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# Spin-Dependent Forces and Scattering Amplitudes for Elastic pp Scattering at $6 \mathbf{G e V} / \boldsymbol{c}$ 

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#### Abstract

We present an improved fit to all the available spin-observables of elastic $p p$ scattering at $6 \mathrm{GeV} / c$ by adding short-range spin-dependent eikonals (range of $0.3 \sim 0.5 \mathrm{fm}$ ) to the intermediate-range ( $\sim 1.5 \mathrm{fm}$ ) and long-range ( $2 \sim 2.5 \mathrm{fm}$ ) eikonals in the eikonal model based on the impact-parameter representation for scattering amplitudes. Natural-parity exchange helicity non-flip amplitude $N_{0}\left(\equiv 1 / 2\left(\phi_{1}+\phi_{3}\right)\right)$ has a zero near $t=-2.5(\mathrm{GeV} / c)^{2}$ in the imaginary part and has no zero in the whole $t$ range up to $\theta_{\mathrm{cm}}=90^{\circ}$ in the real part. Helicity doubleflip amplitude $\phi_{2}$ has a very sharp negative peak around $t=0(\mathrm{GeV} / c)^{2}$ in the real part. Some discussions are given on the behavior of the spin-dependent eikonals.


## § 1. Introduction

Measurements of spin correlation parameters such as $C_{N N}, C_{L L}, C_{S S}, C_{S L}$, etc. for elastic $p p$ scattering at $6 \mathrm{GeV} / c$ with polarized beams and polarized targets at Argonne National Laboratory ${ }^{11}$ have suggested that spin-dependent forces are still rather strong at $6 \mathrm{GeV} \%$ than expected before when Regge pole exchange view was promoted in 1960's. And, moreover, if combined with data on other spinobservables such as depolarization parameters $D_{N N}, D_{S S}$ and $D_{L S}$, polarization transfer tensor $K_{N N}$, etc. which had been obtained earlier at other institutes, it appears to have become possible to get the five independent scattering amplitudes for this process at $6 \mathrm{GeV} / c$ in the wide momentum-transfer squared ( $t$ ) range. ${ }^{\left.2,22^{\prime}, 3^{3}, 3^{\prime}\right)}$

We presented before a method of analysis for elastic $p p$ scattering by taking the five independent spin-dependent eikonals in the impact-parameter representation for scattering amplitudes ${ }^{4}$ (referred to as I hereafter), which was developed from Durand and Halzen's eikonal model which assumed only spin-orbit coupling eikonal as a spin-dependent force. ${ }^{5)}$

In the work I, five eikonals of spin-independent central one, spin-orbit, spinspin, tensor and quadratic spin-orbit couplings were incorporated and it turned out from a fit to the data on $d \sigma / d t, P, C_{N N}, C_{L L}, C_{S S}, C_{S L}, D_{N N}, K_{N N}, \sigma_{T}, \rho, \Delta \sigma_{L}$ and $\Delta \sigma_{T}$ at $6 \mathrm{GeV} / c$ that the spin-spin and tensor coupling eikonals have two components of intermediate-range ( $0.8 \sim 1.2 \mathrm{fm}$ ) and long-range ( $1.5 \sim 2 \mathrm{fm}$ ) and that the longrange component can be considered as originating from the "one-pion-exchange" from its range, sign and effect on the "spike" in $n p \rightarrow p n$. There, however, the

[^0]chi-square value of the fit was fairly large; 541.7 for 125 data points (" 155 data points" was a misprint in p. 1300 of I).

Here, we present an improved fit to all the available kinds of spin-observables by increasing the number of data points in the same model with the addition of a short-range component to the above-mentioned intermediate- and long-range ones for the spin-dependent eikonals. The chi-square value is 408.7 for 225 data points.

The model is briefly summarized in $\S 2$. The results of the fit to the observables are given in $\S 3$. Behavior of the eikonals is discussed in $\S 4$, and the final section is devoted to the discussion on the scattering amplitudes.

## § 2. The model

As stated in I, from the facts that (i) absorption effect is already prominent in several GeV region and (ii) eikonal can be a measure of the scattering force in the impact-parameter since the relativistic effects are integrated out in the eikonal, the eikonal model is used at $6 \mathrm{GeV} / c$. The scattering matrix of elastic proton-proton scattering is written with the eikonal in the impact-parameter plane as

$$
\begin{equation*}
M=\frac{i p}{2 \pi} \int_{0}^{\infty}\left(1-e^{-x(b)}\right) e^{-i \boldsymbol{q} \cdot \boldsymbol{b}} d^{2} \boldsymbol{b}, \tag{1}
\end{equation*}
$$

where $\boldsymbol{q}$ is the momentum-transfer $\boldsymbol{q} \equiv \boldsymbol{p}^{\prime}-\boldsymbol{p}\left(\boldsymbol{q}^{2}=-t\right), \boldsymbol{p}$ and $\boldsymbol{p}^{\prime}$ are the incident and scattered momentum in the center of mass system, respectively, and $b$ is the impact parameter. The eikonal function $\chi(\boldsymbol{b})$ is expressed as the sum of five independent spin-dependent eikonals as

$$
\begin{align*}
\chi(\boldsymbol{b})=\chi_{C}(b) & -i\left(\boldsymbol{\sigma}_{1}+\boldsymbol{\sigma}_{2}\right) \cdot(\boldsymbol{b} \times \hat{\boldsymbol{l}}) \chi_{L \boldsymbol{S}}(b)-i \boldsymbol{\sigma}_{1} \cdot \boldsymbol{\sigma}_{2} \chi_{S S}(b) \\
& -i\left\{\boldsymbol{\sigma}_{1} \cdot(\boldsymbol{b} \times \hat{\boldsymbol{l}})\right\}\left\{\boldsymbol{\sigma}_{2} \cdot(\boldsymbol{b} \times \hat{\boldsymbol{l}})\right\} \chi_{Q}(b)-i S_{12}^{\prime} \chi_{T}(b), \tag{2}
\end{align*}
$$

where $\chi_{C}$ is the spin-independent central eikonal, the second to the fifth terms are spin-orbit, spin-spin, quadratic spin-orbit and tensor coupling eikonals, respectively, $\boldsymbol{\sigma}_{1}$ and $\boldsymbol{\sigma}_{2}$ are the Pauli spin matrices of the two incident protons, $\hat{\boldsymbol{l}} \equiv\left(\boldsymbol{p}+\boldsymbol{p}^{\prime}\right) / \mid \boldsymbol{p}$ $+\boldsymbol{p}^{\prime} \mid, S_{12}$ ' is the "tensor" operator symbolically written in the impact-parameter plane and, here, the impact parameter $\boldsymbol{b}$ is taken to be perpendicular to $\hat{\boldsymbol{l}}$.

The form (2) of eikonal is introduced into Eq. (1) and the resulting equation is expanded in powers of $\delta \chi(\boldsymbol{b}) \equiv \chi(\boldsymbol{b})-\chi_{C}(b)$, since the relations $\left|\chi_{L S}\right|,\left|\chi_{S S}\right|,\left|\chi_{Q}\right|$, $\left|\chi_{T}\right| \ll\left|\chi_{C}\right|$ are expected from the measured values of the spin observables. The expansion is performed up to second order in $\chi_{L S}$ and $\chi_{S S}$ and up to first order in $\chi_{Q}$ and $\chi_{T}$, as the second order terms in $\chi_{Q}$ and $\chi_{T}$ affect the observables at most by $\sim 0.5 \%$ as examined in I. See I for the explicit expressions for the five independent scattering amplitudes for $p p \rightarrow p p$ represented in terms of these five eikonals. ${ }^{4}$

## § 3. Fit to the observables

In order to get the behavior of eikonals and scattering amplitudes at $6 \mathrm{GeV} / c$ from a fit to the spin observables, we used in I the data on $d \sigma / d t, P, C_{N N}, C_{L L}$, $C_{S S}, C_{S L}, D_{N N}, K_{N N}, \sigma_{T}, \rho, \Delta \sigma_{L}$ and $\Delta \sigma_{T}$ with 125 data points. Here, to determine the eikonals as precisely as possible, we use the data on all kinds of observables now available, $d \sigma / d t, P, C_{N N}, C_{L L}, C_{S S}, C_{S L}, ~ ' C C_{S L}$ ', $D_{N N}, D_{S S}, D_{L S}, K_{N N}$ ' $K_{S S}$ ' and ' $H_{S N S}$ ', $\sigma_{T}$,



Fig. 1. (a) Calculated result of $d \sigma / d t$. Data are from Ref. 6).
(b) Calculated result of $P$. Data are from Ref. 7).
(c) Calculated result of $C_{N N}$. Data are from Ref. 8).
(d) Calculated result of $C_{L L}$. Data are from Ref. 9).
(e) Calculated result of $C_{s s}$. Data are from Ref. 10).


(f) Calculated result of $C_{S L}$ and ' $C_{S L}$ '. Data are from Ref. 11).
(g) Calculated result of $D_{N N}$. Data are from Ref. 12).
(h) Calculated result of $D_{\text {ss }}$. Data are from Ref. 13).
(i) Calculated result of $D_{L S}$. Data are from Ref. 14).
( j ) Calculated result of $K_{N N}$. Data are from Ref. 15).
(k) Calculated result of ' $K_{s s}$ '. Data are from Ref. 16).
(1) Calculated result of ' $H_{S N s}$ '. Data are from Ref. 16).
$\rho, \Delta \sigma_{L}$ and $\Delta \sigma_{T}$ and we increase the number of data points from 125 to 225 , where ' $C_{S L}$ ' is combined data of $C_{S L}$ and $C_{S S}$ ' $K_{S S}$ ' is of $K_{S S}, K_{L L}, K_{S L}$ and $K_{L S}$, and ' $H_{S N S}$ ' is of $H_{S N S}, H_{L N L}, H_{S N L}$ and $H_{L N S}$.

We have obtained a greatly improved fit to the observables from the chisquare value per a data point of $4.33(=541.7 / 125)$ for 30 floating parameters of I to that of $1.82(=408.7 / 225)$ for 42 floating ones here: The improvement has been attained mainly by adding a short-range component to the intermediate- and long-range ones for the four spin-dependent eikonals $\chi_{L S}, \chi_{S S}, \chi_{T}$ and $\chi_{Q}$. Modified Fermi distribution functions, which have the Yukawa-type tail at large $b\left(\sim c_{1}\right.$ - $\left(e^{-c_{2} b} / b\right)$ ), are assumed for both the intermediate- and the long-range component of the eikonal as in I. They are made to tend to zero around $b=0$ by multiplying a factor $\left(1-e^{-\left(b / c_{4}\right)^{2}}\right)^{n}$, there the short-range component being given in the form of a Gaussian function. This is because we expect from the analysis in I that longrange and intermediate-range components are originated from one-pion-exchange and one-boson-exchange like $\rho$ or $\omega$ heavier than $\pi$ and that the innermost dynamics might be due to some "constituent" like quarks confined in the proton.*)

The parameters of the eikonals determined by the fit are summarized in the following section and we show the results of the fit to the observables here. In Fig. 1 are given $d \sigma / d t, P, C_{N N}, C_{L L}, C_{S S}, C_{S L},{ }^{\prime} C_{S L}$ ', $D_{N N}, D_{S S}, D_{L S}, K_{N N}, ~ ' K_{S S}$ ' and ' $H_{S V S}$ '. In Fig. 1 (f), the experimental data and the calculated curve are $C_{S L}$ for $0<|t|<0.8(\mathrm{GeV} / c)^{2}$ and are ' $C_{S L}$ ' $=-0.951 C_{S L}+0.308 C_{S S}$ for $0.8<|t|<4.8(\mathrm{GeV} / c) .{ }^{2}$ In this work, we analyze in the whole angular range $0^{\circ} \leq \theta_{\mathrm{CM}} \leq 90^{\circ}$ (i.e., $0 \leq|t|$ $\left.\leq 4.8(\mathrm{GeV} / c)^{2}\right)$, while we did in $0 \leq|t| \leq 3.5(\mathrm{GeV} / c)^{2}$ in I. The calculated forward observables are $\sigma_{T}=40.759 \mathrm{mb}$ (exp.; $\left.40.75 \pm 0.10 \mathrm{mb}^{17}\right), \rho \equiv \operatorname{Re} N_{0} /\left.\operatorname{Im} N_{0}\right|_{t=0}$ $=-0.191\left(-0.32 \pm 0.06^{18)}\right), \Delta \sigma_{T}=0.200 \mathrm{mb}\left(0.33 \pm 0.09 \mathrm{mb}^{19}\right)$ and $\Delta \sigma_{L}=-0.395 \mathrm{mb}$ $\left(-1.04 \pm 0.09 \mathrm{mb}^{207}\right)$.

## § 4. Eikonals obtained by the fit

As stated in §3, the short-range component is introduced into all the four spin-dependent eikonals. As in I, both the intermediate- and the long-range component are given to spin-spin and tensor couplings while only the intermediate-range component is given to spin-orbit and quadratic spin-orbit couplings. In the present analysis, these components are all reduced to zero at $b=0$ by multiplying the factor $\left(1-e^{-\left(b / c_{4}\right)^{2}}\right)^{n}$. The value $n=2$ has proved best by the chi-square least method. The spin-independent central eikonal $1-e^{-x_{G}(b)}$ is assumed to be a sum of three Gaussians in $b$ for both the real and the imaginary part in the same way as in I.

The parameters of the eikonals obtained by the fit are in the following,

$$
\begin{aligned}
& 1-e^{-\chi_{C}(b)}=1.247 e^{-1.71 b^{2}}-0.349 e^{-3.59 b^{2}}+0.181 e^{-12.9 b^{2}} \\
&+i\left(0.781 e^{-2.88 b^{2}}-0.532 e^{-2.57 b^{2}}+0.334 e^{-5.66 b^{2} 2}\right), \\
& \chi_{L S}(b)=6.88 /\left\{1+b \cdot e^{4.32(b+0.230)}\right\} \times\left(1-e^{\left.-(6 / 0.484)^{2}\right)^{2}}-0.801 e^{-7.11 b^{2}},\right.
\end{aligned}
$$

[^1]\[

$$
\begin{align*}
& \chi_{S S}(b)=\left[-0.245 /\left\{1+b \cdot e^{0.76(b+0.304)}\right\}+0.125 /\left\{1+b \cdot e^{1.87(b-0.743)}\right\}\right]  \tag{3}\\
& \times\left(1-e^{-(b / 0.274) 2}\right)^{2}+0.158 e^{-20.762}, \\
& \chi_{T}(b)=\left[-0.142 /\left\{1+b \cdot e^{2.04(b-0.456)}\right\}+0.046 /\left\{1+b \cdot e^{3.14(b-1.152)}\right\}\right] \\
& \times\left(1-e^{-(60 / 0.726)^{2} 2}\right)^{2}+0.400 e^{-44.5 b 2}, \\
& \chi_{Q}(b)=0.280 /\left\{1+b \cdot e^{1.88(b-0.1544)}\right\} \times\left(1-e^{-(6 b 0.572) 2}\right)^{2}+0.379 e^{-3.5562},
\end{align*}
$$
\]

where $b$ is in $\mathrm{fm}, \chi_{L S}(b)$ and $\chi_{Q}(b)$ are in $\mathrm{fm}^{-1}$ and $\mathrm{fm},^{-2}$ respectively, and the other eikonals are non-dimensional. These eikonals are shown in Fig. 2. In Figs. $2(\mathrm{c}) \sim(\mathrm{f}), L, I, S$, and $T$ denote the long-, intermediate-, short-range components and the total of these components, respectively. In the following, discussions are


Fig. 2. Eikonals obtained by the fit to the observables for $p p \rightarrow p p$ at $6 \mathrm{GeV} / c$; (a) Re $\left(1-e^{-x_{C}{ }^{(b)}}\right.$ ), (b) $-\operatorname{Re} \chi_{C}$ and $-\operatorname{Im} \chi_{C}$, (c) $-\chi_{L S}$, (d) $-\chi_{S S}$, (e) $-\chi_{T}$ and (f) $-\chi_{Q}$. In Figs. $2(\mathrm{c}) \sim(\mathrm{f}), L, I, S$ and $T$ denote the long-, intermediate-, short-range components and the total of these components, respectively.
made on the behavior of the five eikonals of Eq. (3), and of Fig. 2.
As seen from Figs. 2 (a) and (b), the absorption proves to have a "peripheral" component centered at 0.5 fm and a "central" component. This is due to that $-\operatorname{Im} \chi_{C}$ is the so-called "absorptive" potential in the impact parameter plane and $\operatorname{Re}\left(1-e^{-X_{C}(0)}\right)$ is the absorption coefficient. It is almost completely absorptive near $b=0$, slightly breaking the unitarity limit there. Here, $-\operatorname{Re} \chi_{C}$ is the spin-independent real part of the scattering "force" (or "potential" in the impact-parameter plane) and it is repulsive in the inner part, $0 \leq b \leq 0.9 \mathrm{fm}$, just like the "soft core" ( $V_{\text {soft-core }}(0) \simeq 750 \mathrm{MeV}$, here) and weakly attractive in the outer part as seen in Fig. $2(\mathrm{~b})$, for the eikonal $\chi$ is proportional to minus of the integral of the "force" over the projectile trajectory as

$$
\begin{equation*}
\chi(\boldsymbol{b})=-\frac{m}{\hbar p} \int_{-\infty}^{\infty} V(\boldsymbol{b}+z \boldsymbol{i}) d z \tag{4}
\end{equation*}
$$

where $\boldsymbol{i}$ is the unit vector in the $z$-direction.
The spin-orbit force has attractive intermediate-range ( $\sim 1 \mathrm{fm}$ ) component and repulsive short-range $(\sim 0.5 \mathrm{fm})$ one, apart from the factor $\left(\boldsymbol{\sigma}_{1}+\boldsymbol{\sigma}_{2}\right) \cdot(\boldsymbol{b} \times \hat{\boldsymbol{l}})$, as seen in Fig. 2 (c). From the range and sign of the intermediate component, it seems to come from the "one-boson-exchange" contribution like $\rho$ or $\omega$. This spin-orbit eikonal has not needed the long-range component, as is expected from the so-called "one-pion-exchange".

The spin-spin force has proved to require three components. As seen from Fig. 2(d), the long-range component has a range of $2 \sim 2.5 \mathrm{fm}$ and is repulsive, the intermediate-range one has a range of $\sim 1.5 \mathrm{fm}$ and is attractive and the shortrange one has a very short range of $0.3 \sim 0.4 \mathrm{fm}$ and is strongly attractive, apart from the factor $\boldsymbol{\sigma}_{1} \cdot \boldsymbol{\sigma}_{2}$. As $\boldsymbol{\sigma}_{1} \cdot \boldsymbol{\sigma}_{2}$ is minus for the spin-singlet state, the short-range force behaves like the "soft core" for ${ }^{1} S_{0}$. Total of the three components for the spin-spin force is drawn by the dashed line in Fig. 2(d) and it happens to be attractive in the wide range of impact-parameter ( $0.4 \leq b \leq 2.5 \mathrm{fm}$ ) for the spinsinglet state. It is speculated that it might be responsible for formation of the diproton resonances of singlet state like ${ }^{1} D_{2}(2.14 \sim 2.17 \mathrm{GeV})$ and ${ }^{1} G_{4}(2.43 \sim 2.50$ GeV ) in the lower energy region, ${ }^{21)}$ since the spin-orbit, tensor and quadratic spin-orbit forces do not make any contribution to the spin-singlet state.

As for the tensor force, the situation is quite similar to the spin-spin force as seen in Fig. $2(\mathrm{e})$, a repulsive long-range $(\sim 2 \mathrm{fm})$ component, an attractive inter-mediate-range $(\sim 1.5 \mathrm{fm})$ one and a strongly attractive short-range $(\sim 0.3 \mathrm{fm})$ one, apart from the factor $S_{12}$. The "repulsive" feature of the long-range component of both the spin-spin and the tensor forces are well reconciled with that of the one-pion-exchange potential established in the low-energy $N N$ scattering and the strength remains nearly the same as that of the potential with $G_{\pi}{ }^{2} / 4 \pi=14.4$ if calculated at $b=3$ and 4 fm for the spin-spin force.

The quadratic spin-orbit force has intermediate-range ( $\sim 1.5 \mathrm{fm}$ ) and short-
range ( $\sim 0.7 \mathrm{fm}$ ) components as seen in Fig. $2(\mathrm{f})$.
The strength of strong short-range force is about $100 \sim 200 \mathrm{MeV}$ in common for the four spin-dependent forces. It would be interesting to study the origin of the force.

## § 5. Scattering amplitudes obtained by the fit

Five independent scattering amplitudes for elastic $p p$ scattering at $6 \mathrm{GeV} / c$ obtained by the fit are depicted in Fig. 3 in the form of natural-parity and un-natural-parity exchange amplitudes defined as

$$
\begin{align*}
& N_{0}=\frac{1}{2}\left(\phi_{1}+\phi_{3}\right), \quad N_{1}=\phi_{5}, \quad N_{2}=\frac{1}{2}\left(\phi_{4}-\phi_{2}\right), \\
& U_{0}=\frac{1}{2}\left(\phi_{1}-\phi_{3}\right), \quad U_{2}=\frac{1}{2}\left(\phi_{2}+\phi_{4}\right), \tag{5}
\end{align*}
$$

where $\phi_{1}, \phi_{2}, \phi_{3}, \phi_{4}$ and $\phi_{5}$ are $s$-channel helicity amplitudes $\langle++\mid++\rangle$, $\langle--\mid++\rangle,\langle+-\mid+-\rangle,\langle+-\mid-+\rangle$ and $\langle++\mid+-\rangle$, respectively. In Fig. 3, the magnitude of the amplitudes is normalized by $d \sigma / d t=\left|N_{0}\right|^{2}+2\left|N_{1}\right|^{2}+\left|N_{2}\right|^{2}$ $+\left|U_{0}\right|^{2}+\left|U_{2}\right| .^{2}$

In the previous analysis of $\mathrm{I}, \operatorname{Re} \phi_{2}$ was positive at $t=0(\mathrm{GeV} / c)^{2}$ in contradiction with the prediction by dispersion relation ${ }^{22)}$ and the result of phase-shift analysis. ${ }^{37}{ }^{\left(3^{\prime}\right)}$ In this analysis, it has turned to a negative value $\operatorname{Re} \phi_{2}(0)=-0.513$ fm , which exhibits itself in the forward negative sharp peak in $\operatorname{Re} U_{2}$ in Fig. 3 (e), and this value is not inconsistent with the predicted one $-0.357 \pm 0.023 \mathrm{fm}$ from the dispersion relation by Grein and Kroll. ${ }^{22)}$ Our Re $\phi_{2}$ reproduces well the "spike" of slope $50 \sim 100(\mathrm{GeV} / c)^{-2}$ in $0 \leq|t| \leq 0.02(\mathrm{GeV} / c)^{2}$ for $n p \rightarrow p n$ at 6 $\mathrm{GeV} / c$. This has happened by the increase of the number of data points and the inclusion of the short-range component in the eikonals. Furthermore, the zero of Re $N_{0}$ at $t \simeq-0.35(\mathrm{GeV} / c)^{2}$ has disappeared and the present solution does not have any zero at least up to $|t|=3(\mathrm{GeV} / c)^{2}$, which is consistent with the result of analysis with phase-modulus dispersion relation by Grein, Guigas and Kroll. ${ }^{23)}$ $\operatorname{Im} N_{0}$ does not have zero at $-t=1 \sim 2(\mathrm{GeV} / c)^{2}$ at $6 \mathrm{GeV} / c$ as seen in Fig. 3(a); the above-mentioned solution by Grein et al. does have one there and this results in the dip of $d \sigma / d t$ in the higher energy region. Ours has a zero around $t=-3(\mathrm{GeV} / c)^{2}$, similar to the solution A from the phase-shift analysis by Matsuda, Suemitsu, Watari and Yonezawa. ${ }^{\left.3), 3^{\prime}\right)}$

Single helicity-flip amplitude $N_{1}$ has a negative imaginary part fairy larger in magnitude than its real part at $|t|=0.1 \sim 0.4(\mathrm{GeV} / c)$, ${ }^{2}$ which is different from the solution for $N_{1}$ from the Regge pole model with $P, f, \omega, \rho, A_{2}, A_{1}, Z, \pi, B, \varepsilon$ and $\omega^{\prime}$ poles by Berger, Irving and Sorensen while their fit to the observables is not so good for their taking too strict constraints on the trajectories and residue functions. ${ }^{2}$ This feature of $N_{1}$ is also different from the solution by Kroll, Leader and von



Fig. 3. Combination of the five helicity amplitudes obtained by the fit for $p p \rightarrow p p$ at $6 \mathrm{GeV} / c$; (a) $N_{0} \equiv\left(\phi_{1}+\phi_{\mathrm{s}}\right) / 2$, (b) $N_{1} \equiv \phi_{\mathrm{s}}$, (c) $N_{2} \equiv\left(\phi_{4}-\phi_{2}\right) / 2, \quad$ (d) $U_{0} \equiv\left(\phi_{1}-\phi_{3}\right) / 2$ and (e) $U_{2} \equiv\left(\phi_{2}+\phi_{4}\right) / 2$. Solid lines are the imaginary parts and dashed lines are the real parts. Truncated values at $t=0$ $(\mathrm{GeV} / c)^{2}$ are $\operatorname{Im} N_{0}(0)=9.21, \operatorname{Re} N_{0}(0)$ $=-1.76, \operatorname{Re} U_{0}(0)=2.09$ and $\operatorname{Re} U_{2}(0)$ $=-0.93 \sqrt{\mathrm{mb}} / \mathrm{GeV} / c$.


Fig. 4. Argand plot of the five scattering amplitudes at (a) $-t=0.115(\mathrm{GeV} / c)^{2}$ and (b) $-t=0.352(\mathrm{GeV} / c)^{2}$. The vertical line is the imaginary part and the horizontal line is the real part.

Schlippe. ${ }^{2 \prime)}$ Our solution resembles the old B solution from the phase-shift analysis by Matsuda et al. ${ }^{3}$ ) Precise measurements of $D_{S S}$ with much smaller errors than now available would be needed to resolve this problem of $N_{1}$, since $D_{s s}$ determines the component of $N_{1}$ parallel to $N_{0}$. Argand plot of the five scattering amplitudes is given at $-t=0.115$ and $0.352(\mathrm{GeV} / c)^{2}$ in Fig. 4.

We have floated the parameters of the eikonals in a wide range, but we have not got the other solutions and the present solution is the only one for the scattering amplitudes as stated above. It might be due to the inherent constraints imposed by the eikonal model or due to the form of eikonals chosen here. It will be examined in detail in the future.

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[^1]:    *) For the short-range component, we also tried a long-tailed function such as the modified Fermi distribution one in the same way as for the other components, but it has not given so good a fit as taking the Gaussian function here for the short-range one.

