Memory Dynamics in Asynchronous Neural Networks

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A model which can perform learning, formation of memory without teacher for successive memory recalls is presented. The philosophical background of the study is summarized. The investigated network consists of two sets both composed of asynchronously firing model neurons. One set of neurons is responsible for the field effect, and the other is introduced as an input/output module. The field effect is given in the form of the system's self-response. It is shown that positive and negative global feedbacks by the field effect play an essential role in the successive recall of stored patterns. The possibility that these proposed mechanisms are implemented in the brain is discussed. We obtained a quasi-deterministic law on the level of a macrovariable concerning a random successive recall of memory representations by taking a Lorenz-plot of this macrovariable. We show that this macroscopic order is deterministic chaos steming from collapse of tori and this type of chaos can be an effective gadget for memory traces.

§ 1. General introduction —philosophical background of the study

The long-range purpose of our research is understanding higher functions of the human brain. The human brain is a highly complicated object which consists of about 10^{10} neurons each of which connects to other $10^3 \sim 10^4$ neurons respectively.[1] In this sense, the brain can be said to be a highly connected "neural network". One should note that the network is not homogeneous as a whole but heterogeneous[2] since there are strong connections between subsystems (for instance, visual cortex, auditory cortex, somatosensory cortex, motor cortex, thalamus, peripheral cortex and brain stem) and even at the level of subsystems many kinds of neurons have a specific function. In this respect, a simple statistical treatment[*] should be restricted in the brain research. The neocortex is subdivided into a mosaic of quasi-discrete space units.[2,3] It is known that the neural networks are plastic in the sense that synaptic connections can be modified depending on the activity of neurons.[33] Physiolosical studies have shown that there are two kinds of plasticity of the networks:[2] One is expressed by the modification of the strength of connectivities between neurons and the other by the modification of the formation of links of the networks. In this way, the brain can be seen as fully plastic and highly connected neural networks.

Although useful physiological and anatomical data concerning the functional characteristics of the networks and its plasticity have been accumulated, it is questionable whether these findings (microscopic version of the system) can explain the mechanisms involved in higher functions of the brain.

On the other hand, the approach from more macroscopic viewpoints has been

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[*] A pioneering work of this type was done by Amari,[15] and Wilson and Cowan.[16]
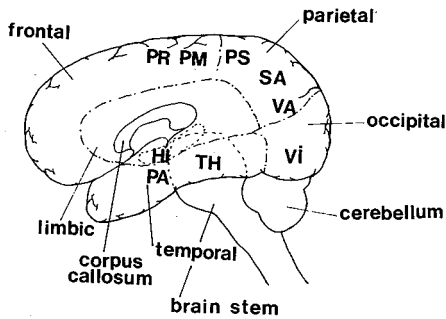
Fig. 1. (a) Rough sketch of the brain. Right hemisphere is viewed from inside. Vi; primary and secondary visual cortex, VA; visual association cortex, SA; sensory association cortex, PS; primary sensory cortex (somato-sensory cortex), PM; primary motor cortex, PR; premotor cortex, PA; primary auditory cortex, TH; thalamus, HI; hippocampus.

tried with different levels of language, which is a so-called psychological description. Although these descriptions have sometimes stronger power than physiological ones in explanation ability, they are insufficient as descriptions for understanding the brain since methodology of physical realization (hardware implementation[9]), namely, of extracting "order on particularity" is unclear only in the scope of such descriptions.

Recently, another approach called "construction approach" has been tried by engineers. For a specific function, neural networks realizing such function are modeled. Although it appears useful, there have been only a few successful models like Marr's cerebeller perceptron model.[4] A weak point of this study is that there is no guarantee that the neural circuit model is a substance constructing the brain.

Taking these situations into account, we propose another kind of approach towards understanding the brain. That is *a story-telling approach,* in other words, *a hermeneutic approach.* Our proposition stresses the importance of "interpretation". On the basis of the physiological and anatomical experimental data, we interpret the activities of neural networks for supposed higher function of the brain. Why do we have to "interpret" the cortical events? Being different from the research object of standard physics, only insufficient knowledge is given for the function to be understood. Insufficient parts of the knowledge is dominant compared with the knowledge required. This tendency is seen not only in the brain research but also in recent nonlinear sciences. One might obtain definite theories in part, but even in this case one would not have *a closed theory.* Despite the theory of localization of the function of the brain, we are obliged to take into account related parts of the cortex which are in most cases immense.

For instance, let us consider "pattern recognition", the purpose of which is understanding what the object is. A possible technology for understanding has been proposed as "Hermeneutics"—the scholar of interpretation.[5] Here let us try to define "interpretation". In the scope of our considerations, the definition of interpretation can be proposed in the following according to the relation between the object to be decided on and the state of knowledge within the system:

Step 1: An interpretation is an action required when information for describing an object is incomplete. Then, one has to guess what the object is by implementing necessary calculations to complete the information and therefore the object. A typical example corresponding to this case is a situation that a part of an object is hidden.

Step 2: If the internal knowledge base is insufficient to solve the problem, descrip-

tions from outside are needed, namely, the description is not closed inside the object. For instance, consider the recognition of "cup". One cannot reach the recoginition only with a possible set of descriptions concerning inherent characters of the matter constructing a cup, say, red, cylindrical shape, glassy, with knob or without knob, etc. Additional description is necessary such like grasping it by the hand and moving it to the mouse. This description comes from outside of inherent characters of the object itself.

We treated this problem on the physiological basis.[6),7)] At the retinal level, a pattern is divided into components. After that, the components are related to each other depending on the semantics through thalamo-cortical interconnections. In this process, one notices that Vorverständnis (preunderstanding)[5)] for an image of the object is necessary since without it initial intriguing of a relation between components does not start.

Here one question arises: Where does Vorverständnis come from? This is a too difficult problem to be treated sufficiently in this paper. We formulated a hypothesis on the decisive role of thalamo-cortical interaction in the formation of a "Vorverständnis".[7)] One important aspect of this scheme is the formation of a sequence of definitely recalled patterns at an input/output neuron pool which communicates with another pool of neurons that is not so strictly controlled by specific inputs like the first one. But no specific memory dynamics was introduced. Here we discuss in which respect such a system could perform a 'random associative' search with high flexibility in response to an arbitrary input pattern sequence.

The problem of the memory should not be restricted to the thalamo-cortical interaction but also to the hippocampal-cortical interaction[34)] and spines of pyramidal cells in the neocortical columns.[3),8)] The importance of the hippocampal function concerning a mechanism of memory consolidations has been recognized.[8)]

It is plausible that memory representations of all or a subset of stored patterns are recalled in a short time range for preparing the Vorverständnis.[7)] In this sense, the study of the mechanism requires that the stored patterns are searched, triggered by a key input before starting the process of recognition of a pattern. We present in this paper a possible model for this searching process of the memory.

Concerning a process of pattern recognition, there are many other problems to be solved. In what form and where is semantics processed? One uses natural language as a final description of a pattern, but how is it done? Our knowledge is too insufficient to solve these problems despite endeavors of many researchers in this field. Our obligation at this stage is to proceed with "interpretation".

We think that this type of approach is useful for other complex systems,[13)] However, one might doubt a relevance of interpretation. There would be several different interpretations conserving at least inner-consistency for one subject (theme). Let us note that there is no basis to choose a specific interpretation among others. So, interpretation itself does not seem to have the possibility of verification. The relevance of interpretation should be established not inside but outside the theory.

Also in physics, an interpretation is necessary even after a rigorous mathematical and physical treatment is obtained, and in this case the relevancy of the interpretation can be verified by more further experiments (which are outside the theory). This is

based on the assumption that since a specific object studied has no strong interactions to other objects, one can discuss the object separated from other ones. On the other hand, in brain research, a part of the phenomena occurring in the brain must be interpreted because the brain is a strong-connected system in the sense of system theory, namely, in the brain, events are mutually related.

One method to obtain the relevancy is to make engineering type simplified system; namely, one can obtain outer-consistency of the interpretation if an algorithm required to implement the story (i.e., the interpretation) on a machine is found. In this respect it is necessary to take into account a field in which higher functions are working. In our case a field is a neural network. A holonic computer[6],[7] (which has a concept similar to a synergetic computer[13]) interpreting patterns was designed[36] based on the hyper-column structure of visual cortex[37] and the thalamo-cortical interactions.

Another method to obtain the relevancy is again to appeal to an experimental verification. As we mentioned above, an interpretation itself is not an object to be verified by experiment at the stage of the present brain research. However, there is still a possibility that such interpretation stimulates a more clear-cut theory or modeling at the level of the subsystems of the underlying problem. Then, that theory or modeling is an object to be verified by experiment. This is a new type of contribution to natural sciences.

We can see that this type of contribution has already been expressed in science although only a little. The Gestalt psychology stimulated brain sciences in giving a new viewpoint (at that times) for studying the mechanism of psychological phenomena. But the Gestalt story itself was not the object to be verified by experiment since it was based on subjectivism.[9] A more clear-cut theory stimulated by Gestalt theory which had a possibility to be verified answered in a negative form the statements proposed by the Gestalt theory. Despite the unfortunate results, one cannot overlook the importnce of contributions of the Gestalt approach to science in the presentation of new viewpoints as one trial among many in order to understand the higher functions of the brain.

Eccles[1],[2],[8] has studied a mechanism of mind on the basis of neural hardware architectures as an interactionist. The interpretation of Eccles concerning brain-mind interaction is beyond the scope of the present natural science. Therefore it is impossible to check it directly by experiment although one might obtain data which seems to show plausibility of the interpretation. However, his interpretation is too much appealing to be overlooked. The subset of his story should be checked by experiment and more precise modeling.

Crick[10] recently interpreted the REM dream by the spinglass model of an associative memory correlated to an unlearning mechanism of Hopfield et al.[11] Again we meet a story-teller whose story cannot be checked by direct experiment. But, actually his interpretation is now stimulating the scientists including physicists in the field of brain research. We believe that more clear subsystems will be found concerning the illogical dream stimulated by Crick's story.

In this way an interpretation becomes *plausible* by an accumulation of experimental results and theory in subsystems.

§ 2. Introductory remarks to memory dynamics

In § 1, we mentioned the mechanism of pattern recognition, where we treated it from the viewpoint of Hermeneutics and inquired by ourselves the origin of Vorverständnis. Although our trial expressing the process of pattern recognition as the Hermeneutics process succeeded in restricted cases (see our provious papers[6),7)], the treatment of memory representation was insufficient.

In order to establish Vorverständnis, stored memory representations must be searched rapidly as mentioned in § 1. For this process, one requires a model for successive recall of memory representations.

Amari and Kohonen[21)] have simulated memory by numbers on the neural networks. Let us assume that memory can be represented by a set of the synaptic connections from neuron i to j; $\{C_{ij}\}$. If a teacher trains the networks initially by an algorithm $C_{ij} = \sum_{\mu=1}^{m} y_i^{\mu} y_j^{\mu+1}$ where the pattern $\boldsymbol{y}^{\mu} = (y_1^{\mu}, \cdots, y_N^{\mu})$, m a number of patterns and $\boldsymbol{y}^{m+1} = \boldsymbol{y}^1$ is assumed, then for a key input, say $\boldsymbol{y}^{\nu}$ the system can produce a sequence of patterns $\boldsymbol{y}^{\nu} \to \boldsymbol{y}^{\nu+1} \to \boldsymbol{y}^{\nu+2} \to \cdots$. Recently, Sompolinsky and Kanter[12)] have treated the same problem and discussed the periodic behavior associated with a sequence similar to the above in recall in relation to the biological locomotions. The algorithm of this type can be called an algorithm with instruction.

In this paper, we propose a model for successive memory recall without instruction. As shown below, our algorithm when storing patterns is simply a Hebbian type,[40)] i.e., $C_{ij} = \sum_{\gamma=1}^{m} y_i^{\gamma} y_j^{\gamma}$. This means that there are no instructions for ordering the patterns. We show that our model can produce a sequence of patterns in spite of no instruction for numbering. Moreover, we show that a random sequence of patterns is governed by a quasi-deterministic law at the level of some macrovariable.

The neural networks treated here have some physiological significance. According to anatomical data obtained by Szentagothai,[3)] the neocortex has a structure suitable to be characterized by the terminology "superstructure". Microscopic parts ($\sim 30~\mu$m in diameter) may be constructed rather regularly, which means rather definite registered structure. Mesoscopic parts ($\sim 300~\mu$m in diameter) are constructed randomly. In this scale, axon-collaterals of pyramidal cells have some spatial distribution. In macroscopic parts (a few mm in diameter) a registered structure is dominant.

Our present model partially contains the structure in these three space scales. As mentioned in § 1, there are many kinds of neurons in cortex. But, it is not possible to take into account the whole kinds of neurons simultaneously in small scale simulations. We restrict our attention to only two kinds of neurons at the starting point. But the hypothesis treated in determination of the possible values of synaptic strength would require the existence of the other kinds of neurons of the cortex. Therefore, another neuron will be introduced operationally. This hypothetical neuron plays an essential role in completion of our aim. We can point out a possible candidate in physiological neurons corresponding to the neuron operationally introduced here. One might criticize that the operation of this type does not make the theory closed. However, it is already known by Haken's contributions[13)] that the understanding of

any complex systems always requires a model which is not closed by itself. He discussed the importance of how operational information influences the system.

For an establishment of Vorverständnis, a memory must be searched rapidly. Furthermore, it has been discussed that the process occurring on the spines of neurons is responsible for an ultra-short-term change of connectivities.[39] On the other hand, since there are many spines on the apical dendrite of the pyramidal cell,[3],[8] it can be stated that those spines give a specific mechanism of the dynamics of memory. As is seen in § 4, our model is suitable for this scheme.

Figure 2 shows our basic model of the neural network. Here we reduced the randomness of the structure in mesoscopic space scale to the randomness of the function of neurons, namely, we regard the dynamics [deterministic threshold dynamics with random and ordered networks] as the dynamics [probabilistic threshold dynamics with ordered networks]. Therefore, in our modelling, neurons may fire asynchronously. In this respect, we call this network asynchronous neural network.

Asynchronous neural networks have recently been retreated by physicists from different viewpoints. Let us review slightly the recent contributions of physicists on this topics.

§ 3. Short review of spinglass model for associative memory

Stimulated by Hopfield's paper of 1982[14] concerning the collective computations in terms of asynchronous neurons, theoretical studies of the memory mechanism of the cerebral cortex[10],[11),[15),[16] and the parallel computation[17),[18] towards new generation computers have been accumulated. Different kinds of approaches to parallel computations have been studied in Bioholonics research project. One is parallel computations in terms of nonlinear oscillators[6),[19] and the other is based on neural networks derived from neuroanatomical data.[7]

Models of associative memory and content-addressable memory were already proposed by Steinbuch,[20] Kohonen,[21] Nakano[22] and others[22] from engineering points of view. This approach is based on the idea that deterministic threshold dynamics can give an associative gadget by using a linear combination of connectivities between the elements (neurons) if they are highly connected.

On the other hand, recent studies of Hopfield[14] and others[15),[23] are engaged with nondeterministic threshold dynamics. They have discussed a similarity of dynamics between the proposed asynchronous neural networks and the spin glasses. Important points of their studies are: ①They discuss the Lyapunov function in the configuration space, ②they show that collective computations are possible even within networks of asynchronously firing neurons, namely, parallel computations are implemented without rigid algorithms, and as the result, ③in contrast to the networks implementing linear algebraic algorithm which relax into an optimum solution these non-deterministic systems relax into a sufficient solution. However, while the first one can be trapped in local optima, the other relax into a global optimum with high probability.

Moreover, it has been shown that the Hopfield model can produce good solutions for combinatorial optimization problems like the traveling salesman and graph

coloring, the transposition code problem and so on.[18]   This type of approach to the well-known difficult problems in engineering is of practical importance.   Namely, even if this approach cannot give the solution of NP complete problems,[*] it may reduce indeed the computational efforts since the parallel computation of this type takes only a very small number of steps of computations even when the system size is so large (several hundreds elements), as far as one can define an appropriate Lyapunov function.

Concerning the pattern recognition, however, a more powerful algorithm was recently introduced by Kinzel.[23]   In the case of the Hopfield model, in order to store patterns, the synaptic connections from neuron i to neuron j, C_{ij} has to be adapted to the patterns by the algorithm

$$C_{ij} = \sum_{\mu=1}^{m} y_i^{\mu} y_j^{\mu} \quad (|C_{ij}| \equiv [0, m]),$$

where the patterns $\boldsymbol{y}^{\mu} = (y_1^{\mu}, y_2^{\mu}, \cdots, y_N^{\mu})$, m a number of patterns and a set of dichotomous variables $y_i^{\mu} = +1$ or -1.   But, this algorithm does not work well for correlated patterns.   Motivated by the fact that many synapses in animal's cortex die out at the early stage (the first few years postnatally) though the learned information is not lost, Kinzel[23] adopted the new algorithm that C_{ij} is taken to be zero if $C_{ij} y_i^{\mu} y_j^{\mu} < 0$ for any patterns $\boldsymbol{y}^{\mu}$, by means of which the system can recognize correlated patterns, too.   By this algorithm representation of patterns in the matrix of connectivities becomes more sharp (see also Kohonen (Ref. 21))).

The dynamical theoretical viewpoint[14],[15],[23]~[26] of the pattern recognition restricts our attention to the tractable definite concepts, namely, the distributions of attractors (fixed points, limit cycles and chaos).   From this viewpoint, one can express stored memories by attractors.   The number of attractors gives the memory capacity of the system.   The size of a basin gives the accessibility to the attractor within the basin.   The depth of the Lyapunov function, if it can be defined, gives the strength of the memory representation.   The strength of the memory representation can be evaluated by learning algorithms, for instance, the Hebbian type learning:

$$\Delta C_{ij}(t) = \varepsilon y_i(t) y_j(t).$$

Although the Hopfield type model is very convenient for the pragmatic pattern recognition in its high memory capacity, rapid association and high ability of self-repair, only one fixed point corresponding to one of the stored patterns is recalled by one input stimulus (a key pattern) in a steady state because of the lack of the mechanism erasing the fixed point once it is reached, or the lack of the mechanism which enables the systems to leave a fixed point by climbing the potential wall.   The purpose of the Hopfieldian model is to find not the optimum but reasonable solutions. Therefore the key point for solving the problem is to define the Lyapunov function. Once the Lyapunov function is obtained, the system should go to the state which

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[*] NP problems are the problems which have no general solutions but can be solved by a specific algorithm with polynomial time.   It is hypothesized that among them there exists a class of the problems in which all NP problems can be solved by an algorithm with polynomial time if one specific algorithm with polynomial time is found.   Problems of this type are called NP complete problems.

makes the function minimum or maximum or at least locally extremum.

The accessibility to attractors has been estimated by Hopfield et al.[11] It has been proved that the accessibility can become high by the algorithm of unlearning. According to them, spurious modes can dissapear by 'unlearning' and the size of a basin of an attractor corresponding to a nominal memory (a relevant mode) becomes large which results in the stabilization of nominal memories. In relation to this topics, as we mentioned in § 1, Crick and Mitchison[10] have discussed a possible mechanism of a dream in the REM (Rapid Eye Movement) sleep period. They hypothesized that the erasing process of spurious modes by the unlearning mechanism of Hopfield et al. is implemented in the REM sleep period and spurious modes are responsible for an illogical dream. In this way, they considered that a spurious mode is an object to be erased for a rational memory recall. However, is it true that a spurious mode is irrelevant for a memory recall? As shown below, in our model a rational successive memory recall can be implemented by triggering spurious modes, namely, a series of recalled modes including spurious modes follows a macroscopic rule generated by the system's dynamics itself.

§ 4.   A model for successive recall of stored memories

A neuron has two states +1 (firing) and −1 (rest). The local dynamics of a neuron is as follows: With a probability p, if the summation of activities of inputs to a neuron exceeds the threshold one time, then at the next time step a neuron takes the
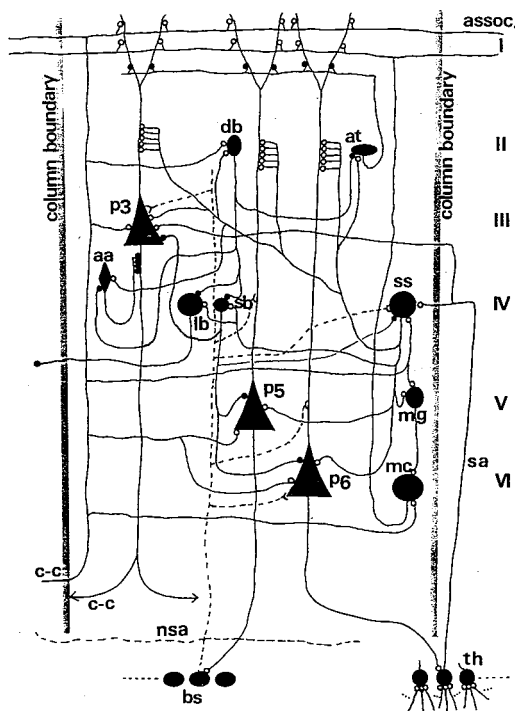

Fig. 1. (b) Quite simplified neocortical neural networks of mammalians (visual or somatosensory cortex) (drawn with knowledges of neuroanatomical data of Szentagothai[3] and of histochemical data of Sano et al.[35]). There are about 2×10^6 columns in neocortex. Only one column is depicted here. In each column there are a few thousands of neurons about a half of which are pyramidal neurons. $P_i(i=3, 5, 6)$; pyramidal cells in layer i (only pyramidal cells in three layers are depicted here), db; double bouque cells, at; axonal tuft cells, aa; axo-axonic cells, ss; spiny stellate cells, sb; small basket cells, lb; large basket cells, mg; neurogliaform cells, mc; Martinotti cells, th; thalamic cells, bs; cells in the brain stem, sa; specific afferents, c-c; cortico-cortical fibres, assoc.; association fibres. Small closed circles indicate inhibitory synapses and small open circles excitatory synapses. However, double bouque cells may change the function from inhibition to excitation and vice versa, and Martinotti cells' function is still an open question. Serotonergic innervations from brain stem to both pyramidal cells and stellate cells are also depicted by dotted lines. Note that a random distribution of axon-collaterals of pyramidal cells is not drawn here.
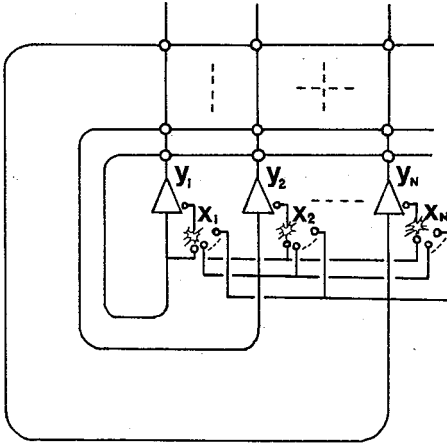
Fig. 2. A model system (I & II). A small circle (○) denotes a synapse. A large triangle (△) denotes a pyramidal neuron symbolized by y and a symbol ☆ a stellate neuron x.

value $+1$, otherwise it takes the value -1, and otherwise the value of the state is kept. So, we use here a symmetric probabilistic neuron motivated from anatomical data of Szentagothai explained in § 2. We assume that p is independent of the input activity level. Similarly, Hopfield adopted a symmetric probabilistic neuron,[14] namely, in his model the average readjusting rate is p. For simplicity, in our model the threshold of the individual neuron is always set to be zero. But the threshold is introduced indirectly into the pyramidal neuron as an effect of the stellate neuron as shown below. These are the most important types of neurons regarding the signal process of the neocortical columns.[3),7),28)] The nomenclature of "pyramidal" and "stellate" comes from its shape of each neuron.

A hypothetical neural network consisting of two kinds of probabilistic neurons is designed as in Fig. 2. Each pyramidal neuron (denoted by y) sends its fibers to both all pyramidal neurons and all stellate neurons (denoted by x), and makes synaptic connections. Each stellate neuron makes synaptic contact with only one corresponding pyramidal neuron. Here, we assume that an input and output are represented by a set of $\boldsymbol{y}$, not $\boldsymbol{x}$. The other assumption is that a memory can be stored only in synaptic connections between pyramidal neurons (recurrent connections of pyramidal neurons to pyramidal neurons).

A model equation of the network of systems I and II is as follows:

$$(\text{I \& II}) \begin{cases} y_i(n+1) = \mathbf{1}^{py}(\sum_{j=1}^{N} C_{ij}(n)y_j(n) + d_i \cdot x_i(n)), \\ x_i(n+1) = \mathbf{1}^{px}(f(y_1(n), y_2(n), \cdots, y_N(n))), \\ C_{ij}(n+1) = C_{ij}(n) + \varepsilon \, y_i(n)y_j(n), \\ C_{ij}(1) = \sum_{\nu=1}^{m} y_i^{\nu}y_j^{\nu}, \end{cases} \tag{1}$$

where we take $f(y_1(n), y_2(n), \cdots, y_N(n)) = 1/N \sum_i e_i y_i(n)$, and $\mathbf{1}^p(z) = \begin{cases} +1 \text{ if } z > 0 \\ -1 \text{ otherwise} \end{cases}$ with probability p. Fixed connections d_i and e_i are predetermined randomly on the intervals $[-\alpha, \alpha]$ and $[-\beta, \beta]$, respectively. By this determination, the effect of inhibitory neurons is implicitly taken into account. From physiological viewpoints basket type inhibitions to the soma (cell body) of both pyramidal cells and stellate cells are the most adequate correspondence to these inhibitory neurons (see Fig. 1(b)). In this sense, the output of our model represents the results of cooperation and

competition between neurons.[27]   Above the threshold, both the pyramidal neurons and the stellate neurons are independently readjusted with an average rate p_y and p_x, respectively.   In the initial connectivities $C(1)$, m patterns are stored by the Hebb type learning algorithm.[40]   According to the Hebbian algorithm, a strength of a synaptic connection depends on both pre- and post-synaptic activities.   This means that besides information by means of electric signals additional information from the post-synaptic part to the pre-synaptic part must be assumed.   Taking into account a long distance from the soma to the apical dendrite of the pyramidal cell (~ 100 μm), it takes much time if a chemical diffusion plays a role.[39]   But if a change of shapes and lengths of dendritic spines is responsible for plasticity of synapses, a very rapid process (about 10 msec) would be possible and therefore the Hebbian algorithm would be justified (see also Malsburg (Ref. 41))).

The stellate neurons embody the external field.   However, in this case the external field is not fixed, but changing according to the total activity of pyramidal neurons. . This also means that the threshold of pyramidal neurons varies according to the activity of stellate neurons which is determined by all pyramidal neurons.

By the Hebbian type learning, the matrix of connectivities C is symmetric. Since the term $\sum_j C_{ij}(n)y_j(n)+d_i x_i(n)$ is a local field of a neuron $y_i(n)$, the following function $W(n)$ can represent the total energy of the system.   If we consider the mathematical expression

$$W(n)=-1/2\sum_i \sum_j C_{ij}(n)y_i(n)y_j(n)-\sum_i d_i x_i(n)y_i(n) , \tag{2}$$

the single neuron's sign change results in the change of the function:

$$\Delta W(n)=-\Delta y_k(n)(\sum_i C_{ki}(n)y_i(n)+d_k x_k(n))$$

$$-\sum_i d_i \cdot \Delta x_i(n)(y_1(n), \cdots, y_N(n))/\Delta y_k(n)\cdot \Delta y_k(n)y_i(n) . \tag{3}$$

The first term on the right-hand side always gives a negative contribution to ΔW.   On the other hand, the second term's contribution fluctuates according to the sign of $\Delta y_k(n)y_i(n)$.   Therefore, in this model the function W may not be a Lyapunov function.

However, for our purpose the above function has some merit.   Initially, we have only a term $\sum_i \sum_j C_{ij}y_i y_j$ since the patterns are stored in $C(1)$.   Then, the function W creates the Lyapunov function at a starting point.   Many local minima, i.e., many fixed points are built in (see Fig. 3).   In the development of the system, both the landscape of the function W and the topology of the phase space vary depending on p_x, p_y and ε.   Therefore, *the asymmetry* in the effective connectivities (created by the second term of W) can be responsible for the formation of relation between memory representations, for instance, both a periodic association by limit cycle and an aperiodic association by chaotic wandering (see Fig. 4).

An algorithm implementing a global negative feedback effect is operationally introduced (into system II).   If system II goes to a steady state, the local field of the neuron i in system II is substracted by $\sum_j^{II} C_{ij}(n)y_j^{ss}$ until the next steady state appears (y_j^{ss} denotes a value of neuron j in a steady state).   In this mechanism to
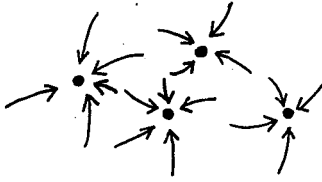
Fig. 3. From dynamical theoretial viewpoints, an event (the lowest order memory) can be represented by a fixed point and a higher order memory by a limit cycle and chaos. A simple example of a structure in memory space is schematically shown in the figure in the case of the lowest order memory. And in Fig. 4 an example of higher order memory is shown.

Fig. 4. A connection between stored events can be organized in the form of limit cycles (left) and chaos (right).

explain the dynamic character of the stored memory representations in the neocortical columns, interest in a pattern being recalled for a long time is lost and a new pattern is generated. A possible correspondence in the cortical columns to a gadget playing a global negative feedback would be axonal tuft cells which may inhibit the apical dendrites of pyramidal cells (see Fig. 1(b)). One cannot overlook that Martinotti cells can be the candidate if they receive a switched signal from neurogliaform cells and inhibit the apical dendrite of pyramidal cells, though Martinotti cell's function is still an open question. Hence the neuron i in system II receives the input $I = \sum_j C_{ij}(n) y_j(n) - \sum_j{}^{\text{II}} C_{ij}(n) y_j{}^{\text{ss}} + d_i x_i(n)$. Although the local field of the neuron i is $\sum_j C_{ij}(n) y_j(n) + d_i x_i(n)$, the global negative feedback should be applied only to the first term since the second term may change the function W or the topology of the phase space by itself only. This means that the global negative feedback acts only on the apical dendrites of pyramidal cells (see Fig. 1(b)). This is the reason why we adopt axonal tuft cells or, if possible, Martinotti cells to this type of inhibitory neurons. A sign of steady state can b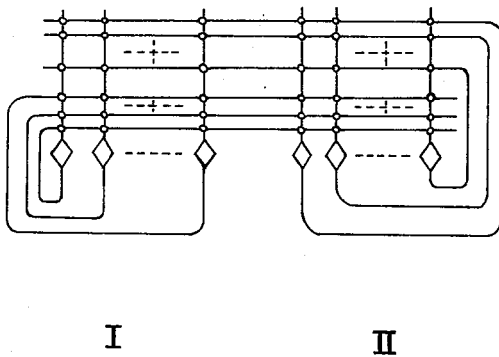e detected by a differential circuit which may be easily found in the neocortex. Another measure is introduced by resetting the values of pyramidal neurons to the initial values (the values for the key input) when the system goes to a steady state. This measure was introduced heuristically. The other assignments of the values of the pyramidal neurons did not result in the recall of all stored patterns. This mechanism seems to be plausible in a sense that a strong injection of the key pattern may reset the cortical activity. However, this should be treated as a hypothetical measure at

I   **II**

Fig. 5. Connected system in terms of the neural networks shown in Fig. 2. For simplicity, a set of the x neuron and the y neuron is denoted by one symbol ◇.

the present.

In order to improve the similarity of the model to the cortical hardware (it is a rare case that all controled negative global feedbacks act simultaneously in the cortex) and to investigate the effect of a global negative feedback introduced to one system (II) to the other system (I), we studied the connected systems. As shown in Fig. 5, the initially equivalent two systems (I & II) are connected:

$$C_{ij}(1) = \sum_{\nu=1}^{m} y_i^{\nu} y_j^{\nu}, \text{ where } y_k^{\nu} = y_{k+N}^{\nu} \text{ for } k = 1 \sim N \tag{4}$$

and

$$\left\{ \begin{array}{l} C_{ij}(n+1) = C_{ij}(n) + \varepsilon\ y_i(n) y_j(n) \\ \qquad \text{for } 1 \le i, j \le N \text{ and } N+1 \le i, j \le 2N, \\ C_{ij}(n+1) = C_{ij}(n) + \varepsilon' y_i(n) y_j(n) \\ \qquad \text{for } 1 \le i \le N \cap N+1 \le j \le 2N \\ \qquad \text{and } N+1 \le i \le 2N \cap 1 \le j \le N, \end{array} \right. \tag{5}$$
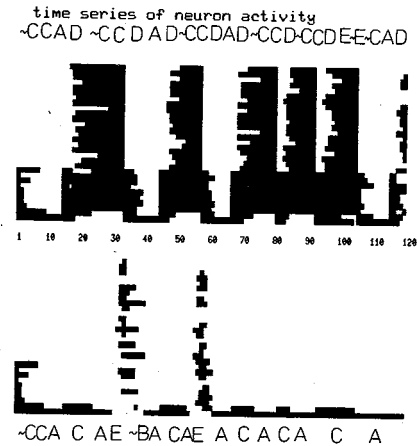
I: $i, j = 1 \sim N$,

II: $i, j = N+1 \sim 2N$.

By the mechanism in system II, system I is highly influenced via connections II to I.

In the next section, we show the results of computation of this model system. The system exhibits an automatic successive recall of stored patterns by a single stimulus.
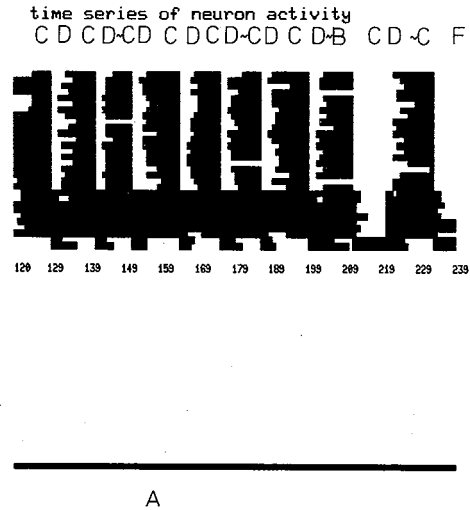
§ 5.   Results of simulations

One typical example is shown in Fig. 6. The resetting algorithm used in Fig. 6 is as follows. The indication of a steady state was given by the condition that the system continues to keep a certain pattern during T time steps. Throughout the simulation here we fixed the value of T to 4. Tr time steps after the decision about a steady state, resetting was done by the procedure discussed in the previous section. In this simulation Tr was taken as 1. We found that a similar phenomenon can appear also in the case without time lag, i.e., Tr=0. However, it should be noted that the number of recalled memory representations and the effect of the feedback from system II to system I are rather sensitive to Tr. Relaxation parameters are $\varepsilon = 0.01$ and $\varepsilon' = 0.0001$, so rather small, readjusting parameters $p_x = p_y = 0.4$, the amplitudes of probability distributions for fixed synapses $\alpha = 10.0$ and $\beta = 1.0$, and the number of elements $N = 30$ for each subsystem. Six patterns denoted by A~F are stored in C by the Hebbian algorithm before starting the dynamics. In the course of the dynamics, system II exhibits the pattern sequence $\sim C \to C \to A \to D \to \sim C \to C \to D \to A \to D$ $\to \sim C \to C \to D \to A \to D \to \sim C \to C \to D \to \sim C \to C \to D \to E \to \sim E \to \sim C \to A \to D \to C \to D \to C \to D$ $\to \sim C \to D \to C \to D \to C \to D \to \sim C \to D \to C \to D \to \sim B \to C \to D \to \sim C \to F \to \sim F \to \sim A \to \sim C \to D$ $\to C \to D \to A \to C \to D \to \sim C \to C \to D \to C \to D \to \sim C \to C \to D \to C \to D \to A \to C \to D \to \cdots$, and sys-

time series of neuron activity
~CCAD ~CC D A D~CCDAD~CCD~CCD E~E~CAD

SYSTEM II

SYSTEM I

~CCA C AE ~BA CAE A C A CA   C   A

time series of neuron activity
C D C D~CD C D CD~CD C D~B   CD ~C F

II

I

A

time series of neuron activity
F~F~A~CD CD AC D~C   CD C D~CCD C DACD
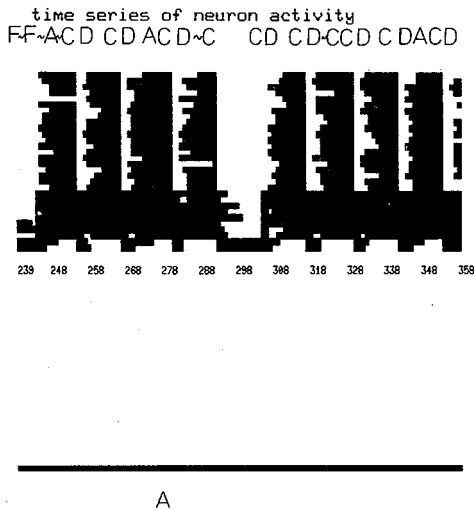

II

I

A

Fig. 6. Stored six patterns denoted by A~F are randomly successively recalled. The symbols depicted in both the upper-side and the lower-side of the figure denote the recalled memory representations. Abscissa denotes time and ordinate pattern. Dynamic behaviours of both systems I and II are shown. System's parameters are set: $p_x = p_y = 0.4$, $\varepsilon = 0.01$, $\varepsilon' = 0.0001$ and $N = 30$. Stored patterns are as follows: $A = \{y_1 = +1, y_i = -1 \text{ for } i \neq 1\}$, $B = \{y_{12} = +1, y_i = -1 \text{ for } i \neq 12\}$, $C = \{y_1 = y_2 = +1, y_i = -1 \text{ for } i \neq 1, 2\}$, $D = \{y_i = +1 \text{ for } 1 \leq i \leq 10, y_i = -1 \text{ for } 11 \leq i \leq 30\}$, $E = \{y_i = +1 \text{ for } 1 \leq i \leq 30\}$ and $F = \{y_1 = y_2 = y_4 = y_5 = +1, y_i = -1 \text{ for } i \neq 1, 2, 4, 5\}$.

tem I, $\sim C \rightarrow C \rightarrow A \rightarrow C \rightarrow A \rightarrow E \rightarrow \sim B \rightarrow A \rightarrow C \rightarrow A \rightarrow E \rightarrow A \rightarrow C \rightarrow A \rightarrow C \rightarrow A \rightarrow C \rightarrow A \rightarrow A \rightarrow A$ $\rightarrow \cdots A \rightarrow$, intermingled with unlearned patterns in both cases. A symbol $\sim X$ indicates that this pattern differs from the pattern X by only one element.

In this system the simultaneous sign change of all neurons ($y_i \rightarrow -y_i$ for all i) keeps the sign of C_{ij} but gives rise to the sign change of x_i, and results in the invariance of the function W. Hence, the dynamical behavior is invariant for the above sign change. In this respect we identify the pattern $\boldsymbol{y} = (y_1, y_2, \cdots, y_N)$ with the pattern $\boldsymbol{y}' = (-y_1, -y_2, \cdots, -y_N)$. This means the identification of the pattern of black figure—white ground, and the pattern of white figure—black ground, namely, the identification of the inverse contrast patterns.

Note that system II continues to reproduce the stored patterns randomly (this randomness is discussed in § 7), but on the other hand system I eventually reproduces only one single pattern. Therefore, in system I the transient phase and the steady phase in motion are clearly distinguished. One might attribute systems I and II to the

visible processing unit and the hidden processing unit[5],[31] respectively in the process of associative recall of the brain. This point will be discussed in more detail in the next section.

The steady state of system I disappears if the relaxation algorithm is not adopted, namely, the case that $\varepsilon = \varepsilon' = 0.0$. In this case, both systems I and II exhibit the random successive recalls of the memory representations (see also § 7). Almost all stored patterns are successfully reproduced (see Table I). Let us stress once again that a resetting mechanism is implemented in system II only. The trajectories in system I jump between the memory representations, influenced by the interaction of system II. One can see that the transition in system I is not always correlated with the resetting of system II. Namely, a time-delay of influence is seen. This comes from the dynamical structure of the phase space formed by the relation of the effects of the relaxation algorithm and those of the self-response via stellate neurons. The possible mechanism will be explained in the next two sections.

As an extreme case of the model, we investigated the deterministic threshold dynamics ($p_x = p_y = 1.0$). An association is often periodic,[22] whose period and the kind of pattern strongly depend on the initial condition of the pyramidal neurons.

In the following two sections, we will discuss a possible mechanism for this successive recall of memory representations.

§ 6. Qualitative explanations for the mechanism of successive recall of memory representations

In this section, we discuss qualitatively a possible mechanism of the aperiodic and periodic recall of memory representations. As discussed in § 4, the function W equals the Lyapunov function without the field effect of x. A set of interneurons x does not always give a consistent inner field to a set of output neurons y, namely, $y_i(n)(\sum_j C_{ij}(n)y_j(n) + d_i x_i(n))$ is not always positive. So, the effect of x is to change the landscape of W. The absolute value of W has no meaning since W is not the Lyapunov function. However, it is meaningful to investigate the relative change in W in order to see the effect of x and ε.

In order to prove this point we investigated the dynamic character of W in the system with the setting the C_{ij} initially according to *tabula rasa*. By *tabula rasa* it is understood that the system has not been trained at the initial stage of the system, i.e., $C_{ij}(1) = c_{ij}$ where c_{ij} is a random number on the interval $[-1, 1]$. The dynamics starts with the initial stimulus submitted appropriately and follows the Hebbian learning algorithm $\Delta C_{ij}(n) = \varepsilon y_i(n)y_j(n)$ subsequently. For a while, only the isolated system is the object to be investigated. In the case that $\varepsilon = 0.01$, $a = 10.0$, $\beta = 1.0$, $p_x = 0.1$ and $p_y = 0.9$ as one example the function W decreases oscillatorily (Fig. 7 (a)). But, the repetitive training by a certain stimulus causes W to decrease monotonically and shortens the relaxation time to a steady state. There are learning effects in this case. We can find the oscillatory decrease of W also in the case of the other values of the parameter set ($\{p_x, p_y\}$). One example is shown in Fig. 7(b): $\varepsilon = 0.01$, $a = 10.0$, $\beta = 1.0$, $p_x = 0.3$ and $p_y = 0.5$. If we increase the value of ε, for instance, $\varepsilon = 0.01 \rightarrow \varepsilon = 0.1$, keeping the same values of the other parameters, W decreases also monotonically in time (Fig. 7(c)). Under this condition of ε the oscillatory behavior of W recovers if
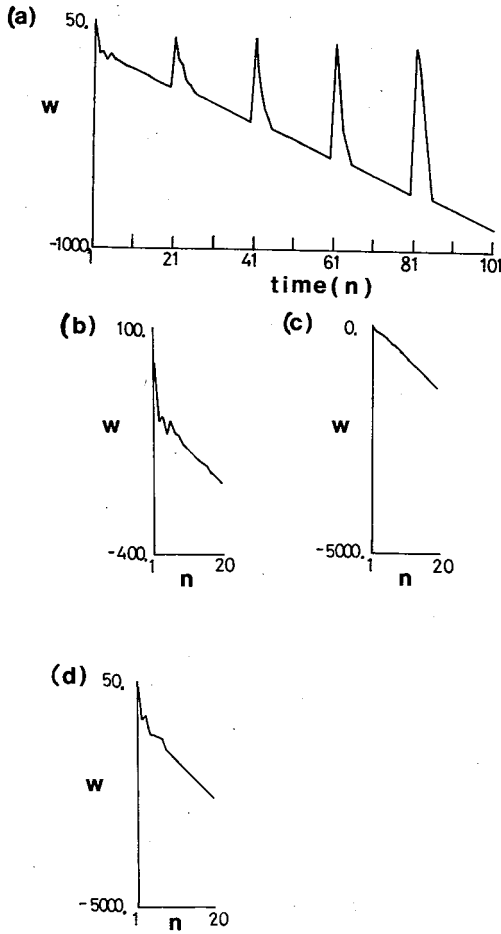
Fig. 7. Time course of the function W in several conditions of the system of *tabula rasa*. Input pattern is A in all figures.

(a) Repetitive training of the system. Pattern A is fed at $t=1, 21, 41, 61, 81$. Training results in smoothing of the potential. Oblique straight lines are signs of the steady state. A time that the system reaches the steady state becomes shorter and shorter. This is also the effect of the training in this system. $p_x=0.1$, $p_y=0.9$ and $\varepsilon=0.01$.

(b) Oscillatory decrease of W: $p_x=0.3$, $p_y=0.5$, $\varepsilon=0.01$, $\beta=1.0$ and $\alpha=10.0$.

(c) Oscillatory behaviour disappears: $p_x=0.3$, $p_y=0.5$, $\varepsilon=0.1$, $\beta=1.0$ and $\alpha=10.0$.

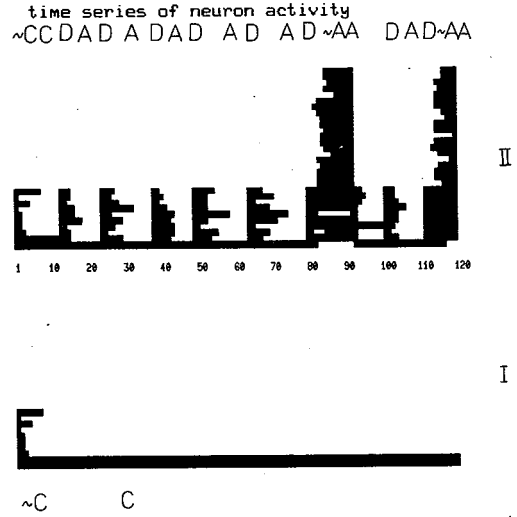(d) Oscillatory behaviour recovers: $p_x=0.3$, $p_y=0.5$, $\varepsilon=0.1$, $\beta=1.0$ and $\alpha=100.0$.

Fig. 8. A few memories are recalled in the case that $\varepsilon=\varepsilon'=0.1$, $p_x=p_y=0.4$, $\beta=1.0$ and $\alpha=10.0$.

the value of α is increased, for instance, $\alpha=10.0 \rightarrow 100.0$ (see Fig. 7(d)). Hence, the stellate neurons x cause the complex change of the landscape of W, which is responsible for an appearance and a disappearance of a spurious mode. On the other hand, the relaxation algorithm $C_{ij}(n+1)=C_{ij}(n)+\varepsilon y_i(n)y_j(n)$ plays a role of relatively deepening a local minimum of W corresponding to a state which the system temporarily reaches and broadening the basin of the attractor, that is, extending the accessibility of the system to this attractor.

Therefore, the relaxation algorithm may strengthen even relatively spurious modes. As the result, if ε increases, then only a few memory representations (a few among the nominal modes) can recover (see Fig. 8).

If the effect of the relaxation algorithm as a function of ε predominates over the effect of stellate interneurons x, the local minimum which the system reached at first is deepened while topology of the neighborhood of such minimum is only slightly affected. Hence, the system proceeds after reset-

Table I. Successive recalls of memory representations in the case that $p_x=p_y=0.4$, $\varepsilon=\varepsilon'=0.0$ and $N=30$.

System II	~C, C, A, D, ~C, C, D, B, A, D, ~A, C, D, D, E, D, ~C, C, ~F, D, C, D, ~C, A, D,
	~C, C, D, ~A, A, A, D, C, D, ~C, D, ~C, C, D, ~B, A, D, ~C, C, F, ~F, ~A, D, A,
	~A, A, D, ~C, D, ~C, C, D, C, D, ~C, C, D, E, A, C, D, ~C, C, D, ~C, D, ~C, C,
	D, C, A, D, A, D, A, C, D, ~C, C, D, ~C, C, D, C, D, ······

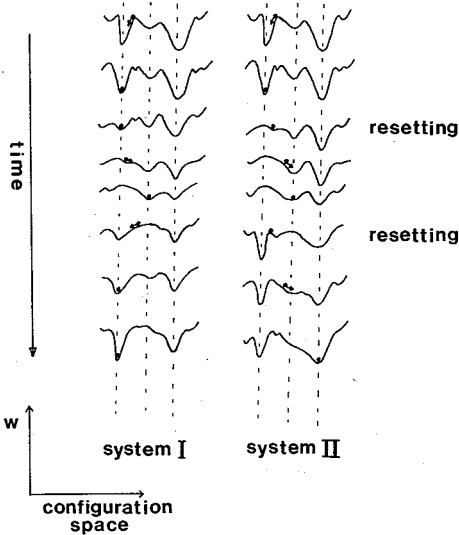| System I	~C, A, C, A, A, E, A, E, ~B, A, C, A, E, B, A, C, A, E, A, E, B, ~F, C, ~A, A,
	E, A, E, A, C, A, E, ~A, C, A, C, ~A, E, A, E, A, E, A, E, A, E, A, E, A, E, A,
	E, ~A, ~C, C, ~B, A, E, A, E, A, ······

Fig. 9. Mechanism of successive recall is schematically drawn.

ting to the other local minimum which is a spurious mode in almost all cases, and it is highly probably that the previous local minimum recovers after the next resetting. In this way, the system has a tendency to exhibit jumping between only a few modes including spurious modes (Fig. 8). On the other hand, if the effect of x predominates over the effect of ε, the change of the landscape of W caused by the appearance and disappearance of spurious modes is stronger than that caused by the relaxation effect. This gives the system a chance of successive recalls of many nominal modes, intermingled with spurious modes (Table I).

Here one question arises: Could it be that the original Hopfield model is a gadget for successive recall of memory representations? The answer is NO! Even if we use the resetting mechanism, the Hopfield model can only produce the jumping phenomenon between only two memory representations for lack of a mechanism to change the topology of the phase space dynamically. Generally, in ordinary associative memory models the depth of the potential of fixed points to be recalled becomes deep. On the other hand, in the present model the landscape of mountains and valleys in the potential changes dynamically like a ripple. We call this a ripple effect.

Including these considerations a possible mechanism of successive recall of memory representations is depicted schematically in Fig. 9. In the case of aperiodic successive recalls the path between the two relevant modes (a series of irrelevant modes) can fluctuate. This results in many different possibilities of path, and on the other hand, in the case of periodic successive recalls only a single path is selected (Fig. 10). This difference between the two cases seems to come from the presence and absence of ergodicity. Spurious modes can be shuffled by randomness both generated and imprinted in the system. This shuffling effect triggers aperiodic successive

Fig. 10. Path between the two relevant modes. Schematical drawings. Coordinates are arbitrary.
(a) nondeterministic case and (b) deterministic case.

recalls. This indicates that fluctuation (deterministic and/or thermal) may have an important role in processing information.

Let us discuss the periodic recall. Although a periodic recall itself is not biological literally (at least not a normal state), it seems to be important if it is used in controlling the periodic behaviour of the movements of the human and animals such as locomotions like swimming and walking, respirations and the hormone change. Furthermore, it seems to be important considering the relation formation between stored items. We have represented as 'item' by a set of neuron activities y. Initially, several items are stored in the networks but these are isolated or not associated with each other because of the learning algorithm. However, the computational results of the network produce periodic recalls of items, namely, periodicity is evoked in the network. Hence the periodicity can be a sign of establishing a higher order concept, that is, the formation of relations.[21]

§ 7.   Quantitative measure of random series of patterns
—genesis of macroscopic order

The problems left are the following: Is there any definite rule in this type of pattern recall?

For the problem we should refer for the first time to the curious structure of the configuration space in spin glasses. Mezard et al.[29] suggested that in spin glasses the hierarchical organization according to the ultrametric topology might occur by changing the control parameter, for instance, temperature (for a nice review of ultrametricity, see Rammal et al.[30]). Moreover, it has been pointed out by Hogg and Huberman[24] that the hierarchical organization characterized by a finite set of attractors in dissipative dynamical systems can exhibit structure of the ultrametric topology. An ultrametricity between hierarchies might be considered as one candidate of a model representing a history of the system's development. We have not yet succeeded in checking whether or not the process of development in our model follows an ultrametric topology.

One might ask the difference of the randomness appeared in the present model from the one in a multi-stable system with finite temperature. The randomness of both cases strongly depends on the structure of a set of stable states. However, in the latter case, only a probabilistic law is obtained at the level of coarse-grained variable. On the other hand, a quasi-deterministic law governs a random sequence in our model.

We calculated the numer of firing pyramidal neurons (N^+_n) at each time when a new value of N^+ appears. Taking a Lorenz-plot of this coarse-grained variable, we obtain a quasi-deterministic law in $N^+_n - N^+_{n+1}$ plane as shown in Fig. 11. Figure 11(a) shows a macroscopic rule which governs a randomness of system II. It can be approximated to the piese-wise linear or slightly wavy (sinusoidal) one-dimensional
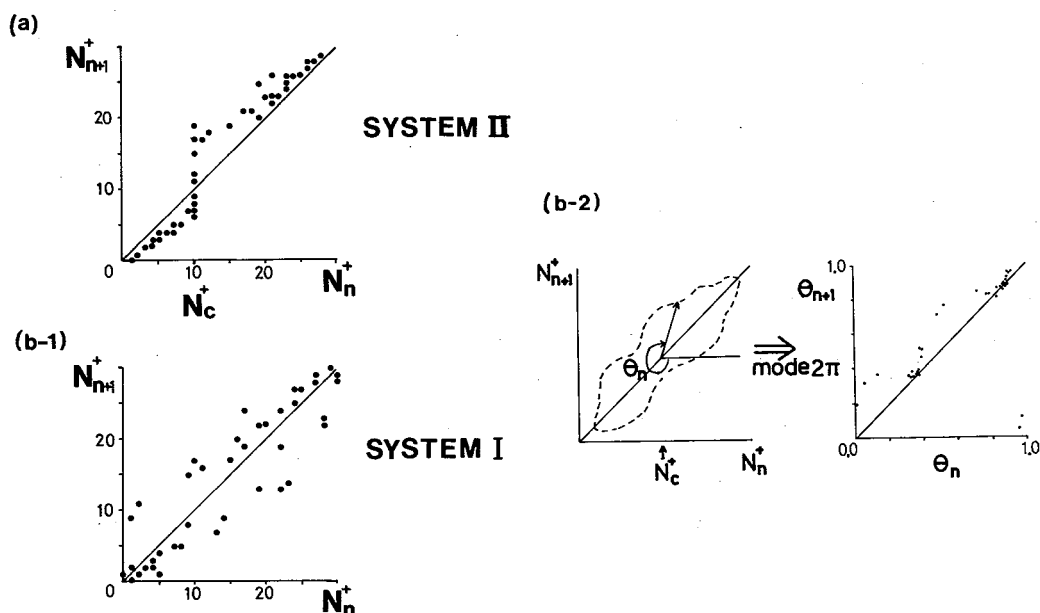
Fig. 11. Lorenz plot of the random series in (a) system II and (b) system I. Construction of a circle map is shown in (b-2). Parameter values: $p_x = p_y = 0.4$, $\varepsilon = \varepsilon' = 0.0$ and $N = 30$.

map which produces stable solutions. A threshold denoted by N^+_c is seen. This value of the threshold is exactly the same as the number N^+ of a key input. Starting from the critical value, the value N^+_2 at the next step is chosen probabilistically. After that, the system moves according to the deterministic rule. Figure 11 (b-1) shows a Lorenz-plot of system I. A slightly fuzzy closed loop is found. Once the point is settled on the line above (or below) the obligue line ($N^+_n = N^+_{n+1}$), the system moves along the upper (or lower) part of the closed loop. Moreover, the system experiences an incommensurable periodic behaviour between upward-directed and downward-directed movements. This means a quasi-periodic or chaotic motion. Then, if we take a new variable Θ_n (rotational angle) in the way shown on the left-hand side of Fig. 11(b-2), we obtain a circle map[42]~[44] approximated with the equation $\Theta_{n+1} = \Theta_n + A \sin 4\pi\Theta_n + C \pmod 1$, where $A \sim 0.0874$ and $C \sim 0.12$. We cannot judge, at the present, what the origin of a few points appeared below the 45° line is, namely, whether it comes from the probability introduced into the system, the choice of the origin for new coordinate, or the twist structure of the manifold of the strange attractor. However, the decreasing part of the map is plausible. If it is true, the system is exhihiting deterministic chaos. Actually, the numerical estimation of $4\pi A$ shows a weak chaos, i.e., $4\pi A \sim 1.098$.

     Nicolis and Tsuda[47] have discussed that the other type of chaos may play a role for short-term memory. On the other hand, the chaos obtained here is useful for memory traces. Preliminary calculations of efficiency defined by an average rate of a complete trace of all stored memory representations showed that the chaotic search is much more effective than the quasi-periodic search.

     The above results mean that deterministic macroscopic chaos is used in the search process of stored memories and might be so in real brain.

In this way, we obtained a macroscopic rule in the case of random series. The neuronal activities generate the 'order parameter' (one-dimensional map) which governs again neuronal activities and therefore the self-production of language. In this respect, a neuron is a synergetic[38] cell. The self-generated rule of this type gives a new role to spurious modes. A spurious mode is not irrelevant and not the object to be erased for a memory recall.

§ 8. Summary and discussion

We have proposed a model for successive recall of stored memory representations in terms of asynchronously firing neurons. Each subsystem (I or II) of our present model has a structure similar to that of the spin glass model proposed by Hopfield[14] and others.[15],[23] Since our model was motivated by the structure of neural assemblies in the cortical columns of mammalian brain,[3] the term corresponding to the external field in the spin glass model was introduced as a variable strongly dependent on the total activities of the output neurons (y), therefore it does not always relax into a specific solution. Instead of giving the Lyapunov function we introduced a potential like function W the landscape of which is subject to variation globally (see Fig. 9). Therefore, after introducing the mechanism (resetting as a global negative feedback) the system can act as an associative gadget for almost all stored patterns. Otherwise, we could not obtain the model of this type. We also discussed the possible assignments of the mechanisms introduced to physiological neurons. Furthermore, we obtained a quasi-deterministic law on the level of a macrovariable, i.e., the number of firing neurons by taking a Lorentz plot of the variable. The expression by a rotational angle Θ showed the chaotic one-dimensional map. This can be seen as "an order on particularity" created by the system itself. According to our theory, a spurious mode which might be irrelevant for a simple association could be relevant for a successive pattern recall. The theory that the pattern recall according to the self-generated macroscopic rule can act as a rapid memory search process for giving Vorverständnis was presented.

Apart from our motivation of the study but related to it, an interesting problem is: Could the proposed model serve as a model for some aspects of thinking? This problem is obviously beyond the present study to get a satisfactory answer. However, one can see an interesting informational structure in our model concerning this topics. The activities of the system when resetting mechanisms work for a pattern $\boldsymbol{P}^i$ can be symbolically represented by $\Psi(\{\boldsymbol{P}^1(k), \cdots, \boldsymbol{P}^m(k)\}/\boldsymbol{P}^i(k-1)) = A(\boldsymbol{P}^1(k), \cdots, \boldsymbol{P}^i(k), \cdots, \boldsymbol{P}^m(k)) - B(\boldsymbol{P}^i(k-1))$, where A is information working in the ordinary learning process and B information coming from the negative global feedback, and k a discrete time. Among much information stored in A, the information which is fully established, $\boldsymbol{P}^i$ is eliminated by the term B. Then, the information $\boldsymbol{P}^i$ seemingly disappears but is conserved qualitatively in the term A. Note that $\Psi(\{\boldsymbol{P}^1(k), \cdots, \boldsymbol{P}^m(k)\}/\boldsymbol{P}^i(k-1)) \neq A(\boldsymbol{P}^1(k), \cdots, \boldsymbol{P}^i(k) - \boldsymbol{P}^i(k-1), \cdots, \boldsymbol{P}^m(k))$. If the equality holds, the stored information $\boldsymbol{P}^i$ disappears from the system. This is one of the reasons that we introduced operationally an inhibitory neuron (the axonal tuft cell or, if possible, the Martinotti cell) whose role is to give a global negative feedback. This considera-

tion seems to be useful for an information structure of thinking processes. However, further development of the model requires the definition of a semantic relation (biological value) of the pattern sequences of recalled items in order to relate it to thinking processes.

Acknowledgements

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