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Covariant String Field Theory on Z_2-Orbifold

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A gauge invariant action is constructed for the closed string compactified on a Z_2-orbifold. Two 3-string vertices which describe two types of interaction, i.e., with and without twisted strings are obtained by the Neumann function method. The $O(g^2)$ gauge invariance of the action requires to modify the 3-string vertices by multiplying cocycle factors. A physical interpretation of the cocycle is given.

§ 1. Introduction

We construct a string field theory for the closed string compactified on a Z_2-orbifold. This simplest case clarifies some of the general features characteristic of the string field theory compactified on the orbifold.[1]

Previously, Hata, Kugo, Ogawa and the present authors have constructed a string field theory compactified on a torus.[2] In order to obtain the 3-string interaction vertex, they used an approach using the Neumann function (Neumann function method).[3),4] However on the resultant vertex, we found the violation of the naive connection conditions of the corresponding internal bosonic coordinates $X^I(\sigma)$. Owing to the presence of this disconnectedness, the gauge invariance of the action at the $O(g^2)$ level requires us to multiply the 3-string vertex by a two-cocycle[5)~7)] factor.

A similar situation occurs in the present case for the closed string compactified on a Z_2-orbifold. In this case, we have two sectors of string states, i.e., the untwisted sector and the twisted sector.[1] Therefore we have two types of 3-string vertices V_u and V_t which describe an interaction between three untwisted strings and that between an untwisted string and two twisted strings, respectively. We will see that the former interaction is realized in terms of the vertex for the torus compactified case mentioned above. For the latter type of interaction, we construct a vertex V_t by the Neumann function method and we find again the disconnectedness for the internal bosonic coordinates. This fact forces us again to multiply the vertex V_t which describes an interaction with the twisted strings by a cocycle factor. This factor is then interpreted from the physical point of view. It is shown to be related to the special feature of the interaction between two sectors: A twisted string at a fixed point jumps to another fixed point by the interaction with an untwisted string.[8]

Our main result in this paper is the gauge invariant action (11·2) with the vertices (6·1) and (6·2) (cf. (4·7) and (5·12)). The physical interpretation of the cocycle in (6·2) is described in § 8.

For an explicit construction of the 3-string vertex V_u and V_t, we take the following strategy. We first consider, on a torus, the closed strings and the open

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strings with a particular boundary condition related to the Z_2-action; then we obtain the vertices of string field theory on the Z_2-orbifold by restricting the states to the Z_2-invariant states.

The remaining part of this paper is organized as follows. In § 2, we review the internal space structure of the torus compactified closed string which corresponds to the untwisted sector. We introduce string fields on the Z_2-orbifold in § 3 and give an expression for the vertex which describes an interaction within the untwisted sector in § 4. A 3-string vertex describing an interaction with the twisted sector is constructed in § 5. We list the identities which are necessary for the proof of gauge invariance of the action in § 6. Then we proceed to the proof of them. The BRS invariance of the vertex with the twisted sector is shown in § 7. In § 8, we explain why we have to modify the vertex with the twisted strings, construct the missing (cocycle) factor and give a physical interpretation of it. We describe some properties of this modified vertex in § 9. The Jacobi identity is discussed with the modified vertex in § 10. We give the gauge invariant action in § 11. Section 12 is devoted to the discussion. Some appendices are added. Appendix A supplements § 2. A Green function on a complex plane, which is related to the 3-string vertex with the twisted strings, is constructed in Appendix B. The singularities of the internal coordinates at an interaction point are evaluated in Appendix C. Finally we prove the connection conditions for the internal bosonic coordinates in Appendix D.

§ 2. Internal space structures of torus compactified string

We consider a closed string compactified on Z_2-orbifold. Before describing our model, let us first consider strings on a D-dimensional torus T_D*) whose external and internal coordinates are denoted as $X^\mu(\sigma)$ ($\mu=0, 1, \cdots, d-1$) and $X^I(\sigma)$ ($I=1, 2, \cdots, D$), respectively. Here σ runs over an interval $[-\pi, \pi]$.

The D dimensional torus T_D is defined by the following identification of points on the D dimensional Euclidean space R_D,

$$\boldsymbol{y}=\boldsymbol{y}+\pi\sum_{I=1}^{D} n_I \boldsymbol{E}_I\,. \qquad (n_I;\ \text{integer}) \tag{2·1}$$

The $\{\boldsymbol{E}_I\}$ is a set of basis vectors of a corresponding lattice Γ_D, i.e., $T_D\equiv R_D/\Gamma_D$. We write the basis of the dual lattice $\tilde{\Gamma}_D$ as $\{\tilde{\boldsymbol{E}}_I\}$,

$$\boldsymbol{E}_I\cdot\tilde{\boldsymbol{E}}_J=\delta_{IJ}\,. \tag{2·2}$$

On any torus T_D, we can define a natural Z_2-action

$$\forall\boldsymbol{y}\in T_D\,, \qquad \boldsymbol{y}\to-\boldsymbol{y}\,. \tag{2·3}$$

Dividing T_D by the action (2·3), we obtain a Z_2-orbifold which we shall consider. As is well known, we have two types of closed strings on the Z_2-orbifold, i.e., the untwisted and the twisted strings. On the torus T_D, the former is a closed string and the latter is an open string with a boundary condition

*) Following Narain[9] et al., we shall consider a torus with a symmetric (G_{IJ}) and an anti-symmetric tensor (B_{IJ}) condensations (see the following discussion).

$$X_t^I(\pi) = -X_t^I(-\pi)\,, \tag{2·4}$$

where the index t indicates that $X_t^I(\sigma)$ is in the twisted sector.

In the remaing part of this section, we shall consider the internal coordinates of an untwisted string $X_u^I(\sigma)$, which are expanded into oscillator modes as

$$\begin{aligned}
&X_u^I(\sigma) = \frac{1}{\sqrt{\pi}}\left[x^I - \frac{1}{2}w^I\sigma + \frac{i}{2}\sum_{n\neq 0}\frac{1}{n}(\alpha_n^{I(+)}e^{in\sigma} + \alpha_n^{I(-)}e^{-in\sigma})\right],\\
&P_u^I(\sigma) = \frac{1}{2\sqrt{\pi}}[p^I + \sum_{n\neq 0}(\alpha_n^{I(+)}e^{in\sigma} + \alpha_n^{I(-)}e^{-in\sigma})]\,,\\
&[\alpha_n^{I(\varepsilon)}, \alpha_m^{J(\varepsilon')}] = n\delta_{n+m,0}\delta^{IJ}\delta^{\varepsilon\varepsilon'}\,,\\
&[x^I, p^J] = i\delta^{IJ}\,. \qquad (\varepsilon, \varepsilon' = \pm)
\end{aligned} \tag{2·5}$$

The zero mode $\boldsymbol{x} = \{x^I\}$ is identified as (2·1) and, correspondingly the winding number $\boldsymbol{w} = \{w^I\}$ should take the values,

$$\boldsymbol{w} = \sum_{I=1}^{D} n_I \boldsymbol{E}_I\,, \qquad (n_I;\ \text{integer}) \tag{2·6}$$

owing to the boundary condition $X_u^I(\pi) = X_u^I(-\pi)$ up to a translation by a lattice vector. Further, in order to operate consistently on the torus T_D, the momentum p^I must take the values,

$$p^I = \sum_{J=1}^{D}(2m_J - \sum_{K=1}^{D} B_{JK}\tilde{\boldsymbol{E}}_K\cdot\boldsymbol{w})(\tilde{\boldsymbol{E}}_J)^I \qquad (m_I;\ \text{integer}) \tag{2·7}$$

(see Eq. (A·17)).

As is shown by Narain et al.,[9] the compactification of the internal space described by $\{X_u^I(\sigma)\}$ to the torus T_D is equivalent to a compactification of the space $\{X_{u+}^I(\sigma), X_{u-}^I(\sigma)\}$ to the Lorentzian even self-dual lattice.[10] Here $X_{u\pm}^I(\sigma)$ are the left and the right moving mode decomposition of the original $X_u^I(\sigma)$,

$$\begin{aligned}
&X_u^I(\sigma) = X_{u+}^I(\sigma) + X_{u-}^I(\sigma)\,, \qquad P_u^I(\sigma) = P_{u+}^I(\sigma) + P_{u-}^I(\sigma)\,,\\
&X_{u\pm}^I(\sigma) = \frac{1}{\sqrt{\pi}}\left[x_\pm{}^I \mp \frac{1}{2}p_\pm{}^I\sigma + \frac{i}{2}\sum_{n\neq 0}\frac{1}{n}\alpha_n^{I(\pm)}e^{\pm in\sigma}\right],\\
&P_{u\pm}^I(\sigma) = \frac{1}{2\sqrt{\pi}}\sum_{n=-\infty}^{\infty}\alpha_n^{I(\pm)}e^{\pm in\sigma} = \mp X_{u\pm}^{I\prime}(\sigma) \equiv \frac{1}{2}A_{u\pm}^I(\sigma)\,,\\
&\alpha_0^{I(\pm)} = p_\pm{}^I\,, \qquad [x_\varepsilon{}^I, p_{\varepsilon'}^J] = \frac{i}{2}\delta^{IJ}\delta^{\varepsilon\varepsilon'}\,. \qquad (\varepsilon, \varepsilon' = \pm)
\end{aligned} \tag{2·8}$$

Note that we are taking the convention[5] that $2p_\pm{}^I$ are the translation operators of $x_\pm{}^I$. Comparing (2·5) and (2·8), we have the relations

$$\begin{aligned}
&x^I = x_+{}^I + x_-{}^I\,,\\
&p^I = p_+{}^I - p_-{}^I\,, \qquad w^I = p_+{}^I - p_-{}^I\,.
\end{aligned} \tag{2·9}$$

The $2D$ dimensional momentum $\boldsymbol{p} \equiv (p_+{}^I, -p_-{}^I)$ is on the lattice $\tilde{\Gamma}_{D,D}$ spanned by the basis vector $\tilde{\boldsymbol{e}}_i (i=1, 2, \cdots, 2D)$

$$\boldsymbol{p}=\sum_{i=1}^{2D} p_i \tilde{\boldsymbol{e}}_i . \qquad (p_i ;\text{ integer}) \tag{2·10}$$

The inner product on $\tilde{\Gamma}_{D,D}$ is defined with the Lorentzian metric,

$$\boldsymbol{p}\cdot\boldsymbol{q}=\sum_I (p_+{}^I q_+{}^I - p_-{}^I q_-{}^I)\,.$$

The $\tilde{\boldsymbol{e}}_i$ is related to the basis vector $\tilde{\boldsymbol{E}}_i$ of $\tilde{\Gamma}_D$ as[11)]

$$\begin{aligned}\tilde{\boldsymbol{e}}_i &= (\boldsymbol{e}_{i+}^K, \boldsymbol{e}_{i-}^K) \\ &= \begin{cases} \left(\frac{1}{2}(G+B)_{IJ}(\tilde{\boldsymbol{E}}_J)^K, \frac{1}{2}(G-B)_{IJ}(\tilde{\boldsymbol{E}}_J)^K\right), & (i=I\le D) \\ ((\tilde{\boldsymbol{E}}_I)^K, -(\tilde{\boldsymbol{E}}_I)^K)\,. & (i=I+D>D) \end{cases}\end{aligned} \tag{2·11}$$

The $G_{IJ}(B_{IJ})$ is a constant symmetric (anti-symmetric) tensor defined on the torus T_D.[9)] By using the relation (2·11) the integers p^i can be identified as

$$p_i=\begin{cases} n_I & i=I\le D\,, \\ m_I & i=I+D>D\,, \end{cases} \tag{2·12}$$

where the n_I and m_I are winding numbers and momenta given in (2·6) and (2·7). Equations (2·11) and (2·12) are derived in Appendix A.

§ 3. String field on Z_2-orbifold

Before considering the string fields on the orbifold, we introduce the oscillator expressions of string coordinates.

The external coordinates $X^\mu(\sigma)$ $(\mu=0, 1, \cdots, d-1)$ $(\sigma\in[-\pi, \pi])$, ghost and anti-ghost coordinates $c(\sigma)$ and $\bar{c}(\sigma)$ respectively are expanded as follows:[*),12),13)]

$$\begin{aligned} X^\mu(\sigma) &= \frac{1}{\sqrt{\pi}}\left[x^\mu + \frac{i}{2}\sum_{n\neq 0}\frac{1}{n}(\alpha_n{}^{\mu(+)}e^{in\sigma}+\alpha_n{}^{\mu(-)}e^{-in\sigma})\right], \\ A_\pm{}^\mu(\sigma) &= \eta^{\mu\nu}P_\nu(\sigma)\mp X^{\mu\prime}(\sigma) = \frac{1}{\sqrt{\pi}}\sum_{n=-\infty}^{\infty}\alpha_n{}^{\mu(\pm)}e^{\pm in\sigma}, \\ C_\pm(\sigma) &= i\pi_{\bar{c}}(\sigma)\mp c(\sigma) = \frac{1}{\sqrt{\pi}}\sum_{n=-\infty}^{\infty} c_n{}^{(\pm)}e^{\pm in\sigma}, \\ \bar{C}_\pm(\sigma) &= \bar{c}(\sigma)\mp i\pi_c(\sigma) = \frac{1}{\sqrt{\pi}}\sum_{n=-\infty}^{\infty}\bar{c}_n{}^{(\pm)}e^{\pm in\sigma}, \\ \bar{c}_0{}^{(\pm)} &= \frac{1}{2}\bar{c}_0 \mp i\pi_c{}^0\,, \qquad c_0{}^{(\pm)} = \frac{\partial}{\partial\bar{c}_0}\pm\frac{i}{2}\frac{\partial}{\partial\pi_c{}^0} \end{aligned} \tag{3·1}$$

with prime denoting $\partial/\partial\sigma$, where $P_\mu(\sigma)$, $\pi_c(\sigma)$ and $\pi_{\bar{c}}(\sigma)$ are the momentum variables $-i\delta/\delta X^\mu(\sigma)$, $-i\delta/\delta c(\sigma)$ and $-i\delta/\delta\bar{c}(\sigma)$, respectively. The oscillator modes $\alpha_n{}^{\mu(\varepsilon)}$,

*) We use the metric $\eta^{\mu\nu}=\text{diag}\,(-1, 1, \cdots 1)$.

$c_n^{(\varepsilon)}$, $\bar{c}_n^{(\varepsilon)}$ ($\varepsilon = \pm$) satisfy the properties

$$[\alpha_n^{\mu(\varepsilon)}, \alpha_m^{\nu(\varepsilon')}] = n\delta_{n+m,0}\eta^{\mu\nu}\delta^{\varepsilon\varepsilon'},$$

$$[x^\mu, p^\nu] = i\eta^{\mu\nu}, \qquad \alpha_0^{\mu(\pm)} = \frac{1}{2}p^\mu,$$

$$\{c_n^{(\varepsilon)}, \bar{c}_m^{(\varepsilon')}\} = \delta_{n+m,0}\delta^{\varepsilon\varepsilon'},$$

$$\alpha_{-n}^{\mu(\varepsilon)} = \alpha_n^{\mu(\varepsilon)\dagger}, \qquad c_{-n}^{(\varepsilon)} = c_n^{(\varepsilon)\dagger}, \qquad \bar{c}_{-n}^{(\varepsilon)} = \bar{c}_n^{(\varepsilon)\dagger}. \tag{3·2}$$

For the internal coordinates $X^I(\sigma)$, we must consider the two sectors corresponding to the boundary conditions. One is the untwisted sector in which $X_u^I(\sigma)$ satisfy the periodic boundary condition and have the same oscillator expressions as the torus compactified string (2·5). The other is the twisted sector which is characteristic of the orbifold compactified string. In the latter sector, the internal coordinate satisfy the anti-periodic boundary condition

$$X_t^I(\pi) = -X_t^I(-\pi) \tag{3·3}$$

and are expanded as

$$X_t^I(\sigma) = \frac{1}{\sqrt{\pi}}\left[x_t^I + \frac{i}{2}\sum_{n' \in \boldsymbol{Z}+(1/2)} \frac{1}{n'}(\alpha_{n'}^{I(+)}e^{in'\sigma} + \alpha_{n'}^{I(-)}e^{-in'\sigma})\right],$$

$$A_{t\pm}^I(\sigma) = P_t^I(\sigma) \mp X_t^{I\prime}(\sigma) = \frac{1}{\sqrt{\pi}}\sum_{n' \in \boldsymbol{Z}+(1/2)} \alpha_{n'}^{I(\pm)}e^{\pm in'\sigma},$$

$$[\alpha_{n'}^{I(\varepsilon)}, \alpha_{m'}^{J(\varepsilon')}] = n'\delta_{n'+m',0}\delta^{IJ}\delta^{\varepsilon\varepsilon'}, \tag{3·4}$$

where n' and m' take the half-integer value ($\boldsymbol{Z}+\frac{1}{2}$) owing to the boundary condition (3·3). Equation (3·3) also restricts the zero mode $\boldsymbol{x}_t = \{x_t^I\}$ to be on the fixed points under the Z_2-action

$$\boldsymbol{x}_t = \frac{\pi}{2}\sum_{I=1}^{D} n_I \boldsymbol{E}_I. \qquad (n_I = 0 \text{ or } 1) \tag{3·5}$$

The twisted strings reside on the fixed points[1] and do not have the center of mass momentum in the internal space.

For later use, we introduce the BRS charge[14] of the orbifold compactified string. The functional form of the BRS charge has the same form as the usual closed string's one,[12),4)]

$$Q_B = Q_{B+} + Q_{B-},$$

$$Q_{B\pm} = \frac{\sqrt{\pi}}{2}\int_{-\pi}^{\pi} d\sigma\, C_\pm(\sigma)\left[-A_{\mu\pm}(\sigma)A_\pm^\mu(\sigma) - A_\pm^I(\sigma)A_\pm^I(\sigma) \mp 2i\frac{dC_\pm(\sigma)}{d\sigma}\bar{C}_\pm(\sigma)\right] \tag{3·6}$$

except that the internal variables $A_\pm^I(\sigma)$ are expanded as (2·8) and (3·4) for the untwisted and the twisted sector respectively. The oscillator expression of the BRS charge in each sector is

$$Q_{B\pm}^{a}=-2\sum_{n=-\infty}^{\infty}:c_{-n}^{(\pm)}(L_n{}^{\mathrm{ext}(\pm)}+L_n{}^{\mathrm{int}(a)(\pm)}+\frac{1}{2}L_n{}^{\mathrm{FP}(\pm)}-\alpha_0(0)\delta_{n,0}):, \tag{3·7}$$

$$L_n{}^{\mathrm{ext}(\pm)}=\frac{1}{2}\sum_{m=-\infty}^{\infty}:\alpha_m{}^{\mu(\pm)}\alpha_{n-m}^{\mu(\pm)}:,$$

$$L_n{}^{\mathrm{int}(a)(\pm)}=\begin{cases}\frac{1}{2}\sum_{m=-\infty}^{\infty}:\alpha_m{}^{I(\pm)}\alpha_{n-m}^{I(\pm)}: & \text{for} \quad a=u\,,\\ \frac{1}{2}\sum_{m=-\infty}^{\infty}:\alpha_{m+(1/2)}^{I(\pm)}\alpha_{n-m-(1/2)}^{I(\pm)}: & \text{for} \quad a=t\,,\end{cases}$$

$$L_n{}^{\mathrm{FP}(\pm)}=\sum_{m=-\infty}^{\infty}(n+m):\bar{c}_{n-m}^{(\pm)}c_m{}^{(\pm)}:, \tag{3·8}$$

where the index $a=u$ or t indicates the untwisted or the twisted sectors and $L_n{}^{\mathrm{ext}}$, $L_n{}^{\mathrm{int}}$ and $L_n{}^{\mathrm{FP}}$ are the Virasoro generators of external, internal and FP-ghost modes respectively. The nilpotency of $Q_B{}^u(Q_B{}^t)$ holds under the conditions $d+D=26$ and $\alpha_u(0)=1$ $(\alpha_t(0)=1-(D/16))$.

3.1. *Untwisted sector*

In order to represent the string field in the untwisted sector, we use the Fock representation for the oscillators constructed on the vacuum

$$(\alpha_n{}^{\mu(\pm)},\ \alpha_n{}^{I(\pm)},\ c_n{}^{(\pm)},\ \bar{c}_n{}^{(\pm)})|0\rangle=0 \qquad (n\geq 1) \tag{3·9}$$

and the momentum representation for the zero-modes p^μ, $\bar{c}_0$ and $\pi_c{}^0$. For the internal momentum zero-modes $p_\pm{}^I$, we take the ket representation by using the following momentum eigenstates,

$$\hat{p}_\pm{}^I|p_\pm{}^I\rangle=p_\pm{}^I|p_\pm{}^I\rangle\,, \tag{3·10}$$

where $\hat{p}_\pm{}^I$ is the zero-mode *operator*. With these representations, the untwisted string field can be written by the ket vector as

$$|\Phi_u(p^\mu,\ \bar{c}_0,\ \pi_c{}^0;\ \alpha)\rangle=-\bar{c}_0|\phi_u(p^\mu;\ \alpha)\rangle+|\psi_u(p^\mu;\ \alpha)\rangle$$
$$+\bar{c}_0\pi_c{}^0|\chi_u(p^\mu;\ \alpha)\rangle+i\pi_c{}^0|\eta_u(p^\mu;\ \alpha)\rangle\,, \tag{3·11}$$

where α is the string length parameter.[15),12)] The component fields $|\phi_u\rangle$, $|\psi_u\rangle$, $|\chi_u\rangle$ and $|\eta_u\rangle$ can be further expanded into the usual local component fields, e.g.,

$$|\phi_u(p^\mu;\ \alpha)\rangle=\sum_{p_\pm{}^I}[\varphi_{p_\pm{}^I}(p^\mu;\ \alpha)+A_{p_\pm{}^I}^{\mu(\pm)}(p^\mu;\ \alpha)\alpha_{-1}^{\mu(\pm)}+\cdots]|0\rangle|p_\pm{}^I\rangle\,. \tag{3·12}$$

The physical modes of the string are contained in the bosonic component $|\phi_u\rangle$. The net ghost numbers N_{FP} of $|\Phi_u\rangle, |\phi_u\rangle, |\psi_u\rangle, |\chi_u\rangle$ and $|\eta_u\rangle$ are $-1, 0, -1, 1$ and 0 respectively. When we consider the gauge-invariant action, $|\Phi_u\rangle$ is further restricted to the sector with internal ghost number $n_{\mathrm{FP}}=0$, which is defined to be the number of excited ghost modes $c_{-n}^{(\pm)}$ $(n\geq 1)$ minus that of excited anti-ghost $\bar{c}_{-n}^{(\pm)}$ $(n\geq 0)$.[13)]

The string field on the Z_2-orbifold must additionally satisfy two constraints. One is the well-known constraint on the closed string field[12),4)]

$$\mathcal{P}|\Phi_u\rangle=|\Phi_u\rangle \qquad \text{or} \qquad (L_+{}^{(u)}-L_-{}^{(u)})|\Phi_u\rangle=0 \tag{3·13}$$

expressing the invariance under the rigid σ-translation, where

$$\mathcal{P}=\int_{-\pi}^{\pi}\frac{d\theta}{2\pi}e^{i\theta(L_+^{(u)}-L_-^{(u)})},$$

$$L_+^{(u)}-L_-^{(u)}=\int_{-\pi}^{\pi}d\sigma(X^{\mu\prime}P_\mu+X^{I\prime}P_I+c'\pi_c+\bar{c}'\pi_{\bar{c}}),$$

$$L_\pm^{(u)}=L_0^{\mathrm{ext}(\pm)}+L_0^{\mathrm{int}(u)(\pm)}+L_0^{\mathrm{FP}(\pm)}-1. \tag{3·14}$$

The other constraint restricts the states to those allowed on the Z_2-orbifold.[1] In order to give an explicit form of this constraint, we consider the Z_2-action on the oscillator modes which follows from Eqs. (2·3) and (2·5):

$$(p_\pm{}^I,\ \alpha_n{}^{I(\pm)})\to(-p_\pm{}^I,\ -\alpha_n{}^{I(\pm)}).$$

Let us introduce operators R and O_u which realize the above operation, i.e.,

$$R|p_\pm{}^I\rangle=|-p_\pm{}^I\rangle,$$

$$O_u\alpha_n{}^{I(\pm)}O_u{}^{-1}=-\alpha_n{}^{I(\pm)}. \tag{3·15}$$

A concrete expression of O_u is given as

$$O_u=\exp i\pi\sum_{I,\pm}\sum_{n\geq 1}\frac{1}{n}\alpha_{-n}^{I(\pm)}\alpha_n{}^{I(\pm)}. \tag{3·16}$$

With these operators, the projection operator to the Z_2-invariant states is written as

$$\mathcal{P}_u{}^{Z_2}=\frac{1}{2}(1+RO_u) \tag{3·17}$$

and the second constraint is now expressed in the form

$$\mathcal{P}_u{}^{Z_2}|\Phi_u\rangle=|\Phi_u\rangle. \tag{3·18}$$

Here we note that this projection operator (3·17) is commutative to the BRS charge $Q_B{}^{(u)}$ given by Eq. (3·7):

$$\mathcal{P}_u{}^{Z_2}Q_B{}^u=Q_B{}^u\mathcal{P}_u{}^{Z_2}. \tag{3·19}$$

This guarantees the closure of Z_2-invariant space under the operation of $Q_B{}^{(u)}$. We further require the reality condition on $|\Phi_u\rangle$:[4]

$$_2\langle\Phi_u(2)|=\int d1\langle\tilde{R}_u(1,2)\|\Phi_u(1)\rangle,$$

$$\langle\tilde{R}_u(1,2)|=(2\pi)^{d+1}\delta(p_1+p_2)\delta(\alpha_1+\alpha_2)\delta(\pi_c{}^{0(1)}-\pi_c{}^{0(2)})\delta(\bar{c}_0{}^{(1)}-\bar{c}_0{}^{(2)})$$
$$\times\sum_{p_{1\pm}{}^I,p_{2\pm}{}^I}\delta_{p_{1\pm}{}^I+p_{2\pm}{}^I,0}\langle p_{1\pm}^I|\langle p_{2\pm}^I|U(\boldsymbol{p}_2)$$
$$\times{}_1\langle 0|_2\langle 0|\exp\Big\{-\sum_\pm\sum_{n\geq 1}\Big(\frac{1}{n}\alpha_{\mu n}{}^{(\pm)(1)}\alpha_n{}^{\mu(\pm)(2)}+\frac{1}{n}\alpha_n{}^{I(\pm)(1)}\alpha_n{}^{I(\pm)(2)}$$
$$+c_n{}^{(\pm)(1)}\bar{c}_n{}^{(\pm)(2)}-\bar{c}_n{}^{(\pm)(1)}c_n{}^{(\pm)(2)}\Big)\Big\},$$

$$U(\mathbf{p}_2)=e^{-(\pi i/4)\mathbf{p}_2^2}=e^{-(\pi i/4)(p_{2+}{}^2-p_{2-}{}^2)}\,. \tag{3·20}$$

The r and dr are a set of zero mode variables and its integration measure of the r-th string, i.e.,

$$r=(p_r{}^\mu,\ \bar{c}_0{}^{(r)},\ \pi_c{}^{0(r)},\ \alpha_r)\,,$$
$$\int dr=\int\frac{dp_r}{(2\pi)^d}d\bar{c}_0{}^{(r)}d\pi_c{}^{0(r)}\frac{d\alpha_r}{2\pi}\,. \tag{3·21}$$

The quantity $\langle\tilde{R}_u(1,2)|$ converts the string coordinates as follows:

$$\langle\tilde{R}_u(1,2)|(Z_u{}^{(1)}-\tilde{Z}_u{}^{(2)})=0\,,$$
$$Z_u=(X^\mu(\sigma),\ X^I_{u\pm}(\sigma),\ c(\sigma),\ \bar{c}(\sigma);\ \alpha)\,,$$
$$\tilde{Z}_u=\left(X^\mu(-\sigma),\ X^I_{u\pm}(-\sigma)\pm\frac{\sqrt{\pi}}{2}p_\pm{}^I,\ -c(-\sigma),\ \bar{c}(-\sigma);\ -\alpha\right). \tag{3·22}$$

3.2. *Twisted sector*

As in the untwisted string case, we can represent the twisted string field by using the Fock representation for the oscillator modes and the momentum representation for the zero-modes p^μ, $\bar{c}_0$ and $\pi_c{}^0$. The oscillator vacuum is defined by

$$(\alpha_n{}^{\mu(\pm)},\ \alpha^{I(\pm)}_{n-(1/2)},\ c_n{}^{(\pm)},\ \bar{c}_n{}^{(\pm)})|0\rangle=0\,.\qquad(n\geq1) \tag{3·23}$$

The twisted string field can be written as

$$|\Phi_t(p^\mu,\ \bar{c}_0,\ \pi_c{}^0;\ n^f,\ \alpha)\rangle=-\bar{c}_0|\phi_t(p^\mu;\ n^f,\ \alpha)\rangle+|\psi_t(p^\mu;\ n^f,\ \alpha)\rangle$$
$$+\bar{c}_0\pi_c{}^0|\chi_t(p^\mu;\ n^f,\ \alpha)\rangle+i\pi_c{}^0|\eta_t(p^\mu;\ n^f,\ \alpha)\rangle\,, \tag{3·24}$$

where $n^f\equiv\{n^I\}$ is related to $\boldsymbol{x}_t$ via Eq. (3·5). In (3·24), we have used the following 2^D-dimensional vector representation for the degree of freedom of the fixed points around which the string is twisted. We express this 2^D-dimensional vector as a direct product of D two-dimensional vectors

$$\underbrace{\begin{pmatrix}\cdot\\\cdot\end{pmatrix}\otimes\begin{pmatrix}\cdot\\\cdot\end{pmatrix}\otimes\cdots\otimes\begin{pmatrix}\cdot\\\cdot\end{pmatrix}}_{D}\,. \tag{3·25}$$

Here $n^I=0$ or 1 corresponds to the upper or lower component of the I-th two-dimensional vector. Again the physical modes of the string are contained in the $|\psi_t\rangle$-component with $N_{\mathrm{FP}}=0$. When we consider the gauge-invariant action, $|\Phi_t\rangle$ is restricted to the internal ghost number $n_{\mathrm{FP}}=0$ sector.

For a twisted string, the projection operator into the Z_2-invariant sector is written as

$$\mathcal{P}_t{}^{Z_2}=\frac{1}{2}(1+O_t) \tag{3·26}$$

with

$$O_t=\exp i\pi\sum_{I,\pm}\sum_{n\geq 1}\frac{1}{n-\frac{1}{2}}\alpha_{-(n-(1/2))}^{I(\pm)}\alpha_{n-(1/2)}^{I(\pm)}\,. \tag{3·27}$$

The operator O_t acts as desired

$$O_t\alpha_{n-(1/2)}^{I(\pm)}O_t^{-1}=-\alpha_{n-(1/2)}^{I(\pm)}\,. \tag{3·28}$$

A corresponding constraint on the string field $|\Phi_t\rangle$ is

$$\mathcal{P}_t^{Z_2}|\Phi_t\rangle=|\Phi_t\rangle\,. \tag{3·29}$$

After restricting Φ_t to the Z_2-invariant states, $|\Phi_t\rangle$ describes the closed string on the Z_2-orbifold and it should satisfy yet another constraint

$$\mathcal{P}|\Phi_t\rangle=|\Phi_t\rangle \qquad \text{or} \qquad (L_+{}^{(t)}-L_-{}^{(t)})|\Phi_t\rangle=0\,,$$

$$L_\pm{}^{(t)}=L_0{}^{\mathrm{ext}(\pm)}+L_0{}^{\mathrm{int}(t)(\pm)}+L_0{}^{\mathrm{FP}(\pm)}-\left(1-\frac{D}{16}\right), \tag{3·30}$$

where $\mathcal{P}$ are defined in (3·14) with the replacement of $L_\pm{}^{(u)}$ by $L_\pm{}^{(t)}$.

In addition, we require the reality condition $|\Phi_t\rangle$ as

$${}_2\langle\Phi_t(2)|=\int d1\langle\tilde{R}_t(1,2)\|\Phi_t(1)\rangle\,,$$

$$\langle\tilde{R}_t(1,2)|=(2\pi)^{d+1}\delta(p_1+p_2)\delta(\alpha_1+\alpha_2)\delta(\pi_c{}^{0(1)}-\pi_c{}^{0(2)})\delta(\bar{c}_0{}^{(1)}-\bar{c}_0{}^{(2)})$$
$$\times\delta_{n_{(1)}{}^f,n_{(2)}{}^f}$$
$$\times{}_1\langle 0|_2\langle 0|\exp\Big\{-\sum_{\pm}\sum_{n\geq 1}\Big(\frac{1}{n}\alpha_n{}^{\mu(\pm)(1)}\alpha_{\mu n}{}^{(\pm)(2)}$$
$$+\frac{1}{n-\frac{1}{2}}\alpha_{n-(1/2)}^{I(\pm)(1)}\alpha_{n-(1/2)}^{I(\pm)(2)}+c_n{}^{(\pm)(1)}\bar{c}_n{}^{(\pm)(2)}-\bar{c}_n{}^{(\pm)(1)}c_n{}^{(\pm)(2)}\Big)\Big\}\,, \tag{3·31}$$

where $\delta_{n_{(1)}{}^f,n_{(2)}{}^f}$ denotes the Kronecker delta for $n^f\equiv\{n^I\}$. The r and dr are a set of zero mode variables and its integration measure of the r-th string, i.e.,

$$r=(p_r{}^\mu,\ \bar{c}_0{}^{(r)},\ \pi_c{}^{0(r)},\ \alpha_r)\,,$$

$$\int dr=\int\frac{dp_r}{(2\pi)^d}d\bar{c}_0{}^{(r)}d\pi_c{}^{0(r)}\frac{d\alpha_r}{2\pi}\,. \tag{3·32}$$

The $\langle\tilde{R}_t(1,2)|$ satisfies an equation similar to Eq. (3·22):

$$\langle\tilde{R}_t(1,2)|(Z_t{}^{(1)}-\tilde{Z}_t{}^{(2)})=0\,,$$

$$Z_t=(X^\mu(\sigma),\ X_t^I(\sigma),\ c(\sigma),\ \bar{c}(\sigma);\ \alpha)\,,$$

$$\tilde{Z}_t=(X^\mu(-\sigma),\ X_t^I(-\sigma),\ -c(-\sigma),\ \bar{c}(-\sigma);\ -\alpha)\,. \tag{3·33}$$

§4. Three string vertex of untwisted strings

The 3-untwisted string vertex can be obtained by multiplying the Z_2-invariant projection operator into the 3-string vertex $|V^{\text{torus}}(1,2,3)\rangle$ in the torus compactified string field theory. The explicit expression of $|V^{\text{torus}}(1,2,3)\rangle$ is[2)]

$$
\begin{aligned}
|V^{\text{torus}}(1,2,3)\rangle &= \varepsilon(\boldsymbol{p}_1,\boldsymbol{p}_2)\mathcal{P}_{123}\frac{\alpha_1\alpha_2}{\alpha_3}\mu_u{}^2(\alpha_1,\alpha_2,\alpha_3)G(\sigma_I)\boldsymbol{\pi}_c|V_0(1,2,3)\rangle\,,\\
|V_0(1,2,3)\rangle &= |V_0{}^{\text{ext-FP}}(1,2,3)\rangle|V_{0u}^{\text{int}}(1,2,3)\rangle\,,\\
&= \exp[E^{\text{ext-FP}}(1,2,3)+E^{\text{int}}(1,2,3)]|0\rangle_{123}\,\bar{\delta}^{\text{ext-FP}}(1,2,3)\delta^{\text{int}}(1,2,3)\,,\\
E^{\text{ext-FP}}(1,2,3) &= \sum_{\pm}\sum_{r,s=1}^{3}\sum_{n,m=0}^{\infty}\bar{N}_{nm}^{rs}\left(\frac{1}{2}\alpha_n{}^{\mu(\pm)r}\alpha_{\mu-m}^{(\pm)(s)}+i\gamma_{-n}^{(\pm)(r)}\bar{\gamma}_{-m}^{(\pm)(s)}\right),\\
E^{\text{int}}(1,2,3) &= \sum_{\pm}\sum_{r,s=1}^{3}\sum_{n,m=0}^{\infty}\bar{N}_{nm}^{rs}\frac{1}{2}\alpha_{-n}^{I(\pm)(r)}\alpha_{-m}^{I(\pm)(s)}\,,\\
\bar{\delta}^{\text{ext-FP}}(1,2,3) &= (2\pi)^{d+1}\delta^d(\sum_r p_r)\delta(\sum_r\alpha_r)\delta(\sum_r\alpha_r{}^{-1}\pi_c{}^{0(r)})\delta(\sum_r\alpha_r{}^{-1}\bar{c}_0{}^{(r)})\,,\\
\delta^{\text{int}}(1,2,3) &= \delta^D(\sum_r p_{r+})\delta^D(\sum_r p_{r-})\,,\\
\gamma_n{}^{(r)} &= in\alpha_r c_n{}^{(r)}\,,\qquad \bar{\gamma}_n{}^{(r)}=\alpha_r{}^{-1}\bar{c}_n{}^{(r)}\,,\\
\boldsymbol{\pi}_c &= \alpha_r\frac{\pi_c{}^{0(r+1)}}{\alpha_{r+1}}-\alpha_{r+1}\frac{\pi_c{}^{0(r)}}{\alpha_r}\,,\\
\mu_u(\alpha_1,\alpha_2,\alpha_3) &= \exp\left[-\tau_0\sum_{r=1}^{3}\frac{1}{\alpha_r}\right],\\
\mathcal{P}_{123} &= \mathcal{P}^{(1)}\mathcal{P}^{(2)}\mathcal{P}^{(3)}\,,\qquad |0\rangle_{123}=|0\rangle_1|0\rangle_2|0\rangle_3\,.
\end{aligned}
\tag{4·1}
$$

The coefficients $\bar{N}_{nm}^{rs}$ are the Fourier components of the Neumann function (Neumann coefficients) for the open-string diagram in Fig. 1. The $G(\sigma_I)$ is the ghost factor at the interaction point

$$G(\sigma_I)=i\sqrt{\pi}\,\alpha_r\pi_{\bar{c}}{}^{(r)}(\sigma_I{}^{(r)})\,.\qquad (r=1,2\text{ or }3) \tag{4·2}$$

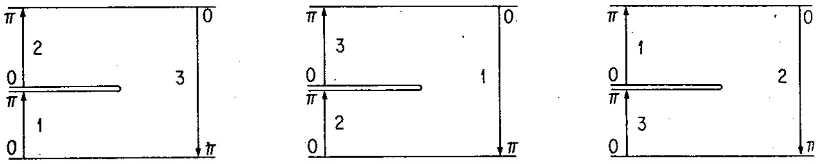
The factor $\varepsilon(\boldsymbol{p}_1,\boldsymbol{p}_2)$ is a two-cocycle factor,[5)~7)] which is an essential ingredient for the field theory of torus compactified closed string.[2)] The ε satisfies the following properties:

$$
\begin{aligned}
&\text{a)}\quad \varepsilon(\boldsymbol{p},\boldsymbol{q})\varepsilon(\boldsymbol{p}+\boldsymbol{q},\boldsymbol{r})=\varepsilon(\boldsymbol{p},\boldsymbol{q}+\boldsymbol{r})\varepsilon(\boldsymbol{q},\boldsymbol{r})\,,\\
&\text{b)}\quad \varepsilon(\boldsymbol{p},\boldsymbol{q})=(-)^{\boldsymbol{p}\cdot\boldsymbol{q}}\varepsilon(\boldsymbol{q},\boldsymbol{p})\,.
\end{aligned}
\tag{4·3}
$$

For a concrete expression of $\varepsilon(\boldsymbol{p},\boldsymbol{q})$, we take[2)]

$$
\begin{aligned}
&\varepsilon(\boldsymbol{p}=\sum_i p_i\tilde{\boldsymbol{e}}_i,\ \boldsymbol{q}=\sum_i q_i\tilde{\boldsymbol{e}}_i)\\
&=\exp\frac{i\pi}{2}(\sum_{i>j}-\sum_{i<j})p_iq_j\tilde{\boldsymbol{e}}_i\cdot\tilde{\boldsymbol{e}}_j\\
&=\exp\frac{i\pi}{2}(\sum_{>}-\sum_{<})\boldsymbol{p}\cdot\boldsymbol{q}\,,
\end{aligned}
\tag{4·4}
$$

where $\{\tilde{\boldsymbol{e}}_i\}$ are the basis vectors of $\tilde{\Gamma}_{D,D}$. Although the solution to (4·3) is unique only



(a) (b) (c)

Fig. 1. The open-string diagrams (or the half-portions of closed-string diagrams with $0 \leq \sigma_r \leq \pi$) to which the Neumann coefficients in the 3-string vertex (4·1) corresponds for cases (a) $|\alpha_1|+|\alpha_2|=|\alpha_3|$, (b) $|\alpha_2|+|\alpha_3|=|\alpha_1|$ and (c) $|\alpha_3|+|\alpha_1|=|\alpha_2|$.

up to the trivial cocycle, this phase factor is determined by the hermiticity and cyclic symmetry of the total vertex (4·1). The connection condition of the internal coordinate on the vertex $|V_0^{\text{int}}(1, 2, 3,)\rangle$[2] is

$$X_\pm{}^{I(1)}(\sigma_1) - X_\pm{}^{I(3)}(\sigma_3) = \pm\frac{\sqrt{\pi}}{2} p_{3\pm}^I ,$$

$$X_\pm{}^{I(2)}(\sigma_2) - X_\pm{}^{I(3)}(\sigma_3) = \mp\frac{\sqrt{\pi}}{2} p_{2\pm}^I = \pm\frac{\sqrt{\pi}}{2}(p_{3\pm}^I + p_{1\pm}^I) \tag{4·5}$$

for the case of $\alpha_1, \alpha_2 > 0$, $\alpha_3 < 0$. Here σ_r are defined as

$$\sigma_1(\sigma) = \frac{\sigma}{\alpha_1}, \quad \sigma_2(\sigma) = \frac{\sigma - \pi\alpha_1 \mathrm{sgn}(\sigma)}{\alpha_2}, \quad \sigma_3(\sigma) = \frac{\sigma + \pi\alpha_3 \mathrm{sgn}(\sigma)}{\alpha_3}. \tag{4·6}$$

The 3-untwisted string vertex in the orbifold compactified string field theory is now given by

$$\begin{aligned} |V_u(1, 2, 3)\rangle &= \mathcal{P}_{123}^{Z_2} | V^{\text{torus}}(1, 2, 3)\rangle \\ &= \mathcal{P}_{123}\Big[\frac{\alpha_2\alpha_2}{\alpha_3}\mu_u{}^2(\alpha_1, \alpha_2, \alpha_3) G(\sigma_I)\boldsymbol{\pi}_c | V_0{}^{\text{ext-FP}}(1, 2, 3)\rangle \\ &\quad \times \mathcal{P}_{123}^{Z_2} | V_{0u}^{\text{int}}(1, 2, 3)\rangle\Big]\varepsilon(\boldsymbol{p}_1, \boldsymbol{p}_2), \end{aligned} \tag{4·7}$$

where

$$\mathcal{P}_{123}^{Z_2} = \mathcal{P}_u{}^{Z_2(1)}\mathcal{P}_u{}^{Z_2(2)}\mathcal{P}_u{}^{Z_2(3)}. \tag{4·8}$$

However, we can neglect an arbitrary one of the three projection operators since the vertex $|V^{\text{torus}}(1, 2, 3)\rangle$ is invariant under a simultaneous operation of Z_2-action RO_u on all the three strings, which is easily seen from Eq. (4·1).

§ 5. 3-string vertex with twisted strings

In § 4, we have introduced the 3-string vertex which describes an interaction between untwisted strings. On the Z_2-orbifold, there is another type of interaction with the twisted strings. As we have done for the former interaction, let us first consider, on T_D, the interaction between a closed string and two open strings with the boundary condition (3·3), which is shown in Fig. 2 with string 1 as the closed string. After constructing a vertex corresponding to this interaction we obtain a 3-string vertex on the Z_2-orbifold by multiplying it by the projection operators $\mathcal{P}_u{}^{Z_2(1)}\mathcal{P}_t{}^{Z_2(2)}$

$\times \mathcal{P}_t^{Z_2(3)}$ defined in Eqs. (3·17) and (3·26). We construct the 3-string vertex on T_D following the method used in Refs. 3) and 4). In Fig. 3 we have shown a 3-string light-cone diagram on the complex ρ-plane for the case of $\alpha_1, \alpha_2>0$ and $\alpha_3<0$. For later convenience, we introduce another complex plane, i.e., z-plane depicted in Fig. 4 which is related to the ρ-plane via the Mandelstam mapping

$$\rho(z)=\alpha_1\ln(z-1)+\alpha_2\ln z\,. \tag{5·1}$$

We parametrize each string region on the ρ-plane by a complex coordinate

$$\zeta_r=\xi_r+i\hat{\sigma}_r\,, \qquad (\xi_r\leq 0,\ -\pi\leq\hat{\sigma}_r\leq\pi)$$

$$\rho=\alpha_r\zeta_r+\tau_0+i\beta_r\,, \qquad \beta_r=\tilde{\beta}-\alpha_r\hat{\sigma}_I{}^{(r)}\,, \tag{5·2}$$

where $\hat{\sigma}_r$ is an intrinsic coordinate of the r-th string defined modulo 2π and $\hat{\sigma}_I{}^{(r)}$ is the value of $\hat{\sigma}_r$ at the interaction point. Here τ_0 and $\tilde{\beta}$ are defined as

$$\tau_0+i\tilde{\beta}=\rho(z_0)=\rho_0 \tag{5·3}$$

for z_0 satisfying

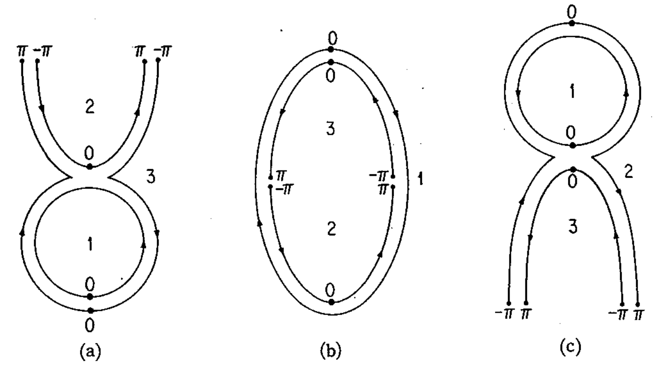


Fig. 2. Interactions on T_D of a closed string and two open strings with the boundary condition (3·3) for cases (a) $|\alpha_1|+|\alpha_2|=|\alpha_3|$, (b) $|\alpha_2|+|\alpha_3|=|\alpha_1|$ and (c) $|\alpha_3|+|\alpha_1|=|\alpha_2|$. String 1 is chosen to be the closed string.

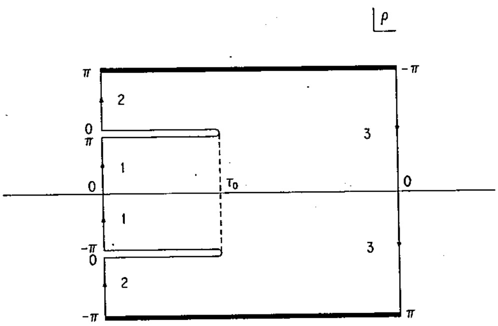


Fig. 3. A 3-string diagram on the ρ-plane for α_1, $\alpha_2>0$ and $\alpha_3<0$. The bold line indicates that strings 2 and 3 satisfy the boundary condition (3·3).

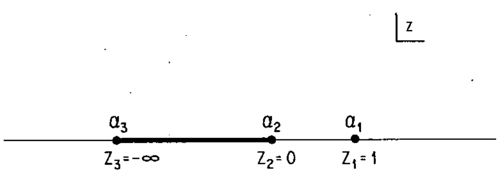


Fig. 4. The z-plane which is related to the ρ-plane in Fig. 3 via a Mandelstam mapping (5·1). The bold line indicates that strings 2 and 3 satisfy the boundary condition (3·3).

$$\frac{d}{dz}\rho(z)|_{z=z_0}=0\,. \tag{5·4}$$

The coordinate $\hat{\sigma}_r$ parametrizes the r-th string as indicated in Fig. 3; origins of the first and the third strings coincide, and end points of open strings correspond to $\hat{\sigma}_r=\pm\pi$. This convention of the intrinsic σ-coordinates is different from that used in (4·6) for the third string. However this is convenient to write the vertex for the interaction with the twisted strings.

We first find a Green function $T(\rho,\tilde{\rho})$ on the ρ-plane ($\rho=\tau+i\sigma$) which satisfies

$$\left[\left(\frac{\partial}{\partial\tau}\right)^2+\left(\frac{\partial}{\partial\sigma}\right)^2\right]T(\rho,\tilde{\rho})=2\pi\delta^2(\rho-\tilde{\rho}) \tag{5·5}$$

with the boundary condition implied by (3·3)

$$T(\rho,\tilde{\rho})|_{\hat{\sigma}_r=\pi}=\begin{cases} T(\rho',\tilde{\rho})|_{\hat{\sigma}_{r'}=-\pi} & (r=1)\\ -T(\rho',\tilde{\rho})|_{\hat{\sigma}_{r'}=-\pi}\,, & (r=2 \text{ or } 3)\end{cases} \tag{5·6}$$

where $\hat{\sigma}_r$ and ρ are related by (5·2). The ρ and ρ' are on the r-th string strip with $\xi_r=\xi_r'$. A solution to Eq. (5·5) with (5·6) is

$$T(\rho,\tilde{\rho})=\ln|\sqrt{z}-\sqrt{\tilde{z}}|-\ln|\sqrt{z}+\sqrt{\tilde{z}}|\,. \tag{5·7}$$

Here z is related to ρ through the Mandelstam mapping (5·1). Note that we have a root cut along $(-\infty,0)$ corresponding to (5·6). In order to construct the vertex we need the coefficients, which are obtained from (5·7) when we expand it in terms of the ζ_r defined in (5·2). Let us first decompose $T(\rho,\tilde{\rho})$ into two parts, which are analytic with respect to $(\rho,\tilde{\rho})$ and its complex conjugate.

$$T(\rho,\tilde{\rho})=\frac{1}{2}[\mathcal{T}(\rho,\tilde{\rho})+\mathcal{T}^*(\rho,\tilde{\rho})]\,,$$

$$\mathcal{T}(\rho,\tilde{\rho})=\ln\left(\frac{\sqrt{z}-\sqrt{\tilde{z}}}{\sqrt{z}+\sqrt{\tilde{z}}}\right). \tag{5·8}$$

The $\mathcal{T}(\rho,\tilde{\rho})$ is expanded as follows,

$$\begin{aligned}\mathcal{T}(\rho,\tilde{\rho})=&-\delta_{r1}\delta_{rs}\Big[\theta(\xi_1-\tilde{\xi}_1)\Big(\sum_{n_1>0}\frac{1}{n_1}e^{n_1(\tilde{\zeta}_1-\zeta_1)}-\zeta_1\Big)\\&\qquad+\theta(\tilde{\xi}_1-\xi_1)\Big(\sum_{n_1>0}\frac{1}{n_1}e^{n_1(\zeta_1-\tilde{\zeta}_1)}-\tilde{\zeta}_1+i\pi\Big)\Big]\\&-\delta_{r2}\delta_{rs}\Big[\theta(\xi_2-\tilde{\xi}_2)\sum_{n_2>0}\frac{1}{n_2}e^{n_2(\tilde{\zeta}_2-\zeta_2)}+\theta(\tilde{\xi}_2-\xi_2)\Big(\sum_{n_2>0}\frac{1}{n_2}e^{n_2(\zeta_2-\tilde{\zeta}_2)}+i\pi\Big)\Big]\\&-\delta_{r3}\delta_{rs}\Big[\theta(\xi_3-\tilde{\xi}_3)\Big(\sum_{n_3>0}\frac{1}{n_3}e^{n_3(\tilde{\zeta}_3-\zeta_3)}+i\pi\Big)+\theta(\tilde{\xi}_3-\xi_3)\sum_{n_3>0}\frac{1}{n_3}e^{n_3(\zeta_3-\tilde{\zeta}_3)}\Big]\\&+\sum_{n_r,m_s\geq 0}T^{rs}_{n_r m_s}e^{n_r\zeta_r+m_s\tilde{\zeta}_s}+i\pi[\delta_{r2}\delta_{s1}+\delta_{r2}\delta_{s3}+\delta_{r1}\delta_{s3}]\,,\end{aligned} \tag{5·9}$$

where ρ and $\tilde{\rho}$ are on the r-th and the s-th strings strip on the light-cone diagram.

From the boundary condition (5·6) $n_1(n_2, n_3)$ are integers (half-integers). Equation (5·9) gives us the definition of $T^{rs}_{n_r m_s}$ whose integral representations are derived in Appendix B. With the coefficients $T^{rs}_{n_r m_s}$ defined in (5·9), we can write a vertex for the internal coordinates $X_t^I(\sigma)$ as follows:

$$e^{-(D/8)\tau_0((1/\alpha_2)+(1/\alpha_3))}\mathcal{P}^{Z_2}_{123}|V^{\text{int}}_{t0}(1,2,3)\rangle\,,$$

$$|V^{\text{int}}_{t0}(1,2,3)\rangle=e^{i\pi(L_+^{(t)(3)}-L_-^{(t)(3)})\text{sgn}(\tilde{\sigma}_3)}e^{E_t(1,2,3)}|0\rangle_{123}\,,$$

$$E_t(1,2,3)=\frac{1}{2}\sum_{\pm}\sum_{r,s=1}^{3}\sum_{n_r,m_s\geq 0}T^{rs}_{n_r m_s}\alpha^{I(\pm)(r)}_{-n_r}\alpha^{I(\pm)(s)}_{-m_s}\,,$$

$$\mathcal{P}^{Z_2}_{123}=\mathcal{P}_u^{Z_2(1)}\mathcal{P}_t^{Z_2(2)}\mathcal{P}_t^{Z_2(3)}\,. \tag{5·10}$$

We have multiplied $e^{i\pi(L_+^{(t)(3)}-L_-^{(t)(3)})\text{sgn}(\tilde{\sigma}_3)}$, since we would like to keep the same convention of choosing the origin of strings in two vertices (4·7) and (5·10). The factor $e^{-(D/8)\tau_0((1/\alpha_2)+(1/\alpha_3))}$ appears corresponding to the difference of the intercepts in two sectors and would be necessary for the Jacobi identity (see § 10). It is also necessary to reproduce an appropriate vertex operator when we let the string length α of a twisted string go to zero.[16] From the invariance of $E_t(1, 2, 3)$ under the simultaneous Z_2-action we may omit any one of the three projection operators in (5·10) as in the 3-untwisted string vertex case. We should mention here that the $T^{rs}_{n_r m_s}$ are real quantities which are invariant under projective transformations on the z-plane (see Appendix B for details). Originally we need a vertex on the ρ-plane so that it should be inert under the projective transformation on the z-plane. Since $T^{rs}_{n_r m_s}$ are real, $E_t(1, 2, 3)$ in (5·10) is symmetric under an exchange of the left-moving and the right-moving modes.

Let us define vertices $|V^{\text{int}}_{t0}(1,2,3)\rangle$ in which the r-th string is in the untwisted sector,

$$|V^{\text{int}}_{t0}(1,2,3)\rangle=e^{i\pi(L_+^{(t)(r+2)}-L_-^{(t)(r+2)})\text{sgn}(\tilde{\sigma}_{r+2})}e^{E_t(r,r+1,r+2)}|0\rangle_{123}\,. \tag{5·11}$$

The full vertex is now obtained as

$$|V_t(1,2,3)\rangle=\mathcal{P}_{123}\bigg[\frac{\alpha_1\alpha_2}{\alpha_3}\mu_t^2(\alpha_1,\alpha_2,\alpha_3)G(\sigma_I)\boldsymbol{\pi}_c|V_0^{\text{ext-FP}}(1,2,3)\rangle$$
$$\times\mathcal{P}^{Z_2}_{123}|V^{\text{int}}_{t0}(1,2,3)\rangle\Gamma(\boldsymbol{p}_r;n^f_{r+1},n^f_{r+2})\bigg]\,,$$

$$\mu_t(\alpha_1,\alpha_2,\alpha_3)=\exp\left[-\tau_0\left\{\frac{1}{\alpha_r}+\left(1-\frac{D}{16}\right)\left(\frac{1}{\alpha_{r+1}}+\frac{1}{\alpha_{r+2}}\right)\right\}\right]\,, \tag{5·12}$$

where $\Gamma(\boldsymbol{p}_r;n^f_{r+1},n^f_{r+2})$ is some $2^D\times 2^D$ matrix corresponding to the fact that the twisted string has the degrees of freedom of the fixed points explained in § 3. This factor Γ cannot be determined by the Neumann function method and will be discussed in § 8.

On $e^{E_t(123)}$ in (5·10) we can prove the following connection condition for $X^I(\tilde{\sigma})$ and $A^I(\tilde{\sigma})$. For definiteness we choose here $\alpha_1, \alpha_2>0$ and $\alpha_3<0$ and the first string as the untwisted string,

$$\begin{cases} X_u^{I(1)}(\hat{\sigma}_1)=\hat{X}_t^{I(3)}(\hat{\sigma}_3)\,, \\ \hat{X}_t^{I(2)}(\hat{\sigma}_2)=\hat{X}_t^{I(3)}(\hat{\sigma}_3)-\dfrac{\sqrt{\pi}}{2}w^{I(1)}\,, \end{cases} \tag{5·13}$$

$$\Theta_1\frac{1}{\alpha_1}A_{u\pm}^{I(1)}(\hat{\sigma}_1)+\Theta_2\frac{1}{\alpha_2}A_{t\pm}^{I(2)}(\hat{\sigma}_2)-\frac{1}{\alpha_3}A_{t\pm}^{I(3)}(\hat{\sigma}_3)=0\,, \tag{5·14}$$

where

$$\Theta_1(\sigma)=\theta(\pi\alpha_1-|\sigma|)\,,\qquad \Theta_2(\sigma)=\theta(|\sigma|-\pi\alpha_1)\,,$$

$$\hat{\sigma}_1(\sigma)=\frac{\sigma}{\alpha_1}\,,\quad \hat{\sigma}_2(\sigma)=\frac{\sigma-\pi\alpha_1\mathrm{sgn}(\sigma)}{\alpha_2}\,,\quad \hat{\sigma}_3(\sigma)=\frac{\sigma}{\alpha_3} \tag{5·15}$$

and $\hat{X}_t^{I(r)}(\hat{\sigma}_r)(r=2,3)$ are the internal coordinates for twisted strings with the zero mode x_t^I omitted. The quantity $w^{I(1)}$ is the winding number of the first string. From (5·15) we see that the ordinary intrinsic coordinate σ_r is related to $\hat{\sigma}_r$ as $\sigma_r=\hat{\sigma}_r-\pi\mathrm{sgn}(\hat{\sigma}_r)\delta_{r3}$ and internal coordinates $\hat{X}_t^{I(3)}(\hat{\sigma}_3)$ are expressed as

$$\hat{X}_t^{I(3)}(\sigma_3)=e^{i\pi(L_+^{(t)(3)}-L_-^{(t)(3)})\mathrm{sgn}(\hat{\sigma}_3)}\hat{X}_t^{I(3)}(\hat{\sigma}_3)e^{-i\pi(L_+^{(t)(3)}-L_-^{(t)(3)})\mathrm{sgn}(\hat{\sigma}_3)}\,. \tag{5·16}$$

These non-trivial relations are due to the presence of root cut in the internal coordinate $X_t^I(\sigma)$ of the twisted strings. In terms of $\hat{X}^{I(r)}(\sigma)$ and $A^{I(r)}(\sigma)$ we obtain the same connection conditions as (5·13) and (5·14) with a replacement of $\hat{\sigma}$ by σ since the σ-translation operator does not change the disconnectedness. Note here that the connection condition for $X^I(\sigma)$ is quite different from that for the external coordinate $X^\mu(\sigma)$, for which the vertex is a naive δ-functional. The situation is similar to the case for the torus compactified closed string. The vertex in that case includes the two-cocycle factor, which is introduced to satisfy the $O(g^2)$ relations, i.e., the Jacobi identity and the commutativity. In the orbifold case, we should take a particular form of $\Gamma(\boldsymbol{p}_r;\,n_{r+1}^f,\,n_{r+2}^f)$ in (5·12) in order for the $O(g^2)$ identities to hold.

§ 6. Identities for gauge invariance

We write the vertices (4·7) and (5·12) in the following general form:

$$|V_a(1,2,3)\rangle=\mathcal{P}_{123}\Big[\frac{\alpha_1\alpha_2}{\alpha_3}\mu_a{}^2(\alpha_1,\alpha_2,\alpha_3)G(\sigma_I)\boldsymbol{\pi}_c|V_0^{\text{ext-FP}}(1,2,3)\rangle$$
$$\times\mathcal{P}_{123}^{Z_2}|V_{a0}^{\text{int}}(1,2,3)\rangle C_a(1,2,3)\Big] \tag{6·1}$$

with $a=u$ or t and

$$C_a(1,2,3)=\begin{cases}\varepsilon(\boldsymbol{p}_1,\boldsymbol{p}_2)\,, & (a=u)\\ \Gamma(\boldsymbol{p}_r;\,n_{r+1}^f,\,n_{r+2}^f)\,. & (a=t)\end{cases} \tag{6·2}$$

Here we understand the r-th string is in the untwisted sector. We now define $*$-product for all possible combinations of sectors to which string fields Φ and Ψ belong,

$$|(\Phi^*\Psi)(3)\rangle=\varepsilon_\Phi\varepsilon_\Psi\int d1d2\langle\Phi(1)|\langle\Psi(2)\|V_a(1,2,3)\rangle\,. \tag{6·3}$$

On the basis of the definitions of the vertices and the $*$-products, we can write down the identities which will be shown, in § 11, to guarantee the gauge invariance of the action as well as the closure of the gauge algebra,

$$Q_B{}^2=0\,, \qquad \text{(nilpotency)} \tag{6·4}$$

$$Q_B(\Phi*\Psi)=Q_B\Phi*\Psi+(-)^{|\Phi|}\Phi*Q_B\Psi\,, \qquad \text{(distributive law)} \tag{6·5}$$

$$(\Phi*\Psi)*\Lambda+(-)^{|\Phi|(|\Psi|+|\Lambda|)}(\Psi*\Lambda)*\Phi+(-)^{|\Lambda|(|\Phi|+|\Psi|)}(\Lambda*\Phi)*\Psi=0\,, \qquad \text{(Jacobi identity)} \tag{6·6}$$

$$\Phi*\Psi=(-)^{1+|\Phi||\Psi|}\Psi*\Phi\,, \qquad \text{(commutativity)} \tag{6·7}$$

where $|\Phi|$ is 0 or 1 if Φ is Grassmann-even or odd respectively. The BRS charge $Q_B=Q_B{}^u$ or $Q_B{}^t$ and $*$-products are taken for all the possible combinations of strings in the untwisted and the twisted sector. In appearance, these identities are in the same forms as those for the non-compactified closed string.[4] The identities (6·4)~(6·7) within the untwisted sector result from similar identities for the torus compactified case[2] if we recall the following facts: As for (6·5), it is sufficient to note that $Q_B{}^u$ is the same as that in the torus compactified case and commutes with $\mathcal{P}_u{}^{Z_2}$ (see (3·19)); the Jacobi identity (6·6) is nothing but the one for the torus case with the external states restricted to Z_2 invariant sector since the $\mathcal{P}^{Z_2}$ for the intermediate string may be omitted (cf. §§ 4 and 5). We emphasize that the identities (6·6) and (6·7) hold only with the two-cocycle factor on the vertex.

§ 7. BRS invariance of twisted vertex

We will prove (6·5) with the twisted vertex in (6·1). Equation (6·5) is equivalently written as

$$[Q_B{}^{u(1)}+Q_B{}^{t(2)}+Q_B{}^{t(3)}]|V_t(1,2,3)\rangle=0\,. \tag{7·1}$$

We should note, here, that the vertex (5·12) can be rewritten into the form,

$$|V_t(1,2,3)\rangle=\mathcal{P}_{123}\frac{\alpha_1\alpha_2}{\alpha_3}\left[\frac{1}{2}\sum_{r,s,t=1}^{3}\varepsilon_{rst}\frac{\pi_c{}^{0(r)}}{\alpha_r}\frac{\pi_c{}^{0(s)}}{\alpha_s}\alpha_t\right]$$

$$\times\mathcal{P}_{123}^{Z_2}\Gamma(\boldsymbol{p}_r;\,n_{r+1}^f,\,n_{r+2}^f)|V_{t+}(1_+,2_+,3_+)\rangle|V_{t-}(1_-,2_-,3_-)\rangle\,,$$

$$|V_{t\pm}(1_\pm,2_\pm,3_\pm)\rangle=\mu_t(\alpha_1,\alpha_2,\alpha_3)\sqrt{\pi}\,\alpha_r C_\pm{}^{(r)}(\sigma_I{}^{(r)})|V_{t0\pm}(1_\pm,2_\pm,3_\pm)\rangle\,. \tag{7·2}$$

$$(r=1,2\text{ or }3)$$

The part $|V_{t0\pm}(1_\pm,2_\pm,3_\pm)\rangle$ in (7·2) is δ-functionals which represent the connection conditions for the left- and the right-moving part of the external, internal Bosonic coordinates and the FP ghost coordinates.

$$|V_{t0\pm}(1_\pm,2_\pm,3_\pm)\rangle=|V_{0\pm}^{\text{ext-FP}}(1_\pm,2_\pm,3_\pm)\rangle|V_{t0}^{\text{int}}(1_\pm,2_\pm,3_\pm)\rangle\,,$$

$$|V_{0\pm}^{\text{ext-FP}}(1_\pm, 2_\pm, 3_\pm)\rangle = e^{E_\pm^{\text{ext-FP}}(1_\pm, 2_\pm, 3_\pm)}|0\rangle_{123}$$
$$\times(2\pi)^{d+1}\delta^d(\sum_r p_r)\delta(\sum_r \alpha_r)\delta(\sum_r \alpha_r^{-1}\bar{c}_0^{(\pm)(r)})\,, \qquad (7\cdot3)$$

where $E_\pm^{\text{ext-FP}}(1_\pm, 2_\pm, 3_\pm)$ ($|V_{t0\pm}^{\text{int}}(1_\pm, 2_\pm, 3_\pm)\rangle$) are the left-moving (upper sign) and the right-moving (lower sign) parts of the external and FP ghost part of $E^{\text{ext-FP}}(1, 2, 3)$ in (4·1) (the internal part vertex $|V_{t0}^{\text{int}}(1, 2, 3)\rangle$ in (5·10)). The α-dependent normalization factor $\mu_t(\alpha_1, \alpha_2, \alpha_3)$ is not determined by the requirement (7·1) but should be fixed so that the Jacobi identity (6·6) to hold.[3] When we take $D=0$, $|V_{t\pm}(1_\pm, 2_\pm, 3_\pm)\rangle$ are reduced to the open string vertices for the left- and right-moving modes. Recall that the 3-string vertex for the ordinary closed string is essentially a product of two open string vertices for the left- and right-moving modes.[12),4)] If we take into the consideration that the quantities in front of $|V_{t+}\rangle|V_{t-}\rangle$ in (7·2) commute with the BRS charge $Q_B{}^u$ or $Q_B{}^t$, Eq. (7·1) holds if the following relations are satisfied,

$$[Q_{B\pm}^{u(1)} + Q_{B\pm}^{t(2)} + Q_{B\pm}^{t(3)}]|V_{t\pm}(1_\pm, 2_\pm, 3_\pm)\rangle = 0\,, \qquad (7\cdot4)$$

where we use the fact that the BRS charge is also decomposed into the left- and right-moving parts as given in (3·6).

In order to prove Eq. (7·4) we use the method developed in Ref. 3). On the ρ-plane in Fig. 3, we define operator-valued functions

$$A_\pm(\rho) = \frac{1}{\alpha_r} A_\pm{}^{(r)}(\mp i\zeta_r) \qquad (7\cdot5\text{a})$$

for both external and internal coordinates and

$$C_\pm(\rho) = \alpha_r C_\pm{}^{(r)}(\mp i\zeta_r)\,, \qquad (7\cdot5\text{b})$$

$$\bar{C}_\pm(\rho) = \alpha_r{}^{-2}\bar{C}_\pm{}^{(r)}(\mp i\zeta_r) \qquad (7\cdot5\text{c})$$

with $\zeta_r = \xi_r + i\sigma_r$, when ρ is on the r-th string region. With these functions the sum of the BRS charge may be rewritten as a contour integration along C_ρ shown in Fig. 5,

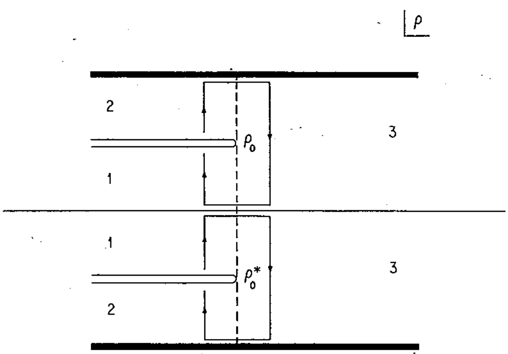


Fig. 5. The original contour C_ρ of the integration (7·6) representing $\sum_{r=1}^3 Q_B{}^{(r)}$ on the ρ-plane.

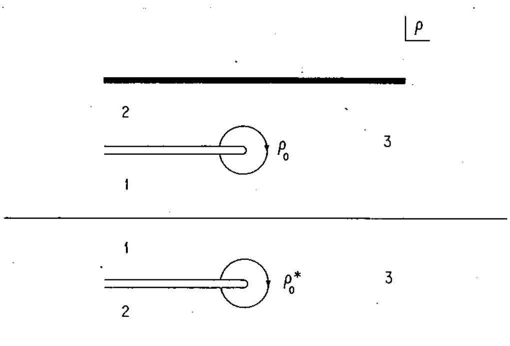


Fig. 6. The contour C_ρ can be deformed to infinitesimal circles enclosing the interaction points ρ_0 and ρ_0^*.

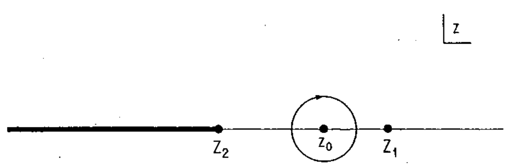


Fig. 7. The contour C_z of the integration (7·8) on the z-plane which corresponds to the reduced contour in Fig. 6.

$$Q_{B\pm}^{u(1)}+Q_{B\pm}^{t(2)}+Q_{B\pm}^{t(3)}$$
$$=-i\frac{\sqrt{\pi}}{2}\oint_{C_\rho}d\rho C_\pm(\rho)\Big[-(A_\pm{}^\mu(\rho))^2-(A_\pm{}^I(\rho))^2+2\frac{dC_\pm(\rho)}{d\rho}\bar{C}_\pm(\rho)\Big]. \tag{7·6}$$

In deriving Eq. (7·6) we used the ξ translational invariance of Q_B. The horizontal part of the contour does not contribute owing to the cancellation between Im$\rho>0$ and Im$\rho<0$ parts. The contour C_ρ on the ρ-plane can be reduced to small circles around the interaction points ρ_0 and $\rho_0{}^*$ depicted in Fig. 6, which is then mapped to a circle around the interaction point z_0 on the z-plane shown in Fig. 7 through the Mandelstam mapping

$$\rho(z)=\sum_{r=1}^{3}\alpha_r\ln(z-Z_r)\,. \tag{7·7}$$

[The $A_\pm{}^I(\rho)A_\pm{}^I(\rho)$ in the BRS charge $Q_{B\pm}$ is analytic, although $A_\pm{}^I(\rho)$ has a root cut shown in Figs. 5 and 7.]

The LHS of (7·4) is now rewritten as

$$-i\frac{\pi}{2}\mu_t(\alpha_1,\alpha_2,\alpha_3)\times\oint_{C_\rho}d\rho C_\pm(\rho)\Big[-(A_\pm{}^\mu(\rho))^2-(A_\pm{}^I(\rho))^2$$
$$+2\frac{dC_\pm(\rho)}{d\rho}\bar{C}_\pm(\rho)\Big]C_\pm(\rho_0)|V_{t0\pm}(1_\pm,2_\pm,3_\pm)\rangle=-i\frac{\pi}{2}\mu_t(\alpha_1,\alpha_2,\alpha_3)$$
$$\times\oint_{C_z}dz\Big(\frac{d\rho(z)}{dz}\Big)^{-1}C_\pm(z)\Big[-(A_\pm{}^\mu(z))^2-(A_\pm{}^I(z))^2+2\frac{dC_\pm(z)}{dz}\bar{C}_\pm(z)\Big]$$
$$\times C_\pm(z_0)|V_{t0\pm}(1_\pm,2_\pm,3_\pm)\rangle\,, \tag{7·8}$$

where new functions on the z-plane are defined as

$$A_\pm(z)=\frac{d\rho(z)}{dz}A_\pm(\rho(z))\,,\quad C_\pm(z)=C_\pm(\rho(z))\,,\quad \bar{C}_\pm(z)=\frac{d\rho(z)}{dz}\bar{C}_\pm(\rho(z))\,. \tag{7·9}$$

As observed in Ref. 3), $A_\pm{}^\mu(\rho)$, $(d/d\rho)C_\pm(\rho)$ and $\bar{C}_\pm(\rho)$ are singular around the interaction point when we let them operate on the vertex. The factor $(dz/d\rho)$ remedies that this singularity and the resulting operator valued functions on the z-plane are regular functions. This is also the case for the internal bosonic coordinates $A_{u\pm}^I(\rho)$ or $A_{t\pm}^I(\rho)$ whose singularity is evaluated in Appendix C.

We calculate the contour integration of (7·8). Pole-singularities in the integrand in (7·8) come from $(d\rho(z)/dz)^{-1}$ or all the possible contractions of $A_\pm(z)$, $C_\pm(z)$ and $\bar{C}_\pm(z)$.[3)] The factor $(d\rho(z)/dz)^{-1}$ has a simple pole

$$\Big(\frac{d\rho(z)}{dz}\Big)^{-1}=-\frac{1}{a}\frac{1}{z-z_0}+\frac{b}{a^2}+\Big(\frac{c}{a^2}-\frac{b^2}{a^3}\Big)(z-z_0)+\cdots, \tag{7·10}$$

where

$$\rho_0-\rho(z)=\frac{1}{2}a(z-z_0)^2+\frac{1}{3}b(z-z_0)^3+\frac{1}{4}c(z-z_0)^4+\cdots,$$
$$\rho_0=\rho(z_0)\,,\quad a=\sum_r\frac{\alpha_r}{(z_0-z_r)^2}\,,\quad b=-\sum_r\frac{\alpha_r}{(z_0-z_r)^3}\,,\quad c=\sum_r\frac{\alpha_r}{(z_0-z_r)^4}\,. \tag{7·11}$$

The caluculation of (7·8) is the same as that for the open bosonic case except for the contribution from the internal bosonic coordinates. Therefore we concentrate on the part coming from them, i.e.,

$$\oint_{C_z} dz\left(\frac{d\rho(z)}{dz}\right)^{-1} C_\pm(z)[-A_\pm{}^I(z)A_\pm{}^I(z)]C_\pm(z_0) \tag{7·12}$$

and discuss whether it gives an extra contribution compared to the external Bosonic coordinates in the open bosonic case. Here the contraction is defined for an arbitrary operator $\mathcal{O}$ as follows,

$$\mathcal{O}_1(\rho)\mathcal{O}_2(\tilde{\rho})=\theta(\tilde{\xi}-\xi)\langle 0|\mathcal{O}_1(\rho)\mathcal{O}_2(\tilde{\rho})|0\rangle_c \pm \theta(\xi-\tilde{\xi})\langle 0|\mathcal{O}_2(\tilde{\rho})\mathcal{O}_1(\rho)|0\rangle_c$$
$$+[\mathcal{O}_1(\rho),[\mathcal{O}_2(\tilde{\rho}), E^{\text{ext-FP}}(1,2,3)+E_t(1,2,3)]]_\mp\,, \tag{7·13}$$

where suffix c denotes the connected part and the lower sign on the RHS should be taken when both of $\mathcal{O}_1$ and $\mathcal{O}_2$ are fermionic. As for the bosonic coordinates we obtain

$$A_\pm{}^\mu(z)A_\pm{}^\nu(\tilde{z})=\frac{1}{\pi}\frac{1}{(z-\tilde{z})^2}\eta^{\mu\nu}\,, \tag{7·14}$$

$$A_\pm{}^I(z)A_\pm{}^J(\tilde{z})=\frac{1}{\pi}\frac{1}{(z-\tilde{z})^2}\delta^{IJ}\left[\frac{1}{2}\frac{z+\tilde{z}}{(z\tilde{z})^{1/2}}\right], \tag{7·15}$$

where the upper equality is given in Ref. 3) and the lower one is obtained from the definition (7·13).

$$A_\pm{}^I(\rho_r)A_\pm{}^J(\tilde{\rho}_s)=\frac{1}{\pi\alpha_r\alpha_s}\frac{\partial}{\partial\zeta_r}\frac{\partial}{\partial\tilde{\zeta}_s}\mathcal{T}(\rho_r,\tilde{\rho}_s)\delta^{IJ}$$
$$=\frac{1}{\pi}\left(\frac{d\rho}{dz}\right)^{-1}\left(\frac{d\tilde{\rho}}{d\tilde{z}}\right)^{-1}\left[\frac{1}{2}\frac{z+\tilde{z}}{(z\tilde{z})^{1/2}}\right]\frac{1}{(z-\tilde{z})^2}\delta^{IJ}\,. \tag{7·16}$$

Thus we have (7·15) using (7·9).

Since there is a singular function $A^I(z)A^I(z)$ in (7·12), we have to regularize it keeping the analyticity of the integrand of the BRS charge. The following regularization has the desired property. On the ρ-plane we separate two $A_\pm{}^I(\rho)$'s of $Q_{B\pm}$ into $A_\pm{}^I(\rho(z))$ and $A_\pm{}^I(\rho(z'))$ where $\rho(z)$ and $\rho(z')$ are related as

$$\rho(z')=\rho(z)-a\delta\,,$$
$$a=\sum_{r=1}^{3}\frac{\alpha_r}{(z_0-Z_r)^3} \tag{7·17}$$

with a real constant δ. Remember that a is also a real constant for our choice $(Z_1, Z_2, Z_3)=(1, 0, -\infty)$ in (5·1) of the Koba-Nielsen variables. This regularization always keeps $A_\pm{}^I(\rho(z))$ and $A_\pm{}^I(\rho(z'))$ on points with the same value of σ and thus retain the analyticity of BRS-charge integrand. The above argument also justifies the cancellation of the contributions from the horizontal parts in Fig. 5.

After this particular regularization*) we have a contribution from the internal coordinates as

$$\oint_{C_z} dz\left(\frac{d\rho(z')}{dz'}\right)^{-1} C_\pm(z)[-\overline{A_\pm{}^I(z)A_\pm{}^I}(z')]C_\pm(z_0) \tag{7·18}$$

instead of (7·12). We evaluate the pole residues and then take the limit of $\delta\to 0$. In order to see the extra contribution to (7·8) due to the difference (7·15) from (7·14), we expand the factor

$$\frac{1}{2}\frac{z+z'}{(zz')^{1/2}}$$

in terms of

$$\varepsilon = z - z_0 \tag{7·19}$$

and δ. By using an expansion

$$z' - z_0 = \varepsilon + \sum_{n\ge 1} f_n(\varepsilon)\delta^n \tag{7·20}$$

a simple calculation gives us the following result:

$$\frac{1}{2}\frac{z+z'}{(zz')^{1/2}} = 1 + O(\varepsilon^2) + O(\delta)\,. \tag{7·21}$$

[The functions $f_n(\varepsilon)$ are given in Appendix C in Ref. 3).] From an expansion (see (3·47a) in Ref. 3)),

$$\left(\frac{d\rho(z')}{dz'}\right)^{-1}\frac{1}{(z-z')^2} = -\frac{1}{4a}\frac{1}{\varepsilon^3} + O\left(\frac{1}{\varepsilon^2}\right), \tag{7·22}$$

we may expect the extra contribution coming from the second term in (7·21). However it leads to a vanishing contribution owing to $C_\pm{}^2(z_0)=0$. Note here that the $O(\delta)$ terms have pole singularities. Nevertheless they will vanish when we take the limit $\delta\to 0$.

In conclusion, the BRS invariance proof of the vertex with the twisted oscillator modes is reduced to that in the ordinary closed string case.[4] Therefore the vertex (6·1) is BRS invariant under the condition of $d+D=26$.

§ 8. Cocycle factor

In this section, we will obtain a condition on Γ in (6·2) in order for the Jacobi identity to hold, in which strings in the twisted sector are included as the external strings. Further we construct the Γ in such a manner that we can make a physical interpretation corresponding to the string interaction.

We may draw P, Q and R configurations shown in Fig. 8 with θ_P, θ_Q and $\theta_R \in [-\pi, \pi]$ and the time interval T set equal to zero corresponding to three terms in (6·6) (let

*) If we take the normal ordering as an alternative regularization, we obtain the correct intercept as well as the critical dimension.[17]

us call them P, Q and R terms). For the Jacobi identity to hold, at least, pairs of string diagrams in P and Q, Q and R, or R and P configurations should coincide when $T \to 0$.[4] As an example, consider P configuration with $\theta_P=0$ and Q configuration with $\theta_Q=\pi$. Let us find the connection conditions on them for the case that strings 1 and 2 are in the untwisted sector, and strings 3 and 4 are in the twisted sector (see Fig. 9). If we disregard the cocycle factors, we obtain an identical connection condition for both configurations,

$$\begin{aligned} &X^{I(1)}(\sigma_1)=\hat{X}^{I(4)}(\sigma_4)\,, \\ &X^{I(2)}(\sigma_2)=\hat{X}^{I(4)}(\sigma_4)+\frac{\sqrt{\pi}}{2}w^{I(1)}\,, \\ &\hat{X}^{I(3)}(\sigma_3)=\hat{X}^{I(4)}(\sigma_4)+\frac{\sqrt{\pi}}{2}(w^{I(1)}+w^{I(2)})\,, \end{aligned} \tag{8·1}$$

by using Eqs. (4·5) and (5·13) [in § 10 we will derive Eq. (8·1)]. Therefore, in order for the P($\theta_P=0$) and Q($\theta_Q=\pi$) configurations to coincide the Γ should satisfy the condition:

$$\varepsilon(\boldsymbol{p}_1, \boldsymbol{p}_2)\Gamma(\boldsymbol{p}_5; n_3{}^f, n_4{}^f)=\Gamma(\boldsymbol{p}_2; n_3{}^f, n_8{}^f)\Gamma(\boldsymbol{p}_1; n_7{}^f, n_4{}^f)\,. \tag{8·2}$$

In the remaining part of this section we construct the Γ which satisfies (8·2). We will first recall how to obtain the two-cocycle $\varepsilon(\boldsymbol{p}, \boldsymbol{q})$ on the Lorentzian even self-dual lattice $\Gamma_{D,D}$ with the metric (A·18).[7] We assume that there is a set of matrices $\gamma_{\tilde{e}_i}$ corresponding to $\tilde{\boldsymbol{e}}_i$ which satisfy

$$\gamma_{\tilde{e}_i}\gamma_{\tilde{e}_j}=(-)^{\tilde{e}_i\cdot\tilde{e}_j}\gamma_{\tilde{e}_j}\gamma_{\tilde{e}_i}\,. \tag{8·3}$$

For an arbitrary vector $\boldsymbol{p}=\sum_{i=1}^{2D}p_i\tilde{\boldsymbol{e}}_i$ on $\Gamma_{D,D}$, we define $\gamma_{\boldsymbol{p}}$ as

$$\gamma_{\boldsymbol{p}}=(\gamma_{\tilde{e}_1})^{p_1}(\gamma_{\tilde{e}_2})^{p_2}\cdots \tag{8·4}$$

and then a two-cocycle $\bar{\varepsilon}(\boldsymbol{p}, \boldsymbol{q})$ by

$$\gamma_{\boldsymbol{p}}\gamma_{\boldsymbol{q}}=\bar{\varepsilon}(\boldsymbol{p}, \boldsymbol{q})\gamma_{\boldsymbol{p}+\boldsymbol{q}}\,. \tag{8·5}$$

One can easily check that this factor actually satisfies the condition (4·3) on the two-cocycle factor. However $\bar{\varepsilon}(\boldsymbol{p}, \boldsymbol{q})$ differs from $\varepsilon(\boldsymbol{p}, \boldsymbol{q})$ in (4·4) by an amount of

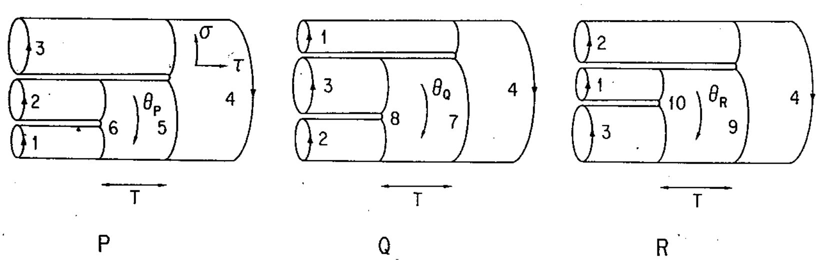


Fig. 8. The closed string diagrams which represent the three configurations corresponding to the first (P), second (Q) and third (R) terms in the Jacobi identity (6·6), respectively, for the case $\alpha_1, \alpha_2, \alpha_3 > 0$. The numbers 1, 2, and 3 correspond to the string fields Φ, Ψ and Λ.

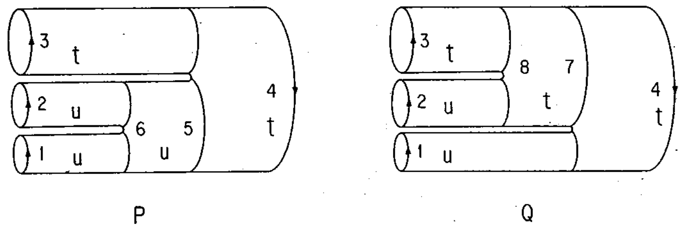


Fig. 9. The P($\theta_P=0$) and Q($\theta_Q=\pi$) configurations which should cancel with each other in the Jacobi identity (10·1). The strings 1 and 2 are in the untwisted sector and 3 and 4 are in the twisted sector.

trivial phase factors,

$$\bar{\varepsilon}(\boldsymbol{p}=\sum_i p_i\tilde{\boldsymbol{e}}_i,\ \boldsymbol{q}=\sum_i q_i\tilde{\boldsymbol{e}}_i)=\exp i\pi\sum_{i>j}p_iq_j\tilde{\boldsymbol{e}}_i\cdot\tilde{\boldsymbol{e}}_j\,, \tag{8·6}$$

$$\varepsilon(\boldsymbol{p},\boldsymbol{q})=\bar{\varepsilon}(\boldsymbol{p},\boldsymbol{q})U^{-1}(\boldsymbol{p})U^{-1}(\boldsymbol{q})U(\boldsymbol{p}+\boldsymbol{q})\,, \tag{8·7}$$

where trivial phase $U(\boldsymbol{p})$ is the one appeared in the hermiticity condition on the untwisted string field (3·20), i.e.,

$$U(\boldsymbol{p})=e^{-(\pi i/4)\boldsymbol{p}^2}\,. \tag{8·8}$$

With (8·7) we can rewrite (8·5) as

$$[U(\boldsymbol{p})\gamma_{\boldsymbol{p}}][U(\boldsymbol{q})\gamma_{\boldsymbol{q}}]=\varepsilon(\boldsymbol{q},\boldsymbol{p})[U(\boldsymbol{p}+\boldsymbol{q})\gamma_{\boldsymbol{p}+\boldsymbol{q}}]\,. \tag{8·9}$$

This relation is quite similar to (8·2). Actually, we will give the Γ in terms of a concrete representation of $\gamma_{\boldsymbol{p}}$ as described below. From the metric (A·18) and (8·3), we see that $\gamma_{\tilde{\boldsymbol{e}}_i}(i\leqq D)$ anticommutes with $\gamma_{\tilde{\boldsymbol{e}}_{i+D}}$ and commutes with all the others. As a solution to (8·3), we may choose

$$\gamma_{\tilde{\boldsymbol{e}}_i}=\mathbf{1}\otimes\cdots\otimes \overset{\overset{i}{\vee}}{\sigma_1}\otimes\cdots\otimes\mathbf{1}\,,$$

$$\gamma_{\tilde{\boldsymbol{e}}_{i+D}}=\mathbf{1}\otimes\cdots\otimes \overset{\overset{i}{\vee}}{\sigma_3}\otimes\cdots\otimes\mathbf{1} \tag{8·10}$$

for $i\leqq D$. Here σ_1 and σ_3 are Pauli matrices. Therefore from (2·12) and (8·4) we have an expression of $\gamma_{\boldsymbol{p}}$ for a general momentum $\boldsymbol{p}=\sum_i p_i\tilde{\boldsymbol{e}}_i$

$$\gamma_{\boldsymbol{p}}=(\sigma_1)^{n_1}(\sigma_3)^{m_1}\otimes\cdots\otimes(\sigma_1)^{n_D}(\sigma_3)^{m_D}\,. \tag{8·11}$$

Since strings 7 and 8 are the same string, the RHS of (8·2) may be regarded as a product of matrices. Thus the relation (8·9) can be identified with Eq. (8·2) if we consider that

$$\Gamma(\boldsymbol{p}_1;\,n_2{}^f,\,n_3{}^f)={}_2[\gamma_{\boldsymbol{p}_1}]_3U(\boldsymbol{p}_1)\,. \tag{8·12}$$

Here subscripts 2 and 3 of ${}_2[\gamma_{\boldsymbol{p}_1}]_3$ indicate the spaces on which the matrix acts.

In order to explain the physical meaning of Γ, let us reconsider the interaction with the twisted strings. In such an interaction, the twisted string changes or does not change its location according to the winding number of the untwisted string.[8] In an interaction of strings 1 (untwisted string), 2 and 3 (twisted strings), we have a relation

$$\boldsymbol{x}_t{}^{(2)}=\boldsymbol{x}_t{}^{(3)}+\frac{\pi}{2}\boldsymbol{w}^{(1)} \tag{8·13}$$

up to a vector on the lattice Γ_D, i.e., $\pi\sum_{I=1}^{D}l_I\boldsymbol{E}_I$ (l_I; integers). Here the $\boldsymbol{w}^{(1)}$ is the winding number of the string 1

$$w^{I(1)}=\sum_{J=1}^{D}n_J{}^{(1)}(\boldsymbol{E}_J)^I=p_{+1}^I-p_{-1}^I\,. \tag{8·14}$$

This type of an interaction may be described by $(\sigma_1)^{n_1}\otimes\cdots\otimes(\sigma_1)^{n_D}$ in Γ if we consider that Γ acts on the space spanned by the fixed points (3·25). One can easily check this operator actually realizes the relation (8·13); in other words, $(\sigma_1)^{n_1}\otimes\cdots\otimes(\sigma_1)^{n_D}$ is a matrix representation of a δ-function on (3·25) $\sum_{\boldsymbol{v}\in\Gamma_D}\delta(\boldsymbol{x}_t{}^{(2)}+\boldsymbol{x}_t{}^{(3)}+(\pi/2)\boldsymbol{w}^{(1)}+\boldsymbol{v})$. The remaining factor could be written on a string field

$$(\sigma_3)^{m_1}\otimes\cdots\otimes(\sigma_3)^{m_D}|\Phi_t(n^f)\rangle=e^{i\hat{\boldsymbol{p}}\cdot\hat{\boldsymbol{x}}_t}|\Phi_t(n^f)\rangle\,, \tag{8·15}$$

$$\hat{\boldsymbol{p}}\cdot\hat{\boldsymbol{x}}_t=\sum_{I=1}^{D}p^Ix_t^I+\sum_{I,J=1}^{D}\hat{x}_t^IB_{IJ}\tilde{\boldsymbol{E}}_J\cdot\boldsymbol{w}\,, \tag{8·16}$$

where

$$\hat{x}_t^I=\boldsymbol{E}_I\cdot\boldsymbol{x}_t=\frac{\pi}{2}n_I\,. \qquad (n_I=0 \text{ or } 1) \tag{8·17}$$

We have used Eq. (A·17) in deriving (8·16).

Next we discuss the properties of Γ in (8·12) and the vertex (6·1) with this Γ. Under the transposition

$$\begin{aligned}{}_3[\gamma_{\boldsymbol{p}_1}^T]_2&=\prod_{I=1}^{D}(-)^{n_Im_I}{}_3[\gamma_{\boldsymbol{p}_1}]_2\\&=e^{(i\pi/2)\hat{\boldsymbol{p}}_1\cdot\hat{\boldsymbol{w}}_1}{}_3[\gamma_{\boldsymbol{p}_1}]_2\\&=U^{-2}(\boldsymbol{p}_1)_3[\gamma_{\boldsymbol{p}_1}]_2\,,\end{aligned} \tag{8·18}$$

where $\hat{\boldsymbol{p}}$ is defined in (A·7), $\hat{\boldsymbol{w}}=(n_1, n_2, \cdots, n_D)$ and the last equality is due to Eq. (A·17). Equation (8·18) implies the commutativity (6·7) extended to the case with Φ or/and Ψ in the twisted sector. In order to see this, let us consider the case of $\alpha_1>0$ (the untwisted string) and α_2, $\alpha_3<0$. The connection conditions on the vertices $|V_{t0}^{\rm int}(1,2,3)\rangle$ and $e^{i\pi(L_+{}^{(w)(1)}-L_-{}^{(w)(1)})}|V_{t0}^{\rm int}(1,3,2)\rangle$ are

$$X_u^{I(1)}(\sigma_1)=\begin{cases}\hat{X}^{I(2)}(\sigma_2)+\dfrac{\sqrt{\pi}}{2}w^{I(1)}\\ \hat{X}^{I(3)}(\sigma_3)\end{cases}\quad\text{on}\quad|V_{t0}^{\rm int}(1,2,3)\rangle\,, \tag{8·19a}$$

$$X_u^{I(1)}(\sigma_1)=\begin{cases}\hat{X}_t^{I(2)}(\sigma_2) \\ \hat{X}_t^{I(3)}(\sigma_3)+\dfrac{\sqrt{\pi}}{2}w^{I(1)}\end{cases} \quad \text{on} \quad e^{i\pi(L_+^{(w)(1)}-L_-^{(w)(1)})}|V_{t0}^{\text{int}}(1,3,2)\rangle\,, \tag{8·19b}$$

which are obtained by applying the method in Appendix D (cf. (5·13)). From Eq. (8·18) the difference of $\Gamma(\boldsymbol{p}_1;n_2^f,n_3^f)$ and $\Gamma(\boldsymbol{p}_1;n_3^f,n_2^f)$ is

$$\Gamma(\boldsymbol{p}_1;n_2^f,n_3^f)=U^{-2}(\boldsymbol{p}_1)\Gamma(\boldsymbol{p}_1;n_3^f,n_2^f)\,, \tag{8·20}$$

which actually converts (8·19a) into (8·19b). Therefore the vertex $\mathcal{P}_{123}^{Z_2}|V_{t0}^{\text{int}}(1,2,3)\rangle \times\Gamma(1,2,3)$ is invariant under the exchange of twisted strings. Since

$$|V_0^{\text{ext-FP}}(1,2,3)\rangle=\prod_{r=1\sim3}e^{i\pi(L_+^{(r)}-L_-^{(r)})}|V_0^{\text{ext-FP}}(2,1,3)\rangle\,, \tag{8·21}$$

we have a relation[4)]

$$|V_t(1,2,3)\rangle=-|V_t(2,1,3)\rangle\,, \tag{8·22}$$

where any one of the strings 1, 2 and 3 is in the untwisted sector. From Eqs. (6·3) and (8·22), it is easy to obtain the commutativity (6·7).

§ 9. Properties of vertices

Let us describe two other properties of the vertices (6·1). We have the hermiticity of the vertex,

$$\int d1'd2'd3'\langle\tilde{R}_u(1',1)|\langle\tilde{R}_t(2',2)|\langle\tilde{R}_t(3',3)\|V_t(1',2',3')\rangle$$
$$=-\langle V_t(1,2,3)|=\langle V_t(2,1,3)|\,. \tag{9·1}$$

This is easily proved by using the following properties: On $\langle R_u(r',r)|$ and $\langle R_t(r',r)|$ defined in (3·20) and (3·31) we have the relations

$$\langle\tilde{R}(1,2)|(\mathcal{A}_{n'}^{(1)}+\mathcal{A}_{-n'}^{(2)})=0\,,$$
$$\mathcal{A}_{n'}=\begin{cases}\alpha_{n'}^{\mu(\pm)},\ \alpha_{n'}^{I(\pm)},\ \gamma_{n'}^{(\pm)},\ \bar{\gamma}_{n'}^{(\pm)}\,, & (n'\neq0)\\ p^\mu,\ p_\pm{}^I,\ \alpha\,, & (n'=0)\end{cases} \tag{9·2}$$

where $n'\in\boldsymbol{Z}$ or $\boldsymbol{Z}+1/2$, and

$$\langle\tilde{R}(1,2)|(\pi_c^{0(1)},\ \bar{c}_0^{(1)},\ x_t^{(1)})=\langle\tilde{R}(1,2)|(\pi_c^{0(2)},\ \bar{c}_0^{(2)},\ x_t^{(2)}) \tag{9·3}$$

and

$$\Gamma(\boldsymbol{p}_1;n_2^f,n_3^f)U^2(\boldsymbol{p}_1)=\Gamma^*(\boldsymbol{p}_1;n_2^f,n_3^f)\,. \tag{9·4}$$

As in the ordinary closed string,[4)] we have a cyclic symmetry,

$$\pi_c^{0(3)}|V_a(1,2,3)\rangle=\pi_c^{0(1)}|V_a(2,3,1)\rangle=\pi_c^{0(2)}|V_a(3,1,2)\rangle\,. \tag{9·5}$$

These properties of the vertices imply the following important relations. First from Eqs. (3·20), (3·21), (6·3), (8·22) and (9·1), we have

$$\langle(\Phi*\Psi)(3)|=(-)^{1+|\Phi||\Psi|}\varepsilon_\Phi\varepsilon_\Psi\int d3'\langle\tilde{R}(3',3)\|(\Phi*\Psi)(3')\rangle$$

$$=\varepsilon_\Phi\varepsilon_\Psi\int d3'\langle\tilde{R}(3',3)\|(\Psi*\Phi)(3')\rangle\,. \tag{9·6}$$

Second the cyclic symmetry (9·5) implies that

$$[\Phi\Psi\Lambda]=(-)^{|\Phi|(|\Psi|+|\Lambda|)}[\Psi\Lambda\Phi]=(-)^{|\Lambda|(|\Phi|+|\Psi|)}[\Lambda\Phi\Psi]\,, \tag{9·7}$$

where trilinear form $[\Phi\Psi\Lambda]$[3] is defined by

$$[\Phi\Psi\Lambda]=\Phi\cdot(\Psi*\Lambda)\,,$$

$$\Phi\cdot\Psi=\int d1d2\pi_c{}^{0(1)}\langle\tilde{R}(2,1)\|\Phi(1)\rangle|\Psi(2)\rangle$$

$$=(-)^{|\Phi||\Psi|}\Psi\cdot\Phi \tag{9·8}$$

with an arbitrary combination of sectors to which Φ, Ψ and Λ belong.

§ 10. Jacobi identity

In this section, we discuss the Jacobi identity (6·6) with the twisted strings. As for the case only with the untwisted strings, it holds trivially[2] (cf. § 6).

Let us write the Jacobi identity (6·6) in the form

$$|(\Phi^{(1)}*\Phi^{(2)})*\Phi^{(3)}\rangle+(-)^{|1|(|2|+|3|)}|(\Phi^{(2)}*\Phi^{(3)})*\Phi^{(1)}\rangle$$

$$+(-)^{|3|(|1|+|2|)}|(\Phi^{(3)}*\Phi^{(1)})*\Phi^{(2)}\rangle=0\,, \tag{10·1}$$

where $|r|=|\Phi^{(r)}|$. Using Eq. (9·6), we can rewrite, e.g., the first term in (10·1) as

$$|((\Phi^{(1)}*\Phi^{(2)})*\Phi^{(3)})(4)\rangle=\varepsilon_1\varepsilon_2\varepsilon_3\int d1d2d3\langle\Phi^{(1)}(1)|\langle\Phi^{(2)}(2)|\langle\Phi^{(3)}(3)|$$

$$\times\int d5d6\langle\tilde{R}(5,6)\|V_a(1,2,6)\rangle|V_b(5,3,4)\rangle\,. \tag{10·2}$$

Substituting the expression of the vertices (6·1), we have[4]

$$|((\Phi^{(1)}*\Phi^{(2)})*\Phi^{(3)})(4)\rangle$$

$$=\frac{\varepsilon_1\varepsilon_2\varepsilon_3}{\alpha_4}\int d1d2d3\langle\Phi^{(1)}|\langle\Phi^{(2)}|\langle\Phi^{(3)}|\int_{-\pi}^{\pi}\frac{d\theta_P}{2\pi}W^{\varepsilon(123)}\mathcal{P}_{1234}$$

$$\times G(\sigma_I{}^{126})G(\sigma_I{}^{534})\mathcal{P}^{Z_2}_{1234}\alpha_5|\Delta(1,2;3,4;\theta_P;a,b)\rangle\,, \tag{10·3}$$

where

$$|\Delta(1,2;3,4;\theta_P;a,b)\rangle\equiv\mu_a{}^2(\alpha_1,\alpha_2,\alpha_6)\mu_b{}^2(\alpha_5,\alpha_3,\alpha_4)$$

$$\times\int d5d6\langle\tilde{R}(5,6)|e^{i(\theta_P\pm\pi)(L_+{}^{(5)}-L_-{}^{(5)})}|V_0{}^{\text{ext-FP}}(1,2,6)\rangle|V_0{}^{\text{ext-FP}}(5,3,4)\rangle$$

$$\times|V_{a0}^{\text{int}}(1,2,6)\rangle|V_{b0}^{\text{int}}(5,3,4)\rangle C_a(1,2,6)C_b(5,3,4)\,,$$

$$W^{\varepsilon(123)}=\frac{1}{2}\sum_{r,s,t=1}^{3}\epsilon^{rst}\alpha_r{}^2\pi_c{}^{0(s)}\pi_c{}^{0(t)}\,. \tag{10·4}$$

The sign of $\pm\pi$ in $e^{i(\theta_P\pm\pi)(L_+{}^{(5)}-L_-{}^{(5)})}$ should be chosen so that both σ_5 and σ_6 fall into $[-\pi,\pi]$.

It is easy to write the other terms of Eq. (10·1) in the same expression by suitable replacements of indices. Let us compare the P configuration ($\theta_P=0$) and the Q configuration ($\theta_Q=\pi$) as an example. The cancellation of the corresponding terms occurs if the following conditions are satisfied:

$$\text{i)}\quad \alpha_5|\Delta(1,2;3,4;\theta_P=0;a,b)\rangle$$
$$=\alpha_7\prod_{r=1,4}e^{i\pi(L_+{}^{(r)}-L_-{}^{(r)})\mathrm{sgn}(\sigma_r)}|\Delta(2,3;1,4;\theta_Q=\pi;a',b')\rangle\,, \tag{10·5}$$

$$\text{ii)}\quad G(\sigma_I{}^{126})G(\sigma_I{}^{534})|_P=-G(\sigma_I{}^{238})G(\sigma_I{}^{714})|_Q\,. \tag{10·6}$$

In the present case, the equality ii) holds without any problem.[4] In order to show equality i) we have to prove the coincidence of the connection conditions and the normalization of the vacuum term.[4]

We first consider the connection conditions. Configurations with the twisted strings are classified into the following cases: 1) Two strings are in the untwisted sector and the others are in the twisted sector; 2) all the strings are in the twisted sector. It is easily found that there are two combinations of vertices in case 1): 1A) $(a,b)=(u,t)$ and $(a',b')=(t,t)$; 1B) $(a,b)=(t,t)$ and $(a',b')=(t,t)$, while there is only one combination in case 2): $(a,b)=(t,t)$ and $(a',b')=(t,t)$. In the following, we explicitly show that the connection conditions coincide for cases 2) and 1A) with strings 1 and 2 in the untwisted sector. The connection conditions in 1B) may be treated similarly to case 2) since case 1B) has essentially the same vertex combination as 2).

For case 1A) we may easily check (8·1), ignoring the cocycles, on the concrete expressions of the 4-string vertices in (10·5) and further the conditions on the cocycles (8·2). Therefore, by construction, the equality (10·5) holds up to the normalization factor. In deriving (8·1) for Q configuration we have used, as the internal part of the 4-string vertex, $|V_{t0}^{\rm int}(2,3,8)\rangle\Gamma(\boldsymbol{p}_2;n_3{}^f,n_8{}^f)|V_{t0}^{\rm int}(1,7,4)\rangle\Gamma(\boldsymbol{p}_1;n_7{}^f,n_4{}^f)$. This is obtained from the one derived by simple replacement of indices in (10·4) by using the equality $|V_{t0}^{\rm int}(1,4,7)\rangle\Gamma(\boldsymbol{p}_1;n_4{}^f,n_7{}^f)=|V_{t0}^{\rm int}(1,7,4)\rangle\Gamma(\boldsymbol{p}_1;n_7{}^f,n_4{}^f)$.

Let us consider case 2) shown in Fig. 10. We first discuss the connection condition on the vertices (10·5) with the cocycles omitted. On the LHS of Eq. (10·5) (P configuration), we have

$$\hat{X}^{I(1)}(\sigma_1)=\hat{X}^{I(4)}(\sigma_4)\,,$$
$$\hat{X}^{I(2)}(\sigma_2)=\hat{X}^{I(4)}(\sigma_4)+\frac{\sqrt{\pi}}{2}w^{I(5)}\,, \qquad \text{(P)}$$
$$\hat{X}^{I(3)}(\sigma_3)=\hat{X}^{I(4)}(\sigma_4)+\frac{\sqrt{\pi}}{2}w^{I(5)}\,. \tag{10·7}$$

Recalling that the factor $e^{\pm i\pi(L_+{}^{(r)}-L_-{}^{(r)})}$ does not change the amount of disconnectedness in Eqs. (4·5) and (5·13), we obtain the connection condition on the RHS of (10·5) (Q configuration)

$$\hat{X}^{I(1)}(\sigma_1)=\hat{X}^{I(4)}(\sigma_4)+\frac{\sqrt{\pi}}{2}w^{I(7)},$$
$$\hat{X}^{I(2)}(\sigma_2)=\hat{X}^{I(4)}(\sigma_4), \qquad \text{(Q)}$$
$$\hat{X}^{I(3)}(\sigma_3)=\hat{X}^{I(4)}(\sigma_4)+\frac{\sqrt{\pi}}{2}w^{I(7)}. \tag{10·8}$$

Next we compare the Γ factor in Eq. (10·5)

$$\Gamma(\boldsymbol{p}_6;\, n_1{}^f,\, n_2{}^f)\Gamma(\boldsymbol{p}_5;\, n_3{}^f,\, n_4{}^f)={}_1[\gamma_{\boldsymbol{p}_6}]_2\, {}_3[\gamma_{\boldsymbol{p}_5}]_4 U(\boldsymbol{p}_5)U(\boldsymbol{p}_6)$$
$$\propto {}_1[(\gamma_{\tilde{e}_1})^{n_6{}^1}(\gamma_{\tilde{e}_2})^{n_6{}^2}\cdots(\gamma_{\tilde{e}_D})^{n_6{}^D}]_2\, {}_3[(\gamma_{\tilde{e}_1})^{n_5{}^1}(\gamma_{\tilde{e}_2})^{n_5{}^2}\cdots(\gamma_{\tilde{e}_D})^{n_5{}^D}]_4$$
$$\times e^{i\boldsymbol{p}_6\cdot \boldsymbol{x}_t{}^{(2)}+i\boldsymbol{p}_5\cdot\boldsymbol{x}_t{}^{(4)}}e^{-(\pi i/2)\boldsymbol{p}_5{}^2}, \tag{10·9a}$$

$$\Gamma(\boldsymbol{p}_8;\, n_2{}^f,\, n_3{}^f)\Gamma(\boldsymbol{p}_7;\, n_1{}^f,\, n_4{}^f)={}_2[\gamma_{\boldsymbol{p}_8}]_3\, {}_1[\gamma_{\boldsymbol{p}_7}]_4 U(\boldsymbol{p}_8)U(\boldsymbol{p}_7)$$
$$\propto {}_2[(\gamma_{\tilde{e}_1})^{n_8{}^1}(\gamma_{\tilde{e}_2})^{n_8{}^2}\cdots(\gamma_{\tilde{e}_D})^{n_8{}^D}]_3\, {}_1[(\gamma_{\tilde{e}_1})^{n_7{}^1}(\gamma_{\tilde{e}_2})^{n_7{}^2}\cdots(\gamma_{\tilde{e}_D})^{n_7{}^D}]_4$$
$$\times e^{i\boldsymbol{p}_8\cdot \boldsymbol{x}_t{}^{(3)}+i\boldsymbol{p}_7\cdot\boldsymbol{x}_t{}^{(4)}}e^{-(\pi i/2)\boldsymbol{p}_7{}^2}, \tag{10·9b}$$

where we used Eqs. (8·8), (8·15) and (8·16). The second term on the RHS of (8·16) is omitted since it commutes with the internal Bosonic coordinates. The parts of Γ written in terms of $\gamma_{\tilde{e}_i}(i\leqq D)$ express δ-functions, e.g., $\sum_{\boldsymbol{v}\in\Gamma_D}\delta(\boldsymbol{x}_t{}^{(1)}+\boldsymbol{x}_t{}^{(2)}+(\pi/2)\boldsymbol{w}^{(6)}+\boldsymbol{v})$ (cf. (8·13)) and the tensor products of them in (10·9a) and (10·9b) turns out to be the same total δ-function $\sum_{\boldsymbol{v}\in\Gamma_D}\delta(\boldsymbol{x}_t{}^{(1)}+\boldsymbol{x}_t{}^{(2)}+\boldsymbol{x}_t{}^{(3)}+\boldsymbol{x}_t{}^{(4)}+\boldsymbol{v})$ after the summation over $\boldsymbol{w}$ of the intermediate strings. With (10·9a) and (10·9b), Eqs. (10·7) and (10·8) are modified to the following connection conditions:

$$\hat{X}^{I(1)}(\sigma_1)=\hat{X}^{I(4)}(\sigma_4)+\frac{\sqrt{\pi}}{2}w^{I(6)}+\frac{1}{\sqrt{\pi}}x_t{}^{I(2)}+\frac{1}{\sqrt{\pi}}x_t{}^{I(4)},$$
$$\hat{X}^{I(2)}(\sigma_2)=\hat{X}^{I(4)}(\sigma_4)+\frac{1}{\sqrt{\pi}}x_t{}^{I(2)}+\frac{1}{\sqrt{\pi}}x_t{}^{I(4)}, \qquad \text{(P)}$$
$$\hat{X}^{I(3)}(\sigma_3)=\hat{X}^{I(4)}(\sigma_4)+\frac{\sqrt{\pi}}{2}w^{I(5)}, \tag{10·10}$$

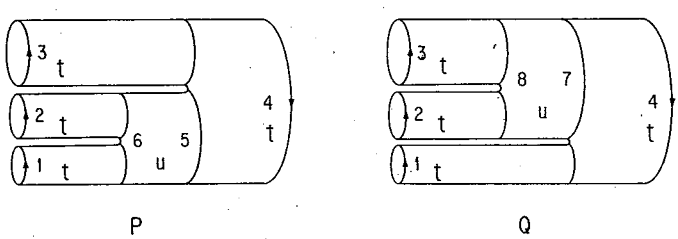


Fig. 10. The P($\theta_P=0$) and Q($\theta_Q=\pi$) configurations which should cancel with each other in the Jacobi identity (10·1). Here all the external strings are in the twisted sector.

$$\hat{X}^{I(1)}(\sigma_1)=\hat{X}^{I(4)}(\sigma_4)+\frac{\sqrt{\pi}}{2}w^{I(7)},$$

$$\hat{X}^{I(2)}(\sigma_2)=\hat{X}^{I(4)}(\sigma_4)+\frac{\sqrt{\pi}}{2}w^{I(7)}+\frac{1}{\sqrt{\pi}}x_t^{I(3)}+\frac{1}{\sqrt{\pi}}x_t^{(4)}, \qquad \text{(Q)}$$

$$\hat{X}^{I(3)}(\sigma_3)=\hat{X}^{I(4)}(\sigma_4)+\frac{1}{\sqrt{\pi}}x_t^{I(3)}+\frac{1}{\sqrt{\pi}}x_t^{I(4)}. \tag{10·11}$$

The signs of x_t^I and w^I are irrelevant since we consider the coordinates on the torus. The above connection conditions are actually identical when we rewrite the winding numbers in terms of the fixed point coordinates by using (8·13). Therefore we have proved that for case 2) the connection conditions are common on both sides of (10·5).

Although we have considered the connection conditions for 1A) and 2) with $\theta_P=0$ and $\theta_Q=\pi$, it is easily understood that the same argument applies for all other configurations as well.

In order to prove the Jacobi identity, it remains only to check the normalization of the vacuum terms in (10·5). When all strings are within the untwisted sector, the generalized Cremmer-Gervais identity[18),4)] justifies the equality of the normalization on both sides of (10·5) when $d+D=26$. Unfortunately it seems difficult to obtain similar identities for the general case with the twisted strings. Nevertheless we expect that the equality of the normalization implies $d+D=26$.

§ 11. Gauge invariant action

In the previous sections, we have shown that the ingredients of our field theory (the BRS operator $Q_B{}^u$ and $Q_B{}^t$, $*$-product and dot-product) have the following properties:

$$(Q_B{}^u)^2=(Q_B{}^t)^2=0\,, \qquad \text{(nilpotency)} \tag{11·1a}$$

$$Q_B(\Phi * \Psi)=Q_B\Phi * \Psi+(-)^{|\Phi|}\Phi * Q_B\Psi\,, \qquad \text{(distributive law)} \tag{11·1b}$$

$$(\Phi * \Psi) * \Lambda+(-)^{|\Phi|(|\Psi|+|\Lambda|)}(\Psi * \Lambda) * \Phi+(-)^{|\Lambda|(|\Phi|+|\Psi|)}(\Lambda * \Phi) * \Psi=0\,,$$
$$\text{(Jacobi identity)} \tag{11·1c}$$

$$\Phi * \Psi=(-)^{1+|\Phi||\Psi|}\Psi * \Phi\,, \qquad \text{(commutativity)} \tag{11·1d}$$

$$\Phi\cdot\Psi=(-)^{|\Phi||\Psi|}\Psi\cdot\Phi\,, \tag{11·1e}$$

$$\Phi\cdot(\Psi * \Lambda)=(-)^{|\Phi|(|\Psi|+|\Lambda|)}\Psi\cdot(\Lambda * \Phi)=(-)^{|\Lambda|(|\Phi|+|\Psi|)}\Lambda\cdot(\Phi * \Psi)\,,$$
$$\text{(cyclic symmetry)} \tag{11·1f}$$

$$Q_B\Phi\cdot\Psi=(-)^{1+|\Phi|}\Phi\cdot Q_B\Psi\,, \qquad \text{(partial derivativity)} \tag{11·1g}$$

where (11·1a)~(11·1c) hold only in $d+D=26$[14),4)] (and correct intercepts in the BRS operators; $\alpha_u(0)=1$ in $Q_B{}^u$ and $\alpha_t(0)=1-D/16$ in $Q_B{}^t$). With these identities, we can easily see that the action

$$S=\Phi_u\cdot Q_B{}^u\Phi_u+\Phi_t\cdot Q_B{}^t\Phi_t+\frac{2}{3}g\Phi_u{}^3+2g\Phi_u\Phi_t{}^2\,,$$

$$\Phi_u{}^3=\Phi_u\cdot(\Phi_u*\Phi_u)\,,\qquad \Phi_u\Phi_t{}^2=\Phi_u\cdot(\Phi_t*\Phi_t) \tag{11·2}$$

is invariant under the following two types of gauge transformations:

$$\begin{cases}\delta(\Lambda_u)\Phi_u=Q_B{}^u\Lambda_u+2g\Phi_u*\Lambda_u\,,\\ \delta(\Lambda_t)\Phi_t=\qquad\qquad 2g\Phi_t*\Lambda_u\,,\end{cases}$$

$$\begin{cases}\delta(\Lambda_u)\Phi_u=\qquad\qquad 2g\Phi_t*\Lambda_t\,,\\ \delta(\Lambda_t)\Phi_t=Q_B{}^t\Lambda_t+2g\Phi_u*\Lambda_t\,,\end{cases} \tag{11·3}$$

where parameter functionals Λ_u and Λ_t satisfy the constraints: $\mathcal{P}^{Z_2}|\Lambda_{u(t)}\rangle=|\Lambda_{u(t)}\rangle$ and $\mathcal{P}|\Lambda_{u(t)}\rangle=|\Lambda_{u(t)}\rangle$. The former is the usual gauge transformation as in the torus case[2] (with an appropriately broken gauge group). The latter, characteristic of the orbifold case, mixes the untwisted and the twisted fields.

We can also show the group structure of gauge transformations (11·3) by using the identities (11·1)

$$[\delta(\Lambda_a{}^{(1)}),\,\delta(\Lambda_b{}^{(2)})]=\delta(2g\Lambda_a{}^{(1)}*\Lambda_b{}^{(2)})\qquad (a,\,b=u\text{ or }t)\,. \tag{11·4}$$

§ 12. Discussion

We have constructed the string field theory for closed string compactified on the Z_2-orbifold. We obtain two 3-string vertices which correspond to two types of interactions. The vertex only with the untwisted strings is essentially the same one which describes the interaction of the closed strings compactified on a torus.[2] It should contain the two-cocycle factor in order to satisfy the $O(g^2)$ requirement for the gauge invariance of the action, especially the Jacobi identity. By the Neumann function method we construct the vertex which describes the interaction between two sectors and have shown the BRS invariance of the vertex. From the $O(g^2)$ requirement, however, the vertex obtained from the Neumann function method should be modified with being multiplied by the cocycle factor. This factor is then naturally understood in relation to the characteristic feature of the interaction; the twisted string changes the location of its center of mass corresponding to the winding number of the untwisted string. In the context of the operator formalism, the necessity of the cocycle has been also observed.[19]

The gauge-fixed and BRS invariant action $\hat{S}$ is obtained in a similar manner to the ordinary closed string case; in the action (11·2) we only retain the ϕ component of Φ_u and Φ_t with discarding the restriction on the internal ghost number of ϕ,

$$\hat{S}=\left[\Phi_u\cdot Q_B{}^u\Phi_u+\Phi_t\cdot Q_B{}^t\Phi_t+\frac{2}{3}g\Phi_u{}^3+2g\Phi_u\Phi_t{}^2\right]_{\psi=\chi=\eta=0}. \tag{12·1}$$

The BRS transformation $\hat{\delta}_B\phi_u$ and $\hat{\delta}_B\phi_t$ is also obtained as follows:

$$\hat{\delta}_B\phi_a=\int d\bar{c}_0\delta_B\Phi_a|_{\psi=\chi=\eta=0}\,,$$

$$\delta_B\Phi_u=Q_B{}^u\Phi_u+g(\Phi_u*\Phi_u+\Phi_t*\Phi_t)\,,$$

$$\delta_B\Phi_t=Q_B{}^t\Phi_t+2g\Phi_u*\Phi_t\,. \tag{12·2}$$

The α-independence of the on-shell physical amplitudes can be also discussed to the same level as in the ordinary closed string.[16] Therefore we may obtain, from the string vertex V_t, an operator formalism vertex which describes the emission of the string states in the untwisted or the twisted sector. If we take the $\alpha\to0$ limit for the untwisted string, we obtain an untwisted emission vertex,

$$V(\sigma;p_+,p_-)=:e^{2\sqrt{\pi}i(p_{+1}{}^I\hat{X}_+{}^I(\sigma)+p_{-1}{}^I\hat{X}_-{}^I(\sigma))}:e^{i\hat{p}_1\cdot\hat{x}_t}\sum_{v\in\Gamma_D}\delta(\boldsymbol{x}_t{}^{(2)}+\boldsymbol{x}_t{}^{(3)}+\frac{\pi}{2}\boldsymbol{w}^{(1)}+\boldsymbol{v}) \tag{12·3}$$

(see Eqs. (8·13), (8·15) and (8·16)). This is the vertex constructed by Hamidi and Vafa[8] multiplied by a *non-trivial cocycle factor*.[19] As described in § 8, we naturally obtain a concrete expression for the non-trivial cocycle factor from the requirement of gauge invariance. The twisted string emission vertex for the model with a fixed point is constructed by Kazama and Suzuki.[20] The relation of this vertex to the path integral is also discussed.[21] By taking an appropriate limit in which one of the α parameters of the twisted string goes to zero, we have found that, as for the non-zero oscillator part, our vertex (6·1) reduces to the one given by Kazama and Suzuki. The difference between two vertices is the cocycle factor and zero mode bilinear part in the exponent,

$$\Gamma(\boldsymbol{p}_1;n_2{}^f,n_3{}^f)\times2^{-(1/2)[(p_1{}^I)^2+(w_1{}^{I(1)})^2]}\,. \tag{12·4}$$

The former appears since we have considered the model with several fixed points. The latter is the damping factor which was obtained by an argument on duality.[8]

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Appendix A
—— *Derivation of Eqs. (2·10) and (2·11)* ——

Following Narain et al.,[9] we consider the first quantization of a string described by the coordinates $\hat{X}_I(\tau,\sigma)$ $(I=1,2,\cdots,D)$ in the constant symmetric and anti-symmetric background fields G_{IJ} and B_{IJ}. The action is

$$I=\frac{1}{2}\int d\tau\int_{-\pi}^{\pi}d\sigma[G_{IJ}\eta_{\alpha\beta}\partial_\alpha\hat{X}_I\partial_\beta\hat{X}_J+B_{IJ}\epsilon_{\alpha\beta}\partial_\alpha\hat{X}_I\partial_\beta\hat{X}_J]\,. \tag{A·1}$$

The $\hat{X}_I(\tau,\sigma)$ parametrizes the string in reference to the oblique coordinate system, in which the metric is

$$G_{IJ}=\boldsymbol{E}_I\cdot\boldsymbol{E}_J\,. \tag{A·2}$$

The $X_u^I(\sigma)$ is related to the $\hat{X}_I(\tau,\sigma)$ as

$$X_u^I(\sigma)=\sum_{J=1}^{D}\hat{X}_J(\tau=0,\sigma)(\boldsymbol{E}_J)^I\,. \tag{A·3}$$

In (A·1) $\eta_{\alpha\beta}=\mathrm{diag}(1,-1)$ and $\epsilon_{\alpha\beta}$ is an antisymmetric tensor with $\epsilon_{01}=1$. The constant background fields G_{IJ} and B_{IJ} only affect the quantization of zero modes. We write an ansatz for zero-modes

$$\hat{X}_I(\tau,\sigma)=\frac{1}{\sqrt{\pi}}\left[\hat{x}_I(\tau)-\frac{1}{2}n_I\sigma\right]\equiv\frac{1}{\sqrt{\pi}}\left[\hat{x}_I(\tau)-\frac{1}{2}\hat{w}_I\sigma\right],$$

$$\hat{x}_I(\tau=0)=\hat{x}_I=\sum_{J=1}^{D}(\tilde{\boldsymbol{E}}_I)_J x^J\,,$$

$$\hat{\boldsymbol{w}}=\boldsymbol{n}=(n_1,n_2,\cdots,n_D)\,, \tag{A·4}$$

where the n_I is related to the winding number $\boldsymbol{w}$ by (2·6). Inserting (A·4) into (A·1), we obtain

$$I=\int d\tau[\dot{\hat{\boldsymbol{x}}}G\dot{\hat{\boldsymbol{x}}}-\frac{1}{4}\boldsymbol{n}G\boldsymbol{n}-\dot{\hat{\boldsymbol{x}}}B\boldsymbol{n}]\,, \tag{A·5}$$

where $\hat{\boldsymbol{x}}=(\hat{x}_1,\hat{x}_2,\cdots,\hat{x}_D)$ and $\boldsymbol{n}=(n_1,n_2,\cdots,n_D)$ are D-dimensional vectors; $G\equiv(G_{IJ})$ and $B\equiv(B_{IJ})$ are $D\times D$ matrices. The conjugate momentum

$$\hat{\boldsymbol{p}}=-i\frac{\partial}{\partial\hat{\boldsymbol{x}}}=2G\dot{\hat{\boldsymbol{x}}}-B\boldsymbol{n} \tag{A·6}$$

is a set of even integers

$$\hat{\boldsymbol{p}}=2\boldsymbol{m}\,,\qquad \boldsymbol{m}=(m_1,m_2,\cdots,m_D)\,, \tag{A·7}$$

since it should generate a translation on the lattice Γ_D

$$\hat{\boldsymbol{x}}\to\hat{\boldsymbol{x}}+\pi\boldsymbol{k}\,,\qquad \boldsymbol{k}=(k_1,k_2,\cdots,k_D)\,. \tag{A·8}$$

From Eqs. (A·4), (A·6) and (A·7) we have

$$\begin{aligned}\hat{X}_I(\tau,\sigma)&=\frac{1}{\sqrt{\pi}}\left[\hat{x}_I+\dot{\hat{x}}_I\tau-\frac{1}{2}n_I\sigma\right]\\&=\frac{1}{\sqrt{\pi}}\left[\hat{x}_I+\frac{1}{2}\{L_{I-}(\tau+\sigma)+L_{I+}(\tau-\sigma)\}\right],\end{aligned} \tag{A·9}$$

$$\boldsymbol{L}_+=\frac{1}{2}\boldsymbol{n}+\dot{\hat{\boldsymbol{x}}}=\frac{1}{2}\boldsymbol{n}+G^{-1}\left(\boldsymbol{m}+\frac{1}{2}B\boldsymbol{n}\right),$$

$$\boldsymbol{L}_-=-\frac{1}{2}\boldsymbol{n}+\dot{\hat{\boldsymbol{x}}}=-\frac{1}{2}\boldsymbol{n}+G^{-1}\left(\boldsymbol{m}+\frac{1}{2}B\boldsymbol{n}\right). \tag{A·10}$$

Going back to the orthogonal coordinate $X_u^I(\sigma)$, we have a relation (see Eqs. (2·8) and (A·3))

$$X^I_{u\pm}(\sigma)=\frac{1}{\sqrt{\pi}}\left[x_\pm{}^I\mp\frac{1}{2}p_\pm{}^I\sigma+(\text{non-zero modes})\right]$$

$$=\frac{1}{\sqrt{\pi}}\sum_{J=1}^{D}(\boldsymbol{E}_J)^I\left[\hat{x}_{J\pm}\mp\frac{1}{2}L_{J\pm}\sigma+(\text{non-zero modes})\right]. \qquad (\text{A}\cdot 11)$$

Therefore

$$\boldsymbol{p}=(p_+{}^I,\ -p_-{}^I)=(\sum_{J=1}^{D}L_{J+}(\boldsymbol{E}_J)^I,\ -\sum_{J=1}^{D}L_{J-}(\boldsymbol{E}_J)^I). \qquad (\text{A}\cdot 12)$$

From Eqs. (A·10) and (A·12), an (Lorentzian) inner product of arbitrary momenta $\boldsymbol{p}$ and $\boldsymbol{p}'$ is

$$\boldsymbol{p}\cdot\boldsymbol{p}'=\boldsymbol{L}_+G\boldsymbol{L}_+'-\boldsymbol{L}_-G\boldsymbol{L}_-'=\sum_{I=1}^{D}(n_Im_I'+n_I'm_I),$$

which implies that $\boldsymbol{p}$ is on the Lorentzian even lattice $\tilde{\Gamma}_{D,D}$ and can be written in terms of the basis vectors $\tilde{\boldsymbol{e}}_i=(\tilde{e}^I_{i+},\ \tilde{e}^I_{i-})$ $(i=1,2,\cdots,2D)$ of $\tilde{\Gamma}_{D,D}$

$$\boldsymbol{p}=\sum_{i=1}^{2D}p_i\tilde{\boldsymbol{e}}_i. \qquad (p_i\text{: integer}) \qquad (\text{A}\cdot 13)$$

From (A·12) and (A·13), we have

$$p_i=\sum_{I,J=1}^{D}[L_{J+}(\boldsymbol{E}_J)^Ie^I_{i+}+L_{J-}(\boldsymbol{E}_J)^Ie^I_{i-}]. \qquad (\text{A}\cdot 14)$$

Since the p_i $(i=1,2,\cdots 2D)$ are integers, the RHS should give n^I or m^I which are integers included in $\boldsymbol{L}_\pm$ through (A·10). From this observation, we have a relation between $\{\boldsymbol{E}_I\}$ and $\{\boldsymbol{e}_i\}$[11]

$$\boldsymbol{e}_i=\begin{cases}(\tilde{\boldsymbol{E}}_I,\ -\tilde{\boldsymbol{E}}_I) & (\text{for}\quad i=I\leq D)\\[2ex] \left(\frac{1}{2}(G-B)_{IJ}\tilde{\boldsymbol{E}}_J,\ \frac{1}{2}(G+B)_{IJ}\tilde{\boldsymbol{E}}_J\right) & (\text{for}\quad i=I+D>D)\end{cases} \qquad (\text{A}\cdot 15)$$

and a relation of $\{p_i\}$ to n_I and m_I

$$p_i=\begin{cases}n_I & (\text{for}\quad i\leq D),\\ m_I & (\text{for}\quad i>D).\end{cases} \qquad (\text{A}\cdot 16)$$

Using (A·7), (A·10), (A·12) and (2·6), we have

$$\hat{p}_I=2m_I=\sum_{J=1}^{D}(\boldsymbol{E}_I)^Jp^J-\sum_{J,K=1}^{D}B_{IJ}(\tilde{\boldsymbol{E}}_J)^Kw^K. \qquad (\text{A}\cdot 17)$$

From (A·15) we can see that the lattice $\tilde{\Gamma}_{D,D}$ is even and self-dual:

$$g_{ij}=\boldsymbol{e}_i\cdot\boldsymbol{e}_j=\begin{pmatrix}\boldsymbol{0}_D & \boldsymbol{1}_D\\ \boldsymbol{1}_D & \boldsymbol{0}_D\end{pmatrix},$$

$$\det g_{ij}=1\,, \tag{A·18}$$

where $\mathbf{0}_D$ and $\mathbf{1}_D$ are $D\times D$ null and unit matrix respectively. Equation (A·18) implies that

$$\tilde{\boldsymbol{e}}_i=\begin{cases}\boldsymbol{e}_{i+D} & (\text{for} \quad i\le D)\,,\\ \boldsymbol{e}_{i-D} & (\text{for} \quad i>D)\,.\end{cases} \tag{A·19}$$

Equation (A·15) and (A·19) lead to Eq. (2·11).

Appendix B
—— Green Function on ρ-Plane ——

In this appendix, we discuss a Green function on the 3-string diagram in Fig. 3 which corresponds to the interaction on the Z_2-orbifold with two twisted strings.

A general Mandelstam mapping which transform a 3 closed string diagram (ρ-plane) onto the whole complex z-plane is

$$\rho(z)=\sum_{r=1}^{3}\alpha_r\ln(z-Z_r)\,. \tag{B·1}$$

Here α_r are string length parameter satisfying $\sum_{r=1}^{3}\alpha_r=0$ and Z_r are Koba-Nielsen variables onto which the external r-th strings are transformed. In general the KN variables are complex valued and different choices of them are related to one another by the projective transformation on the z-plane

$$z\to\frac{Az+B}{Cz+D}\,. \qquad (AD-BC=1) \tag{B·2}$$

We need a Green function which satisfies (5·5) with the boundary condition (5·6). We construct it by solving the Laplace equation on the z-plane with a proper boundary condition and transform it into a Green function on the ρ-plane by (B·1). The Mandelstam mapping (B·1) gives an interval (Z_3, Z_2) on the z-plane corresponding to the bold line on the ρ-plane in Fig. 3. As a boundary condition on the z-plane, we require the same condition as (5·6) on both sides of the line. Therefore the Green function on the z-plane is

$$T(z,\,\tilde{z})=\ln\left|\left(\frac{z-Z_2}{z-Z_3}\right)^{1/2}-\left(\frac{\tilde{z}-Z_2}{\tilde{z}-Z_3}\right)^{1/2}\right|-\ln\left|\left(\frac{z-Z_2}{z-Z_3}\right)^{1/2}+\left(\frac{\tilde{z}-Z_2}{\tilde{z}-Z_3}\right)^{1/2}\right|\,. \tag{B·3}$$

The Green function on the ρ-plane is simply

$$T(\rho,\,\tilde{\rho})=T(z(\rho),\,\tilde{z}(\tilde{\rho}))\equiv\frac{1}{2}\{\mathcal{T}(\rho,\,\tilde{\rho})+\mathcal{T}^*(\rho,\,\tilde{\rho})\}\,, \tag{B·4}$$

where

$$\mathcal{T}(\rho,\,\tilde{\rho})\equiv\ln\left[\left(\frac{z-Z_2}{z-Z_3}\right)^{1/2}-\left(\frac{\tilde{z}-Z_2}{\tilde{z}-Z_3}\right)^{1/2}\right]-\ln\left[\left(\frac{z-Z_2}{z-Z_3}\right)^{1/2}+\left(\frac{\tilde{z}-Z_2}{\tilde{z}-Z_3}\right)^{1/2}\right]$$

$$=-\delta_{rs}\delta_{r1}\left[\theta(\xi_1-\tilde{\xi}_1)\left(\sum_{n_1\ge1}\frac{1}{n_1}e^{n_1(\tilde{\xi}_1-\zeta_1)}-\zeta_1\right)\right.$$

$$+\theta(\tilde{\xi}_1-\xi_1)\Big(\sum_{n_1\geq 1}\frac{1}{n_1}e^{n_1(\zeta_1-\tilde{\zeta}_1)}-\tilde{\zeta}_1+i\pi\Big)\Big]$$

$$-\delta_{rs}\delta_{r2}\Big[\theta(\xi_2-\tilde{\xi}_2)\sum_{n_2>0}\frac{1}{n_2}e^{n_2(\tilde{\zeta}_2-\zeta_2)}+\theta(\tilde{\xi}_2-\xi_2)\Big(\sum_{n_2>0}\frac{1}{n_2}e^{n_2(\zeta_2-\tilde{\zeta}_2)}+i\pi\Big)\Big]$$

$$-\delta_{rs}\delta_{r3}\Big[\theta(\xi_3-\tilde{\xi}_3)\Big(\sum_{n_3>0}\frac{1}{n_3}e^{n_3(\tilde{\zeta}_3-\zeta_3)}+i\pi\Big)+\theta(\tilde{\xi}_3-\xi_3)\sum_{n_3>0}\frac{1}{n_3}e^{n_3(\zeta_3-\tilde{\zeta}_3)}\Big]$$

$$+\sum_{n_r,m_s\geq 0}T^{rs}_{n_r m_s}e^{n_r\zeta_r+m_s\tilde{\zeta}_s}-i\pi(\delta_{r1}\delta_{s3}+\delta_{r2}\delta_{s1}+\delta_{r2}\delta_{s3})\,. \qquad (\mathrm{B}\cdot 5)$$

In the above, we have expanded $\mathcal{T}(\rho, \tilde{\rho})$, defined by the 1st line, in terms of ζ defined in (5·2). The last equality gives definition of the Fourier coefficients $T^{rs}_{n_r m_s}$. Here we understand that ρ and $\tilde{\rho}$ belong to the r-th and s-th string regions, and n_t is integer (half-integer) for $t=1$ ($t=2, 3$) corresponding to the untwisted (twisted) string. The last term in the expansion (B·5) is necessary for the consistency when $z\to Z_r$ and $\tilde{z}\to Z_s$. It is easy to see that the function $\mathcal{T}(\rho, \tilde{\rho})$ is invariant under the projective transformation (B·2) and ζ_r is also invariant under (B·2) by its definition. Therefore the Fourier coefficients $T^{rs}_{n_r m_s}$ defined in (B·5) are projective invariant quantities. Using this invariance, we may fix (Z_1, Z_2, Z_3) to $(1, 0, \infty)$ and then the Green function is reduced to

$$T(\rho, \tilde{\rho})=\ln|\sqrt{z}-\sqrt{\tilde{z}}|-\ln|\sqrt{z}+\sqrt{\tilde{z}}|\,, \qquad (\mathrm{B}\cdot 6)$$

where ρ and z are related via the Mandelstam mapping (5·1).

The integral representation of $T^{rs}_{n_r m_s}$ can be derived in the same manner as that in Ref. 3).

$$T^{11}_{00}=-2\ln 2+\frac{\tau_0}{\alpha_1}\,, \qquad \tau_0=\sum_{r=1}^{3}\alpha_r\ln|\alpha_r|\,, \qquad (\mathrm{B}\cdot 7)$$

$$T^{r1}_{n_r 0}=T^{1r}_{0 n_r}=\frac{1}{n_r}\oint_{Z_r}\frac{dz}{2\pi i}\frac{1}{\sqrt{z}(z-1)}e^{-n_r\zeta_r(z)}\,, \qquad (\mathrm{B}\cdot 8)$$

$$T^{rs}_{n_r m_s}=\frac{1}{2n_r m_s}\oint_{Z_r}\frac{dz}{2\pi i}\oint_{Z_s}\frac{d\tilde{z}}{2\pi i}\frac{z+\tilde{z}}{(z\tilde{z})^{1/2}}\frac{1}{(z-\tilde{z})^2}e^{-n_r\zeta_r(z)-m_s\zeta_s(\tilde{z})}\,. \qquad (\mathrm{B}\cdot 9)$$

In (B·7)~(B·9), we have fixed $(Z_1, Z_2, Z_3)=(1, 0, \infty)$ and use the Mandelstam mapping (5·1). We see that $T^{rs}_{n_r m_s}$ is symmetric under a simultaneous exchange of (r, s) and (n_r, m_s).

Equation (B·7) is derived from Eq. (B·5) by first letting $\tilde{z}\to Z_1=1$ (which implies $\tilde{\xi}_r\to-\infty$) and then taking the limit $z\to Z_1=1$ ($\xi_r\to-\infty$). It is necessary to substitute the following expression for $\zeta_{r=1}$ obtained from the definition (5·2):

$$\zeta_1=\frac{1}{\alpha_1}[\alpha_1\ln(z-1)+\alpha_2\ln z-\tau_0]\,. \qquad (\mathrm{B}\cdot 10)$$

In order to derive Eq. (B·8) we differentiate (B·5) with respect to ζ_r keeping $\xi_r>\tilde{\xi}_s$,

$$\frac{\partial}{\partial \zeta_r}\mathcal{I}(\rho, \tilde{\rho})=\frac{\partial z}{\partial \zeta_r}\left(\frac{\tilde{z}}{z}\right)^{1/2}\frac{1}{z-\tilde{z}}$$

$$=\delta_{rs}\delta_{r1}\left(\sum_{n_1\geq 1} e^{n_1(\tilde{\zeta}_1-\zeta_1)}+1\right)+\delta_{rs}(\delta_{r2}+\delta_{r3})\sum_{n_r>0} e^{n_r(\tilde{\zeta}_r-\zeta_r)}$$

$$+\sum_{n_r, m_s\geq 0} n_r T^{rs}_{n_r m_s} e^{n_r\zeta_r+m_s\tilde{\zeta}_s}. \tag{B·11}$$

In the limit that $\tilde{z}\to Z_1=1$, we have

$$\frac{\partial z}{\partial \zeta_r}\frac{1}{\sqrt{z}(z-1)}=\delta_{rs}\delta_{r1}+\sum_{n_r\geq 0} n_r T^{r1}_{n_r 0} e^{n_r\zeta_r}$$

$$=\delta_{rs}\delta_{r1}+\sum_{n_r\geq 0} n_r T^{r1}_{n_r 0}\omega_r^{n_r}, \tag{B·12}$$

where

$$\omega_r=e^{\zeta_r}. \tag{B·13}$$

The 1st equality of Eq. (B·8) is obtained from (B·12) by multiplying $\omega_r^{-n_r-1}$ on both sides and integrating it around a point $\omega_r=0$. The integration representation for $T^{1r}_{0n_r}$ is obtained in a similar manner to the above derivation and found to be expressed in the same integration as that for $T^{r1}_{n_r 0}$.

For the derivation of (B·9), we differentiate (B·11) with respect to $\tilde{\zeta}_s$,

$$\frac{1}{2}\frac{\partial z}{\partial \zeta_r}\frac{\partial \tilde{z}}{\partial \tilde{\zeta}_s}\frac{z+\tilde{z}}{(z-\tilde{z})^2}\left(\frac{1}{z\tilde{z}}\right)^{1/2}$$

$$=\delta_{rs}\sum_{n_r\geq 0} n_r e^{n_r(\tilde{\zeta}_r-\zeta_r)}+\sum_{n_r, m_s\geq 0} n_r m_s T^{rs}_{n_r m_s} e^{n_r\zeta_r+m_s\tilde{\zeta}_s}. \tag{B·14}$$

When we multiply $\omega_r^{-n_r-1}\times\tilde{\omega}_s^{-m_s-1}$ and integrate it around the origins, we obtain (B·9).

We will show that the coefficients $T^{rs}_{n_r m_s}$ are real quantities. From (5·2)

$$\zeta_1=\ln(z-1)+\frac{1}{\alpha_1}[\alpha_2\ln z-\tau_0],$$

$$\zeta_2=\ln z+\frac{1}{\alpha_2}[\alpha_1\ln(1-z)-\tau_0],$$

$$\zeta_3=\frac{1}{\alpha_3}[\alpha_1\ln(z-1)+\alpha_2\ln z-\tau_0]$$

$$=\ln w+\frac{1}{\alpha_3}[\alpha_1\ln(1-w)-\tau_0] \tag{B·15}$$

with

$$w=\frac{1}{z}. \tag{B·16}$$

The root cut in (B·8) and (B·9) disappears when we substitute (B·15) into these equations. It is rather easy to see from the substituted equations that the coefficients are actually real. Note that this reality is due to our convention of $\tilde{\sigma}_3$ in (5·2).

Appendix C
— Singularity of $A_u(\sigma)$ or $A_t(\sigma)$ —

We evaluate the singularity of the internal Bosonic coordinates when they act on the vertex. Let us write the internal coordinate as

$$A_{\pm}{}^{I(r)}(\sigma_r)=\frac{1}{\sqrt{\pi}}\sum_{n_r}\alpha_{n_r}^{I(\pm)(r)}e^{\pm in_r\sigma_r}, \tag{C·1}$$

where n_r takes integer (half-integer) values if the r-th string is in the untwisted (twisted) sector. An operator valued function on the ρ-plane in (7·5a) is

$$A_{\pm}{}^{I}(\rho)=\frac{1}{\alpha_r}A_{\pm}{}^{I(r)}(\mp i\zeta_r)=\frac{1}{\alpha_r\sqrt{\pi}}\sum_{n_r}\alpha_{n_r}^{I(\pm)(r)}e^{n_r\zeta_r}, \tag{C·2}$$

when ρ is on the r-th plane. We let this operator act on $|V_{t0}^{\text{int}}(1,2,3)\rangle$ in (5·10) and write the result solely in terms of creation operator

$$\alpha_r\sqrt{\pi}A_{\pm}{}^{I}(\rho)=\sum_{m_s\geq 0}(\delta_{rs}e^{-m_s\zeta_r}+\sum_{n_r\geq 0}n_rT_{n_rm_s}^{rs}e^{n_r\zeta_r})\alpha_{-m_s}^{I(\pm)(s)}. \tag{C·3}$$

The coefficients on the RHS are singular around the interaction point as is evident from the expression

$$\begin{aligned}&\delta_{rs}e^{-m_s\zeta_r}+\sum_{n_r\geq 0}n_rT_{n_rm_s}^{rs}e^{n_r\zeta_r}\\&=\frac{\alpha_r}{\alpha_s}\left(\frac{d\rho}{dz}\right)^{-1}\frac{1}{\sqrt{z}}\oint_{z_s}\frac{d\tilde{z}}{2\pi i}\left(\frac{d\tilde{\rho}}{d\tilde{z}}\right)\frac{\sqrt{\tilde{z}}}{z-\tilde{z}}e^{-m_s\tilde{\zeta}_s}.\end{aligned} \tag{C·4}$$

This singularity is remedied in the expression for $A_{\pm}(z)$ due to the factor $d\rho/dz$. In order to derive (C·4), we multiply $\tilde{\omega}_s{}^{-m_s-1}(\tilde{\omega}_s\equiv e^{\zeta_s})$ to (B·11) and integrate it around the origin of $\tilde{\omega}_s$.

Appendix D
— Connection Conditions (5·13) and (5·14) —

The internal coordinates of a twisted and an untwisted string are expanded as (2·8) and (3·4). We let them operate on $e^{E_t(1,2,3)}|0\rangle$ defined in (5·10) and rewrite them in terms of creation operators; for an untwisted string (string 1)

$$\begin{aligned}\sqrt{\pi}X_{u\pm}^{I(1)}(\hat{\sigma}_1)=-\frac{i}{2}\sum_{\substack{n_s\geq 0\\ s}}\alpha_{-n_s}^{I(\pm)(s)}\Big\{&\delta_{s1}\left[\mp i\hat{\sigma}_1\delta_{n_10}+(1-\delta_{n_10})\frac{1}{n_1}e^{\mp in_1\hat{\sigma}_1}\right]\\&-\sum_{m\geq 0}T_{mn_s}^{1s}e^{\pm im\hat{\sigma}_1}\Big\}\end{aligned} \tag{D·1}$$

for twisted strings (strings 2 or 3)

$$\sqrt{\pi}X_t^{I(r)}(\hat{\sigma}_r)=x_t^{I(r)}-\frac{i}{2}\sum_{\substack{n_s\geq 0\\ s,\pm}}\alpha_{-n_s}^{I(\pm)(s)}\left[\delta^{rs}\frac{1}{n_r}e^{\mp in_r\hat{\sigma}_r}-\sum_{m_r>0}T_{m_rn_s}^{rs}e^{\pm im_r\hat{\sigma}_r}\right]. \tag{D·2}$$

For definiteness, we will consider the case of α_1, $\alpha_2>0$ and $\alpha_3<0$ in which we have an identity

$$\mathcal{I}(\rho_3,\,\tilde{\rho})|_{\xi_3=0}=\mathcal{I}(\rho_r,\,\tilde{\rho})|_{\xi_r=0}\qquad (r=1\text{ or }2) \tag{D·3}$$

with $\rho_3=\rho_r$ on the ρ-plane. From Eq. (B·5) we have

$$\begin{aligned}
&\mathcal{I}(\rho,\,\tilde{\rho})|_{\xi_r=0}\\
&=-\delta_{rs}\delta_{r1}\left(\sum_{n_1\geq 1}\frac{1}{n_1}e^{n_1(\tilde{\zeta}_1-i\hat{\sigma}_1)}-i\hat{\sigma}_1\right)\\
&\quad-\delta_{rs}\delta_{r2}\sum_{n_2>0}\frac{1}{n_2}e^{n_2(\tilde{\zeta}_2-i\hat{\sigma}_2)}-\delta_{rs}\delta_{r3}\left(\sum_{n_3>0}\frac{1}{n_3}e^{n_3(\tilde{\zeta}_3-i\hat{\sigma}_3)}+i\pi\right)\\
&\quad+\sum_{n_r,m_s\geq 0}T_{n_rm_s}^{rs}e^{in_r\hat{\sigma}_r+m_s\tilde{\zeta}_s}-i\pi(\delta_{r1}\delta_{s3}+\delta_{r2}\delta_{s1}+\delta_{r2}\delta_{s3})\\
&=-\sum_{n_s\geq 0}e^{n_s\tilde{\zeta}_s}\left\{\delta_{rs}\left[-i\hat{\sigma}_r\delta_{r1}\delta_{n_s0}+(1-\delta_{n_r0})\frac{1}{n_r}e^{-in_r\hat{\sigma}_r}\right]\right.\\
&\qquad\qquad\left.-\sum_{m_r\geq 0}T_{m_rn_s}^{rs}e^{im_r\hat{\sigma}_r}\right\}-i\pi(\delta_{s3}+\delta_{r2}\delta_{s1})\,.
\end{aligned} \tag{D·4}$$

From the above equation, it is easy to derive the connection conditions (5·13) for the internal coordinates

$$\begin{cases}
X_u^{I(1)}(\hat{\sigma}_1)=\hat{X}_t^{I(3)}(\hat{\sigma}_3)\,,\\
\hat{X}_t^{I(2)}(\hat{\sigma}_2)=\hat{X}_t^{I(3)}(\hat{\sigma}_3)-\dfrac{\sqrt{\pi}}{2}w^{I(1)}\,,
\end{cases} \tag{D·5}$$

where $\hat{X}_t^I(\hat{\sigma})$ is the internal coordinate with the zero mode x_t omitted.

In the same way we can prove (5·14). On the vertex we have

$$\sqrt{\pi}A_\pm{}^{I(r)}(\hat{\sigma}_r)=\sum_{\substack{m_s\geq 0\\ s}}\alpha_{-m_s}^{I(\pm)(s)}\Big(\delta_{rs}e^{\mp im_s\hat{\sigma}_r}+\sum_{n_r>0}n_rT_{n_rm_s}^{rs}e^{\pm in_r\hat{\sigma}_r}\Big)\,, \tag{D·6}$$

where $A_\pm{}^{I(r)}(\hat{\sigma}_r)$ corresponds to either the untwisted string ($r=1$) or the twisted string ($r=2$ or 3). This should be compared to the equality

$$\frac{1}{\alpha_3}\frac{\partial}{\partial\zeta_3}\mathcal{I}(\rho_3,\,\tilde{\rho})|_{\xi_3=0}=\frac{1}{\alpha_r}\frac{\partial}{\partial\zeta_r}\mathcal{I}(\rho_r,\,\tilde{\rho})|_{\xi_r=0}\,.\quad (r=1\text{ or }2) \tag{D·7}$$

From the expression

$$\frac{\partial}{\partial\zeta_r}\mathcal{I}(\rho_r,\,\tilde{\rho})|_{\xi_r=0}=\sum_{n_s\geq 0}e^{n_s\tilde{\zeta}_s}\Big(\delta_{rs}e^{-in_r\hat{\sigma}_r}+\sum_{m_r\geq 0}m_rT_{m_rn_s}^{rs}e^{im_r\hat{\sigma}_r}\Big) \tag{D·8}$$

one can easily derive Eq. (5·14).

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