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P-Wave Charmed Mesons

Katsuo NAKASHIMA,*) Toshiaki ITO, Yoshimitsu MATSUI,
Shoji SAWADA and Tsuyoshi B. SUZUKI

Department of Physics, Nagoya University, Nagoya 464-01

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We study the *P*-wave charmed meson spectroscopy. We analyze the mixing of two 1^+ states, the masses and decay widths of the *P*-wave charmed mesons by use of the Breit-Fermi Hamiltonian to account for the $1/m_Q$ effects. Though the possibility of the case that $D_1(2420)$ observed by experiments belongs to the $s_l^P=(3/2)^+$ dominant multiplet is high but not yet decisive. Therefore we also examine the case where the $D_1(2420)$ belongs to the $s_l^P=(1/2)^+$ dominant multiplet. For these two cases the relation between the masses and decay widths of the *P*-wave charmed mesons is obtained. When $D_1(2420)$ is assigned into the member of the $s_l^P=(3/2)^+$ dominant multiplet, the decay widths of unfound D_1 and D_0^* become very large, which is consistent to the fact that these states are not found yet by the experiments.

§ 1. Introduction

Experimentally the two *P*-wave charmed mesons which are $D_2^*(2460)$ and $D_1(2420)$ have been observed. Recently new experimental data for the spectroscopy of these excited states are given by CLEO Collaboration[5] and E687 Collaboration.[6] In particular, the data of angular distribution of two dacay pions from $D_1(2420)$ are expected to bring an important information for the assignment of s_l^P.

Heavy flavored hadrons are approximately classified by use of the spin-parity of the light degrees of freedom s_l^P.[10] Since $D_2^*(2460)$ has $J^P=2^+$, we can assign this meson uniquely to the $s_l^P=(3/2)^+$ state. On the other hand, there are possibilities that the $D_1(2420)$ meson with $J^P=1^+$ belongs to the $s_l^P=(3/2)^+$ dominant doublet or the $s_l^P=(1/2)^+$ dominant one. Usually $D_1(2420)$ has been assigned to a member of the $s_l^P=(3/2)^+$ dominant doublet since the mass and decay width of this meson are nearly equal to those of $D_2^*(2460)$. However we should note that experiments do not negate definitely the possibility that $D_1(2420)$ belongs to the $s_l^P=(1/2)^+$ dominant doublet as we discuss in § 3.

In this paper, we study the masses and decay widths of unknown *P*-wave charmed mesons for the two cases:

Case I. The $D_1(2420)$ belongs to the $s_l^P=(3/2)^+$ dominant doublet, denoted by $s_l^P\simeq(3/2)^+$.

Case II. The $D_1(2420)$ belongs to the $s_l^P=(1/2)^+$ dominant doublet, denoted by $s_l^P\simeq(1/2)^+$.

In the next section, we discuss the masses, decay widths and D_1 mixing angle. In § 3 we analyze the angular distribution of two final pions in the $D_1(2420)$ decay. A summary and discussion are given in § 4.

*) Address after April 1, 1995: Software Development Center, Hitachi Ltd., Yokohama 244.

§ 2. Masses, decay widths and mixing angle of the *P*-wave charmed mesons

By taking into account the $1/m_Q$ corrections from the heavy quark limit, we consider in this section masses and strong decay widths of the P-wave charmed mesons and the mixing angle between two D_1 mesons. In order to estimate the effects of the $1/m_Q$ corrections, we use the Breit-Fermi Hamiltonian,[7)]

$$\Delta H = c_q \boldsymbol{L}\cdot\boldsymbol{S}_q + c_Q \boldsymbol{L}\cdot\boldsymbol{S}_Q + c_T[3(\boldsymbol{S}_q\cdot\hat{\boldsymbol{r}})(\boldsymbol{S}_Q\cdot\hat{\boldsymbol{r}}) - (\boldsymbol{S}_q\cdot\boldsymbol{S}_Q)]\,. \tag{1}$$

In Hamiltonian (1), $\boldsymbol{L}$ is the angular momentum between a heavy quark Q and a light anti-quark $\bar{q}$, $\boldsymbol{S}_Q$ and $\boldsymbol{S}_q$ are spins of heavy quark and light anti-quark, and $\hat{\boldsymbol{r}}$ is the unit vector in the direction from the heavy quark to the light anti-quark. We ignore $\boldsymbol{S}\cdot\boldsymbol{S}$ force in Eq. (1) because it affects mainly the S-wave states and not the P-wave states.

First we discuss the two D_1 meson mixing due to the $\boldsymbol{L}\cdot\boldsymbol{S}$ force which causes the mixing between $s_l^P=(3/2)^+$ and $s_l^P=(1/2)^+$ states. We shall call the lower-mass eigenstate D_1^α and the higher-mass one D_1^β for physical $J^P=1^+$ mesons. Since we assume that mesons in the $s_l^P\simeq(3/2)^+$ doublet are heavier than those in the $s_l^P\simeq(1/2)^+$ doublet, we may assign D_1^α into $s_l^P\simeq(1/2)^+$ state and D_1^β into $s_l^P\simeq(3/2)^+$ state. So $D_1(2420)=D_1^\beta$ in Case I and $D_1(2420)=D_1^\alpha$ in Case II. The eigenstates D_1^α and D_1^β are written as

$$|D_1^\alpha\rangle = \cos\Delta\phi\left|J^P=1^+, s_l^P=\frac{1}{2}^+\right\rangle - \sin\Delta\phi\left|J^P=1^+, s_l^P=\frac{3}{2}^+\right\rangle\,, \tag{2}$$

$$|D_1^\beta\rangle = \sin\Delta\phi\left|J^P=1^+, s_l^P=\frac{1}{2}^+\right\rangle + \cos\Delta\phi\left|J^P=1^+, s_l^P=\frac{3}{2}^+\right\rangle\,, \tag{3}$$

where $\Delta\phi$ is the mixing angle and specified by

$$\tan^2(\phi_0-\Delta\phi) = \frac{M_\beta-\bar{M}}{\bar{M}-M_\alpha}\,. \tag{4}$$

Here M_α and M_β are the masses of the D_1^α and D_1^β mesons, respectively, and $\bar{M}$ is the average of the spin independent part of the Hamiltonian for $L=1$ excited states given by

$$\bar{M} = \frac{5M_2+3(M_\alpha+M_\beta)+M_0}{12}\,, \tag{5}$$

M_0 and M_2 being the masses of D_0^* and D_2^*. The mass deviations of M_α and M_β from $\bar{M}$ can be derived straightforwardly by using Eqs. (1)~(3).[7)] In Eq. (4) ϕ_0 is the mixing angle between $|^1P_1\rangle$ and $|^3P_1\rangle$ states for heavy quark mass limit given by

$$\tan\phi_0 \equiv \sqrt{\frac{1}{2}}\,, \tag{6}$$

and $\Delta\phi$ is the angle indicating the deviation from the heavy quark limit.

If we consider $m_Q \gg m_q$ in Eq. (1), since c_q is a quantity of $1/m_q$ effects and c_Q, c_T

are quantities of $1/m_Q$ effects, then

$$|c_q| \gg |c_Q| \simeq |c_T| \tag{7}$$

is derived when we take into account a Hamiltonian including linear confinement and Coulomb-type interactions.[8] Therefore the masses of P-wave charmed mesons are parametrized as

$$M_2 = \bar{M} + \frac{1}{2}c_q + \frac{2}{5}c_Q\,, \tag{8}$$

$$M_\beta = \bar{M} + \frac{1}{2}c_q - \frac{2}{3}c_Q\,, \tag{9}$$

$$M_\alpha = \bar{M} - c_q + \frac{2}{3}c_Q\,, \tag{10}$$

$$M_0 = \bar{M} - c_q - 2c_Q\,. \tag{11}$$

So these lead to

$$\{M_2 - M_\beta\} : \{M_\alpha - M_0\} = 2:5\,. \tag{12}$$

Then we give the relations of the mixing angle $\Delta\phi$ and the mass of unknown D_1 by use of Eq. (4) and

$$\bar{M} = \frac{5M_2 + 8M_\alpha + 11M_\beta}{24}\,. \tag{13}$$

In the following calculation we treat the unknown D_1 mass M_1 as a parameter ($M_1 = M_\alpha$ for Case I and $M_1 = M_\beta$ for Case II). If the mass difference between two D_1 states is too small and the right-hand side of Eq. (4) becomes negative, we obtain no solution of the mixing angle.[7] Therefore we have to take $M_1 \le 2400$ in Case I and $2435 \le M_1 \le 2460$ in Case II. Furthermore if $M_1 < 2150$ MeV, this meson cannot decay into $D^*\pi$, and is stable for the strong interaction. Therefore for the unknown D_1 mass we take $2150 \le M_1 \le 2400$ MeV in Case I and $2435 \le M_1 \le 2460$ MeV in Case II.

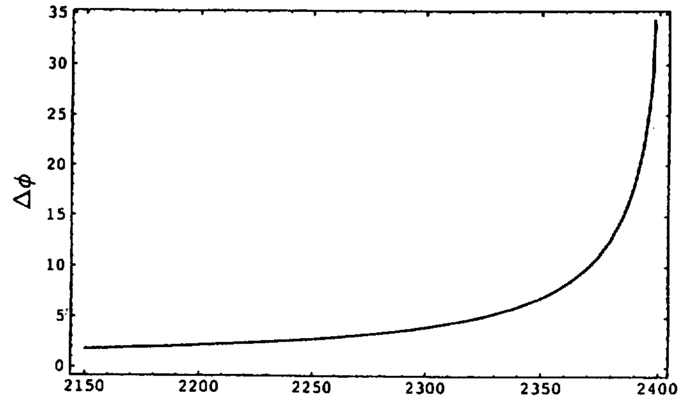
Relations between M_1 and $\Delta\phi$ both for Cases I and II are shown in Fig. 1. The deviation of mixing angle from heavy quark mass limit $\Delta\phi$ almost 0° to $\phi_0 \simeq 35°$ by varying the unknown D_1 meson mass M_1 from 2510 MeV to 2400 MeV for Case I and from 2460 MeV to 2436 MeV for Case II.

Next, we consider decay widths for the P-wave mesons. In order to extract the decay amplitude from experimental decay widths, the kinematical factors are removed by use of the following equation:

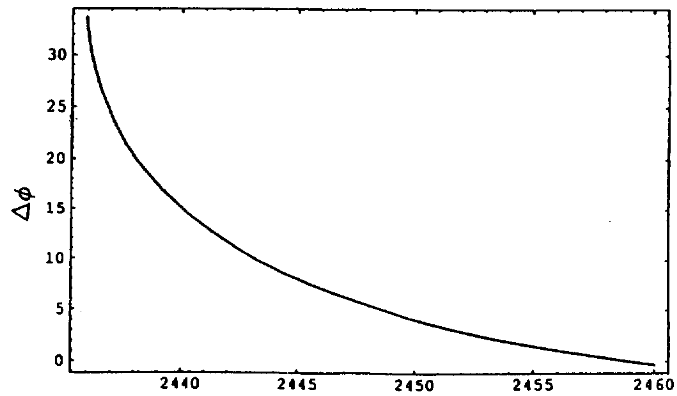
$$\Gamma = \frac{1}{8\pi M_{D^{**}}} p^{2L+1} |A|^2 |f(p)|^2 \tag{14}$$

$$= \frac{1}{8\pi M_{D^{**}}} p \tilde{\Gamma} |f(p)|^2\,, \tag{15}$$

where Γ, $M_{D^{**}}$ and $f(p)$ are the decay width, the initial hadron mass and the form factor, respectively. Reduced partial decay widths $\tilde{\Gamma}$ are given by the following



(a)



(b)

Fig. 1. Relations between the mass of unknown D_1 meson and the mixing angle $\Delta\phi$ between two D_1 mesons (a) for Case I and (b) for Case II.

equations:[7)]

$$\tilde{\Gamma}(D_2^* \to D^*\pi) = \frac{3}{10}(p/\kappa)^4 D^2 , \tag{16}$$

$$\tilde{\Gamma}(D_2^* \to D\pi) = \frac{1}{5}(p/\kappa)^4 D^2 , \tag{17}$$

$$\tilde{\Gamma}(D_1{}^\alpha \to D^*\pi) = \frac{1}{2}(S^2 \cos^2 \Delta\phi + (p/\kappa)^4 D^2 \sin^2 \Delta\phi) , \tag{18}$$

$$\tilde{\Gamma}(D_1{}^\beta \to D^*\pi) = \frac{1}{2}(S^2 \sin^2 \Delta\phi + (p/\kappa)^4 D^2 \cos^2 \Delta\phi) , \tag{19}$$

$$\tilde{\Gamma}(D_0^* \to D\pi) = \frac{1}{2} S^2 , \tag{20}$$

where S and D are the reduced S- and D-wave amplitudes, p is the magnitude of three momentum of decay pion in the center of mass frame for each decay process and κ is a momentum scale characteristic of the decay. The coefficients are determined by the Clebsh-Gordan coefficient for each decay process. We consider the relation of the mass and decay width for the unknown D_1 and D_0^* states by use of the amplitude for the D-wave decay of $D_2^*(2460)$.

In the following analyses, we assume the form factor of the decay vertex $f(p)$ is a Gaussian type given by

$$f(p)=\exp\left(-\frac{p^2}{2a^2}\right), \tag{21}$$

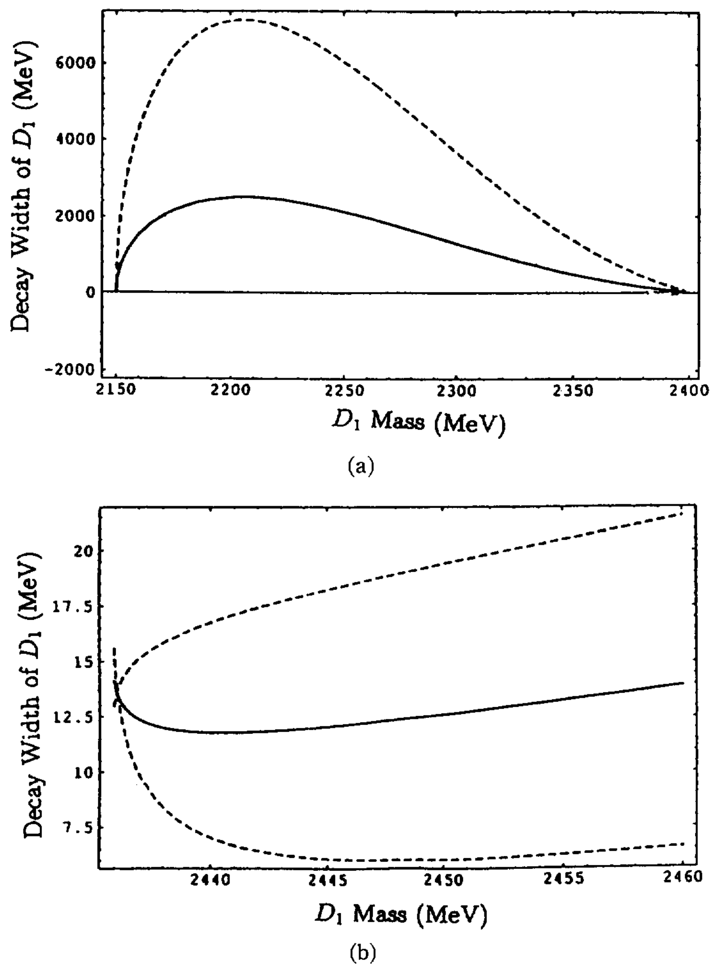


Fig. 2. Relations between the mass and decay width of unknown D_1 (a) for Case I and (b) for Case II. Solid lines show results obtained from experimental central values of the $D_2^*(2460)$ and $D_1(2420)$ decay widths. Dashed lines show experimental uncertainties.

where a is a parameter with the dimension of the mass.*) The parameter a is determined so as to reproduce the D-wave decay ratio,

$$\frac{\tilde{\Gamma}([D_2^* \to D\pi]_D)}{\tilde{\Gamma}([D_2^* \to D^*\pi]_D)} = \frac{2}{3} \tag{22}$$

which is derived from Eqs. (16) and (17). From the experimental value of the ratio of $D_2^*(2460)$ decay $\Gamma(D_2^* \to D\pi)/\Gamma(D_2^* \to D^*\pi) = 2.2 \pm 0.7 \pm 0.6$, we obtain $a = 2.0$ GeV. Although we use this value of a, the ratio of two amplitudes Eq. (22) is insensitive for the value of a.

We use the reduced D-wave amplitude D/κ^2 obtained from Eq. (16) and the

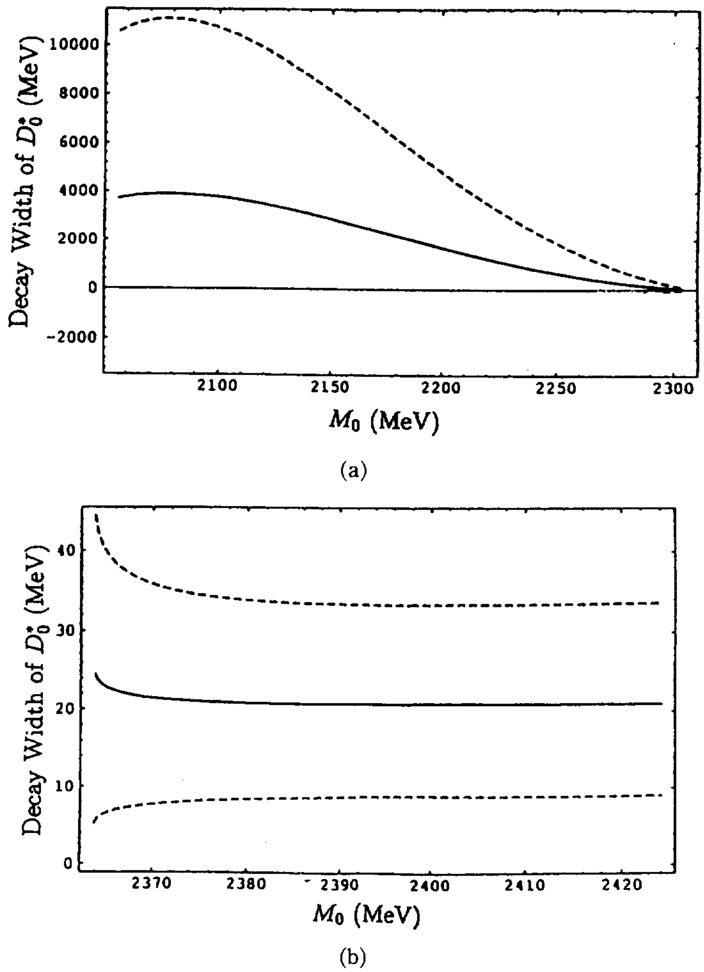


Fig. 3. Relations between mass and decay width of unknown D_0^* state (a) for Case I and (b) for Case II. Solid lines and dashed lines denote the same as in Fig. 2.

*) There is ambiguity of function form of the form factor. It may also be possible to take $f(p) = (p^2/a^2)\exp(-p^2/2a^2)$ for the D-wave decay form factor according to Ref. 9). However, the difference may be absorbed into the parameter a and the effect of ambiguity of form factor does not affect the analyses in this paper.

experimental value of $\Gamma(D_2^*(2460)\to D^*\pi)$. In Case I where $D_1(2420)$ is assigned to the $D_1{}^\beta$ state, we can determine the reduced S-wave decay amplitude by use of Eq. (19),

$$S^2=\frac{1}{\sin^2\Delta\phi}\left[\frac{16\pi M_{D_1(2420)}}{p|f(p)|^2}\Gamma(D_1(2420)\to D^*\pi)-\left(\frac{p}{\kappa}\right)^4 D^2\cos^2\Delta\phi\right], \tag{23}$$

where

$$\Delta\phi=\phi_0-\tan^{-1}\sqrt{\frac{M_{D_1(2420)}-\bar{M}}{\bar{M}-M_{D_1}}}. \tag{24}$$

The reduced decay width of the unknown D_1 is given by Eq. (18) in Case I. Calculated results for the decay width and mass of the unknown D_1 are shown in Fig. 2(a). The solid line shown in Fig. 2(a) is obtained from experimental central values of the decay widths of $D_2^*(2460)$ and $D_1(2420)$. Dashed lines show the upper bound due to the experimental uncertainties. Though the experimental error is rather large and almost zero decay width for the unknown D_1 meson is not rejected, the decay width is expected very large in Case I.

In Case II the reduced decay width of the unknown D_1 is given by Eq. (19). In Fig. 2(b) we show the relation between the mass and decay width of unknown D_1 for Case II. In this case the expected decay width lies between 6 and 20 MeV almost independent of unknown D_1 mass.

We obtained the reduced S-wave decay amplitude in Eq. (23) in Case I. By use of this S-wave amplitude we calculate the mass and decay width of D_0^* meson which are shown in Fig. 3(a) for Case I. By a similar method we calculate the S-wave amplitude in Case II and obtain the relation between mass and decay width of the D_0^* meson. The results for the D_0^* meson in Case II are shown also in Fig. 3(b). The decay width of D_0^* meson is expected to be very large in Case I, while about 20 MeV irrespective of its mass in Case II.

§ 3. Angular distribution of two decay pions from $D_1(2420)$

Since $D_1(2420)$ decays into $D^*\pi$ and D^* decays successively into $D\pi$ through the strong interactions, the final states of $D_1(2420)$ are $D\pi\pi$. Recently the angular distribution of the final two decay pions from the $D_1(2420)$ was measured by E687 Collaboration.[6] This experiment gives the important information for the mixing of S- and D-wave decay amplitudes. The angular distribution of pions at θ which is the angle between directions of two pions in the D^* rest frame is given by

$$\frac{1}{N}\frac{dN}{d\cos\theta}=\frac{1}{2}\left(R+(1-R)\frac{1+3\cos^2\theta}{2}+\sqrt{2R(1-R)}\cos\varphi(1-3\cos^2\theta)\right), \tag{25}$$

where φ is the relative phase between the S- and D-wave amplitudes. In Eq. (25) $R=\Gamma_S/\Gamma$ and Γ_S is the S-wave contribution to the total decay width Γ. Then $R=S^2\sin^2\Delta\phi/2\tilde{\Gamma}$ in Case I and $R=S^2\cos^2\Delta\phi/2\tilde{\Gamma}$ in Case II.

The distribution function in the form of $A(1+B\cos^2\theta)$ is fitted to the angular distribution of decay pions from the neutral $D_1(2420)$ measured by E687 Collaboration. The result is $B=2.74^{+1.40}_{-0.93}$.[5] If $D_1(2420)$ is the pure eigenstate with $s_l{}^P=(3/2)^+$ or with

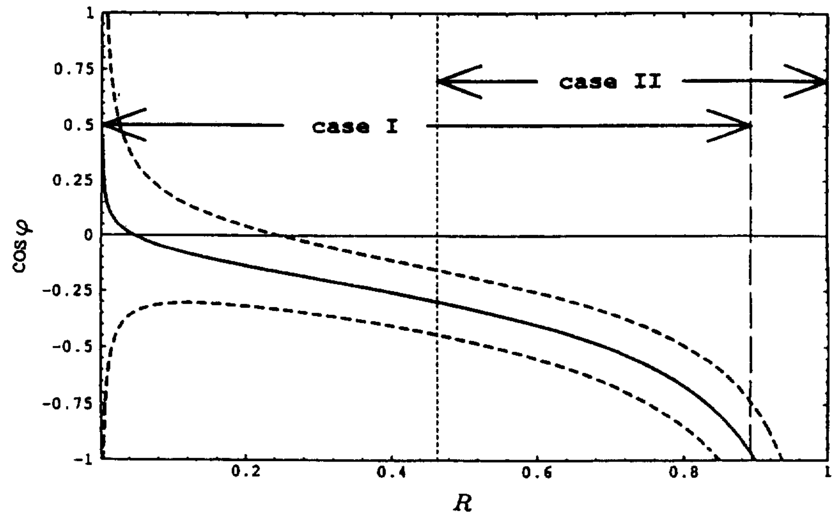


Fig. 4. Allowed area of the S-wave contribution to the $D_1(2420)$ decay width, denoted as R, and the relative phase φ of between the S- and D-wave decay amplitudes of $D_1(2420)$ meson. The experimental limits of R are shown for Cases I and II.

$s_l^P=(1/2)^+$, $B=3$ or 0, respectively. From this result the value B is consistent with the pure D-wave decay of $D_1(2420)$. Then one may conclude that $D_1(2420)$ belongs to the $s_l^P\simeq(3/2)^+$ state. However if we include the effects of relative phase between the S-and D-wave decay amplitudes there remains a possibility that $D_1(2420)$ belongs to the $s_l^P\simeq(1/2)^+$ doublet. The relation between the relative phase φ and the S-wave contribution R is given by

$$\cos\varphi=\frac{C-R}{2\sqrt{2R(1-R)}}, \tag{26}$$

where $C=(3-B)/(3+B)$. In Fig. 4 we show the plot of R versus $\cos\varphi$ by using the experimental B. The area between the two dashed lines is the allowed region determined by the E687 experiment.

The allowed region of R is also restricted from both the unknown D_1 mass and the decay width of $D_1(2420)$. For Cases I and II, we obtain the allowed regions for solutions from these constraints,

$$0\leq R\leq 0.893 \quad \text{(Case I)} \tag{27}$$

$$0.464\leq R\leq 1.0 \quad \text{(Case II)} \tag{28}$$

which are shown in Fig. 4.

As is seen from this result the E687 experiment does not reject the possibility that $D_1(2420)$ is a member of $s_l^P\simeq(1/2)^+$doublet, though the possibility to belong to $s_l^P\simeq(3/2)^+$ is much higher than to $s_l^P\simeq(1/2)^+$.

§ 4. Summary and discussion

In the preceding sections we studied the spectroscopy of P-wave charmed mesons from the Breit-Fermi Hamiltonian given in § 2. The two 1^+ states mixing, the masses and decay widths of unfound D_1 and D_0^* are evaluated. It is believed usually that the mixing angle will be small in the charmed mesons. In Case I where the $D_1(2420)$ meson belongs to $s_l^P=(3/2)^+$ dominant states, the decay widths of unknown D_1 and D_0^* are expected to be very large though the experimental uncertainties are rather large. The large decay widths of these mesons come from the very large S-wave amplitude which is needed to reproduce the experimental ratio $\Gamma(D_2^*\to D\pi)/\Gamma(D_2^*\to D^*\pi)=2.2\pm0.7\pm0.6$. The large decay widths may be the reason why these mesons are not discovered yet by experiments. On the other hand, in Case II, where $D_1(2420)$ is a member of the $s_l^P=(1/2)^+$ dominant doublet, the decay widths of unknown D_0^* and D_1 become small then these mesons will be easily observed by experiments.

We also analyzed the experimental angular distribution of final two pions of the $D_1(2420)$ decay by E687 Collaboration. Our results are that the possibility that $D_1(2420)$ belongs to a member of the $s_l^P=(3/2)^+$ dominant doublet is much higher than the possibility that $D_1(2420)$ belongs to $s_l^P=(1/2)^+$ dominant doublet, but the latter case is not completely rejected.

We are eager for further experiments to decide the spectroscopy of the P-wave charmed mesons including the $D_1(2420)$ more clearly and decay widths.

References

1) N. Isgur and M. B. Wise, Phys. Lett. **B232** (1989), 113; **B237** (1990), 527.
2) M. Neubert, Phys. Rep. **245** (1994), 259.
3) T. Mannel, W. Robert and Z. Ryzak, Phys. Lett. **B254** (1990), 274; **B259** (1991), 359.
4) M. Neubert, Phys. Lett. **B264** (1991), 455.
5) CLEO Collaboration, P. Avery et al., Phys. Lett. **B331** (1994), 236.
6) E687 Collaboration, P. L. Frabetti et al., Phys. Rev. Lett. **72** (1994), 324.
7) J. L. Rosner, Comment Nucl. Part. Phys. **16** (1986), 109.
8) S. Godfrey and N. Isgur, Phys. Rev. **D32** (1985), 189.
   H. J. Schnitzer, Phys. Rev. **D18** (1978), 3482.
9) N. Isgur, D. Scora, B. Grinstein and M. B. Wise, Phys. Rev. **D39** (1989), 799.
   T. B. Suzuki, T. Ito, S. Sawada and M. Matsuda, Prog. Theor. Phys. **91** (1994), 757.
10) N. Isgur and M. B. Wise, Phys. Rev. Lett. **66** (1991), 1130.