Progress of Theoretical Physics, Vol. 89, No. 6, June 1993

Implications of the COBE-DMR and South-Pole Experiments on CMB Anisotropies to Inflationary Cosmology

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(Received February 26, 1993)

We discuss implications of the discovery of the cosmic microwave anisotropy at 10° scale by COBE-DMR and the null result at 1° scale from the South-Pole experiment to inflationary universe models. In doing so, we derive an approximate analytic formula which relates the anisotropies at 10° and at 1° without a heavy numerical computation. The formula, when tested against the known numerical results, turns out to reproduce those results quite accurately, and gives us a clear insight into cause and effect of the intermediate and large angular CMB anisotropies. Then applying the formula to models with adiabatic density perturbations based on the inflationary universe scenario, we find that the only model compatible with the claimed observational data is either (i) a power-law inflation model which predicts the power spectrum of density fluctuations with the power-law index $n \lesssim 0.8$, in which case the gravitational wave contribution to the CMB anisotropies on $\theta > 5°$ FWHM is significant, or (ii) a natural inflation model with $n \lesssim 0.7$, in which case the gravitational wave contribution is negligible on all angular scales. In both of these cases, if the universe is dominated by cold dark matter, the resulting bias factor turns out to be $b \gtrsim 2$, i.e., a relatively large bias is unavoidable.

§ 1. Introduction

The first detection of cosmic microwave background (CMB) anisotropies by COBE-DMR[1] has given much excitement in the field of cosmology.[2] From the analysis of the data, Smoot et al.[1] concluded that the rms value of the anisotropy at 10° scale is

$$(\Delta T/T)_{10° \mathrm{FWHM}} = (1.1 \pm 0.2) \times 10^{-5}\,, \tag{1·1}$$

and the autocorrelation function is consistent with a power-law spectrum of the primordial density fluctuation; $P(k) \propto k^n$ with $n = 1.1 \pm 0.5$. Smoot et al. also claimed a finite detection of the quadrupole moment; $(\Delta T/T)_Q = (6 \pm 1.5) \times 10^{-6}$. However, here we disregard the reported value, since it may be an overestimate[3] or at least it is subject to a large cosmic variance. At any rate, unless one considers a rather eccentric scenario, Eq.(1·1) implies the existence of density fluctuations on super-horizon scales in the very early universe with an almost scale-invariant spectrum, a natural explanation of which is possible only in the context of the inflationary universe scenario.

However, soon after the discovery of CMB anisotropy by COBE-DMR, Gaier et al.[4] reported no detection of CMB anisotropy above $(\Delta T/T)_{\mathrm{noise}} = 10^{-5}$ at 1° scale in the sky near the South Pole. This result is rather controversial, since the predicted rms temperature fluctuation at 1° scale for the scale-invariant spectrum, which is the case of the standard exponential inflationary scenario, is $\sim 2 \times 10^{-5}$. Hence one would naively expect that the probability of no signal detection is small. In fact, assuming $(\Delta T/T)_{10° \mathrm{FWHM}} = 1 \times 10^{-5}$, Gorski et al.[5] performed a detailed statistical analysis and concluded that all conceivable $\Omega=1$ universe models with $n=1$ spectrum are excluded at 95% C.L.

In this respect, the possible dominance of the tensor mode contribution to the CMB anisotropy and its relation to the scalar mode power spectrum has been discussed already by Davis et al.[6] and Lucchin et al.[7] However, the analysis by Davis et al. relies on the value of quadrupole moment detected by COBE, which is subject to large cosmic variations as mentioned above. Further, they seem to place too much emphasis on the tensor mode contribution to the CMB anisotropy. On the other hand, Lucchin et al. give a detailed analysis but only for power-law inflation models.

In this paper, we investigate all conceivable inflation models with adiabatic curvature perturbations having the spectral index $n\leq 1$ in a semi-quantitative but analytically tractable way and derive the constraint on these models from the results of 10° COBE-DMR and 1° South-Pole experiments. By doing so, we clearly demonstrate the essential factors that determine the intermediate and large angular CMB anisotropies.

§ 2. Scalar and tensor contributions to CMB anisotropy

If both of the results of COBE-DMR and South-Pole experiments should be taken seriously, one has to abandon at least the simplest scenario of exponential inflation. It is then a matter of great concern if this means the death of the inflationary universe scenario. Fortunately, it is not so but indeed there are other viable inflationary scenarios in which the power spectrum of density perturbations differs appreciably from $n=1$. Among them are the scenarios of power-law inflation[8)~9)] and natural inflation.[11)] In particular, the former predicts not only the matter density fluctuations but also a significant amplitude of fluctuations in the transverse-traceless part of the metric, i.e., the gravitational wave perturbations, which may be actually the ones detected by COBE-DMR.[6),7)]

In what follows, we briefly review the origin of these fluctuations in the inflationary universe and give an estimate of the predicted amplitude of CMB anisotropy. Concerning the possible types of density perturbations, there is yet another possibility, namely isocurvature perturbations. However, here we focus on the curvature, (i.e., the so-called adiabatic type) perturbations.

2.1. *Origin of fluctuations*

In almost all of the inflationary scenarios, inflationary expansion of the universe is driven by the potential energy of some scalar field ϕ. At the stage when the universe is under inflation, the quantum fluctuations of the scalar field are rapidly redshifted to a macroscopic scale and give rise to super-horizon scale fluctuations in the matter density and hence the scalar-type perturbations in the metric which eventually turns into large scale structures of the universe.

The power spectrum of thus generated perturbation is given by[12),13)]

$$\frac{4\pi k^3}{(2\pi)^3}P_{\mathcal{R}}(k)\equiv\langle\mathcal{R}^2\rangle_k=\frac{H^4}{4\pi^2\dot{\phi}^2}\bigg|_{t=t_k}, \tag{2·1}$$

where $\mathcal{R}$ is the spatial curvature perturbation measured on the comoving hypersurface and t_k is the time at which the comoving scale k leaves the Hubble horizon scale H^{-1} during the inflation. After the inflation, this in turn gives rise to perturbations in the Newtonian potential Ψ as

$$\langle\Psi^2\rangle_k=\begin{cases}\left(\frac{3}{5}\right)^2\langle\mathcal{R}^2\rangle_k\text{: matter-dominated stage;}\\ \left(\frac{2}{3}\right)^2\langle\mathcal{R}^2\rangle_k\text{: matter-dominated stage.}\end{cases} \tag{2·2}$$

On the other hand, the quantum fluctuations of the gravitational wave modes are also redshifted to a macroscopic scale and give rise to the transverse-traceless (i.e., tensor-type) perturbations in the metric h_{ij}^{TT}. The power spectrum is given by[14)]

$$\frac{4\pi k^3}{(2\pi)^3}P_T(k)\equiv\langle(h_{ij}^{TT})^2\rangle_k=64\pi\frac{H^2}{4\pi^2 m_{pl}^2}\bigg|_{t=t_k}, \tag{2·3}$$

where m_{pl} is the planck mass.

2.2. *CMB anisotropy*

The CMB anisotropy induced by the perturbations discussed above can be calculated by solving the photon propagation equation in the perturbed Friedmann universe. Assuming the universe is spatially flat, matter-dominated after the baryon-photon decoupling, it is approximately given by[15)]

$$\frac{\Delta T}{T}(\gamma^i;\ \eta_0)\approx\frac{1}{2}\int_{\eta_d}^{\eta_0}d\eta\left(\frac{\partial h_{ij}^{TT}}{\partial\eta}\gamma^i\gamma^j\right)+\frac{1}{3}\Psi(\eta_d)+\gamma^i(V_i(\eta_d)-V_i(\eta_0)), \tag{2·4}$$

where $\eta=\int^t dt/a(t)$ is the conformal time, η_d and η_0 are the decoupling time and the present time, respectively, γ^i is the direction cosine and V_i is the matter velocity perturbation. As it is clear, the first term on the right-hand side of Eq.(2·4) is due to tensor-type perturbations, and the second and third to scalar-type ones. Following conventional terminology, we call the second the Sachs-Wolfe effect and the third the Doppler effect.

When discussing the CMB anisotropy, it is often convenient to express it in terms of its multipoles with respect to the spherical harmonics:

$$\frac{\Delta T}{T}=\sum_{l,m}a_{lm}Y_{lm}(\Omega_\gamma). \tag{2·5}$$

<u>*Quadrupole*</u>

Among the multipoles, the lowest non-trivial one is the dipole, but it is generally believed (and is true for most of viable cosmological models) that the dipole is dominated by our peculiar velocity at present, i.e., $V_i(\eta_0)$. Hence, the lowest nontrivial multipole which carries the direct information of the early universe is the quadrupole. The mean square of it can be estimated from Eqs.(2·1)~(2·3), and (2·4) as

$$4\pi Q_S{}^2 \equiv \sum_m \langle a_{2m}^2 \rangle_S \approx \frac{H^4}{60\pi\dot{\phi}^2}\bigg|_{t_k},$$

$$4\pi Q_T{}^2 \equiv \sum_m \langle a_{2m}^2 \rangle_T \approx 0.93\frac{H^2}{m_{pl}^2}\bigg|_{t_k}, \quad (2\cdot6)$$

where $k = H_0$, H_0 is the present value of the Hubble parameter and the scale factor at present has been normalized to unity; $a_0 = 1$. From Eq.(2·6), one finds

$$\frac{Q_T{}^2}{Q_S{}^2} \approx 56\pi\frac{\dot{\phi}^2}{m_{pl}^2 H^2} = 42\frac{\frac{1}{2}\dot{\phi}^2}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}\Bigg|_{t_k}. \quad (2\cdot7)$$

Thus, the tensor mode contribution can dominate the quadrupole only if the kinetic energy of the scalar field is comparable to the potential energy of it at the inflationary stage.

Intermediate scales

At intermediate angular scales ($1^\circ \sim 10^\circ$), it is convenient to express the degree of anisotropy in terms of the temperature auto-correlation function,

$$C(\theta) = \left\langle \frac{\Delta T}{T}(\boldsymbol{\gamma} + \boldsymbol{\theta})\frac{\Delta T}{T}(\boldsymbol{\gamma}) \right\rangle = \frac{1}{4\pi}\sum_l \langle a_l{}^2 \rangle P_l(\cos\theta),$$

where $\langle a_l{}^2 \rangle = \sum_m \langle a_{lm}^2 \rangle$. In the present case, $\langle a_l{}^2 \rangle$ can be approximately expressed as

$$\langle a_l{}^2 \rangle \approx \langle a_l{}^2 \rangle_{\mathrm{GW}} + \langle a_l{}^2 \rangle_{\mathrm{SW}} + \langle a_l{}^2 \rangle_{\mathrm{Doppler}}, \quad (2\cdot8)$$

where the first, second and third terms are the contributions from the gravitational waves, the Sachs-Wolfe effect and the Doppler effect, respectively. For inflation models considered later, the primordial power spectrum is well approximated by a power-law. In such a case, the Sachs-Wolfe contribution can be analytically evaluated.[16] Further, for models of power-law (or extended) inflation, in which case the tensor contribution can become significant, one may assume $P_{\mathfrak{R}}(k) \propto P_T(k) \propto k^{n-4}$.[6),7)] In this case, one approximately has

$$\langle a_l{}^2 \rangle_{\mathrm{GW}} \propto \langle a_l{}^2 \rangle_{\mathrm{SW}} \simeq \frac{12}{5}\langle a_2{}^2 \rangle_{\mathrm{SW}}\, l^{n-2}; \quad l \gg 1. \quad (2\cdot9)$$

As for the Doppler term, noting that the Fourier component is given by

$$(\gamma^i V_i)_{\boldsymbol{k}} = \frac{1}{3}\mu_k k \eta_d \Psi_{\boldsymbol{k}}, \quad \mu_k = \frac{\boldsymbol{k}\cdot\boldsymbol{\gamma}}{k}, \quad (2\cdot10)$$

it may be approximated as

$$\langle a_l{}^2 \rangle_{\mathrm{Doppler}} \sim \frac{l^2}{1+z_d}\langle a_l{}^2 \rangle_{\mathrm{SW}} = \left(\frac{l}{l_d}\right)^2 \langle a_l{}^2 \rangle_{\mathrm{SW}}, \quad (2\cdot11)$$

where $z_d \sim 10^3$ is the redshift at decoupling and hence $l_d \sim 30$, which corresponds to the angular scale $\theta_d \sim 2^\circ (=5^\circ \text{FWHM})$ or to the horizon scale at decoupling.

From Eqs. (2·9) and (2·11), one finds:

(a) $(\Delta T/T)_{10^\circ}$ is dominated by $\langle a_l^2 \rangle_{\rm GW}$ and/or $\langle a_l^2 \rangle_{\rm SW}$, and

(b) $(\Delta T/T)_{1^\circ}$ is dominated by $\begin{cases} \langle a_l^2 \rangle_{\rm Doppler} & \text{if } Q_T \ll Q_S\,, \\ \langle a_l^2 \rangle_{\rm GW} \text{ and/or } \langle a_l^2 \rangle_{\rm Doppler} & \text{if } Q_T \gtrsim Q_S\,. \end{cases}$

§ 3. $(\Delta T/T)$-test of power spectrum

In order to evaluate the CMB anisotropy predicted in a given cosmological model in a quantitatively accurate way, one of course has to do numerical integrations. However, it is always useful to have an analytic formula to do a semi-quantitative analysis, not only because we may gain clearer understanding of cause and effect, but also we can obtain predictions of other models without a heavy numerical computation by simply varying the parameters in the formula. Furthermore, considering uncertainties in the observational data, it is sometimes meaningless to require a high accuracy in the resulting numbers. In this section, we give such a formula for cosmological models with a power-law spectrum discussed above and compare the result with the observations.

3.1. $(\Delta T/T)_{10^\circ}$ *versus* $Q_{\rm rms}$

According to Smoot et al.,[1] the relevant theoretical formula which corresponds to the mean square temperature fluctuation at 10° discovered by COBE-DMR is

$$\left(\frac{\Delta T}{T}\right)^2_{10^\circ} = C_{\rm COBE}(0) = \frac{1}{4\pi}\sum_{l>2}\langle a_l^2\rangle e^{-l^2/l_0^2}$$

$$\approx Q_{\rm rms}^2 \times \frac{12}{5}\int_3^\infty \frac{dl}{l} l^{n-1} e^{-l^2/l_0^2}$$

$$\approx Q_{\rm rms}^2 \times \frac{12}{5} e^{-3^2/l_0^2} \frac{3^{n-1}}{1-n}\left(1-\left(\frac{l_0}{3}\right)^{n-1}\right), \tag{3·1}$$

where $Q_{\rm rms} = \sqrt{Q_T{}^2 + Q_S{}^2}$ is the rms quadrupole moment, $l_0 = 17.8$ is the smearing scale relevant for COBE-DMR, and Eq.(2·9) has been used to approximate $\langle a_l^2 \rangle$. The above formula gives

$$\left(\frac{\Delta T}{T}\right)_{10^\circ} \approx \begin{cases} 2.0\,Q_{\rm rms}; & n=1\,, \\ 1.6\,Q_{\rm rms}; & n=0.8\,, \\ 1.4\,Q_{\rm rms}; & n=0.6\,, \end{cases} \tag{3·2}$$

which are in good agreement with more accurate numerical results.[1] This shows our approximation is reasonable after all.

3.2. $(\Delta T/T)_{10^\circ}$ *versus* $(\Delta T/T)_{1^\circ}$

Given the evidence that our approximation works fairly well, let us now turn to the anisotropies on intermediate angular scales.

The Sachs-Wolfe and tensor contributions

First consider contributions from the Sachs-Wolfe effect and gravitational waves. Again, from Eq.(2·9), one has

$$\left(\frac{\Delta T}{T}\right)^2_{1^\circ,\mathrm{SW+GW}}(n)\approx Q_{\mathrm{rms}}^2\times\frac{12}{5}\int_0^\infty\frac{dl}{l}l^{n-1}e^{-l^2\sigma^2}f(l;\,\theta_0)\,;$$

$$f(l;\,\theta_0)=2(1-P_l(\cos\theta_0))\approx2(1-J_0(l\theta_0))\,;\quad\theta_0\ll1,\ l\gg1\,,\tag{3·3}$$

where $\theta_0=0.037(2.1^\circ)$ and $\sigma=0.011(0.63^\circ)$ are the beam separation and smearing angles, respectively, and f is the filter function, relevant for the South-Pole experiment.[4] Unfortunately, the above integral cannot be done exactly. However, for $n=1$, it may be evaluated as

$$\left(\frac{\Delta T}{T}\right)^2_{1^\circ,\mathrm{SW+GW}}\approx\frac{12}{5}Q_{\mathrm{rms}}^2\int_0^{\theta_0^2/(4\sigma^2)}\frac{1-e^{-x}}{x}dx$$

$$\approx\frac{12}{5}Q_{\mathrm{rms}}^2\left(\ln\frac{\theta_0^2}{4\sigma^2}+\gamma\right)\lesssim(10^{-5})^2\,,\tag{3·4}$$

where $\gamma=0.577\cdots$ is the Euler constant and the last figure is obtained from Eqs. (1·1) and (3·2) with $n=1$. Now for a fixed value of $(\Delta T/T)_{10^\circ}$, $(\Delta T/T)_{1^\circ}(n)$ is apparently a decreasing function of n since the dominant contribution to the integral (3·3) comes from $l\gtrsim\theta_0^{-1}=27>l_0=17.8$. Hence we may conclude that the Sachs-Wolfe and gravitational wave contributions to the CMB anisotropy at 1° scale will not make a conflict with the observational upper bound for all values of $n\le1$.

The Doppler contribution

The above result tells us that the only contribution to the 1° anisotropy which we have to worry about is the one due to the Doppler effect. Using Eq.(2·11), it is approximately given by

$$\left(\frac{\Delta T}{T}\right)^2_{1^\circ}(n)\approx\alpha Q_S{}^2\int_0^\infty\frac{dl}{l}\left(\frac{l}{l_d}\right)^2l^{n-1}e^{-l^2\sigma^2}f(l;\,\theta_0)$$

$$\approx\sigma^{1-n}\left(\frac{\Delta T}{T}\right)^2_{1^\circ}(1)\,,\tag{3·5}$$

where α is a constant of $O(1)$. Since the approximation (2·11) for the Doppler contribution is very crude, we should regard l_d as an adjustable parameter rather than a given one, by absorbing the factor α. The adjustment can be done by using the fact that $(\Delta T/T)_{1^\circ}(1)\approx3Q_S$:[5]

$$\left(\frac{\Delta T}{T}\right)^2_{1^\circ}(1)\approx\frac{Q_S{}^2}{(l_d\sigma)^2}(1-e^{-\theta_0^2/(4\sigma^2)})=3^2Q_S{}^2\,.\tag{3·6}$$

This happens to give $l_d\sim30$; an interesting coincidence that errors due to the crudeness of Eq.(2·11) are canceled somehow by that of Eq.(3·5).

Combining the formulas (3·1) and (3·5) with Eq.(3·6), we obtain the final formula,

$$\left(\frac{\Delta T}{T}\right)_{1^\circ}(n) \approx 1.5RF(n)\left(\frac{\Delta T}{T}\right)_{10^\circ}; \quad R=\sqrt{\frac{S}{T+S}}, \tag{3·7}$$

where

$$F(n)=\sqrt{\frac{(1-n)(3\sigma)^{1-n}}{1-(3/l_0)^{1-n}}\ln(l_0/3)}=\begin{cases} 1.00; & n=1.0\,, \\ 0.77; & n=0.8\,, \\ 0.60; & n=0.6\,. \end{cases} \tag{3·8}$$

For the COBE-normalization; $(\Delta T/T)_{10^\circ}=1.1\times10^{-5}$, the above formula gives

$$\left(\frac{\Delta T}{T}\right)_{1^\circ} \approx \begin{cases} 1.6\times10^{-5}R; & n=1.0\,, \\ 1.3\times10^{-5}R; & n=0.8\,, \\ 1.0\times10^{-5}R; & n=0.6\,. \end{cases} \tag{3·9}$$

Comparing these numbers with the rms noise level of the South-Pole experiment; $(\Delta T/T)_{\rm noise}=10^{-5}$,[4] and taking into account the inaccuracy of our approximation (which we estimate as $\lesssim 10\%$), we conclude that (i) if $Q_T \gtrsim Q_S$, all models with $n \leq 1$ are consistent with the observational bound, but (ii) if $Q_T \ll Q_S$, even a model with $n=0.8$ seems marginally excluded and a moderate allowable range would be $n \lesssim 0.7$.

§ 4. Models of inflation with $n<1$

Now, let us examine models of inflation which can explain the observational data.

4.1. *Power-law inflation*

Two typical models of power-law inflation are extended inflation[9] and soft inflation.[10] Although the extended inflation is basically different from the soft inflation in that the former is based on the Brans-Dicke type gravity theory while the latter on the conventional Einstein gravity, both of them give effectively the same law of inflationary expansion, i.e., the power-law expansion of the universe. Hence, here we focus on the proto-type power-law inflationary model.[8]

The power-law inflation is driven by a scalar field with an exponential potential,[8]

$$V(\phi)=V_0\exp(\lambda\kappa\phi); \quad \kappa=\frac{\sqrt{8\pi}}{m_{pl}}. \tag{4·1}$$

The scalar field and the cosmic scale factor then approach asymptotically to

$$\phi(t)=\frac{2}{\lambda\kappa}\ln\left[\left(\frac{6-\lambda^2}{2\lambda^2 V_0}\right)^{1/2}\frac{2}{\lambda\kappa t}\right], \quad a(t)\propto t^p: \quad p=\frac{2}{\lambda^2}\,(>1)\,. \tag{4·2}$$

At this asymptotic stage of inflation, both fluctuations in the scalar and tensor parts of the metric are produced with the same power-law index $n=(p-3)/(p-1)$. The ratio of the kinetic energy to the total energy density is given by

$$\frac{\frac{1}{2}\dot{\phi}^2}{\frac{1}{2}\dot{\phi}^2+V(\phi)}=\frac{6}{\lambda^2}=\frac{1}{3p}=\frac{1-n}{3(3-n)}, \tag{4·3}$$

which, together with Eq.(2·7), implies

$$\frac{Q_T^2}{Q_S^2} \approx \frac{14}{p} = 14\frac{1-n}{3-n}. \tag{4·4}$$

Hence $Q_T \gtrsim Q_S$ for $p \lesssim 14$, or $n \lesssim 0.85$.

Thus, according to the power-law inflationary scenario, the conclusion of the previous section implies that the detected anisotropy at 10° by COBE-DMR must be dominated by the tensor mode contribution with the power-law index $n \lesssim 0.8$. For cold dark matter (CDM) models, this implies the bias factor $b \gtrsim 2$ as mentioned in Davis et al.[6] and Lucchin et al.[7]

4.2. *Natural inflation*

In the scenario of natural inflation,[11] the potential has the form,

$$V(\phi) = M^4\left(\frac{1+\cos(\phi/f)}{2}\right), \tag{4·5}$$

and inflation is assumed to occur when the scalar field is at $|\phi| \ll f$. In this case, the expansion is exponential,

$$a(t) \propto e^{Ht}; \quad H = \sqrt{\frac{8\pi}{3}}\frac{M^2}{m_{pl}}, \tag{4·6}$$

and $\dot{\phi}^2 \ll V(\phi)$ so that the tensor mode contribution is negligible. But the slow rolling approximation for the scalar field does not necessarily hold and the power-law index n of the density perturbation can differ appreciably from unity:[11),13]

$$\phi(t) \propto e^{\delta Ht}, \quad n \approx 1-2\delta; \quad \delta(\delta+3) = \frac{3m_{pl}^2}{16\pi f^2}. \tag{4·7}$$

For example, for a typical GUT scale; $M \sim 10^{15}$GeV, a reasonable amplitude of the scalar perturbation; $\Psi \lesssim 10^{-4}$, is realized for $f \sim 5\times 10^{18}$GeV. This value implies $\delta \sim 0.1$, hence $n \sim 0.8$.

Since the tensor perturbation must be necessarily negligible in this scenario, we conclude that $n \lesssim 0.7$ if the natural inflation should be adopted. In this case, the corresponding bias factor for CDM models is again $b \gtrsim 2$.[17]

§ 5. Summary

In this paper, we discussed the implications of the results of COBE-DMR and South-Pole experiments to inflationary universe models. For this purpose we derived a semi-analytic formula which relates the CMB anisotropies at 10° and 1° scales. In the context of inflationary cosmology, we found that only power-law inflation models with the spectral index $n \lesssim 0.8$ and natural inflation models with $n \lesssim 0.7$ are allowed, provided that the primordial power spectrum is of adiabatic type. In both of these cases, the resulting bias factor b for CDM models is found to be $b \gtrsim 2$. This coincidence happens because the difference in the scalar mode amplitude at 10°, which corresponds to a scale $\sim 200h^{-1}$Mpc, is compensated by the difference in the rate of increase in the power spectrum as the scale goes down to $8h^{-1}$Mpc. Of course, to justify the above conclusion in the strict sense, we have to perform detailed numerical computations. In particular, the effects neglected in the present approximation, such as the intrinsic photon density fluctuations at decoupling era, may be important on 1° scale in certain models. However, as noted before, errors due to the crudeness of our approximation are expected to be buried well under uncertainties of the observational data. Thus our conclusion should be taken as a general, semi-quantitative constraint on models of inflation.

Finally we note that if the existence of large-scale flow; $V(40h^{-1}\mathrm{Mpc})\sim 400$km, is proved and if it is found to be a typical value everywhere in the universe, it will be a clear contradiction to the null result of the South-Pole experiment, provided one relies on the standard gravitational instability scenario of large-scale structure formation.[18] A possible resolution is to assume reionization of the universe soon after decoupling until quite recently which smears out the CMB anisotropies on 1° scales. For this to occur, it seems necessary to have baryon isocurvature perturbations of a large amplitude on small scales,[19] and interestingly enough there exists such a scenario in the context of power-law inflation.[20] Therefore it may be worthwhile to investigate the predictions of such a model in more detail, or a hybrid model which includes *everything* necessary, though one cannot deny the feeling that the latter is too *ad hoc*.

The author would like to thank K. M. Gorski and E. D. Stewart for stimulating conversations and J. Yokoyama for useful comments. This work was supported in part by the Grant-in-Aid for Scientific Research on Priority Areas of Ministry of Education, Science and Culture, No.04234104.

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