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Microscopic Description of Nuclear Collective Rotation by Means of the Self-Consistent Collective Coordinate Method

—— Occurrence Mechanism of Collective Rotation ——

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On the basis of the concept of spontaneous breakdown of the rotation symmetry in the deformed Hartree-Fock-Bogoliubov minimum as the origin of the nuclear collective rotation, the self-consistent collective coordinate (SCC) method is applied to disclosing occurrence mechanism of the collective rotation. Through the intermediary of the SCC method, a manifest relationship between Marshalek and Weneser's full quantum theory of rotational motion and the conventional cranking model approach is given.

In order to reveal how the so-called rotation-vibration coupling effects are coherently organized so as to construct a global optimum subspace of collective rotation, a set of basic equations of the SCC method is solved for the low-spin ground-state rotational bands of Er isotopes. Systematic features of microscopic structure of rotation-vibration couplings are investigated in detail.

§ 1. Introduction

In accordance with rapidly expanding experimental observations on nuclear collective phenomena, in the past decades it has become an inevitable theoretical subject to develop a *microscopic theory of nuclear collective dynamics* which enables us to describe global aspects of collectivity, e. g., occurrence, persistency, transfiguration, dissipation and termination of collective modes of motion.

Since the nucleus is an isolated finite many-body quantum system in which the self-consistent mean field is realized, the collective modes of motion (associated with time evolution of the mean field) are of large amplitude and highly involved with non-collective (intrinsic) modes of motion in a strong self-consistent way. Thus, the first task toward the microscopic theory of nuclear collective dynamics is to define an optimum "global" collective subspace and "global" collective variables specifying the subspace. The self-consistent collective coordinate (SCC) method[1),2)] has been proposed for this purpose. When once the optimum collective subspace is properly determined by the SCC method, the intrinsic modes of motion can be precisely defined in a compatible way with the SCC method.[3)] Thus, the whole nuclear dynamics is *optimally* described in terms of the collective and intrinsic modes of motion.[4)] The transfiguration and dissipation of the collective modes are then described *as results* due to couplings with the intrinsic modes.[4),5)] The above scenario for the nuclear collective dynamics has been so far formulated, step by step (as INS-TSUKUBA joint research project on a large-amplitude collective motion), by examining its applicability in each step with the employment of simple numerically-solvable models.

This is the first one of a series of papers with the purpose of applying the new microscopic theory in describing various phenomena associated with the nuclear

collective rotation. Since the nuclear collective rotation is one of the most typical modes of *large-amplitude* collective motion, various methods to describe the rotational motion have been so far developed successfully,[6),7)] and experimental investigations on high-spin states are now in rapid developments. For these reasons, it seems desirable to describe the collective rotational phenomena by means of the new microscopic theory and compare its results with those of the conventional methods: This may provide us with another examination of the usefulness and applicability of the theory and may inform us of important clues to further development of it.

The focus in this first paper is on the problem of *occurrence of the collective rotation* as a typical large-amplitude collective mode. Let us suppose that a rotational-invariant nuclear Hamiltonian with an effective inter-nucleon interaction for an even-even nucleus is given, in which rotational spectra appear very regularly. In this case, one usually starts by supposing that the nucleus has a well-defined intrinsic *deformed* shape, which may be obtained by a stationary solution of the Hartree-Fock (-Bogoliubov) approximation. The concept of the intrinsic deformation leads us to a localization of orientation of the system, which inevitably needs the concept of *spontaneous breakdown* of the rotation symmetry. Thus, the very occurrence of collective rotational degrees of freedom may be said to originate in restoring the broken rotation symmetry.

The earliest method[8)] for restoring the broken rotation symmetry was proposed by the random-phase approximation (RPA), which describes *small oscillations* about the deformed Hartree-Fock (-Bogoliubov) stationary solution by definition. In this method, a zero-energy mode appears for each broken symmetry in analogy with the Goldstone theorem. Thus, the rotations are regarded as the Goldstone bosons associated with the broken rotation symmetry. Up to this stage, however, we encounter the following related difficulty: The simultaneous eigenfunctions of the Hamiltonian and the angular momentum operator within the RPA turn out to be unnormalizable and they form a continuum instead of the discrete set appropriate to angular momentum eigenfunctions.[9)] The difficulty arises, of course, from the fact that the RPA is *valid* only in a small neighborhood of the Hartree-Fock (-Bogoliubov) minimum. It is, therefore, crucial to go one step further to understand *mechanism of transmutation* of the "vibrational" Goldstone modes into the "rotational" modes. In the present case, the transmutation has to be induced with mixing of the residual interaction which has been neglected under the RPA (and generally involves the Goldstone bosons in a complicated way), by properly taking account of the rotation symmetry of the total Hamiltonian.

The first successful method to understand the mechanism of transmutation of the "vibrational" Goldstone mode into the "rotational" mode has been proposed by Marshalek and Weneser.[9)~11)] In the Marshalek-Weneser (MW) method the generalized Holstein-Primakoff boson mapping[12),13)] of fermion pairs (i.e., particle-hole pairs or quasiparticle pairs) is employed, and the rotational invariant Hamiltonian (of the even-even nucleus) is expressed in terms of the bosons (i.e., canonically conjugate coordinates and momenta) based on the deformed equilibrium shape. In this mapping the fermion-pairs can be expanded by the boson operators in such a way that the correct commutation relations for the fermion pairs are satisfied in each order of the

expansion. The expansion is characterized by a perturbation parameter $\Omega^{-1/2}$ where $\Omega \sim \langle 2j+1 \rangle$ represents the number of levels available to the active particles, and the diagonalized form of the lowest order Hamiltonian (of order Ω^2) is simply the RPA Hamiltonian. Since the MW method performs the transmutation of the Goldstone mode into the rotational mode by explicitly depending on this perturbation order, its excellent ingredients are often concealed by cumbersome perturbation arguments. This fact also makes it uneasy to give a straightforward link between the MW method and the constrained cranking-model approach which is practically used to describe the large-amplitude rotational motion.

The main purpose of this paper is to clarify the basic ingredients of the MW method in a transparent way and to demonstrate the transmutation mechanism of the Goldstone mode into the large-amplitude "rotational" one, by employing the SCC method. The quantum theory of SCC method is formulated in order to define the optimum global collective subspace and the global collective variables specifying it, in such a way that the global operators can be connected with the "local" RPA operators in the small-amplitude region. When the SCC method is applied to the collective rotational motion, therefore, the method itself manifests the transmutation mechanism of the "local" Goldstone mode into the "global" rotational motion. In § 2, we show the transmutation mechanism in terms of the SCC method, with the use of a two-dimensional system of particles adopted in the MW papers[9),10)] for didactic simplicity. The essential ingredients of the MW method become then transparent. It is known[2),3)] that a *classical image* of the global collective boson operators employed in the quantum theory of SCC method simply corresponds to the collective variables in the semi-classical theory of SCC method formulated within the time-dependent Hartree-Fock (-Bogoliubov) theory. Through the intermediary of this fact in the SCC method, in § 3, we demonstrate the manifest relationship between the MW method and the conventional cranking model approach. In order to visualize the occurrence mechanism of the low-spin ground-state rotational band by the SCC method, in § 4 we solve the basic equations of the SCC method for Er isotopes with the use of the pairing-plus-quadrupole-force model and analyze how the so-called rotation-vibration coupling effects are coherently organized so as to construct the optimum rotational subspace. Concluding remarks are given in § 5.

§ 2. Description of collective rotation by means of SCC method

2.1. *Quantum description of collective rotation in boson representation*

Following to the MW paper,[9)] let us suppose for the sake of simplicity that a two-dimensional system consisting of even fermions is given with a rotational invariant Hamiltonian $\hat{H}$ with an effective (smooth) inter-particle force. We further suppose that the Hartree-Fock minimization gives a stationary solution with an intrinsic deformation which leads us to the concept of spontaneous breakdown of the rotation symmetry. In the finite quantal system under consideration, the broken symmetry has to be restored by proper inclusion of the residual interaction which has been neglected under the Hartree-Fock approximation. Thus, microscopic structure

of the collective rotation is essentially related to the problem of how to treat the residual interaction so as to restore the broken symmetry. The first order inclusion of the residual interaction is known to be treated by means of the RPA, and the collective rotation in this case manifests itself as a Goldstone (boson) mode. Although it was shown by Thouless and Valatin[8] that the moment of inertia thus obtained turned out to be the same in the leading order as had been obtained from the cranking model, there remained the difficulties[9] associated with the fact that the rotational mode is not of the "vibrational" one such as the Goldstone mode. In order to clarify the occurrence mechanism of the rotational motion, we therefore have to go one step further to display the mechanism of transmutation of the vibrational Goldstone mode into the large-amplitude rotational mode.

Essential advantage of the MW method for this problem is to employ the generalized Holstein-Primakoff boson mapping of the fermion pairs and to explicitly demonstrate a possibility of describing the rotational invariant Hamiltonian in terms of canonically conjugate coordinates and momenta (i.e., bosons) based on the deformed equilibrium shape.*) According to the generalized Holstein-Primakoff boson mapping, the fermion-pair operators**) $\{\hat{c}_\alpha^\dagger \hat{c}_i,\ \hat{c}_i^\dagger \hat{c}_\alpha,\ \hat{c}_\alpha^\dagger \hat{c}_\beta,\ \hat{c}_i \hat{c}_j^\dagger\}$ are represented in terms of bosons $\{\boldsymbol{B}_{\alpha i}^\dagger, \boldsymbol{B}_{\beta j}\}$ (satisfying $[\boldsymbol{B}_{\alpha i}, \boldsymbol{B}_{\beta j}^\dagger]=\delta_{\alpha\beta}\delta_{ij}$ and $[\boldsymbol{B}_{\alpha i}, \boldsymbol{B}_{\beta j}]=0$) so as to keep the commutation relations among the fermion-pair operators in the following manner,

$$\begin{aligned}
&\hat{c}_\alpha^\dagger \hat{c}_\beta \to (\hat{c}_\alpha^\dagger \hat{c}_\beta)_{\mathrm{B}}=\sum_j \boldsymbol{B}_{\alpha j}^\dagger \boldsymbol{B}_{\beta j} \equiv \boldsymbol{A}_{\beta\alpha}\,,\\
&\hat{c}_i \hat{c}_j^\dagger \to (\hat{c}_i \hat{c}_j^\dagger)_{\mathrm{B}}=\sum_\alpha \boldsymbol{B}_{\alpha i}^\dagger \boldsymbol{B}_{\alpha j}\,,\\
&\hat{c}_\alpha^\dagger \hat{c}_i \to (\hat{c}_\alpha^\dagger \hat{c}_i)_{\mathrm{B}}=[\boldsymbol{B}^\dagger \sqrt{\boldsymbol{I}-\boldsymbol{A}}\,]_{i\alpha}\,,\\
&\hat{c}_i^\dagger \hat{c}_\alpha \to (\hat{c}_i^\dagger \hat{c}_\alpha)_{\mathrm{B}}=[\sqrt{\boldsymbol{I}-\boldsymbol{A}}\,\boldsymbol{B}]_{\alpha i}\,,
\end{aligned} \tag{2·1}$$

where $\boldsymbol{I}$ is the unit matrix in the unoccupied space, $(\boldsymbol{I})_{\alpha\beta}=\delta_{\alpha\beta}$. Thus, with the aid of the generalized Holstein-Primakoff boson mapping, our starting Hamiltonian $\hat{H}(\hat{c}^\dagger, \hat{c})$ is given in terms of the bosons as

$$\hat{H}(\hat{c}^\dagger, \hat{c}) \to \boldsymbol{H}(\boldsymbol{B}^\dagger, \boldsymbol{B})\,, \tag{2·2}$$

which is expressed by the RPA bosons $\{\boldsymbol{O}_\mu^\dagger, \boldsymbol{O}_\mu, \boldsymbol{J}_{\mathrm{RPA}}, \boldsymbol{\Phi}_{\mathrm{RPA}}\}$ as***)

$$\begin{aligned}
\boldsymbol{H}(\boldsymbol{B}^\dagger, \boldsymbol{B}) &= \boldsymbol{H}_{\mathrm{RPA}}+\boldsymbol{H}_{\mathrm{resid}}(\boldsymbol{O}^\dagger, \boldsymbol{O}, \boldsymbol{J}_{\mathrm{RPA}}, \boldsymbol{\Phi}_{\mathrm{RPA}})\\
&= \boldsymbol{H}(\boldsymbol{O}^\dagger, \boldsymbol{O}, \boldsymbol{J}_{\mathrm{RPA}}, \boldsymbol{\Phi}_{\mathrm{RPA}})\,,\\
\boldsymbol{H}_{\mathrm{RPA}} &= \mathrm{const}+\sum_\mu \omega_\mu \boldsymbol{O}_\mu^\dagger \boldsymbol{O}_\mu+\frac{1}{2\mathcal{J}_0}\boldsymbol{J}_{\mathrm{RPA}}^2\,.\text{****)}
\end{aligned} \tag{2·3}$$

*) An extension of such a boson description to the non-Aberian case of three dimensional rotation is complicated but essentially capable, as has been shown by Marshalek.[14]

**) Throughout §§ 2 and 3 we use the convention of denoting occupied single-particle states of the deformed Hartree-Fock state $|\phi_0\rangle$ by indices $i, j, \cdots$, and unoccupied single-particle states by indices $\alpha, \beta, \cdots$. Thus we have $\hat{c}_\alpha|\phi_0\rangle=\hat{c}_i^\dagger|\phi_0\rangle=0$.

***) For simplicity, Goldstone modes other than the rotational one are omitted here.

****) Throughout the formulation in this paper, we employ the convention $\hbar=1$.

Here, the operators $\{\boldsymbol{O}_\mu^\dagger, \boldsymbol{O}_\mu\}$ represent the vibrational RPA bosons given by

$$\boldsymbol{O}_\mu^\dagger=\sum_{\alpha i}\{\psi_\mu(\alpha i)\boldsymbol{B}_{\alpha i}^\dagger+\varphi_\mu(\alpha i)\boldsymbol{B}_{\alpha i}\},$$

$$\boldsymbol{O}_\mu=\sum_{\alpha i}\{\varphi_\mu^*(\alpha i)\boldsymbol{B}_{\alpha i}^\dagger+\psi_\mu^*(\alpha i)\boldsymbol{B}_{\alpha i}\}, \tag{2·4}$$

and satisfy

$$[\boldsymbol{H}_{\mathrm{RPA}}, \boldsymbol{O}_\mu^\dagger]=\omega_\mu\boldsymbol{O}_\mu^\dagger, \quad (\omega_\mu>0)$$

$$[\boldsymbol{O}_\mu, \boldsymbol{O}_\nu^\dagger]=\delta_{\nu\mu}, \quad [\boldsymbol{O}_\mu, \boldsymbol{O}_\nu]=0. \tag{2·5}$$

The operator $\boldsymbol{J}_{\mathrm{RPA}}$ is the RPA-order part of the angular momentum operator $\boldsymbol{J}$ $(=(\tilde{J})_{\mathrm{B}})$ in the boson representation and corresponds to the Goldstone boson with zero energy, and $\boldsymbol{\Phi}_{\mathrm{RPA}}$ is the angle operator canonically conjugate to $\boldsymbol{J}_{\mathrm{RPA}}$. They commute with the RPA bosons, i.e., $[\boldsymbol{O}_\mu, \boldsymbol{J}_{\mathrm{RPA}}]=[\boldsymbol{O}_\mu, \boldsymbol{\Phi}_{\mathrm{RPA}}]=0$, and satisfy

$$[\boldsymbol{H}_{\mathrm{RPA}}, \boldsymbol{J}_{\mathrm{RPA}}]=0, \quad [\boldsymbol{H}_{\mathrm{RPA}}, \boldsymbol{\Phi}_{\mathrm{RPA}}]=-i\frac{\boldsymbol{J}_{\mathrm{RPA}}}{\mathcal{J}_0}, \quad [\boldsymbol{\Phi}_{\mathrm{RPA}}, \boldsymbol{J}_{\mathrm{RPA}}]=i. \tag{2·6}$$

The set of Eq. (2·6) is sufficient to determine $\boldsymbol{\Phi}_{\mathrm{RPA}}$ and the Thouless-Valatin moment of inertia $\mathcal{J}_0$.

In the quantum theory of SCC method, the representation with the use of the set of local bosons $\{\boldsymbol{O}_\mu^\dagger, \boldsymbol{O}_\mu, \boldsymbol{J}_{\mathrm{RPA}}, \boldsymbol{\Phi}_{\mathrm{RPA}}\}$ is called the *initial representation*.[2] It has to be noted that the residual interaction $\boldsymbol{H}_{\mathrm{resid}}$ in Eq. (2·3) involves the Goldstone boson $\boldsymbol{J}_{\mathrm{RPA}}$ as well as the angle operator $\boldsymbol{\Phi}_{\mathrm{RPA}}$ in a very complicated way, and the introduction of the collective operators $\{\boldsymbol{J}_{\mathrm{RPA}}, \boldsymbol{\Phi}_{\mathrm{RPA}}\}$ and the non-collective operators $\{\boldsymbol{O}_\mu^\dagger, \boldsymbol{O}_\mu\}$ in the initial representation is only valid in the "small-amplitude" RPA region in the neighborhood of the stable Hartree-Fock minimum. In the SCC method, the precise definition of the global collective operators (specifying the optimum collective subspace) and the "intrinsic" (non-collective) operators is given in the *dynamical representation*.[2]

The dynamical representation is obtained by a non-linear unitary transformation from the initial representation,

$$\boldsymbol{J}=\boldsymbol{V}(\boldsymbol{O}^\dagger, \boldsymbol{O}, \boldsymbol{J}_{\mathrm{RPA}}, \boldsymbol{\Phi}_{\mathrm{RPA}})\boldsymbol{J}_{\mathrm{RPA}}\boldsymbol{V}^{-1}(\boldsymbol{O}^\dagger, \boldsymbol{O}, \boldsymbol{J}_{\mathrm{RPA}}, \boldsymbol{\Phi}_{\mathrm{RPA}}),$$

$$\boldsymbol{\Phi}=\boldsymbol{V}(\boldsymbol{O}^\dagger, \boldsymbol{O}, \boldsymbol{J}_{\mathrm{RPA}}, \boldsymbol{\Phi}_{\mathrm{RPA}})\boldsymbol{\Phi}_{\mathrm{RPA}}\boldsymbol{V}^{-1}(\boldsymbol{O}^\dagger, \boldsymbol{O}, \boldsymbol{J}_{\mathrm{RPA}}, \boldsymbol{\Phi}_{\mathrm{RPA}}),$$

$$\boldsymbol{\beta}_\mu^\dagger=\boldsymbol{V}(\boldsymbol{O}^\dagger, \boldsymbol{O}, \boldsymbol{J}_{\mathrm{RPA}}, \boldsymbol{\Phi}_{\mathrm{RPA}})\boldsymbol{O}_\mu^\dagger\boldsymbol{V}^{-1}(\boldsymbol{O}^\dagger, \boldsymbol{O}, \boldsymbol{J}_{\mathrm{RPA}}, \boldsymbol{\Phi}_{\mathrm{RPA}}) \quad \text{and h.c.},$$

$$\boldsymbol{V}(\boldsymbol{O}^\dagger, \boldsymbol{O}, \boldsymbol{J}_{\mathrm{RPA}}, \boldsymbol{\Phi}_{\mathrm{RPA}})=e^{i\boldsymbol{F}(\boldsymbol{O}^\dagger, \boldsymbol{O}, \boldsymbol{J}_{\mathrm{RPA}}, \boldsymbol{\Phi}_{\mathrm{RPA}})}, \quad \boldsymbol{F}^\dagger=\boldsymbol{F}, \tag{2·7a}$$

where $\boldsymbol{J}$ is the exact angular momentum operator and the set $\{\boldsymbol{J}, \boldsymbol{\Phi}\}$ are the global collective operators optimally describing the collective rotational motion, while the set $\{\boldsymbol{\beta}_\mu^\dagger, \boldsymbol{\beta}_\mu\}$ describes the intrinsic degrees of freedom. By definition, we have

$$\boldsymbol{J}_{\mathrm{RPA}}=\boldsymbol{V}^{-1}(\boldsymbol{\beta}^\dagger, \boldsymbol{\beta}, \boldsymbol{J}, \boldsymbol{\Phi})\boldsymbol{J}\boldsymbol{V}(\boldsymbol{\beta}^\dagger, \boldsymbol{\beta}, \boldsymbol{J}, \boldsymbol{\Phi})$$

$$=\boldsymbol{J}+[\boldsymbol{J}, i\boldsymbol{F}(\boldsymbol{\beta}^\dagger, \boldsymbol{\beta}, \boldsymbol{J}, \boldsymbol{\Phi})]+\frac{1}{2!}[[\boldsymbol{J}, i\boldsymbol{F}], i\boldsymbol{F}]+\cdots,$$

$$\boldsymbol{\Phi}_{\mathrm{RPA}}=\boldsymbol{V}^{-1}(\boldsymbol{\beta}^{\dagger},\boldsymbol{\beta},\boldsymbol{J},\boldsymbol{\Phi})\boldsymbol{\Phi}\boldsymbol{V}(\boldsymbol{\beta}^{\dagger},\boldsymbol{\beta},\boldsymbol{J},\boldsymbol{\Phi})$$

$$=\boldsymbol{\Phi}+[\boldsymbol{\Phi},i\boldsymbol{F}(\boldsymbol{\beta}^{\dagger},\boldsymbol{\beta},\boldsymbol{J},\boldsymbol{\Phi})]+\frac{1}{2!}[[\boldsymbol{\Phi},i\boldsymbol{F}],i\boldsymbol{F}]+\cdots,$$

$$\boldsymbol{O}_{\mu}^{\dagger}=\boldsymbol{V}^{-1}(\boldsymbol{\beta}^{\dagger},\boldsymbol{\beta},\boldsymbol{J},\boldsymbol{\Phi})\boldsymbol{\beta}_{\mu}^{\dagger}\boldsymbol{V}(\boldsymbol{\beta}^{\dagger},\boldsymbol{\beta},\boldsymbol{J},\boldsymbol{\Phi})$$

$$=\boldsymbol{\beta}_{\mu}^{\dagger}+[\boldsymbol{\beta}_{\mu}^{\dagger},i\boldsymbol{F}(\boldsymbol{\beta}^{\dagger},\boldsymbol{\beta},\boldsymbol{J},\boldsymbol{\Phi})]+\frac{1}{2!}[[\boldsymbol{\beta}_{\mu}^{\dagger},i\boldsymbol{F}],i\boldsymbol{F}]+\cdots \quad \text{and h.c.},$$

$$\boldsymbol{V}(\boldsymbol{\beta}^{\dagger},\boldsymbol{\beta},\boldsymbol{J},\boldsymbol{\Phi})=e^{i\boldsymbol{F}(\boldsymbol{\beta}^{\dagger},\boldsymbol{\beta},\boldsymbol{J},\boldsymbol{\Phi})}=e^{i\boldsymbol{F}(\boldsymbol{O}^{\dagger},\boldsymbol{O},\boldsymbol{J}_{\mathrm{RPA}},\boldsymbol{\Phi}_{\mathrm{RPA}})}. \tag{2·7b}$$

The Hamiltonian in the dynamical representation is generally expressed as

$$\boldsymbol{H}(\boldsymbol{O}^{\dagger},\boldsymbol{O},\boldsymbol{J}_{\mathrm{RPA}},\boldsymbol{\Phi}_{\mathrm{RPA}})=\boldsymbol{V}^{-1}(\boldsymbol{\beta}^{\dagger},\boldsymbol{\beta},\boldsymbol{J},\boldsymbol{\Phi})\boldsymbol{H}(\boldsymbol{\beta}^{\dagger},\boldsymbol{\beta},\boldsymbol{J},\boldsymbol{\Phi})\boldsymbol{V}(\boldsymbol{\beta}^{\dagger},\boldsymbol{\beta},\boldsymbol{J},\boldsymbol{\Phi})$$

$$=\mathcal{H}_{\mathrm{coll}}(\boldsymbol{J})+\Delta\mathcal{H}(\boldsymbol{\beta}^{\dagger},\boldsymbol{\beta},\boldsymbol{J})\equiv\mathcal{H}(\boldsymbol{\beta}^{\dagger},\boldsymbol{\beta},\boldsymbol{J}), \tag{2·8}$$

where the Hamiltonian $\mathcal{H}(\boldsymbol{\beta}^{\dagger},\boldsymbol{\beta},\boldsymbol{J})$ does not include the angle operator $\boldsymbol{\Phi}$ because of the rotational invariance of the Hamiltonian. It is now self-evident that the occurrence mechanism of the rotational motion, i.e., the transmutation mechanism of the Goldstone mode into the rotational mode is given by the non-linear unitary transformation $\boldsymbol{V}$. The first term $\mathcal{H}_{\mathrm{coll}}(\boldsymbol{J})$ in Eq. (2·8) has no intrinsic bosons $\{\boldsymbol{\beta}_{\mu}^{\dagger},\boldsymbol{\beta}_{\mu}\}$ and consists of only the angular momentum $\boldsymbol{J}$, so that it is identified to be the collective rotational Hamiltonian. The remaining part is expressed as

$$\Delta\mathcal{H}(\boldsymbol{\beta}^{\dagger},\boldsymbol{\beta},\boldsymbol{J})\equiv\sum_{n\geq 1}\mathcal{H}^{[n]}(\boldsymbol{\beta}^{\dagger},\boldsymbol{\beta},\boldsymbol{J}), \tag{2·9}$$

where n denotes a power of the intrinsic bosons $\{\boldsymbol{\beta}_{\mu}^{\dagger},\boldsymbol{\beta}_{\mu}\}$ contained in $\mathcal{H}^{[n]}(\boldsymbol{\beta}^{\dagger},\boldsymbol{\beta},\boldsymbol{J})$. Hereafter the superscript n is called rank.

An essential idea of the theory of nuclear collective dynamics based on the SCC method is to determine the non-linear unitary transformation $\boldsymbol{V}(\boldsymbol{\beta}^{\dagger},\boldsymbol{\beta},\boldsymbol{J},\boldsymbol{\Phi})$, by successive unitary transformations depending on the rank.[2),4),5)] The first one is decisively important and determines the collective Hamiltonian $\mathcal{H}_{\mathrm{coll}}(\boldsymbol{J})$ as well as the optimum collective rotational subspace. This first unitary transformation, denoted by $\boldsymbol{V}_{\mathrm{SCC}}(\boldsymbol{\beta}^{\dagger},\boldsymbol{\beta},\boldsymbol{J},\boldsymbol{\Phi})$, is determined by the SCC method, by demanding the following two conditions:

i) The optimum collective subspace should satisfy the maximal decoupling condition

$$\mathcal{H}^{[n=1]}(\boldsymbol{\beta}^{\dagger},\boldsymbol{\beta},\boldsymbol{J})=0. \tag{2·10}$$

ii) The resultant collective Hamiltonian $\mathcal{H}_{\mathrm{coll}}(\boldsymbol{J})$ depends only on the angular momentum $\boldsymbol{J}$ and never includes the angle operator $\boldsymbol{\Phi}$.

In solving the basic equations of the SCC method derived from the two conditions, we need boundary conditions to specify the global operators $\{\boldsymbol{\beta}_{\mu}^{\dagger},\boldsymbol{\beta}_{\mu},\boldsymbol{J},\boldsymbol{\Phi}\}$. This is chosen in such a way that these operators have to be connected with the local bosons $\{\boldsymbol{O}_{\mu}^{\dagger},\boldsymbol{O}_{\mu},\boldsymbol{J}_{\mathrm{RPA}},\boldsymbol{\Phi}_{\mathrm{RPA}}\}$ in the small-amplitude RPA region,

$$\boldsymbol{J}\rightarrow\boldsymbol{J}_{\mathrm{RPA}},\quad\boldsymbol{\Phi}\rightarrow\boldsymbol{\Phi}_{\mathrm{RPA}},\quad\boldsymbol{\beta}_{\mu}^{\dagger}\rightarrow\boldsymbol{O}_{\mu}^{\dagger}\quad\text{and h.c.} \tag{2·11}$$

Although we do not intend to go further into details of the solution in this paper, as shown in Ref. 2), the generator $\boldsymbol{F}_{\rm SCC}(\boldsymbol{\beta}^\dagger, \boldsymbol{\beta}, \boldsymbol{J}, \boldsymbol{\Phi})$ defined by

$$\boldsymbol{V}_{\rm SCC}(\boldsymbol{\beta}^\dagger, \boldsymbol{\beta}, \boldsymbol{J}, \boldsymbol{\Phi})=e^{i\boldsymbol{F}_{\rm SCC}(\boldsymbol{\beta}^\dagger,\boldsymbol{\beta},\boldsymbol{J},\boldsymbol{\Phi})}, \quad \boldsymbol{F}_{\rm SCC}^\dagger=\boldsymbol{F}_{\rm SCC}, \tag{2·12}$$

is determined by the SCC method, as well as the collective rotational Hamiltonian $\mathcal{H}_{\rm coll}(\boldsymbol{J})$ specifying the optimum stationary collective subspace.

Now, let us express the local bosons $\{\boldsymbol{O}_\mu^\dagger, \boldsymbol{O}_\mu, \boldsymbol{J}_{\rm RPA}, \boldsymbol{\Phi}_{\rm RPA}\}$ in Eq. (2·7b) in terms of the rank n for the intrinsic bosons $\{\boldsymbol{\beta}_\mu^\dagger, \boldsymbol{\beta}_\mu\}$,

$$\begin{aligned}
&\boldsymbol{J}_{\rm RPA}=\sum_{n\geq 0}\boldsymbol{J}_{\rm RPA}^{[n]}(\boldsymbol{\beta}^\dagger, \boldsymbol{\beta}, \boldsymbol{J}, \boldsymbol{\Phi}),\\
&\boldsymbol{\Phi}_{\rm RPA}=\sum_{n\geq 0}\boldsymbol{\Phi}_{\rm RPA}^{[n]}(\boldsymbol{\beta}^\dagger, \boldsymbol{\beta}, \boldsymbol{J}, \boldsymbol{\Phi}),\\
&\boldsymbol{O}_\mu^\dagger=\sum_{n\geq 0}\boldsymbol{O}_{\rm RPA}^{[n]\dagger}(\boldsymbol{\beta}^\dagger, \boldsymbol{\beta}, \boldsymbol{J}, \boldsymbol{\Phi}) \qquad \text{and h.c.}
\end{aligned} \tag{2·13}$$

Since the unitary transformation $\boldsymbol{V}_{\rm SCC}(\boldsymbol{\beta}^\dagger, \boldsymbol{\beta}, \boldsymbol{J}, \boldsymbol{\Phi})$ is the first one of the successive unitary transformations constructing $\boldsymbol{V}(\boldsymbol{\beta}^\dagger, \boldsymbol{\beta}, \boldsymbol{J}, \boldsymbol{\Phi})$, the SCC method determines the lowest rank ($n=0$) parts in Eq. (2·13) through

$$\begin{aligned}
&\boldsymbol{J}_{\rm RPA}^{[n=0]}(\boldsymbol{J}, \boldsymbol{\Phi})=[\,\boldsymbol{V}_{\rm SCC}^{-1}\boldsymbol{J}\boldsymbol{V}_{\rm SCC}]^{[n=0]},\\
&\boldsymbol{\Phi}_{\rm RPA}^{[n=0]}(\boldsymbol{J}, \boldsymbol{\Phi})=[\,\boldsymbol{V}_{\rm SCC}^{-1}\boldsymbol{\Phi}\boldsymbol{V}_{\rm SCC}]^{[n=0]}.\\
&(\boldsymbol{O}_\mu^{[n=0]\dagger}(\boldsymbol{J}, \boldsymbol{\Phi})=[\,\boldsymbol{V}_{\rm SCC}^{-1}\boldsymbol{\beta}_\mu^\dagger\,\boldsymbol{V}_{\rm SCC}]^{[n=0]}=0)
\end{aligned} \tag{2·14}$$

Once the lowest rank relations (2·14) and the collective Hamiltonian $\mathcal{H}_{\rm coll}(\boldsymbol{J})$ are determined by the SCC method, the higher rank relations of Eq. (2·13) and $\mathcal{H}^{[n]}(\boldsymbol{\beta}^\dagger, \boldsymbol{\beta}, \boldsymbol{J})$ in Eq. (2·9) are determined by the successive unitary transformations which are obtained by a perturbation theory with respect to the rank n of the intrinsic bosons.[2),4),5)]

2.2. *Classical image of boson representation by time-dependent Hartree-Fock theory in canonical-variable representation*

In the previous subsection we have seen that the essential dynamics of the occurrence mechanism of the rotational motion is given by the non-linear unitary transformation $\boldsymbol{V}_{\rm SCC}(\boldsymbol{\beta}_\mu^\dagger, \boldsymbol{\beta}_\mu, \boldsymbol{J}, \boldsymbol{\Phi})$. In order to visualize the occurrence dynamics in a transparent way, it is convenient to employ a classical image of the boson representation of the generalized Holstein-Primakoff type. The classical image is directly obtained by means of the time-dependent Hartree-Fock (TDHF) theory in the canonical-variable representation.[3),15),16)]

The basic equation of the TDHF theory is

$$\delta\langle\phi(t)|\left\{\left(i\frac{\partial}{\partial t}-\hat{H}\right)|\phi(t)\rangle\right\}=0, \tag{2·15}$$

where the time-dependent Slater determinant $|\phi(t)\rangle$ is given by

$$|\phi(t)\rangle = e^{i\hat{G}(t)}|\phi_0\rangle \equiv \hat{U}(t)|\phi_0\rangle\,, \quad i\hat{G}(t) = \sum_{ai}\{f_{ai}(t)\,\hat{c}_a^\dagger\,\hat{c}_i - f_{ai}^*(t)\,\hat{c}_i^\dagger\,\hat{c}_a\} \tag{2·16}$$

with $|\phi_0\rangle$ being the deformed Hartree-Fock stationary solution. Instead of the variables $\{f_{ai}^*(t), f_{ai}(t)\}$, we introduce a new set of variables $\{C_{ai}^*, C_{ai}\}$ through a variable transformation

$$\{f_{ai}^*(t), f_{ai}(t)\} \to \{C_{ai}^*, C_{ai}\}\,. \tag{2·17}$$

The symplectic structure of the TDHF theory[3] always enables us to choose the variables $\{C_{ai}^*, C_{ai}\}$ to be of the canonical-variable representation, in which the TDHF equation (2·15) can be expressed as the canonical equations of motion in classical mechanics,

$$i\dot{C}_{ai}^* = -\partial H/\partial C_{ai}\,, \quad i\dot{C}_{ai} = \partial H/\partial C_{ai}^*\,, \tag{2·18a}$$

i.e.,

$$\dot{q}_{ai} = \partial H/\partial p_{ai}\,, \qquad \dot{p}_{ai} = -\partial H/\partial q_{ai}\,,$$

$$q_{ai} = (C_{ai}^* + C_{ai})/\sqrt{2}\,, \quad p_{ai} = i(C_{ai}^* - C_{ai})/\sqrt{2}\,, \tag{2·18b}$$

where

$$H \equiv \langle\phi(t)|\hat{H}|\phi(t)\rangle = \langle\phi_0|\hat{U}^\dagger\hat{H}\hat{U}|\phi_0\rangle\,. \tag{2·19}$$

The necessary condition for the canonical-variable representation is that the local infinitesimal generators, defined by

$$\hat{Y}_{ai}^\dagger = (\partial\hat{U}/\partial C_{ai})\hat{U}^\dagger\,, \quad \hat{Y}_{ai} = -(\partial\hat{U}/\partial C_{ai}^*)\hat{U}^\dagger\,, \tag{2·20}$$

have to satisfy the "weak" boson-like commutation relations

$$\langle\phi|[\hat{Y}_{ai}, \hat{Y}_{\beta j}^\dagger]|\phi\rangle = \delta_{a\beta}\delta_{ij}\,, \quad \langle\phi|[\hat{Y}_{ai}, \hat{Y}_{\beta j}]|\phi\rangle = 0\,. \tag{2·21}$$

Since the TDHF equation (2·15) is written as

$$\delta\langle\phi|\hat{H} - i\sum_{ai}\{\dot{C}_{ai}\hat{Y}_{ai}^\dagger - \dot{C}_{ai}^*\hat{Y}_{ai}\}|\phi\rangle = 0\,, \tag{2·22}$$

and the generators $\{\hat{Y}_{ai}^\dagger, \hat{Y}_{ai}\}$ are one-body operators by definition, the canonical equations of motion (2·18) are easily derived from Eq. (2·22) by taking $|\delta\phi\rangle \propto : \hat{Y}_{ai}^\dagger : |\phi\rangle$ and $: \hat{Y}_{ai} : |\phi\rangle$, and using Eq. (2·21).

It can be proved[1] that the generators $\{\hat{Y}_{ai}^\dagger, \hat{Y}_{ai}\}$ which satisfy Eq. (2·21) are generally determined by the relations called the canonical-variable conditions

$$\langle\phi|\hat{Y}_{ai}^\dagger|\phi\rangle = \frac{1}{2}C_{ai}^* - i\frac{\partial}{\partial C_{ai}}S(C^*, C)\,,$$

$$\langle\phi|\hat{Y}_{ai}|\phi\rangle = \frac{1}{2}C_{ai} + i\frac{\partial}{\partial C_{ai}^*}S(C^*, C)\,, \tag{2·23}$$

where $S(C^*, C)$ is an arbitrary real function of $\{C_{ai}^*, C_{ai}\}$. The form of Eq. (2·18) is invariant under canonical transformations of the variables $\{C_{ai}^*, C_{ai}\}$, and so the function $S(C^*, C)$ expresses the freedom in choosing a set of canonical variables.

Let us choose the variable transformation in Eq. (2·17) so as to be of the form

$$C=\frac{\sin\sqrt{GG^\dagger}}{\sqrt{GG^\dagger}}G\,,\quad C^\dagger=G^\dagger\frac{\sin\sqrt{GG^\dagger}}{\sqrt{GG^\dagger}}\,, \tag{2·24}$$

with C, $C^\dagger$, G and $G^\dagger$ being matrices whose elements are defined by

$$(C)_{\alpha i}=C_{\alpha i}\,,\quad (C^\dagger)_{i\alpha}=C^*_{\alpha i}\,,$$
$$(G)_{\alpha i}=f_{\alpha i}\,,\quad (G^\dagger)_{i\alpha}=f^*_{\alpha i}\,. \tag{2·25}$$

Then, we can see [1),3)] that the choice of the canonical variables $\{C^*_{\alpha i}, C_{\alpha i}\}$ satisfy Eq. (2·23) with $S(C^*, C)=0$, and leads us to the relations

$$\langle\phi|\hat{c}^\dagger_\alpha\hat{c}_\beta|\phi\rangle=\sum_j C^*_{\alpha j}C_{\beta j}\equiv A_{\beta\alpha}\,,\quad \langle\phi|\hat{c}_i\hat{c}^\dagger_j|\phi\rangle=\sum_\alpha C^*_{\alpha i}C_{\alpha j}\,,$$
$$\langle\phi|\hat{c}^\dagger_\alpha\hat{c}_i|\phi\rangle=(C^\dagger\sqrt{I-A})_{i\alpha}\,,\quad \langle\phi|\hat{c}^\dagger_i\hat{c}_\alpha|\phi\rangle=(\sqrt{I-A}C)_{\alpha i}\,. \tag{2·26}$$

Comparing Eq. (2·26) with Eq. (2·1), we can easily see that the TDHF theory in the canonical-variable representation with the choice (2·24) precisely corresponds to a classical image of the boson representation of the fermion-pair operators $\{\hat{c}^\dagger_\alpha\hat{c}_i, \hat{c}^\dagger_i\hat{c}_\alpha, \hat{c}^\dagger_\alpha\hat{c}^\dagger_\beta, \hat{c}_i\hat{c}^\dagger_j\}$, with the correspondence

$$\boldsymbol{B}^\dagger_{\alpha i}(t)\leftrightarrow C^*_{\alpha i}(t)\,,\quad \boldsymbol{B}_{\alpha i}(t)\leftrightarrow C_{\alpha i}(t)\,,\quad (\boldsymbol{B}^\dagger_{\alpha i}(t)\equiv e^{iHt}\boldsymbol{B}^\dagger_{\alpha i}e^{-iHt})$$
$$[\boldsymbol{B}_{\alpha i}, \boldsymbol{B}^\dagger_{\beta j}]=\delta_{\alpha\beta}\delta_{ij}\leftrightarrow\{C_{\alpha i}, C^*_{\beta j}\}_{\mathrm{PB}}=\delta_{\alpha\beta}\delta_{ij}\,,$$
$$[\boldsymbol{B}_{\alpha i}, \boldsymbol{B}_{\beta j}]=0\leftrightarrow\{C_{\alpha i}, C_{\beta j}\}_{\mathrm{PB}}=0\,, \tag{2·27}$$

where the symbol $\{A, B\}_{\mathrm{PB}}$ denotes the Poisson bracket defined by

$$\{A, B\}_{\mathrm{PB}}\equiv\sum_{\alpha i}\left\{\frac{\partial A}{\partial C_{\alpha i}}\frac{\partial B}{\partial C^*_{\alpha i}}-\frac{\partial B}{\partial C_{\alpha i}}\frac{\partial A}{\partial C^*_{\alpha i}}\right\}=i\sum_{\alpha i}\left\{\frac{\partial A}{\partial q_{\alpha i}}\frac{\partial B}{\partial p_{\alpha i}}-\frac{\partial B}{\partial q_{\alpha i}}\frac{\partial A}{\partial p_{\alpha i}}\right\}. \tag{2·28}$$

Thus, the classical correspondence $\{O^*_\mu, O_\mu, J_{\mathrm{RPA}}, \Phi_{\mathrm{RPA}}\}$ of the RPA boson operators $\{\boldsymbol{O}^\dagger_\mu, \boldsymbol{O}_\mu, \boldsymbol{J}_{\mathrm{RPA}}, \boldsymbol{\Phi}_{\mathrm{RPA}}\}$, which have been defined by Eq. (2·4) and by

$$\boldsymbol{J}_{\mathrm{RPA}}=\sum_{\alpha i}\{J_{\alpha i}\boldsymbol{B}^\dagger_{\alpha i}+J^*_{\alpha i}\boldsymbol{B}_{\alpha i}\}\,,\quad \boldsymbol{\Phi}_{\mathrm{RPA}}=\sum_{\alpha i}\{\Phi^{(0)}_{\alpha i}\boldsymbol{B}^\dagger_{\alpha i}+\Phi^{(0)*}_{\alpha i}\boldsymbol{B}_{\alpha i}\}\,, \tag{2·29}$$

is simply obtained by the RPA canonical transformation of $\{C^*_{\alpha i}, C_{\alpha i}\}$,

$$O^*_\mu=\sum_{\alpha i}\{\psi_\mu(\alpha i)C^*_{\alpha i}+\varphi_\mu(\alpha i)C_{\alpha i}\}\text{ and c.c.,}$$
$$J_{\mathrm{RPA}}=\sum_{\alpha i}\{J_{\alpha i}C^*_{\alpha i}+J^*_{\alpha i}C_{\alpha i}\}\,,\quad \Phi_{\mathrm{RPA}}=\sum_{\alpha i}\{\Phi^{(0)}_{\alpha i}C^*_{\alpha i}+\Phi^{(0)*}_{\alpha i}C_{\alpha i}\}\,. \tag{2·30}$$

2.3. *Collective rotation in TDHF phase space*

Corresponding to the non-linear unitary transformation (2·7), let us consider a non-linear canonical transformation in the TDHF phase space,

$$\{O^*_\mu, O_\mu, J_{\mathrm{RPA}}, \Phi_{\mathrm{RPA}}\}\rightarrow\{\xi^*_\mu, \xi_\mu, J, \Phi\}\,, \tag{2·31}$$

where $\{\xi^*_\mu, \xi_\mu\}$ and $\{J, \Phi\}$ are classical correspondents of the global intrinsic and collective operators, $\{\boldsymbol{\beta}^\dagger_\mu, \boldsymbol{\beta}_\mu\}$ and $\{\boldsymbol{J}, \boldsymbol{\Phi}\}$, respectively. In a parallel way with the full

quantum theory, we can express the canonical transformation with the use of a Lie derivative[5] $\mathcal{L}_F$ with a real generating function $F(\xi^*, \xi, J, \Phi)$, which is defined by

$$\mathcal{L}_F * = -i\{F, *\}_{\mathrm{PB}} . \tag{2·32}$$

As the correspondents to Eq. (2·7b), we thus have

$$\begin{aligned}
&J_{\mathrm{RPA}}=(\exp\mathcal{L}_F)J=J+\{J, iF(\xi^*, \xi, J, \Phi)\}_{\mathrm{PB}}+\frac{1}{2!}\{\{J, iF\}_{\mathrm{PB}}, iF\}_{\mathrm{PB}}+\cdots ,\\
&\Phi_{\mathrm{RPA}}=(\exp\mathcal{L}_F)\Phi=\Phi+\{\Phi, iF(\xi^*, \xi, J, \Phi)\}_{\mathrm{PB}}+\frac{1}{2!}\{\{\Phi, iF\}_{\mathrm{PB}}, iF\}_{\mathrm{PB}}+\cdots ,\\
&O^*_\mu=(\exp\mathcal{L}_F)\xi^*_\mu=\xi^*_\mu+\{\xi^*_\mu, iF(\xi^*, \xi, J, \Phi)\}_{\mathrm{PB}}+\frac{1}{2!}\{\{\xi^*_\mu, iF\}_{\mathrm{PB}}, iF\}_{\mathrm{PB}}+\cdots \text{ and c.c.}
\end{aligned} \tag{2·33}$$

Since the transformation (2·33) is canonical, the canonical equations of motion in the TDHF phase space, (2·18), is transformed into the canonical form with respect to $\{\xi^*_\mu, \xi_\mu, J, \Phi\}$, i.e.,

$$\dot{\Phi}=\partial\mathcal{H}/\partial J , \quad \dot{J}=-\partial\mathcal{H}/\partial\Phi=0 , \quad i\dot{\xi}_\mu=\partial\mathcal{H}/\partial\xi^*_\mu , \quad i\dot{\xi}^*_\mu=-\partial\mathcal{H}/\partial\xi_\mu , \tag{2·34}$$

where the Hamiltonian $\mathcal{H}(\xi^*, \xi, J)$ is given by

$$H(O^*, O, J_{\mathrm{RPA}}, \Phi_{\mathrm{RPA}})=(\exp\mathcal{L}_F)H(\xi^*, \xi, J, \Phi)\equiv\mathcal{H}(\xi^*, \xi, J) , \tag{2·35}$$

corresponding to Eq. (2·8).

The generators with respect to the new variables $\{\xi^*_\mu, \xi_\mu, J, \Phi\}$ are expressed in terms of those of Eq. (2·20) as

$$\begin{aligned}
&\hat{J}_\Phi\equiv\left(i\frac{\partial}{\partial\Phi}\hat{U}\right)\hat{U}^\dagger=i\sum_{\alpha i}\left\{\frac{\partial C_{\alpha i}}{\partial\Phi}\hat{Y}^\dagger_{\alpha i}-\frac{\partial C^*_{\alpha i}}{\partial\Phi}\hat{Y}_{\alpha i}\right\} ,\\
&\hat{\Phi}\equiv\left(-i\frac{\partial}{\partial J}\hat{U}\right)\hat{U}^\dagger=-i\sum_{\alpha i}\left\{\frac{\partial C_{\alpha i}}{\partial J}\hat{Y}^\dagger_{\alpha i}-\frac{\partial C^*_{\alpha i}}{\partial J}\hat{Y}_{\alpha i}\right\} ,\\
&\hat{X}^\dagger_\mu\equiv\left(-\frac{\partial}{\partial\xi^*_\mu}\hat{U}\right)\hat{U}^\dagger=-\sum_{\alpha i}\left\{\frac{\partial C_{\alpha i}}{\partial\xi^*_\mu}\hat{Y}^\dagger_{\alpha i}-\frac{\partial C^*_{\alpha i}}{\partial\xi^*_\mu}\hat{Y}_{\alpha i}\right\} \text{ and h.c.}
\end{aligned} \tag{2·36}$$

By definition, they have to satisfy the canonical-variable conditions with the same form as Eq. (2·23),[1),*]

$$\begin{aligned}
&\langle\phi|\hat{J}_\Phi|\phi\rangle=J+\frac{\partial}{\partial\Phi}S(\xi^*, \xi, J, \Phi) ,\\
&\langle\phi|\hat{\Phi}|\phi\rangle=-\frac{\partial}{\partial J}S(\xi^*, \xi, J, \Phi) ,\\
&\langle\phi|\hat{X}^\dagger_\nu|\phi\rangle=\frac{1}{2}\xi^*_\nu-i\frac{\partial}{\partial\xi_\nu}S(\xi^*, \xi, J, \Phi) ,
\end{aligned}$$

*) As will be seen from Eqs. (2·45), (2·46) and (4·8), it turns out that $S(\xi^*, \xi, J, \Phi)=0$ is actually convenient for describing the collective rotation under consideration.

$$\langle\phi|\hat{X}_\nu|\phi\rangle=\frac{1}{2}\xi_\nu+i\frac{\partial}{\partial\xi_\nu^*}S(\xi^*,\xi,J,\Phi)\,, \tag{2·37}$$

which guarantee the "weak" canonical commutation relations

$$\langle\phi|[\hat{\Phi},\hat{J}_\Phi]|\phi\rangle=i\,,\qquad \langle\phi|[\hat{\Phi},\hat{X}_\mu]|\phi\rangle=\langle\phi|[\hat{J}_\Phi,\hat{X}_\mu]|\phi\rangle=0\,,$$
$$\langle\phi|[\hat{X}_\mu,\hat{X}_\nu^\dagger]|\phi\rangle=\delta_{\mu\nu}\,,\qquad \langle\phi|[\hat{X}_\mu,\hat{X}_\nu]|\phi\rangle=0\,. \tag{2·38}$$

Corresponding to Eq. (2·8) with Eq. (2·9), the Hamiltonian (2·35) is expressed as a Taylor expansion with respect to the intrinsic variables $\{\xi_\mu^*,\xi_\mu\}$,

$$\mathcal{H}(\xi^*,\xi,J)=\sum_{n\geq 0}\mathcal{H}^{[n]}(\xi^*,\xi,J)\,,$$
$$\mathcal{H}^{[n=0]}=[\mathcal{H}]\equiv\mathcal{H}_{\rm coll}(J)\,,$$
$$\mathcal{H}^{[n=1]}=\sum_\mu\left\{\xi_\mu^*\left[\frac{\partial\mathcal{H}}{\partial\xi_\mu^*}\right]+\xi_\mu\left[\frac{\partial\mathcal{H}}{\partial\xi_\mu}\right]\right\},\ \cdots, \tag{2·39}$$

where the symbol $[A]$ for any function $A(\xi^*,\xi,J,\Phi)$ denotes a function on a collective submanifold (surface) Σ_R, i.e.,

$$[A]\equiv A(\xi^*=0,\xi=0,J,\Phi)\,. \tag{2·40}$$

In the same way as the full quantum case, the canonical transformation $\exp\mathcal{L}_F$ in Eq. (2·33) is determined by successive transformations depending on the rank n. The first one is expressed by $\exp\mathcal{L}_{F\rm SCC}$ with

$$\mathcal{L}_{F\rm SCC}*=-i\{F_{\rm SCC},*\}_{\rm PB}\,,$$
$$F_{\rm SCC}=F_{\rm SCC}^{[n=0]}(J,\Phi)+F_{\rm SCC}^{[n=1]}(\xi^*,\xi,J,\Phi)\,,\quad F_{\rm SCC}^*=F_{\rm SCC}\,,$$
$$F_{\rm SCC}^{[n=1]}\equiv\sum_\mu\{\xi_\mu^*F_{\rm SCC}^\mu(J,\Phi)+\xi_\mu F_{\rm SCC}^{\mu*}(J,\Phi)\}\,, \tag{2·41}$$

and determines the collective Hamiltonian $\mathcal{H}_{\rm coll}(J)$ and the optimum collective submanifold Σ_R. Thus, the essential dynamics of the occurrence mechanism of rotational motion (described by the global collective variables $\{J,\Phi\}$) is given by the first canonical transformation with $F_{\rm SCC}$. The aim of the SCC method formulated within the TDHF theory is to determine the functional form $F_{\rm SCC}$, so as to extract the optimum collective submanifold Σ_R out of the huge-dimensional TDHF phase space and to obtain the optimum collective Hamiltonian $\mathcal{H}_{\rm coll}(J)$.

The basic equations of the SCC method are the following.[1] The canonical equations of motion (2·34) have expressions in the rank $n=0$,

$$\text{[I]}\qquad \dot{\Phi}=\partial[\mathcal{H}]/\partial J\,,\quad \dot{J}=-\partial[\mathcal{H}]/\partial\Phi=0\,, \tag{2·42}$$

which is called the *equations of collective motion* of the SCC method. The condition that the optimum collective submanifold Σ_R should be stationary is simply written as

$$\text{[II]}\qquad \mathcal{H}^{[n=1]}=0\,,\ \text{i.e.,}\ \left[\frac{\partial\mathcal{H}}{\partial\xi_\mu^*}\right]=\left[\frac{\partial\mathcal{H}}{\partial\xi_\mu}\right]=0\,, \tag{2·43}$$

which is called the *equation of collective submanifold* or the maximal decoupling

condition. The canonical-variable conditions (2·37) have expressions in the rank $n=0$,

$$\text{[III]} \qquad \langle\phi_0|[\hat{U}^\dagger][\hat{J}_\Phi][\hat{U}]|\phi_0\rangle = J + \frac{\partial}{\partial \Phi}[S]\,, \quad \langle\phi_0|[\hat{U}^\dagger][\hat{\Phi}][\hat{U}]|\phi_0\rangle = -\frac{\partial}{\partial J}[S]\,, \tag{2·44}$$

which is called the *canonical-variable condition for the collective variables* $\{J, \Phi\}$ in the SCC method. When we choose

$$\frac{\partial}{\partial \Phi}[S]=0\,, \tag{2·45}$$

we obtain $\langle\phi_0|[\hat{U}^\dagger][\hat{J}_\Phi][\hat{U}]|\phi_0\rangle=J$, so that the generator $[\hat{J}_\Phi]\equiv(i\partial[\hat{U}]/\partial\Phi)[\hat{U}^\dagger]$ may be identified with the angular momentum operator in terms of the fermion operators, i.e.,

$$[\hat{J}_\Phi]=\hat{J}\,. \tag{2·46}$$

The set of basic equations [I], [II] and [III] of the SCC method, together with the choice (2·45) and the boundary condition in the small-amplitude RPA region,

$$J\to J_{\mathrm{RPA}}\,, \quad \Phi\to\Phi_{\mathrm{RPA}}\,, \quad \xi^*_\mu\to O^*_\mu\,, \tag{2·47}$$

enables us to determine the generating function F_{SCC} as well as the collective rotational Hamiltonian $\mathcal{H}_{\mathrm{coll}}(J)$.

§ 3. SCC method and cranking model

The classical image of the boson representation by means of the TDHF theory discussed in the previous section enables us to establish a manifest relationship between the MW full quantum theory of collective rotation and the conventional cranking model description through the mediation of the SCC method.

Let us introduce a class of TDHF states

$$|\phi_{\mathrm{SCC}}(t)\rangle=[\hat{U}]|\phi_0\rangle\,, \quad [\hat{U}][\hat{U}^\dagger]=1\,, \tag{3·1}$$

which corresponds to the rank $n=0$ part of Eq. (2·16) with

$$[\hat{U}(t)]\equiv\hat{U}_{\mathrm{SCC}}=e^{i[\hat{G}(t)]}\,, \quad [\hat{G}(t)]=\sum_{ai}\{g_{ai}(J,\Phi)\hat{c}^\dagger_a\hat{c}_i-g^*_{ai}(J,\Phi)\hat{c}^\dagger_i\hat{c}_a\}\,. \tag{3·2}$$

It satisfies

$$\delta\langle\phi_{\mathrm{SCC}}(t)|\left\{\left(i\frac{\partial}{\partial t}-\hat{H}\right)|\phi_{\mathrm{SCC}}(t)\rangle\right\}=0\,, \tag{3·3}$$

which is called the *invariance principle of the time-dependent Schrödinger equation* in the SCC method. Since $\hat{U}_{\mathrm{SCC}}$ is a function of only the collective variables $\{J, \Phi\}$, Eq. (3·3) is formally written as

$$\delta\langle\phi_{\mathrm{SCC}}(t)|\hat{H}-\dot{\Phi}\hat{J}+\dot{J}[\hat{\Phi}]|\phi_{\mathrm{SCC}}(t)\rangle=0\,, \tag{3·4}$$

where $[\hat{\Phi}]$ is the rank $n=0$ part of the generator $\hat{\Phi}$ defined in Eq. (2·36) and $\hat{J}$ is the angular momentum operator in Eq. (2·46).

Advantage of the use of the invariance principle of the time-dependent Schrödinger equation (3·3) is that the basic equations [I], [II] and [III] can be derived from it without the explicit use of the intrinsic variables $\{\xi_\mu^*, \xi_\mu\}$.

They are derived in the following way. By taking $|\delta\phi_{\rm SCC}\rangle \propto :[\hat{\Phi}]: |\phi_{\rm SCC}\rangle$ and $:\hat{J}: |\phi_{\rm SCC}\rangle$ in Eq. (3·4) and by using the rank $n=0$ parts of the commutation relations (2·38), we obtain the equations of collective motion

$$\text{[I]} \qquad \dot{\Phi}=\partial\mathcal{H}_{\rm coll}/\partial J\,, \quad \dot{J}=-\partial\mathcal{H}_{\rm coll}/\partial\Phi=0\,,$$
$$\mathcal{H}_{\rm coll}=\langle\phi_{\rm SCC}|\hat{H}|\phi_{\rm SCC}\rangle\equiv[\mathcal{H}]\,. \tag{3·5}$$

By taking $|\delta_\perp\phi_{\rm SCC}\rangle \propto :[\hat{X}_\mu^\dagger]: |\phi_{\rm SCC}\rangle$ and $:[\hat{X}_\mu]: |\phi_{\rm SCC}\rangle$ in Eq. (3·4), we obtain the equation of collective submanifold*)

$$\text{[II]} \qquad \delta_\perp\langle\phi_{\rm SCC}|\hat{H}|\phi_{\rm SCC}\rangle=0\,, \tag{3·6}$$

which means that the optimum collective submanifold Σ_R should be extracted in such a way that the expectation value of the Hamiltonian with the TDHF states (3·1) is stationary at each point on the surface Σ_R with respect to the variations perpendicular to the surface. The canonical-variable conditions for the collective variables (2·44) with Eq. (2·45) are simply written as

$$\text{[III]} \qquad \langle\phi_{\rm SCC}|\hat{J}|\phi_{\rm SCC}\rangle=J\,, \quad \langle\phi_{\rm SCC}|[\hat{\Phi}]|\phi_{\rm SCC}\rangle=-\frac{\partial}{\partial J}[S]\,. \tag{3·7}**)$$

Inserting Eq. (3·5) into Eq. (3·4), we obtain

$$\delta\langle\phi_{\rm SCC}|\hat{H}-\omega(J)\hat{J}|\phi_{\rm SCC}\rangle=0\,, \quad \omega(J)\equiv\frac{\partial}{\partial J}\mathcal{H}_{\rm coll}(J)=\frac{\partial}{\partial J}\langle\phi_{\rm SCC}|\hat{H}|\phi_{\rm SCC}\rangle\,. \tag{3·8}$$

Thus, the set of basic equations [I], [II] and [III] of the SCC method (Eqs. (2·42) ~(2·44)) is simply reduced to Eq. (3·8) with the canonical-variable conditions (3·7).

Now the relationship of the above formulation of the SCC method to the cranking model is self-evident.[17] With Eqs. (3·8) and (3·7) we may have

$$\delta\langle\phi_{\rm SCC}(\omega)|\hat{H}-\omega\hat{J}|\phi_{\rm SCC}(\omega)\rangle=0\,, \quad \langle\phi_{\rm SCC}(\omega)|\hat{J}|\phi_{\rm SCC}(\omega)\rangle=J \tag{3·9}$$

with the Lagrange multiplier ω. This is nothing but the basic equation of the constrained cranking model. For lower values of J, the equation is conventionally solved by the perturbation method with respect to ω. For higher values of J, it is well known that the equation is successfully solved self-consistently by making use of the steepest decent method.[18]

*) Equivalence of the expression (3·6) to that of Eq. (2·43) for the equation of collective submanifold is proved in Ref. 3).

**) See the footnote on p. 1244.

§ 4. Structure of low-spin states in ground-state rotational band

4.1. *Excitation energies of ground-state rotational band*

In order to visualize the occurrence dynamics of the collective rotation, in this section we analyze structure of low-spin states in the ground-state rotational bands of axially symmetric even-even deformed nuclei, by solving the basic equations of the SCC method. The excitation energies of the low-spin states in the ground bands of these nuclei are conventionally expressed, in an expansion form with respect to the angular momentum I,[6),*)]

$$E(I)=\mathcal{A}I(I+1)+\mathcal{B}I^2(I+1)^2+\cdots . \tag{4·1}$$

Provided that the coefficient $\mathcal{A}$ is given by the cranking model value $\mathcal{A}=1/2\mathcal{J}_0$ with the Thouless-Valatin moment of inertia $\mathcal{J}_0$, various investigations of the structure of the coefficient $\mathcal{B}$ have been done. Marshalek[20] treated the Coriolis force as a perturbation, and calculated $E(I)$. He separated $\mathcal{B}$ into some coupling terms by paying attention to the variation of the self-consistent field caused by the rotation. Ma et al.[21] separated $\mathcal{B}$ into some terms with the aid of the variable moment of inertia model.[22] Pavlichenkov[23] and Mikoshiba et al.[24] also separated $\mathcal{B}$ into some coupling terms by representing the variation of the self-consistent field in terms of the RPA operators and treating them as a perturbation. Semi-macroscopic approach should be also noted. One can obtain the contribution to $\mathcal{B}$ which comes from the coupling with the γ-vibration and the β-vibration from the experimental data of the transition probability based on the model of Bohr and Mottelson.[6] Faessler et al. also performed calculations by means of the rotation-vibration coupling model.[25] Physically the term with $\mathcal{B}$ expresses leading effects due to variations of the deformed mean field by the rotation.

In the SCC method, the dynamics of the occurrence of the optimum collective rotation is self-consistently specified by the basic equations [I], [II] and [III]. Especially the equation of collective submanifold [II] given in Eq. (3·6) is decisively important in determining the optimum collective rotation. It is, therefore, quite interesting to investigate the structure of the coefficient $\mathcal{B}$ by means of the SCC method: We then can clarify how the rotation-vibration coupling effects are self-consistently organized so as to construct the optimum collective rotation.

4.2. *Perturbative solution of basic equations of SCC method*

Since we are considering the low-spin states in the ground band of an axially symmetric deformed even-even nucleus, let us suppose that a rotational invariant Hamiltonian is given with the pairing-plus-quadrupole force, which is adequately treated by the Hartree-Fock-Bogoliubov quasiparticles $\{\tilde{a}_\alpha^\dagger, \tilde{a}_\alpha\}$[**)] rather than the

*) In the low-spin region we can use this expression, although the expression is worse than that of the ω-expansion in the high-spin region.

**) Hereafter, we use the convention of denoting the single-quasiparticle states of the deformed Hartree-Fock-Bogoliubov state $|\phi_0\rangle$ by indices α, β, ···. Thus, we have $\tilde{a}_\alpha|\phi_0\rangle=0$.

Hartree-Fock particles and holes $\{\hat{c}_a^\dagger, \hat{c}_i\ ;\ \hat{c}_a, \hat{c}_i^\dagger\}$ employed in §§ 2 and 3. For simplifying the problem, we further restrict ourselves to the collective rotation around an axis (1-axis) perpendicular to the symmetry axis (3-axis). The invariance principle of the time-dependent Schrödinger equation in the SCC method (in Eq. (3·3)) is then expressed as

$$\delta\langle\phi_{\rm SCC}|\left\{i\frac{\partial}{\partial t}-\hat{H}|\phi_{\rm SCC}\rangle\right\}=0\,,\quad |\phi_{\rm SCC}\rangle=\hat{U}_{\rm SCC}|\phi_0\rangle\,,$$
$$\hat{U}_{\rm SCC}\equiv[\hat{U}]=e^{-i\Phi\hat{J}}e^{i\hat{G}_{\rm SCC}(J)}\,, \tag{4·2}$$

where $\hat{J}$ is the component of the angular momentum to the 1-axis and $|\phi_0\rangle$ is the deformed Hartree-Fock-Bogoliubov ground state. The form of the unitary transformation $\hat{U}_{\rm SCC}$ in Eq. (4·2) is adopted so as to satisfy Eq. (2·46).[17] Then $i\hat{G}_{\rm SCC}(J)$ is an anti-Hermitian operator written in terms of the quasiparticles and can be expressed by means of the quasiparticle RPA operators as

$$i\hat{G}_{\rm SCC}(J)=iJ\hat{\Phi}_{\rm RPA}+\sum_{r\geq 2}J^r\{if_A(r)\hat{\Phi}_{\rm RPA}-if_M(r)\hat{J}_{\rm RPA}+\sum_\mu(g_\mu(r)\hat{O}_\mu^\dagger-g_\mu^*(r)\hat{O}_\mu)\}\,, \tag{4·3}$$

where the quasiparticle RPA operators $\{\hat{O}_\mu^\dagger, \hat{O}_\mu, \hat{J}_{\rm RPA}, \hat{\Phi}_{\rm RPA}\}$ satisfy

$$[\hat{H}, \hat{O}_\mu^\dagger]_{\rm RPA}=\omega_\mu\hat{O}_\mu^\dagger\,,\quad(\omega_\mu>0)$$
$$[\hat{O}_\mu, \hat{O}_\nu^\dagger]_{\rm RPA}=\delta_{\mu\nu}\,,\quad [\hat{O}_\mu, \hat{O}_\nu]_{\rm RPA}=0\,,$$
$$[\hat{H}, \hat{J}_{\rm RPA}]_{\rm RPA}=0\,,\quad [\hat{H}, \hat{\Phi}_{\rm RPA}]_{\rm RPA}=-i\hat{J}_{\rm RPA}/\mathcal{J}_0\,,$$
$$[\hat{\Phi}_{\rm RPA}, \hat{J}_{\rm RPA}]_{\rm RPA}=i\,,$$
$$[\hat{O}_\mu, \hat{\Phi}_{\rm RPA}]_{\rm RPA}=[\hat{O}_\mu, \hat{J}_{\rm RPA}]_{\rm RPA}=0\,, \tag{4·4}$$

corresponding to Eqs. (2·5) and (2·6), and the symbol $[\hat{A}, \hat{B}]_{\rm RPA}$ denotes the commutation relation under the quasiparticle RPA. The linear term in J of $i\hat{G}_{\rm SCC}(J)$ is chosen so as to satisfy the boundary condition that, in the small-amplitude RPA region, the collective variables $\{J, \Phi\}$ are reduced to

$$J\to\langle\phi_{\rm SCC}|\hat{J}_{\rm RPA}|\phi_{\rm SCC}\rangle\,,\quad \Phi\to\langle\phi_{\rm SCC}|\hat{\Phi}_{\rm RPA}|\phi_{\rm SCC}\rangle\,. \tag{4·5}$$

The unknown expansion coefficients, $f_A(r)$, $f_M(r)$ and $g_\mu(r)$ in $i\hat{G}_{\rm SCC}(J)$ are determined self-consistently so as to satisfy the basic equations (3·8) and (3·7) in each order of J.

It is shown[1] that the set of basic equations (3·5) and (3·6) can be reduced to an equation

$$\langle\phi_0|[e^{-i\hat{G}_{\rm SCC}(J)}\{\hat{H}-\omega(J)\hat{J}\}e^{i\hat{G}_{\rm SCC}(J)}, \hat{O}_\mu^\dagger]|\phi_0\rangle=0\,,$$
$$\omega(J)=\frac{d\mathcal{H}_{\rm coll}(J)}{dJ}\,, \tag{4·6}$$

accompanied with the canonical-variable condition (3·7). Provided that the Hartree-Fock-Bogoliubov ground state $|\phi_0\rangle$ has a definite time-reversal symmetry and satisfies $\langle\phi_0|\hat{J}|\phi_0\rangle=0$, we obtain a self-consistent solution of Eqs. (4·6) and (3·7) which are necessary for obtaining the expression (4·1), in the form

$$f_A(2)=0\,,$$

$$f_M(2)=0\,,$$

$$g_\mu(2)=\frac{1}{\omega_\mu}\left\{\frac{1}{2}\langle\phi_0|[\hat{H},\, i\hat{\Phi}_{\mathrm{RPA}},\, i\hat{\Phi}_{\mathrm{RPA}},\, \hat{O}_\mu]|\phi_0\rangle-\frac{1}{\mathcal{J}_0}\langle\phi_0|[\hat{J},\, i\hat{\Phi}_{\mathrm{RPA}},\, \hat{O}_\mu]|\phi_0\rangle\right\},$$

$$f_A(3)=-\langle\phi_0|[\hat{J},\, i\hat{\Phi}_{\mathrm{RPA}},\, \sum_\mu\{g_\mu(2)\hat{O}_\mu^\dagger-g_\mu^*(2)\hat{O}_\mu\}]|\phi_0\rangle$$

$$-\frac{1}{3!}\langle\phi_0|[\hat{J},\, i\hat{\Phi}_{\mathrm{RPA}},i\hat{\Phi}_{\mathrm{RPA}},\, i\hat{\Phi}_{\mathrm{RPA}}]|\phi_0\rangle\,,$$

$$f_M(3)=0 \tag{4·7}$$

with the choice of

$$[S]=0\,. \tag{4·8}$$

In Eq. (4·7) the symbol $[\hat{A},\hat{B},\hat{C}]$ denotes a multi-commutation relation

$$[\hat{A},\hat{B},\hat{C}]\equiv[[\hat{A},\hat{B}],\hat{C}]\,. \tag{4·9}$$

Evaluating the expectation value $\langle\phi_{\mathrm{SCC}}|\hat{H}|\phi_{\mathrm{SCC}}\rangle$ and replacing J^2 by $I(I+1)$, we obtain

$$\mathcal{B}_0=-\frac{1}{3!\mathcal{J}_0}\langle\phi_0|[\hat{J},\, i\hat{\Phi}_{\mathrm{RPA}},\, i\hat{\Phi}_{\mathrm{RPA}},\, i\hat{\Phi}_{\mathrm{RPA}}]|\phi_0\rangle$$

$$+\frac{1}{4!}\langle\phi_0|[\hat{H},\, i\hat{\Phi}_{\mathrm{RPA}},\, i\hat{\Phi}_{\mathrm{RPA}},\, i\hat{\Phi}_{\mathrm{RPA}},\, i\hat{\Phi}_{\mathrm{RPA}}]|\phi_0\rangle-\sum_\mu\omega_\mu|g_\mu(2)|^2, \tag{4·10}$$

which corresponds to $\mathcal{B}$ in Eq. (4·1).*)

The first and second terms in $\mathcal{B}_0$ are independent of the RPA vibrational modes $\{\hat{O}_\mu^\dagger,\hat{O}_\mu\}$, and the third term in $\mathcal{B}_0$ just corresponds to the effects due to the conventional rotation-vibration coupling with the RPA modes. It is however noted that, in the conventional rotation-vibration coupling approach, only the couplings with the "collective" vibrational modes (with the lowest eigenvalue for each K-quantum number) are explicitly taken into account. One of the characteristic features of the SCC method is that not only the coupling with the "collective" vibrational modes but also the coupling with the other many non-collective modes contribute to $\mathcal{B}_0$ explicitly. The formulation of Pavlichenkov[23] also has this feature.

4.3. *Numerical calculations for Er isotopes*

In order to visualize the various contributions to the coefficient $\mathcal{B}_0$ given in Eq. (4·10), we perform the numerical calculation for Er isotopes by using the model Hamiltonian with the pairing-plus-quadrupole force, which is treated by the Hartree-Fock-Bogoliubov quasiparticles. Since the quasiparticle representation based on the deformed Hartree-Fock-Bogoliubov ground state $|\phi_0\rangle$ breaks down the particle (proton and neutron) number conservations, there also occur the Goldstone modes associated with the broken symmetry. The fluctuations of the pairing field induced

*) We have used the notation $\mathcal{B}_0$ in Eq. (4·10), since we will use the notation $\mathcal{B}$ in Eq. (4·23) for an extended case.

by the rotation are thus interwoven with such Goldstone modes. In obtaining the expression (4·10), we have not taken account of such Goldstone modes. We therefore make the numerical calculation in the following way: First of all, we perform the numerical calculation of Eq. (4·10) by employing a model Hamiltonian without the pairing residual interaction. In this case the correlations for the RPA modes arise from the quadrupole force. In the next subsection, we treat the effects of the pairing fluctuations by carefully taking into account the Goldstone modes associated with the broken particle-number conservations.

The first model Hamiltonian thus consists of a one-body Hamiltonian with the quadrupole-force two-body interaction. It is rotationally invariant but breaks the particle-number conservations, and is given by

$$\hat{H}_{\rm I}=\hat{h}_{\rm sho}+v_{ls}\,\hat{\boldsymbol{l}}\cdot\hat{\boldsymbol{s}}+\sum_{\tau={\rm p,n}}v_{ll}(\tau)(\hat{\boldsymbol{l}}^2-\sum_k\langle\hat{\boldsymbol{l}}^2\rangle_{N_{\rm osc}}\hat{c}_k^\dagger\hat{c}_k)$$
$$-\sum_{\tau={\rm p,n}}\{\Delta_\tau(\hat{P}_\tau+\hat{P}_\tau^\dagger)+\lambda_\tau\hat{N}_\tau\}-\frac{1}{2}\chi\sum_{M=-2}^{2}\hat{Q}_M^\dagger\hat{Q}_M\,, \tag{4·11}$$

where $\hat{h}_{\rm sho}$ describes the single-particle Hamiltonian based on the spherical harmonic oscillator,*)

$$\hat{h}_{\rm sho}=\sum_k\hbar\omega_0(N_{\rm osc}+3/2)\,\hat{c}_k^\dagger\hat{c}_k \tag{4·12}$$

with $\hat{c}_k^\dagger$ and $\hat{c}_k$ being the nucleon creation and annihilation operators of the single-particle state k of the oscillator, respectively. For simplifying the problem, two major shells are used for the calculation, i.e., $N_{\rm osc}=4$ and 5 for the proton and $N_{\rm osc}=5$ and 6 for the neutron. The terms $\hat{\boldsymbol{l}}\cdot\hat{\boldsymbol{s}}$ and $\hat{\boldsymbol{l}}^2$ in Eq. (4·11) are the conventional one-body operators defined by

$$\hat{\boldsymbol{l}}\cdot\hat{\boldsymbol{s}}\equiv\sum_{kk'}(\boldsymbol{l}\cdot\boldsymbol{s})_{kk'}\hat{c}_k^\dagger\hat{c}_{k'}\,,\quad \hat{\boldsymbol{l}}^2\equiv\sum_{kk'}(\boldsymbol{l}^2)_{kk'}\hat{c}_k^\dagger\hat{c}_{k'}\,, \tag{4·13}$$

where $\hat{\boldsymbol{l}}$ and $\hat{\boldsymbol{s}}$ are the single-particle orbital angular momentum and its spin, respectively, and $\langle\hat{\boldsymbol{l}}^2\rangle_{N_{\rm osc}}=N_{\rm osc}(N_{\rm osc}+3)/2$ is the expectation value of $\hat{\boldsymbol{l}}^2$ averaged over each major shell with a definite value of $N_{\rm osc}$. The quantities $\hat{Q}_M(M=0,\pm1,\pm2)$, $\hat{P}_\tau(\tau={\rm p,n})$ and $\hat{N}_\tau$ are the quadrupole operators, the conventional monopole pair-operators and the number operators for the proton (p) and the neutron (n), respectively. The adopted values of the parameters in Eq. (4·11) are the following:

$$\hbar\omega_0=41/A^{1/3}\ {\rm MeV}\,,^{6)}\qquad v_{ls}=-0.1270\hbar\omega_0\ {\rm MeV}\,,$$
$$v_{ll}({\rm n})=-0.0268\hbar\omega_0\ {\rm MeV}\,,\qquad v_{ll}({\rm p})=-0.0382\hbar\omega_0\ {\rm MeV}\,,^{6)}$$
$$\chi=240/A^{5/3}(M\omega_0/\hbar)^2\times\alpha\ {\rm MeV}\,,$$

where A and M are the mass number and the nucleon mass, respectively, and α is a correction factor with the value of 1.02–1.03 by which the experimental energy of the β-vibration is reproduced by the RPA within the difference of about 10 %. For derivation of the value $\chi=240/A^{5/3}(M\omega_0/\hbar)^2$ MeV, see, for example, Ref. 26). The

*) In the numerical calculation, we do not employ the convention $\hbar=1$ which is used throughout the formulation in this paper.

Table I. Parameters used in the Hamiltonian (4·11).

	χ $\left[\left(\frac{M\omega_0}{\hbar}\right)^2 \text{MeV}\right]$	Δ_p [MeV]	Δ_n [MeV]	λ_p [MeV]	λ_n [MeV]
^{160}Er	0.050895	1.0544	1.1287	44.37	48.47
^{162}Er	0.051348	1.0470	1.0522	44.20	48.50
^{164}Er	0.050309	0.9855	1.0378	44.04	48.74
^{166}Er	0.048824	0.8762	0.9640	43.89	49.01
^{168}Er	0.048328	0.8305	0.7755	43.71	49.26

values of the pairing fields Δ_τ are determined from the experimental odd-even mass differences,[27] and the chemical potentials λ_τ are fixed so as to satisfy the particle-number constraints for the ground state $|\phi_0\rangle$. These values are given in Table I together with the values of χ.

We summarize the procedure of the numerical calculation. At first, we solve the Hartree-Bogoliubov equation and determine the deformed ground state $|\phi_0\rangle$, by which the single-particle (quasiparticle) base with the signature representation[28] is constructed. The exchange term of the quadrupole interaction has been neglected. Under this condition, the (quasiparticle) RPA modes with the negative signature have no contribution to $\mathcal{B}_0$. The contributions of the RPA modes with higher energies are very small, so we have adopted about 250 RPA modes in the order of the lowest in energy, except the special modes which have zero matrix elements of the quadrupole operators. The highest excitation energy of the adopted RPA modes is 7.0–7.5 MeV.

In Figs. 1 and 2, the calculated values of $\mathcal{A}$ and $\mathcal{B}_0$ in Eq. (4·10) for Er isotopes are shown, respectively. In order to compare with the experimental values, we also show the values of $\mathcal{A}$ deduced from the energy of the lowest $I^\pi=2^+$ state[29] and $\mathcal{B}$ obtained by the 4-parameter fitting with the expression

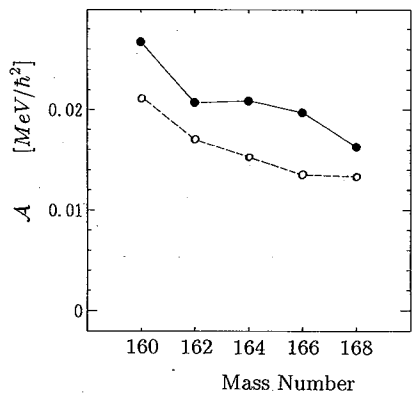

Fig. 1. Parameter $\mathcal{A}$ for Er isotopes as a function of the mass number. Solid circles indicate the results of our calculation with the Hamiltonian (4·11). Open circles are deduced from the experimental energy at $I^\pi=2^+$.[29]

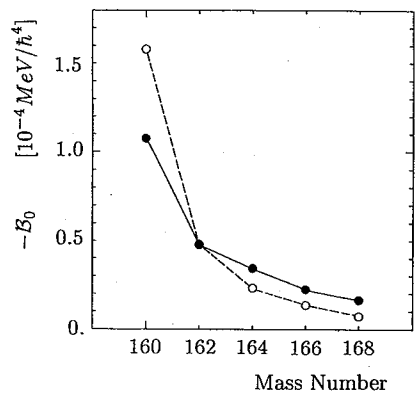

Fig. 2. Parameter $-\mathcal{B}_0$ for Er isotopes as a function of the mass number. Solid circles indicate the results of our calculation with the Hamiltonian (4·11). Open circles are the results of the 4-parameter fitting with Eq. (4·14) to the experimental energies up to $I^\pi=8^+$ state.

$$E(I)=\mathcal{A}I(I+1)+\mathcal{B}(I(I+1))^2+\mathcal{C}(I(I+1))^3+\mathcal{D}(I(I+1))^4 \qquad (4\cdot14)$$

to the experimental energies up to $I^\pi=8^+$ state.

The magnitudes of both $\mathcal{A}$ and $\mathcal{B}_0$ of our calculations are larger than those of the experimental ones.*) The problem of the larger $\mathcal{A}$ values, which means small moments of inertia, has been studied as a problem of the effective interaction. The introduction of the quadrupole-pairing interaction[30] may be one of the possibilities to revise the problem.

Figure 3 shows the contribution of the rotation-vibration coupling by each RPA mode, separately. In Fig. 4, the sum of the contribution of the rotation-vibration couplings (the third term in Eq. (4·10)) and the contribution of the first and second

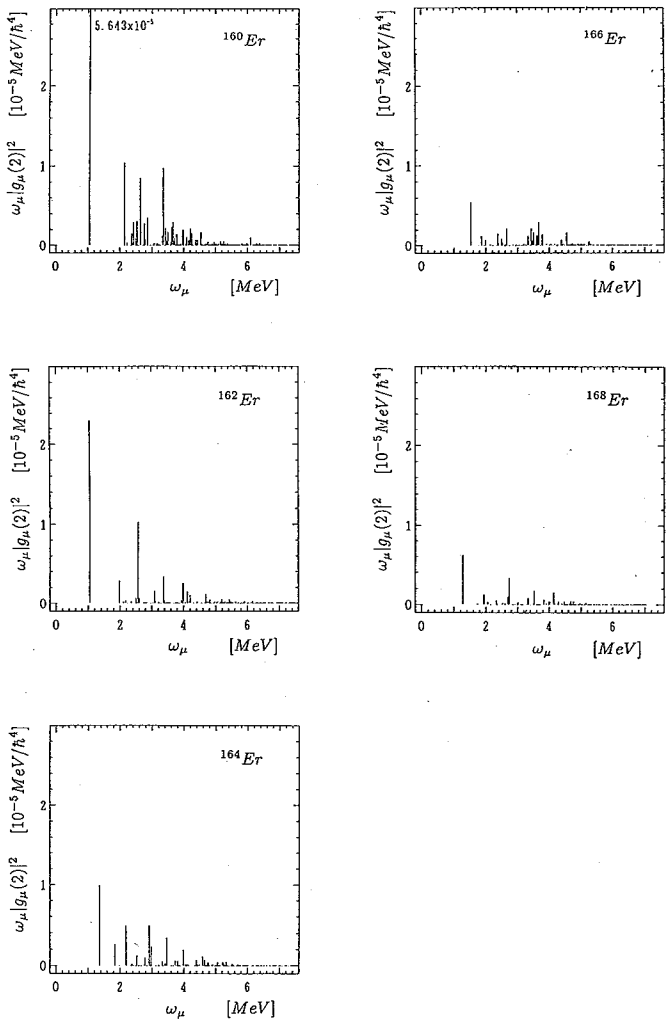

Fig. 3. The rotation-vibration coupling $\omega_\mu|g_\mu(2)|^2$ with the RPA eigenenergy ω_μ. Note that the contribution to $\mathcal{B}_0$ is $-\omega_\mu|g_\mu(2)|^2$.

*) Although the reduction of the magnitude of the pairing fields by about 20-30% brings much better result, such reduction becomes inconsistent with the odd-even mass difference.

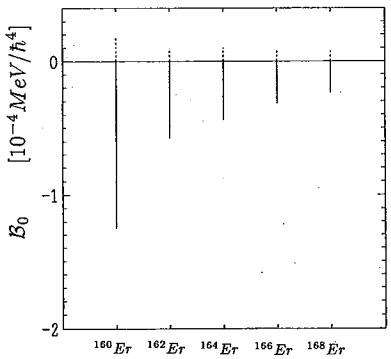

Fig. 4. Contribution of the sum of the rotation-vibration couplings and the other terms to the $\mathcal{B}_0$ parameter. Solid line indicates the sum of the rotation-vibration couplings and dotted line shows the other terms. Pairing residual interaction is not included.

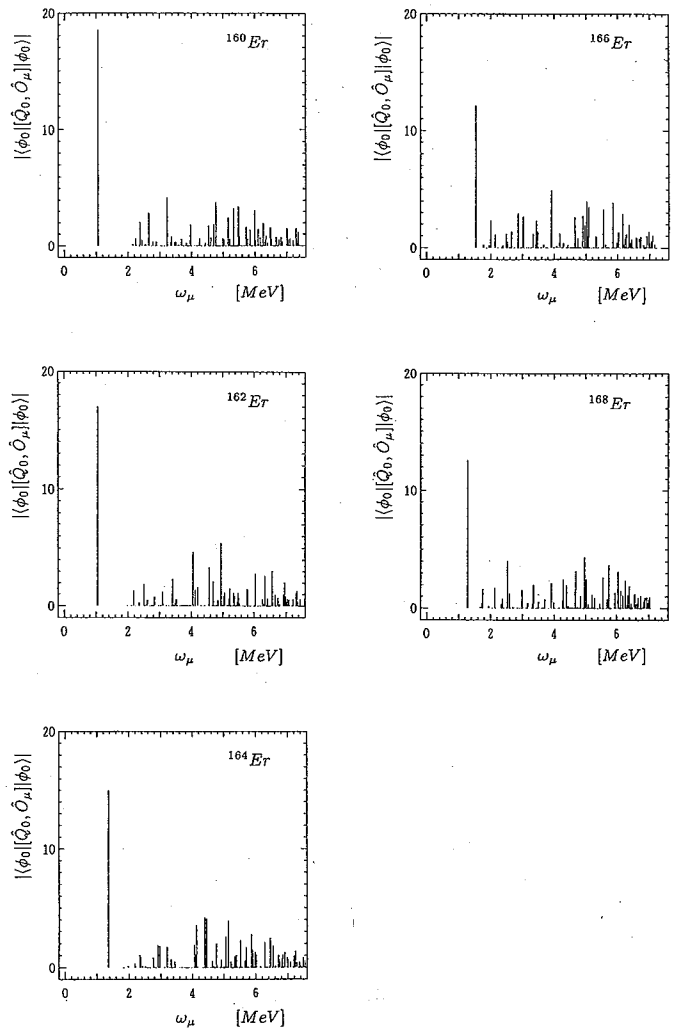

Fig. 5. The strength function $|\langle\phi_0|[\tilde{Q}_0, \tilde{O}_\mu]|\phi_0\rangle|$ in a free scale. The abscissa denotes the RPA eigenenergy ω_μ.

terms are shown by the solid line and the dashed line, respectively. It is clear that the contribution of the first and second terms is always positive, and its isotope depen-

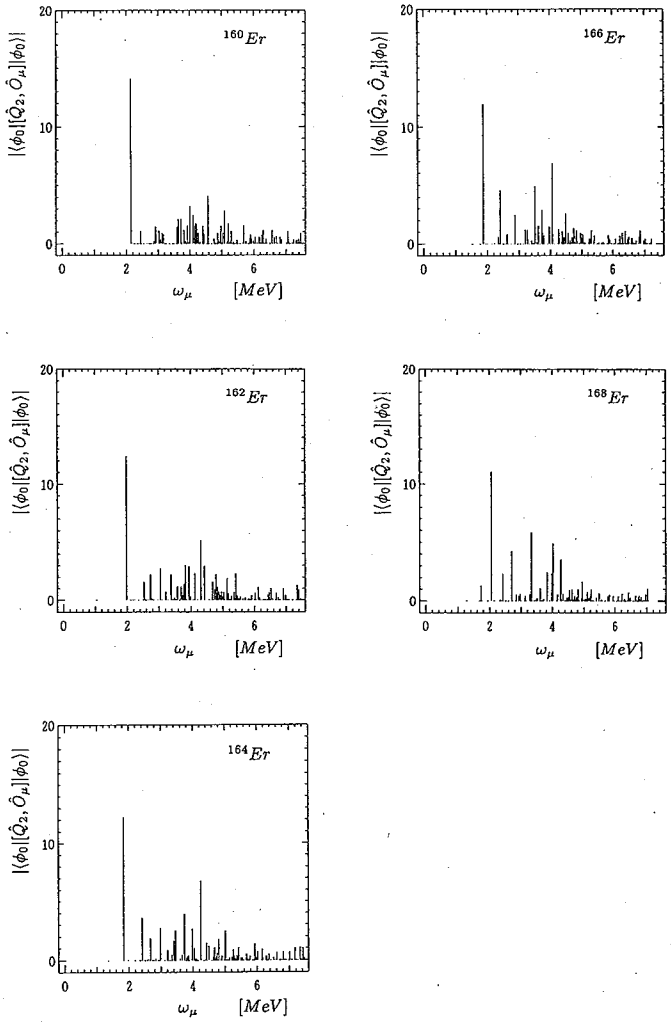

Fig. 6. The strength function $|\langle\phi_0|[\hat{Q}_2, \hat{O}_\mu]|\phi_0\rangle|$ in a free scale as a function of the RPA eigenenergy ω_μ.

dence is less prominent than that of the rotation-vibration couplings. In Figs. 5 and 6, the strength functions with $\hat{Q}_{M=0}$ and $\hat{Q}_{M=2}$ are shown, respectively. By comparing Fig. 3 with Figs. 5 and 6, it is clearly seen that the rotation-vibration couplings with $K=2$ modes are smaller than those of $K=0$ modes,*) and the largest coupling, which is the one with the β-vibration, shows especially strong neutron number dependence: The magnitude increases with the neutron number decreasing. Here, it should be remembered that the transition from the rotational nuclei to the vibrational nuclei is known to occur at the neutron number $N\sim 88$–90,[31] and ^{154}Er is clearly a vibrational nucleus.[32]

*) The energy of the γ-vibration in our calculation is overestimated compared with the experimental value due to the fact that the adopted strength of the QQ-force in the spherical base is K-independent because of the rotational invariance. This may make the magnitude of the rotation-γ-vibration coupling underestimated to some extent.

4.4. *Contribution of pairing residual interaction*

We are now at the stage to investigate the contribution of the pairing fluctuation to $\mathcal{B}$ in Eq. (4·1) by carefully taking account of the Goldstone modes associated with the broken particle-number conservations. The Hamiltonian adopted is that of the pairing-plus-quadrupole force model:

$$\hat{H}=\hat{h}_{\rm sho}+v_{ls}\hat{\boldsymbol{l}}\cdot\hat{\boldsymbol{s}}+\sum_{\tau={\rm p,n}}v_{ll}(\tau)(\hat{\boldsymbol{l}}^2-\sum_k\langle\hat{\boldsymbol{l}}^2\rangle_{N_{\rm osc}}\hat{c}_k^\dagger\hat{c}_k)-\sum_{\tau={\rm p,n}}\lambda_\tau\hat{N}_\tau$$
$$-\frac{\chi}{2}\sum_{M=-2}^{2}\hat{Q}_M^\dagger\hat{Q}_M-\sum_{\tau={\rm p,n}}G_\tau\hat{P}_\tau^\dagger\hat{P}_\tau\,, \qquad (4\cdot15)$$

which is rotationally invariant and conserves the neutron and proton numbers. The strength of the pairing force $G_\tau(\tau={\rm p, n})$ are determined so as to reproduce the pairing field Δ_τ of the ground state, and are given in Table II. The other parameters are the same as those in $\hat{H}_{\rm I}$ in Eq. (4·11). Physically the coupling of the rotational motion with the Goldstone modes associated with the broken particle-number conservation originates from the coupling between the spatial rotation and the so-called pairing rotation. The coupling can be precisely treated by the SCC method in the following way:[33] In addition to the variables $\{J, \Phi\}$, we introduce the collective variables $\{N_\tau, \Theta_\tau;\ \tau={\rm p, n}\}$ which describes the particle numbers and their gauge angles. Then, the state $|\phi_{\rm SCC}\rangle$ in Eq. (4·2) is extended as

$$|\phi_{\rm SCC}\rangle=\hat{U}_{\rm SCC}|\phi_0\rangle\,,\quad \hat{U}_{\rm SCC}=e^{-i\Phi\hat{J}}e^{-i\Theta\hat{N}}e^{i\hat{G}_{\rm SCC}(J,N)}\,, \qquad (4\cdot16)$$

where we have used the $\hbar=1$ convention employed in the formulation, and, for simplicity, an abbreviation such as $i\Theta\hat{N}=i\sum_\tau\Theta_\tau\hat{N}_\tau$ is adopted hereafter by dropping the subscript τ. The equations of collective motion (3·5) are then written as

$$\text{[I]}\quad \dot{\Phi}=\frac{\partial\mathcal{H}_{\rm coll}}{\partial J}\,,\quad \dot{J}=-\frac{\partial\mathcal{H}_{\rm coll}}{\partial\Phi}=0\,,$$
$$\dot{\Theta}=\frac{\partial\mathcal{H}_{\rm coll}}{\partial N}\,,\quad \dot{N}=-\frac{\partial\mathcal{H}_{\rm coll}}{\partial\Theta}=0\,,$$
$$\mathcal{H}_{\rm coll}(J,N)\equiv\langle\phi_{\rm SCC}|\hat{H}|\phi_{\rm SCC}\rangle\,. \qquad (4\cdot17)$$

The equation of collective submanifold (3·6) is expressed as

$$\text{[II]}\quad \delta\langle\phi_{\rm SCC}|\Big(\hat{H}-\frac{\partial\mathcal{H}_{\rm coll}}{\partial J}\hat{J}-\frac{\partial\mathcal{H}_{\rm coll}}{\partial N}\hat{N}\Big)|\phi_{\rm SCC}\rangle=0\,, \qquad (4\cdot18)$$

which simply means $\delta_\perp\langle\phi_{\rm SCC}|\hat{H}|\phi_{\rm SCC}\rangle=0$. The canonical-variable condition can be adopted as

$$\text{[III]}\quad \langle\phi_{\rm SCC}|\hat{J}|\phi_{\rm SCC}\rangle=J\,,$$
$$\langle\phi_{\rm SCC}|\frac{\partial}{\partial J}|\phi_{\rm SCC}\rangle=0\,,$$
$$\langle\phi_{\rm SCC}|\hat{N}|\phi_{\rm SCC}\rangle=N+N_0\,,$$

Table II. Strength of the pairing interaction used in the Hamiltonian (4·15).

	$G_{\rm p}$ [MeV]	$G_{\rm n}$ [MeV]
^{160}Er	0.17225	0.14565
^{162}Er	0.17421	0.14729
^{164}Er	0.17128	0.14413
^{166}Er	0.16283	0.14038
^{168}Er	0.16089	0.13141

$$\langle\phi_{\rm SCC}|\frac{\partial}{\partial N}|\phi_{\rm SCC}\rangle=0 \tag{4·19}$$

with $N_0\equiv\langle\phi_0|\hat{N}|\phi_0\rangle$.

As immediately seen from [I]~[III], the treatment of the pairing rotation is parallel to that of the rotation. Thus, we put $i\hat{G}_{\rm SCC}(J,N)$ as

$$\begin{aligned}i\hat{G}_{\rm SCC}(J,N)=&iJ\hat{\Phi}_{\rm RPA}+iN\hat{\Theta}_{\rm RPA}+\sum_{n\geq 2}\sum_{r+s=n}J^rN^s\{if_A(r,s)\hat{\Phi}_{\rm RPA}-if_M(r,s)\hat{J}_{\rm RPA}\\&+if_{NA}(r,s)\hat{\Theta}_{\rm RPA}-if_{NM}(r,s)\hat{N}_{\rm RPA}+\sum_\mu(g_\mu(r,s)\hat{O}_\mu^\dagger-g_\mu^*(r,s)\hat{O}_\mu)\},\end{aligned} \tag{4·20}$$

where $\{\hat{O}_\mu^\dagger, \hat{O}_\mu, \hat{J}_{\rm RPA}, \hat{\Phi}_{\rm RPA}\}$ are the quasiparticle RPA operators defined in Eq. (4·4), and $\hat{N}_{\rm RPA}$ and $\hat{\Theta}_{\rm RPA}$ are the number operator (corresponding to the Goldstone mode) and the gauge-angle operator in the quasiparticle RPA, respectively. The quasiparticle RPA operators $\hat{N}_{\rm RPA}$ and $\hat{\Theta}_{\rm RPA}$ satisfy

$$[\hat{H}, i\hat{\Theta}_{\rm RPA}]_{\rm RPA}=\hat{N}_{\rm RPA}/\mathcal{J}_{\rm N}, \quad [\hat{N}_{\rm RPA}, i\hat{\Theta}_{\rm RPA}]_{\rm RPA}=1. \tag{4·21}$$

The unknown coefficients $f_A(r,s)$, $f_M(r,s)$, $f_{NA}(r,s)$, $f_{NM}(r,s)$ and $g_\lambda(r,s)$ are determined self-consistently so as to satisfy the basic equations (4·18) and (4·19) in each order of the expansion. Since the variable N means the difference of particle numbers between the ground state and the excited states in the pairing rotation, in fact, we can put $N=0$ after the desired quantities have been calculated. We thus obtain the following results with the replacement of J by $\{I(I+1)\}^{1/2}$,

$$\mathcal{H}_{\rm coll}(J,N=0)={\rm const}+\mathcal{A}I(I+1)+\mathcal{B}I^2(I+1)^2+\cdots, \tag{4·22}$$

where $\mathcal{A}=1/2\mathcal{J}_0$ is determined by Eq. (4·4) and

$$\begin{aligned}\mathcal{B}=&\mathcal{B}_0-\frac{1}{\mathcal{J}_0}f_{NA}(2,0)\\&\times\langle\phi_0|[\hat{J}, i\hat{\Phi}_{\rm RPA}, i\hat{\Theta}_{\rm RPA}]|\phi_0\rangle\\&+\frac{1}{2\mathcal{J}_{\rm N}}f_{NA}^2(2,0)+\frac{1}{2}f_{NA}(2,0)\\&\times\langle\phi_0|[\hat{H}, i\hat{\Phi}_{\rm RPA}, i\hat{\Phi}_{\rm RPA}, i\hat{\Theta}_{\rm RPA}]|\phi_0\rangle\end{aligned} \tag{4·23}$$

with

$$f_{NA}(2,0)=-\frac{1}{2}\langle\phi_0|[\hat{N}, i\hat{\Phi}_{\rm RPA}, i\hat{\Phi}_{\rm RPA}]|\phi_0\rangle. \tag{4·24}$$

In Eq. (4·23) $\mathcal{B}_0$ has the same expression as that given in Eq. (4·10). Thus, we

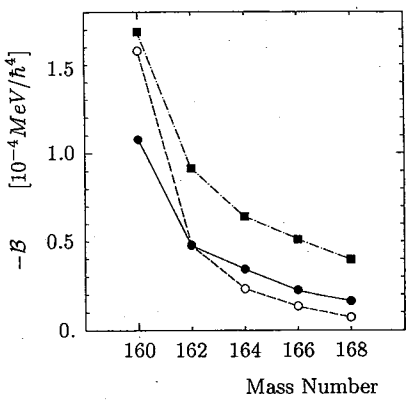

Fig. 7. The coefficients of $I^2(I+1)^2$ in the excitation energy with the inverted sign. Solid square denotes $-\mathcal{B}$ obtained with the pairing residual interaction. Solid circles and open circles are the same as those in Fig. 2.

have two types of contribution of the pairing fluctuations to $\mathcal{B}$. The second, third and fourth terms in Eq. (4·23) represent the contribution of the Goldstone modes associated with the broken particle-number conservations, and the contribution of the pairing fluctuations through the correlations producing the RPA modes are given in the form of $\mathcal{B}_0$.

The calculated values of $\mathcal{B}$ in Eq. (4·23) for Er isotopes are plotted in Fig. 7 together with those values obtained in the previous subsection as well as the experimental values. The magnitude of $\mathcal{B}$ is appreciably larger than the $\mathcal{B}_0$ value calculated in the previous subsection. This demonstrates that the contribution of the pairing fluctuation to the rotation is quite large. In order to analyze the contribution of the pairing fluctuations through the rotation-vibration coupling term, which corre-

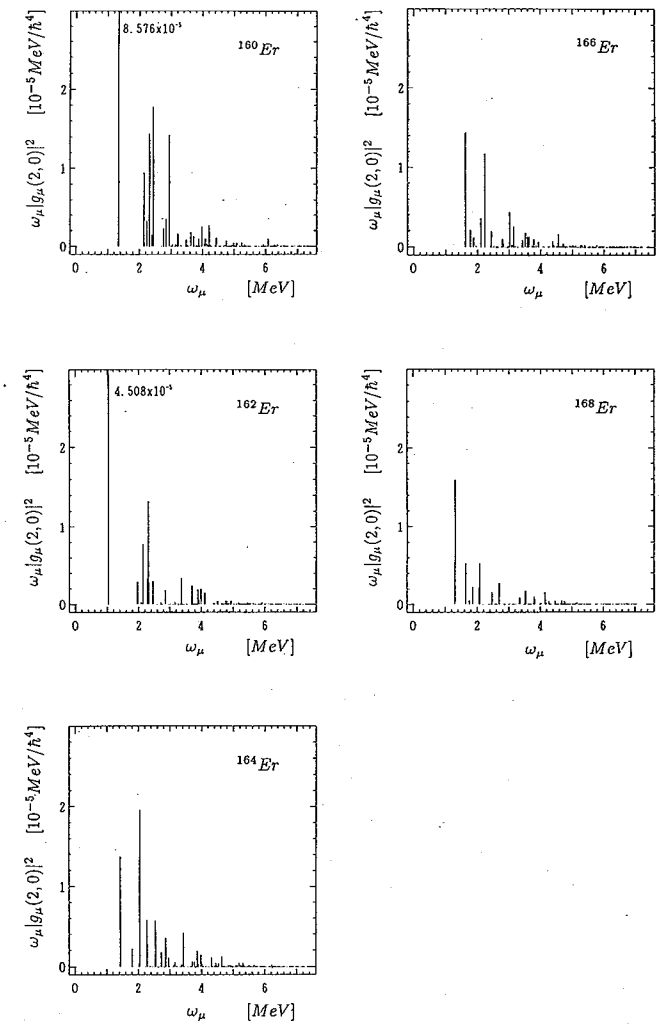

Fig. 8. The rotation-vibration coupling $\omega_\mu|g_\mu(2,0)|^2$ calculated from the Hamiltonian (4·15). Note that contribution to $\mathcal{B}$ is $-\omega_\mu|g_\mu(2,0)|^2$.

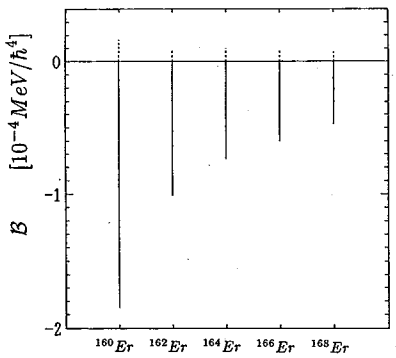

Fig. 9. The same as Fig. 4 but the pairing residual interaction is included.

sponds to the third term of $\mathcal{B}_0$ in the expression (4·10) with

$$g_\mu(2,0)=\frac{1}{\omega_\mu}\times\left\{\frac{1}{2}\langle\phi_0|[\hat{H}, i\hat{\Phi}_{\mathrm{RPA}}, i\hat{\Phi}_{\mathrm{RPA}}, \hat{O}_\mu]|\phi_0\rangle - \frac{1}{\mathcal{J}_0}\langle\phi_0|[\hat{J}, i\hat{\Phi}_{\mathrm{RPA}}, \hat{O}_\mu]|\phi_0\rangle\right\}, \tag{4·25}$$

we show in Fig. 8 the contribution of the rotation-vibration coupling by each RPA

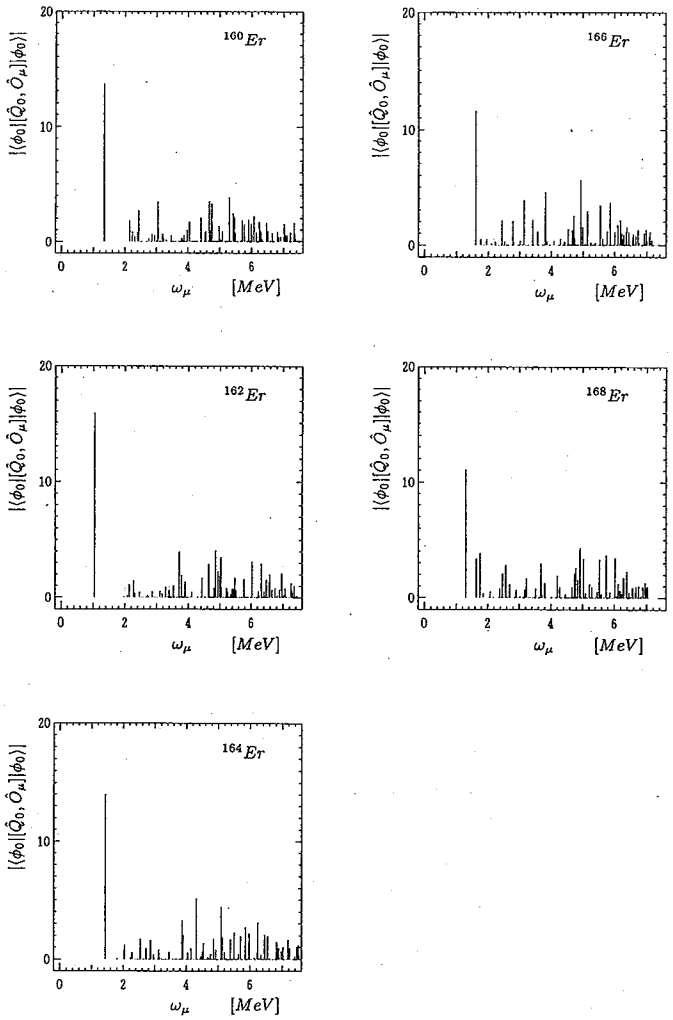

Fig. 10. The strength function $|\langle\phi_0|[\hat{Q}_0, \hat{O}_\mu]|\phi_0\rangle|$ in a free scale, $\hat{O}_\mu$ is derived from the RPA equation with the Hamiltonian (4·15).

mode with $\omega_\mu \neq 0$. By comparing Fig. 8 with Fig. 3 and also from Fig. 7, we can see how the pairing residual interaction contributes to the rotation through the correlations producing the RPA modes: The effect of the pairing residual interaction is not concentrated on a single coupling, but rather distributed to many couplings. The sum of the couplings and the other contribution are shown in Fig. 9.

The coupling with the Goldstone mode associated with the pairing rotation, which is represented by the second, third and fourth terms in Eq. (4·23), is smaller than the other terms by a factor of $\sim 10^{-2}$. In Figs. 10 and 11, the strength functions with $\hat{Q}_{M=0}$ operator and the pair operator $\hat{P}^\dagger + \hat{P}$ are shown, respectively. It is seen that the coupling with the β-vibration is important for the lighter nuclei, while the contribution of the pairing correlation is relatively important in the heavier nuclei, as shown by Fig. 7. This point is consistent with the analysis of Mikoshiba et al.[24]

4.5. *Microscopic structure of rotation-vibration coupling*

In this subsection, in order to understand microscopic structure of the rotation-

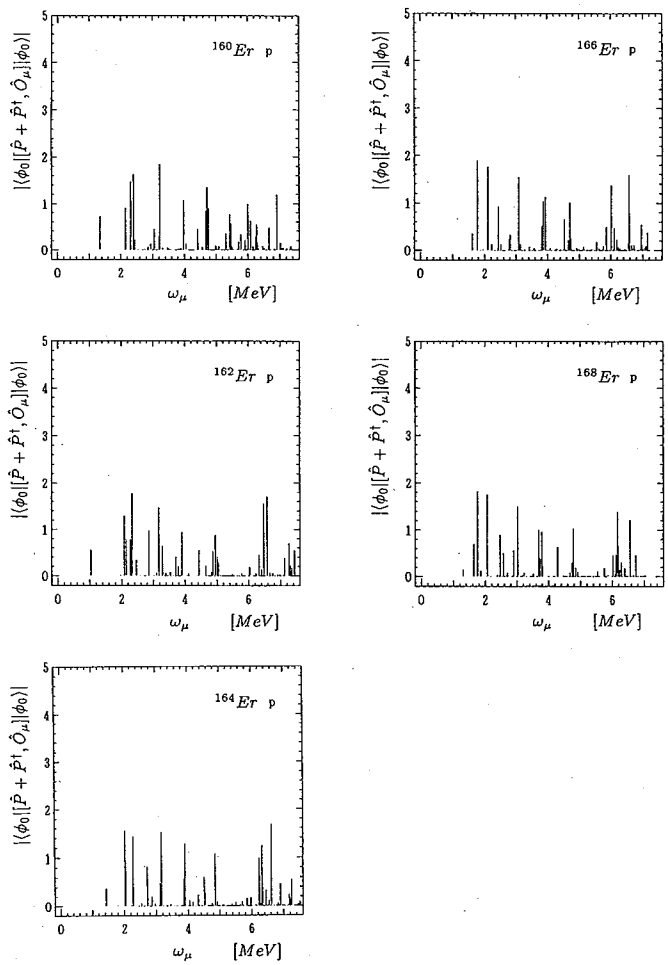

Fig. 11. (a) The strength function $|\langle\phi_0|[\hat{P}^\dagger + \hat{P}, \hat{O}_\mu]|\phi_0\rangle|$ in a free scale for the proton.

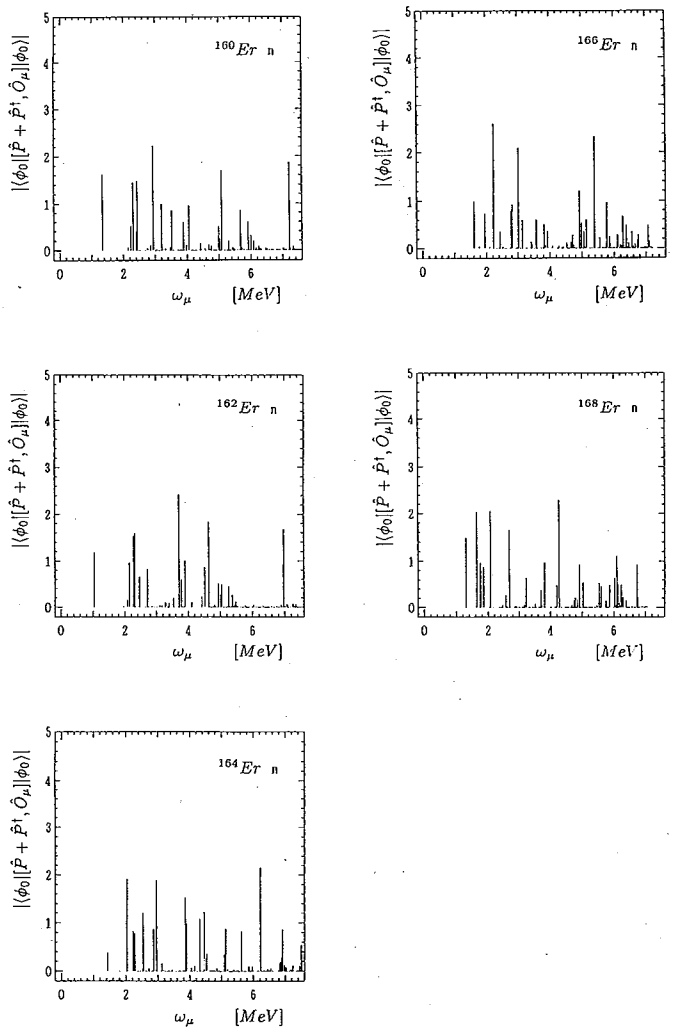

Fig. 11. (b) The strength function $|\langle\phi_0|[\hat{P}^\dagger+\hat{P}, \hat{O}_\mu]|\phi_0\rangle|$ in a free scale for the neutron.

vibration coupling, we investigate what components of the quasiparticle excitations are effective for the rotation-vibration coupling. The quantity $g_\mu(2, 0)$ in Eq. (4·25) can be separated into

$$g_\mu(2,0)=g_\mu(P_\mathrm{p})+g_\mu(P_\mathrm{n})+g_\mu(Q)+g_\mu(R)\,,$$

$$g_\mu(P_\tau)=\frac{1}{\omega_\mu}\frac{1}{2}\langle\phi_0|[-G_\tau{:}\hat{P}_\tau^\dagger\hat{P}_\tau{:}, i\hat{\Phi}_\mathrm{RPA}, i\hat{\Phi}_\mathrm{RPA}, \hat{O}_\mu]|\phi_0\rangle\,,\quad(\tau=\mathrm{p,\,n})$$

$$g_\mu(Q)=\frac{1}{\omega_\mu}\frac{1}{2}\langle\phi_0|\left[-\frac{\chi}{2}\sum_\nu{:}\hat{Q}_\nu^\dagger\hat{Q}_\nu{:}, i\hat{\Phi}_\mathrm{RPA}, i\hat{\Phi}_\mathrm{RPA}, \hat{O}_\mu\right]|\phi_0\rangle\,,$$

$$g_\mu(R)=-\frac{1}{\omega_\mu}\frac{1}{\mathcal{J}_0}\langle\phi_0|[\hat{J}, i\hat{\Phi}_\mathrm{RPA}, \hat{O}_\mu]|\phi_0\rangle\,. \qquad (4\cdot26)$$

The $g_\mu(R)$ term originates from $(\partial\mathcal{H}_{\rm coll}/\partial J)\hat{J}$ in the equation of submanifold (4·18), and we call it a direct term hereafter. The quantities $g_\mu(P_\tau)$ and $g_\mu(Q)$ are due to the coupling through the pairing residual interaction and the quadrupole residual interaction, respectively.

In order to investigate these coupling effects microscopically, we explicitly employ the quasiparticles $\{\hat{a}^\dagger_\alpha, \hat{a}^\dagger_{\bar{\alpha}}, \hat{a}_\alpha, \hat{a}_{\bar{\alpha}}\}$ in the signature representation,*) which we have used so far. Then we have

$$\hat{O}^\dagger_\mu=\sum_{\alpha\beta}\{\psi^\mu_{\alpha\beta}\hat{a}^\dagger_\alpha\hat{a}^\dagger_{\bar{\beta}}+\varphi^\mu_{\alpha\beta}\hat{a}_\alpha\hat{a}_{\bar{\beta}}\},$$

$$i\hat{\Phi}_{\rm RPA}=\sum_{\alpha\beta}\Phi_{\alpha\beta}(\hat{a}^\dagger_\alpha\hat{a}^\dagger_{\bar{\beta}}+\hat{a}_\alpha\hat{a}_{\bar{\beta}}), \tag{4·27}$$

and the angular momentum operator $\hat{J}$ is explicitly written in the form

$$\hat{J}=\sum_{\alpha\beta\alpha'\beta'}(\hat{a}^\dagger_\alpha,\ \hat{a}_{\bar{\beta}})\begin{pmatrix}J^{11}_{\alpha\alpha'} & J^{12}_{\alpha\beta'}\\ J^{21}_{\beta\alpha'} & J^{22}_{\beta\beta'}\end{pmatrix}\begin{pmatrix}\hat{a}_{\alpha'}\\ \hat{a}^\dagger_{\bar{\beta}'}\end{pmatrix}, \tag{4·28}$$

where

$$J^{11}_{\alpha\alpha'}=J_{\alpha\alpha'}(u_\alpha u_{\alpha'}+v_\alpha v_{\alpha'}),\qquad J^{12}_{\alpha\beta'}=J_{\alpha\beta'}(-u_\alpha v_{\beta'}s_{\beta'}+u_{\beta'}v_\alpha s_{\beta'}),$$
$$J^{21}_{\beta\alpha'}=J_{\beta\alpha'}(-v_\beta s_\beta u_{\alpha'}+v_{\alpha'}s_\beta u_\beta),\qquad J^{22}_{\beta\beta'}=J_{\beta\beta'}(v_\beta v_{\beta'}+u_\beta u_{\beta'})s_\beta s_{\beta'}. \tag{4·29}$$

Here, s_α is a phase factor which connects the time-reversal inversion with the signature inversion,**) and $J_{\alpha\beta}$ are the matrix elements of the angular momentum operator with the deformed single-particle base, which is assumed to be connected with the quasiparticle base by a BCS transformation. In the lowest approximation, $i\hat{\Phi}_{\rm RPA}$ is given by

$$\Phi_{\alpha\beta}\simeq\frac{1}{\mathcal{J}_0}\frac{J^{12}_{\alpha\beta}}{E_\alpha+E_{\bar{\beta}}}, \tag{4·30}$$

where E_α is the quasiparticle energy. Using Eqs. (4·27)~(4·30), we obtain

$$\omega_\mu g_\mu(R)=\frac{1}{\mathcal{J}_0}\sum_{\alpha\beta}(\psi^\mu_{\alpha\beta}+\varphi^\mu_{\alpha\beta})\tilde{g}(R)_{\alpha\beta},$$

$$\tilde{g}(R)_{\alpha\beta}\equiv\sum_\gamma(-\Phi_{\alpha\gamma}J^{22}_{\beta\gamma}+J^{11}_{\alpha\gamma}\Phi_{\gamma\beta})$$
$$=\frac{1}{\mathcal{J}_0}\sum_\gamma J_{\alpha\gamma}J_{\gamma\beta}\left\{\frac{1}{E_{\bar{\alpha}}+E_\gamma}(-u_\alpha v_\gamma+u_\gamma v_\alpha)(v_\gamma v_\beta+u_\gamma u_\beta)\right.$$
$$\left.+\frac{1}{E_{\bar{\gamma}}+E_\beta}(u_\alpha u_\gamma+v_\alpha v_\gamma)(u_\gamma v_\beta-u_\beta v_\gamma)\right\}s_\beta. \tag{4·31}$$

From this expression, we can see that $\tilde{g}(R)_{\alpha\beta}$ has the following properties:

i) From the structure of $J_{\alpha\beta}$-dependence, the diagonal components, which couples with signature-pair excitations of the quasiparticles in $\hat{O}_\mu$ with $\Delta K=0$, and also the

*) α and $\bar{\alpha}$ denote single-quasiparticle states with the positive signature and the negative signature, respectively.

**) This phase factor is given by $\hat{a}^\dagger_{\tilde{\alpha}}=s_\alpha\hat{a}^\dagger_{\bar{\alpha}}$, where $\tilde{\alpha}$ denotes the time-reversed state of α.

components $\tilde{g}(R)_{\alpha\beta}$ with $\Delta K=2$ may be dominant. Among them, especially, the components associated with the high-j and low-K single-particle orbits may be large.

ii) From the uv-factor dependence, quasiparticles far from the Fermi surface and those very close to the Fermi surface may make small contributions.

In Table III, the quantities $\omega_\mu g_\mu(Q)$, $\omega_\mu g_\mu(P_\tau)(\tau=\mathrm{p, n})$, $\omega_\mu g_\mu(R)$ and $\omega_\mu g_\mu(2, 0)$ are shown. One can see that the direct term $g_\mu(R)$ makes the main contribution to $g_\mu(2, 0)$. We show in Table IV some components of $\tilde{g}(R)$ associated with low-energy levels for ^{166}Er. From these tables, it is clearly seen that the components in $\hat{O}_\mu$ which are effective in making the coupling large are the signature-pair excitations not far

Table III.(a) Components of the rotation-vibration couplings of ^{168}Er. ω_μ is a RPA eigenenergy.

ω_μ [MeV]	$\omega_\mu g_\mu(Q)$ $\left[\frac{\mathrm{MeV}}{\hbar^2}10^{-3}\right]$	$\omega_\mu g_\mu(P_\mathrm{p})$ $\left[\frac{\mathrm{MeV}}{\hbar^2}10^{-3}\right]$	$\omega_\mu g_\mu(P_\mathrm{n})$ $\left[\frac{\mathrm{MeV}}{\hbar^2}10^{-3}\right]$	$\omega_\mu g_\mu(R)$ $\left[\frac{\mathrm{MeV}}{\hbar^2}10^{-3}\right]$	$\omega_\mu g_\mu(2, 0)$ $\left[\frac{\mathrm{MeV}}{\hbar^2}10^{-3}\right]$
1.299	−0.716	0.031	0.610	4.57	4.50
1.639	−0.197	−0.167	−0.823	−1.70	−2.89
1.745	−0.038	0	0	0.168	0.130
1.755	0.235	−0.427	0.384	−1.02	−0.833
1.857	−0.025	0.029	−0.366	−1.62	−1.99
2.055	0.0001	−0.423	0.001	−1.57	−2.00
2.063	−0.310	0	0	1.19	0.884
2.073	−0.007	−0.038	−0.780	−2.39	−3.21

(b) The same as (a) for ^{166}Er.

ω_μ [MeV]	$\omega_\mu g_\mu(Q)$ $\left[\frac{\mathrm{MeV}}{\hbar^2}10^{-3}\right]$	$\omega_\mu g_\mu(P_\mathrm{p})$ $\left[\frac{\mathrm{MeV}}{\hbar^2}10^{-3}\right]$	$\omega_\mu g_\mu(P_\mathrm{n})$ $\left[\frac{\mathrm{MeV}}{\hbar^2}10^{-3}\right]$	$\omega_\mu g_\mu(R)$ $\left[\frac{\mathrm{MeV}}{\hbar^2}10^{-3}\right]$	$\omega_\mu g_\mu(2, 0)$ $\left[\frac{\mathrm{MeV}}{\hbar^2}10^{-3}\right]$
1.603	−0.897	0.137	0.462	5.15	4.85
1.768	−0.030	−0.611	0.020	−1.33	−1.95
1.870	−0.356	0	0	1.92	1.56
1.944	−0.033	0.005	−0.350	−0.256	−0.634
2.103	−0.009	−0.583	0.014	−2.22	−2.80
2.219	0.051	−0.041	−0.501	−1.43	−1.92
2.223	0.013	−0.010	−1.27	−3.78	−5.04
2.231	−0.001	0	0	−0.046	−0.047

(c) The same as (a) for ^{164}Er.

ω_μ [MeV]	$\omega_\mu g_\mu(Q)$ $\left[\frac{\mathrm{MeV}}{\hbar^2}10^{-3}\right]$	$\omega_\mu g_\mu(P_\mathrm{p})$ $\left[\frac{\mathrm{MeV}}{\hbar^2}10^{-3}\right]$	$\omega_\mu g_\mu(P_\mathrm{n})$ $\left[\frac{\mathrm{MeV}}{\hbar^2}10^{-3}\right]$	$\omega_\mu g_\mu(R)$ $\left[\frac{\mathrm{MeV}}{\hbar^2}10^{-3}\right]$	$\omega_\mu g_\mu(2, 0)$ $\left[\frac{\mathrm{MeV}}{\hbar^2}10^{-3}\right]$
1.431	−1.08	0.124	0.193	5.21	4.44
1.807	−0.351	0	0	2.42	2.07
2.017	0.054	−0.498	0.547	0.699	0.802
2.041	0.108	−0.302	−1.01	−5.13	−6.33
2.223	−0.003	0.004	−0.413	0.739	0.328
2.272	0.042	−0.334	−0.426	−2.96	−3.68
2.275	−0.023	−0.470	0.202	−0.937	−1.23
2.378	−0.105	0	0	0.524	0.419

(continued)

(d) The same as (a) for ^{162}Er.

ω_μ [MeV]	$\omega_\mu g_\mu(Q)$ $\left[\frac{\text{MeV}}{\hbar^2}10^{-3}\right]$	$\omega_\mu g_\mu(P_p)$ $\left[\frac{\text{MeV}}{\hbar^2}10^{-3}\right]$	$\omega_\mu g_\mu(P_n)$ $\left[\frac{\text{MeV}}{\hbar^2}10^{-3}\right]$	$\omega_\mu g_\mu(R)$ $\left[\frac{\text{MeV}}{\hbar^2}10^{-3}\right]$	$\omega_\mu g_\mu(2,0)$ $\left[\frac{\text{MeV}}{\hbar^2}10^{-3}\right]$
1.035	−1.15	0.172	0.641	7.20	6.87
1.962	−0.344	0	0	2.66	2.31
2.075	0.014	−0.406	0.080	−0.461	−0.774
2.137	0.080	−0.248	−0.541	−3.25	−3.96
2.212	0	0	0	0	0
2.269	−0.086	0.240	−0.800	−2.07	−2.72
2.307	0.050	−0.174	−0.798	−4.64	−5.56
2.323	0.002	−0.569	−0.009	−2.08	−2.26

(e) The same as (a) for ^{160}Er.

ω_μ [MeV]	$\omega_\mu g_\mu(Q)$ $\left[\frac{\text{MeV}}{\hbar^2}10^{-3}\right]$	$\omega_\mu g_\mu(P_p)$ $\left[\frac{\text{MeV}}{\hbar^2}10^{-3}\right]$	$\omega_\mu g_\mu(P_n)$ $\left[\frac{\text{MeV}}{\hbar^2}10^{-3}\right]$	$\omega_\mu g_\mu(R)$ $\left[\frac{\text{MeV}}{\hbar^2}10^{-3}\right]$	$\omega_\mu g_\mu(2,0)$ $\left[\frac{\text{MeV}}{\hbar^2}10^{-3}\right]$
1.340	−1.45	0.448	1.03	10.8	10.77
2.139	−0.492	0	0	5.05	4.56
2.144	−0.162	−0.398	0.031	−0.544	−1.07
2.232	0.090	−0.012	−0.369	−2.44	−2.73
2.301	−0.017	0.148	−1.05	−2.84	−3.76
2.305	0	0	0	0	0
2.312	−0.006	−0.682	−0.433	−4.56	−5.68
2.400	−0.061	−0.725	0.305	−1.41	−1.89

from the Fermi surface. This means that rotation-β-vibration coupling is large. Moreover, when the neutron number approaches the transition region, the Fermi surface becomes close to the high-j and low-K levels, so that the coupling is enhanced. The levels which largely contribute to the coupling in ^{166}Er are seen to be (in terms of the asymptotic quantum number) [642 5/2], [624 9/2] and [512 5/2] of the neutron and [541 1/2], [523 7/2], [514 9/2] and [404 7/2] of the proton. The main components of the β-vibration in ^{166}Er are the signature-pair excitations to the levels [541 1/2] and [404 7/2] of the proton, while the main components of the γ-vibration are the excitations to [521 1/2] [523 5/2] of the neutron and to [411 1/2] [411 3/2] of the proton.

Tables III and IV also indicate that the couplings induced by the $\Delta K=2$ excitations are smaller than those of the $\Delta K=0$ excitations except for the γ-vibration, as pointed out in the previous subsection. For these two types of excitations, Eq. (4·31) is expressed as

$$\begin{aligned}\tilde{g}(R)_{\mu K\ \mu K}=\Big\{&(J_{\mu K\ \mu K+1})^2\frac{2}{E_{\overline{\mu K}}+E_{\mu K+1}}(u_{\mu K}v_{\mu K}(v_{\mu K+1}{}^2-u_{\mu K+1}{}^2)\\&-u_{\mu K+1}v_{\mu K+1}(v_{\mu K}{}^2-u_{\mu K}{}^2))\\&+(J_{\mu K\ \mu K-1})^2\frac{2}{E_{\overline{\mu K}}+E_{\mu K-1}}(u_{\mu K}v_{\mu K}(v_{\mu K-1}{}^2-u_{\mu K-1}{}^2)\\&-u_{\mu K-1}v_{\mu K-1}(v_{\mu K}{}^2-u_{\mu K}{}^2))\Big\}(-s_{\mu K}),\end{aligned}$$

$$\tilde{g}(R)_{\mu K+1\ \mu K-1}=J_{\mu K+1\ \mu K}J_{\mu K\ \mu K-1}$$
$$\times\Big\{\frac{1}{E_{\overline{\mu K+1}}+E_{\mu K}}(u_{\mu K+1}v_{\mu K}{}^2v_{\mu K-1}+u_{\mu K+1}v_{\mu K}u_{\mu K}u_{\mu K-1}$$
$$-u_{\mu K}v_{\mu K}v_{\mu K+1}v_{\mu K-1}-u_{\mu K}{}^2v_{\mu K+1}u_{\mu K-1})$$
$$-\frac{1}{E_{\overline{\mu K}}+E_{\mu K-1}}(u_{\mu K+1}u_{\mu K}{}^2v_{\mu K-1}-u_{\mu K+1}v_{\mu K}u_{\mu K}u_{\mu K-1}$$
$$+u_{\mu K}v_{\mu K}v_{\mu K+1}v_{\mu K-1}-v_{\mu K}{}^2v_{\mu K+1}u_{\mu K-1})\Big\}(-s_{\mu K-1}), \qquad (4\cdot32)$$

where the indices K and μ denote the K-quantum number and the other quantum numbers, respectively. In $\tilde{g}(R)_{\mu K\ \mu K}$, only two quasiparticle-levels with K and $K+1$ or K and $K-1$ contribute. In $\tilde{g}(R)_{\mu K+1\ \mu K-1}$, however, three levels with $K-1$, K and $K+1$ contribute simultaneously. Thus, the main difference in magnitude of the above two components comes from the fact that three quasiparticle-levels which have the K-quantum numbers with $K-1$, K and $K+1$ are not energetically close to one

Table IV.(a-1) Matrix elements of $\tilde{g}(R)$ in the space of the neutron $N_{\rm osc}=6$ of ^{166}Er. E denotes a quasiparticle energy.

E[MeV]	1.75	1.84	2.03	2.67	3.03
1.75	0.004	0	0	0	0.051
1.84	0	0.088	0.011	−0.005	0
2.03	0	0.011	0.087	0	0
2.67	0	0.005	0	−0.010	−0.004
3.03	0.051	0	0	−0.004	−0.018

(a-2) Main components of the quasiparticle states in terms of the Nilsson orbit.

E[MeV]	$[Nn_3\Lambda\Omega]$
1.75	$\left[633\frac{7}{2}\right]$
1.84	$\left[642\frac{5}{2}\right]$
2.03	$\left[624\frac{9}{2}\right]$
2.67	$\left[651\frac{1}{2}\right]$
3.03	$\left[651\frac{3}{2}\right]$

(b-1) The same as (a-1) in the space of the neutron $N_{\rm osc}=5$.

E[MeV]	1.25	1.27	1.54	1.67	1.73
1.25	0.006	0.002	0.008	0	0.006
1.27	0.002	−0.034	−0.0001	0	0
1.54	−0.008	0.0001	−0.030	0	0
1.67	0	0	0	−0.008	0
1.73	0.006	0	0	0	0.04

(continued)

(b-2) The same as (a-2).

E[MeV]	$[Nn_3\Lambda\Omega]$
1.25	$\left[521\frac{1}{2}\right]$
1.27	$\left[512\frac{5}{2}\right]$
1.54	$\left[523\frac{5}{2}\right]$
1.67	$\left[505\frac{11}{2}\right]$
1.73	$\left[521\frac{3}{2}\right]$

(c-1) The same as (a-1) in the space of the proton $N_{\rm osc}=5$.

E[MeV]	1.19	1.33	2.09	2.34	2.46
1.19	0.037	0	0	−0.007	0.007
1.33	0	0.046	0	−0.001	0
2.09	0	0	−0.066	0	0
2.34	0.007	0.001	0	0.019	0.017
2.46	0.007	0	0	0.017	−0.009

(c-2) The same as (a-2).

E[MeV]	$[Nn_3\Lambda\Omega]$
1.19	$\left[541\frac{1}{2}\right]$
1.33	$\left[523\frac{7}{2}\right]$
2.09	$\left[514\frac{9}{2}\right]$
2.34	$\left[532\frac{3}{2}\right]$
2.46	$\left[530\frac{1}{2}\right]$

(d-1) The same as (a-1) in the space of the proton $N_{\rm osc}=4$.

E[MeV]	0.88	1.05	1.45	1.70	2.03
0.88	0.002	0	−0.003	−0.005	0.003
1.05	0	−0.022	0	0.001	0
1.45	0.003	0	0.017	0	0.001
1.70	0.005	−0.001	0	−0.014	0
2.03	0.003	0	−0.001	0	0.019

(d-2) The same as (a-2).

E[MeV]	$[Nn_3\Lambda\Omega]$
0.88	$\left[411\frac{1}{2}\right]$
1.05	$\left[404\frac{7}{2}\right]$
1.45	$\left[402\frac{5}{2}\right]$
1.70	$\left[411\frac{3}{2}\right]$
2.03	$\left[413\frac{5}{2}\right]$

another, while two quasiparticle-levels with K and $K-1$ or $K+1$ have a chance to be close to each other, as can be seen from Table IV. Of course, this fact depends on the distribution of the single-particle levels.

Now let us discuss quantity $g_\mu(P_\tau)$ due to the coupling through the pairing residual interaction, which is smaller than the direct term but not negligible. With the use of the matrix elements of the pair operators $\hat{P}_\tau^\dagger$ and $\hat{P}_\tau$ defined in the same way as Eq. (4·28), i.e.,

$$\hat{P}_\tau^\dagger = \sum_{\alpha\beta\alpha'\beta'} (\hat{a}_\alpha^\dagger, \hat{a}_{\bar{\beta}}) \begin{pmatrix} P_\tau^{11}{}_{\alpha\alpha'} & P_\tau^{12}{}_{\alpha\beta'} \\ P_\tau^{21}{}_{\beta\alpha'} & P_\tau^{22}{}_{\beta\beta'} \end{pmatrix} \begin{pmatrix} \hat{a}_{\alpha'} \\ \hat{a}_{\bar{\beta}'}^\dagger \end{pmatrix} + \text{const}\,,$$

$$P_\tau^{11}{}_{\alpha\alpha'} = u_\alpha v_\alpha \delta_{\alpha\alpha'}\,, \qquad P_\tau^{12}{}_{\alpha\beta'} = u_\alpha{}^2 s_\alpha \delta_{\alpha\beta'}\,,$$

$$P_\tau^{21}{}_{\beta\alpha'} = -v_\beta{}^2 s_\beta \delta_{\alpha'\beta}\,, \quad P_\tau^{22}{}_{\beta\beta'} = -v_\beta u_\beta \delta_{\beta\beta'}\,, \tag{4·33}$$

we obtain

$$\omega_\mu g_\mu(P_\tau) = -G_\tau \langle\phi_0|[\hat{P}_\tau^\dagger + \hat{P}_\tau, \hat{O}_\mu^\dagger]|\phi_0\rangle \sum_{\lambda\nu\kappa} (\Phi_{\lambda\nu}\Phi_{\kappa\nu} P_\tau^{11}{}_{\kappa\lambda} - \Phi_{\nu\kappa}\Phi_{\nu\lambda} P_\tau^{22}{}_{\kappa\lambda})$$

$$= -G_\tau \sum_{\alpha\beta} (\psi_{\alpha\beta}^\mu + \varphi_{\alpha\beta}^\mu)(P_\tau^{12}{}_{\alpha\beta} + P_\tau^{\dagger 12}{}_{\alpha\beta}) \sum_{\lambda\nu\kappa} (\Phi_{\lambda\nu}\Phi_{\kappa\nu} P_\tau^{11}{}_{\kappa\lambda} - \Phi_{\nu\kappa}\Phi_{\nu\lambda} P_\tau^{22}{}_{\kappa\lambda})\,, \tag{4·34}$$

[23]

where we have used the axial symmetry property of the single-particle base. This form of the expression demonstrates that the RPA mode $\hat{O}_\mu$ which has the pairing-vibrational character with the signature-pair excitations has the remarkable coupling with the rotation. In ^{166}Er, the RPA mode with the energy $\omega_\mu = 2.223$ MeV mainly consists of the signature-pair excitations to the levels [512 5/2], [523 5/2] and [505 11/2] of the neutron, so that it has the remarkable coupling with the rotation.

The factor in Eq. (4·34) which does not depend on the character of the RPA operator $\hat{O}_\mu$ can be written

$$\tilde{g}(P_\tau) \equiv \sum_{\lambda\nu\kappa} (\Phi_{\lambda\nu}\Phi_{\kappa\nu} P_\tau^{11}{}_{\kappa\lambda} - \Phi_{\nu\kappa}\Phi_{\nu\lambda} P_\tau^{22}{}_{\kappa\lambda})$$

$$= \frac{2}{g_0{}^2} \sum_{\nu\kappa} \frac{1}{(E_\nu + E_{\bar{\kappa}})^2} (J_{\nu\kappa})^2 (u_\nu v_\kappa - u_\kappa v_\nu)^2 u_\kappa v_\kappa\,. \tag{4·35}$$

The quasiparticle excitations to the levels far from the Fermi surface do not have large contributions because of the factors $1/(E_\nu + E_{\bar{\kappa}})^2$ and $u_\kappa v_\kappa$. The high-j and low-K levels are important due to the factor $(J_{\nu\kappa})^2$, and this results in the mass-number dependence of the rotation- pairing-vibration coupling. We show in Table V the neutron-number dependence of $\tilde{g}(P_n)$. It is noted that the magnitude of the strength $|\langle\phi_0|[\hat{P}_\tau^\dagger + \hat{P}_\tau, \hat{O}_\mu]|\phi_0\rangle|$ does not have strong neutron-number dependence, so that $\tilde{g}(P_n)$ carries the main neutron-number dependence of the $g_\mu(P_n)$ in Eq. (4·34).

Table V. Neutron-number dependence of $\tilde{g}(P_n)$.

	$\tilde{g}(P_n)[1/\hbar^2]$
^{160}Er	-0.7857×10^{-2}
^{162}Er	-0.5518×10^{-2}
^{164}Er	-0.5487×10^{-2}
^{166}Er	-0.5103×10^{-2}
^{168}Er	-0.4492×10^{-2}

Concerning the rotation-vibration

coupling through the quadrupole force with the use of the matrix elements of $\hat{Q}_M$ defined in the same way as Eq. (4.28), we have

$$\omega_\mu g_\mu(Q) = -\chi \sum_{M=-2}^{2} \sum_{\alpha\beta} (\psi^\mu_{\alpha\beta} + \varphi^\mu_{\alpha\beta}) Q_M{}^{12}{}_{\alpha\beta} \tilde{g}(Q_M) ,$$

$$\tilde{g}(Q_M) \equiv \sum_{\lambda\nu\kappa} (\Phi_{\kappa\nu}\Phi_{\lambda\nu} Q_M{}^{11}{}_{\lambda\kappa} - \Phi_{\nu\lambda}\Phi_{\nu\kappa} Q_M{}^{22}{}_{\lambda\kappa})$$

$$= \frac{2}{\mathcal{J}_0{}^2} \sum_{\lambda\nu\kappa} \frac{1}{(E_\kappa + E_{\bar{\nu}})(E_\lambda + E_{\bar{\nu}})}$$

$$\times J_{\kappa\nu} J_{\lambda\nu} Q_{M\ \lambda\kappa} (u_\kappa v_\nu - u_\nu v_\kappa)(u_\lambda v_\nu - u_\nu v_\lambda)(u_\lambda u_\kappa - v_\lambda v_\kappa) . \qquad (4\cdot36)$$

One should note that $\tilde{g}(Q_M)$ contains the factor $(u_\lambda u_\kappa - v_\lambda v_\kappa)$ instead of $u_\kappa v_\kappa$ in $\tilde{g}(P_\tau)$. This makes the contribution of low energy levels suppressed, so that $\omega_\mu g_\mu(Q)$ does not become large.

§ 5. Concluding remarks

In this paper, emphasizing the concept of the broken symmetry as the origin of the collective rotation, we have shown the main concept and scenario to understand the occurrence mechanism of the nuclear collective rotation by means of the SCC method: The initial representation which is based on the RPA is transformed into the dynamical representation which is beyond the RPA-order. This scenario may be realized in both the full quantum theory and the semi-classical one in a parallel way.

In order to visualize how the rotation-vibration coupling effects are coherently organized so as to construct the optimum rotational subspace, we have performed the numerical calculations for Er isotopes with the pairing-plus-quadrupole force model. In the case with the fixed pairing field and no pairing residual interaction, it has been shown that the coupling between the collective rotation and the β-vibration especially increases with the neutron number approaching the transition region. It has been also found that the couplings between the collective rotation and the other many non-collective modes also increase. Such an analysis is one of the advantages of our approach, because the initial representation is based on the RPA so that the microscopic definition of the rotation-vibration coupling becomes clear. Furthermore, the couplings with many non-collective modes are explicitly included in a coherent way so as to construct the optimum collective subspace. As has been shown in the end of § 3, our equations used for the numerical calculations are essentially equivalent to those of the constraint cranking model. It should be noted, therefore, that the $\mathcal{A}$-parameter and the $\mathcal{B}$-parameter in our calculation ought to be the same as those obtained by means of the perturbation calculation within the constraint cranking model without making use of the RPA operators.

By the calculation with the pairing residual interaction, it has been confirmed that the influence of the pairing fluctuation due to the pairing residual interaction on the structure of the collective rotation is appreciably large: The calculation shows that the conventional values of parameters for the pairing-plus-quadrupole force lead us to the overestimated contributions of the rotation-vibration couplings when the pairing

fluctuation due to the pairing residual interaction is properly taken into account.

In our opinion, the following two points should be examined in order to solve this problem: The first is an investigation of effects of $\mathcal{H}^{[n=2]}(\boldsymbol{\beta}^{\dagger}, \boldsymbol{\beta}, \boldsymbol{J})$ in Eq. (2·9) which are neglected in the numerical calculations in this paper. The second is an investigation of the improvement of the effective interaction. The quadrupole-pairing force may be one of the candidates as mentioned in § 4.

We have also investigated the microscopic structure of the rotation-vibration coupling. It has been found that the low-energy signature-pair excitations of quasiparticles play an important role for the coupling.

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