remember that Einstein's gravity theory is very nonlinear. An important point may be the simplicity of the principle but not that of the expression. 3) The presence of the mass-dimensional parameter (or coupling constant) μ and the need of the momentum cutoff in our model Lagrangian also seem to be very suggestive. Since subquarks may constitute the ultimate structure of matter, the true theory of subquark dynamics, if any, can be the final theory. If this is the case, it should contain the mass parameter as a natural constant and explain how the momentum cutoff (or short distance cutoff) appears in nature, probably at the Planck mass (or Planck length). The possible size of leptons and quarks must be related to this cutoff. In this sense, it would look as revolutionary as relativity and quantum theories. Until such true theory becomes available, a relation between the cutoff and the mass-dimensional parameter such as (17) may perhaps be useful as a temporary hypothesis.

Needless to say, a solution similar to the one discussed in this paper could be a candidate for quark dynamics if baryons were made of a spinor quark and a scalar quark, if they had the Dirac moment and if no other fields such as gluons were involved.

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