# Size and shape measurement in contemporary cephalometrics

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SUMMARY The traditional method of analysing cephalograms—conventional cephalometric analysis (CCA)—involves the calculation of linear distance measurements, angular measurements, area measurements, and ratios. Because shape information cannot be determined from these 'size-based' measurements, an increasing number of studies employ geometric morphometric tools in the cephalometric analysis of craniofacial morphology. Most of the discussions surrounding the appropriateness of CCA, Procrustes superimposition, Euclidean distance matrix analysis (EDMA), thin-plate spline analysis (TPS), finite element morphometry (FEM), elliptical Fourier functions (EFF), and medial axis analysis (MAA) have centred upon mathematical and statistical arguments. Surprisingly, little information is available to assist the orthodontist in the clinical relevance of each technique. This article evaluates the advantages and limitations of the above methods currently used to analyse the craniofacial morphology on cephalograms and investigates their clinical relevance and possible applications.

## Introduction

The methods currently available to evaluate craniofacial form include anthropometry, (stereo)photogrammetry, cephalometry, ultrasound, computed tomographic (CT) scanning, magnetic resonance imaging (MRI), and optical surface scanning. Arguably, cephalometry continues to be the most versatile technique in the investigation of the craniofacial skeleton because of its validity and practicality. Despite the inherent cephalometric distortion and differential magnification of the craniofacial complex, in comparison with newer imaging techniques, the cephalogram produces a high diagnostic yield at a low physiological cost (Melsen and Baumrind, 1995). Nevertheless, there are problems in deriving a numerical representation of craniofacial form using cephalometry (Chen *et al.*, 2000). This is because 'form' is the combination of 'size' and 'shape' (Sprent, 1972) and separating shape from size is complex (Hennessy and Moss, 2001). Perhaps the most important limitation of cephalometry relates to the errors inherent with the identification and recording of the structures therein. Interestingly, although the errors associated with the cephalometric technique have been extensively quantified (Battagel, 1993), few studies have investigated the errors associated with non-cephalometric imaging formats.

Because each cephalogram involves the exposure to a small, but not insignificant dose of ionizing radiation, they 'must be appropriately analysed in order to obtain the maximum clinical information' (Isaacson and Thom, 2001). The traditional method of analysing cephalograms (conventional cephalometric analysis, CCA) has, in recent years, been supplemented with a variety of sophisticated morphometric methods. Although these newer methods possess mathematical and statistical advantages, each has limitations, which may not be clear to the reader of study reports. This article investigates the techniques currently available for the analysis of cephalograms, their advantages and drawbacks, clinical relevance, and possible applications.

## Analysis of the cephalogram

There are two distinct groups of scientifically valid analytical methods used in cephalometry: landmark-based techniques and boundary outline methods. Landmark-based techniques are dependent on cephalometric landmarks: discrete points defined intrinsically in terms of the surrounding anatomy to represent the craniofacial form. As such, landmarks do not define the form of the object they represent; they lie upon it (Moyers and Bookstein, 1979). Landmarks convey information relating only to their location, providing no information either about the interlandmark or surrounding anatomy. In particular, landmarks cannot represent curving anatomy (Lavelle, 1989), and all are not equally valid and reproducible.

Landmark-based techniques include CCA, Procrustes superimposition techniques, Euclidean distance matrix analysis (EDMA), thin-plate spline analysis (TPS), biorthogonal grids (BOG), and finite element morphometry/finite element scaling analysis (FEM/FESA). CCA, Procrustes techniques, EDMA, TPS, and FEM/FESA are currently used in cephalometry and will be described. BOG has been superseded by FEM and is effectively redundant.

Boundary outline techniques do not require cephalometric landmarks to represent the craniofacial form. As their generic term suggests, they only investigate the shape of the perimeter of a structure. Medial axis

analysis (MAA), resistant-fit theta rho analysis, eigen shape analysis, and elliptical Fourier functions (EFF) are considered under the boundary outline technique umbrella. MAA and EFF are both of relevance in cephalometry and are described below.

## Conventional cephalometric analyses (CCA)

The use of algebraic measurements in traditional cephalometric analyses is now known as conventional cephalometric analysis. The simplicity of CCA ensures its universal clinical and research use. The four parameters employed in CCA are:

1. Linear distance measurements between two landmarks, such as articulare–gnathion, measuring mandibular length on the lateral cephalogram.
2. Angles, calculated from triplicate measurement of landmarks, e.g. SNA. Importantly, the size of angles varies with the relative spatial location of the landmarks (e.g. changes in the location of nasion).
3. Areas of triangles can be measured and summed, e.g. maxillary area on lateral cephalograms.
4. Ratios: usually of linear distance measurements. These can be compared between images obtained at different magnification factors. Spurious correlation can arise when several ratios are calculated using the same denominator.

### Statistical analysis

Usually a specific series of measurements conforming to a particular analysis [Steiner (1960), Downs (1956), Ricketts (1981), Eastman (Mills, 1970), McNamara (1984), Opal (Orthognathic Planning and Analysis, http://www.cix.co.uk/~felix/opal/)] is compared with appropriate referent data in the management of individual patients. When using CCA to compare groups of cephalograms, frequently univariate statistics such as two-sample *t*-tests are applied to CCA variables. Although such an approach may be valid when only one variable is under consideration (for example the angle ANB), comparing two morphologies using multiple linear, angular, area, and ratio variables is inappropriate. This is because these variables may not be independent and may be highly correlated. Moreover, multiple univariate tests may produce statistically significant results purely by 'chance'. Despite the availability of methods such as the Bonferroni correction, Monte-Carlo methods, or the Simes procedure to correct the level of significance with multiple testing, a more appropriate technique involves the application of 'traditional morphometrics' (Marcus, 1988). This is the application of classical multivariate statistics to a series of CCA measurements, such that simultaneous testing of these multiple variables creates a 'single-best composite' as an overall estimate of morphology.

The commonly used multivariate techniques include principal component analysis (PCA), principal co-ordinate analysis, factor analysis, canonical variates analysis, Mahalanobis distance analysis, and discriminant analysis (Marcus, 1988).

### Limitations of CCA

CCA relies on the use of a reference structure for orientation and superimposition: the anterior cranial base (sella–nasion) in lateral cephalometry. This is assumed to be biologically constant. Apparent changes occur only in relation to this plane (Richtsmeier and Cheverud, 1986). Even small changes in the anterior cranial base diminish its validity as a reference structure, rendering the localization of form differences between cephalograms difficult. Importantly, the use of a reference plane for the comparison of forms may be biologically meaningless.

CCA is an excellent method of describing a regular object (Lestrel, 1989a,b), however the craniofacial complex is an irregular biological structure. Although angles are size independent and have been coveted with having some relevance to shape, they cover large aspects of the craniofacial complex, failing to describe the information within the included angle (Lestrel, 1989a,b). As a result, CCA cannot adequately produce the shape detail demonstrated by the cephalogram, and is therefore not capable of fully evaluating craniofacial form.

Measurements calculate the magnitude of vectors between landmarks, ignoring their *direction* (Cheverud *et al.*, 1983). In the evaluation of anteroposterior growth of the maxilla, an increase in the ANS–PNS measurement cannot localize the region of change within the maxilla, or detect positional change in relation to the surrounding anatomical structures due to growth.

Some landmarks used in CCA (e.g. menton) are neither 'co-ordinate-free' nor invariant (Lestrel, 1989a), being dependent on a method of registration and superimposition. The location of many landmarks, e.g. Downs points 'A' and 'B' on the lateral cephalogram, is related to the subject's head posture during recording of the image.

One of the most significant limitations of CCA is the lack of objectivity. Thus investigators can choose the landmarks to be recorded and select the variables to be measured. On occasion, these may be selected to demonstrate the results desired by the investigator.

Despite the numerous drawbacks associated with CCA, this user-friendly simple technique is likely to continue in routine clinical use to determine an individual patient's response to treatment or the effect of growth. Above all, the comparison of cephalometric data of individual patients to referent data can only be conducted using CCA.

## Geometric morphometrics

The development of morphometrics has been accompanied by the introduction of a number of terms that are unfamiliar to most orthodontists. 'Morphometrics' is derived from the Greek words 'morph', shape, and 'mentron', measurement, used in contemporary investigations to define *size and shape* (Lele and Richtsmeier, 1990). Size change refers to a proportional increase or decrease in all dimensions of the form under examination, often accompanied by a change in shape. Changes in shape require a change in the outline of the form under examination, often resulting from localized size changes. Shape was defined by Kendall (1989), 'as the information remaining when location, size, and rotational factors are all removed'. This definition was advanced by Lele (1991) to encompass: 'that which remains invariant under scaling, translation, rotation, and reflection'. In light of the visual aspect, Chen *et al.* (2000) advocated the boundary of the form as part of the shape premise. Allometry is defined as the study of shape differences associated with size differences (Sprent, 1972; Slice *et al.*, 1998a), often prefixed by *growth-* or *size-*. Growth allometry refers to size and shape relationships in the same individual over time, whereas size allometry is reserved for studies involving different individuals.

The use of geometric morphometric tools in the analysis of form is also known as statistical shape analysis (Rohlf and Bookstein, 1988). The sophisticated morphometric techniques of Procrustes superimposition, EDMA, TPS analysis, FEM/FESA, and EFF produce unambiguous shape information *if the forms under comparison are scaled to an equivalent size beforehand*. The mathematical elegance and rigour of these techniques avoids the necessity for registration and superimposition—a prerequisite when using CCA. Therefore, any changes in the relative spatial relationship of the landmarks are solely due to shape changes. Furthermore, morphometric techniques allow the integration of the distinct information present in cephalometry: geometric *location* and biological *homology* (Bookstein, 1982), regardless of whether the information is collected using landmarks or outlines. [In craniofacial morphometrics, homology *per se* embodies biological correspondence (Bookstein, 1991), the same locus of a biological feature in different subjects.]

### Procrustes superimposition

Procrustes was a legendary Greek mythical character who regarded his iron bed as the standard of length. It had the unique property that its length exactly matched whoever lay on it. Procrustes' unique 'one-size-fits-all' method involved either stretching his victims or shortening them by cutting off their legs until they were able to fit his bed. Thus everyone was converted to an identical size. Procrustes superimposition programmes compute, visualize, and test the significance of the quantitative and qualitative difference between morphologies. Each form is represented by a series of landmark co-ordinates forming a figure, known as a configuration. For visualization purposes only, the landmarks can be linked by straight lines. Links have no effect on computations—they are only included to aid the spatial location of the landmarks. Procrustes software can be found at ftp://life.bio.sunysb.edu/morphmet/grf-ndz.exe (GRF-ND programme), ftp://life.bio.sunysb.edu/morphmet/tpssuperw32.exe (tpsSuper programme), and http://www.cpod.com/monoweb/aps (APS programme).
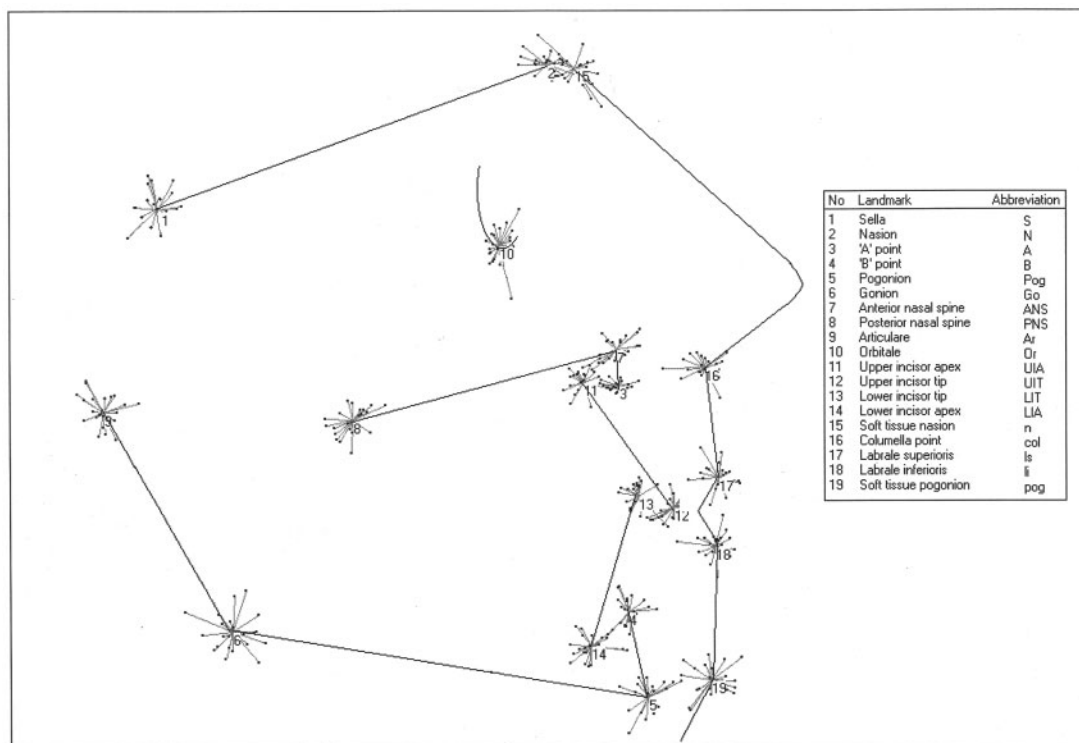
The configurations are firstly scaled to the same size. The Procrustes superimposition algorithms translate the configurations to superimpose the centroids and iteratively rotate the configurations to minimize the squared differences between the landmarks of the configurations (Auffray *et al.*, 1999). This is essentially the position of 'best-fit'. After the superimposition, the mean configuration called the *consensus* is computed. For each landmark, the Procrustes *residual* is calculated as the difference between the location of the landmarks of each form, and the position of the landmark in the consensus. These can be plotted to display the shape variance of a configuration of landmarks (Figure 1). The Procrustes residuals matrix can be used for further statistical procedures such as PCA to investigate shape variance. Thus an *F*-test can be used to statistically test the shape variance between the forms under investigation.

Procrustes analysis has been used for the evaluation of normal and syndromic craniofacial growth (Richtsmeier and Lele, 1990; Dean *et al.*, 2000). Singh *et al.* (1997a,b,c,d,e,f, 1998a,b,c,d,e, 1999a,b,c,d, 2000a,b), Singh and Hay (1999), Hay *et al.* (2000), Hay and Singh (2000), and Singh and Clark (2001) utilized Procrustes superimposition to uniformly scale their experimental groups as a precursor to further morphometric analyses.

### Euclidean distance matrix analysis (EDMA)

EDMA (Lele and Richtsmeier, 1991) quantitatively compares biological shapes using landmark co-ordinate data by mathematically localizing the morphological difference between two forms using a proportionate technique. The numerical output from EDMA is a series of Euclidean distance ratios between the two averaged forms. EDMA software can be found at Richtsmeier's laboratory website: http://faith.med.jhmi.edu/edma.html.

All Euclidean distances between the landmark pairs for the numerator and denominator morphologies are calculated and a mean form matrix is generated for each morphology. By systematically comparing pairs of homologous linear distances as ratios, an ordered form-difference matrix (FDM) can be produced (Table 1).

| No | Landmark | Abbreviation |
|---|---|---|
| 1 | Sella | S |
| 2 | Nasion | N |
| 3 | 'A' point | A |
| 4 | 'B' point | B |
| 5 | Pogonion | Pog |
| 6 | Gonion | Go |
| 7 | Anterior nasal spine | ANS |
| 8 | Posterior nasal spine | PNS |
| 9 | Articulare | Ar |
| 10 | Orbitale | Or |
| 11 | Upper incisor apex | UIA |
| 12 | Upper incisor tip | UIT |
| 13 | Lower incisor tip | LIT |
| 14 | Lower incisor apex | LIA |
| 15 | Soft tissue nasion | n |
| 16 | Columella point | col |
| 17 | Labrale superioris | ls |
| 18 | Labrale inferioris | li |
| 19 | Soft tissue pogonion | pog |

**Figure 1** Procrustes superimposition demonstrating the shape variance around landmarks digitized from a series of lateral cephalograms.

This allows the numerator and denominator morphologies to be compared by identifying the linear distances that differ most and least between the forms (Richtsmeier *et al.*, 1991). The FDM can be interpreted as follows. If all the elements in the FDM equal 1, both morphologies are identical. Consistent differences are attributable to size differences only. For ratios less than 1, the denominator distance is larger, the converse being true for ratios greater than 1. A variety of values greater and less than 1 in the FDM means the morphological differences involve size and shape. If Procrustes superimposition is conducted before EDMA, only shape is under investigation.

The $T$ statistic (ratio of the maximum/minimum elements) represents the total range of shape differences between the two forms. The statistical significance of $T$ is calculated by comparing the observed value with an empirical distribution of $T$ values from a non-parametric bootstrap procedure (Richtsmeier and Lele, 1993). If the observed value of $T$ is in the extreme right hand tail of the null distribution, the null hypothesis is rejected at the appropriate level of significance (Lele and Richtsmeier, 1991), producing a $P$ value. The median ratio estimates the general difference between the forms. Where the data are characterized by a few large or small values, the median ratio is an appropriate measure of the central tendency (Corner and Richtsmeier, 1991). The $T$ statistic and the median ratio

summarize the FDM (Table 1), and should be reported along with selected elements of significance.

Comparing interlandmark distances as ratios avoids the need for registration (O'Higgins and Jones, 1998), permitting the determination of 'influential' landmarks in the difference between two series of cephalograms (optimized when uniformly scaled by Procrustes superimposition). In contrast to the subjective individually measured variables in CCA, the process of calculating and comparing all the possible Euclidean distances simultaneously means that EDMA is co-ordinate invariant, and ensures geometric integrity of the forms under consideration (Corner and Richtsmeier, 1991). In contrast to TPS, no attempt is made to generate interlandmark data, as the Euclidean distances that are calculated are 'as the crow flies'. There are, however, drawbacks with EDMA. EDMA allows 'influential' landmarks to be discerned, but does not allow relative landmark movements to be addressed. Moreover, because the FDM is computed from a mean form matrix for each morphology, 'outliers' can adversely influence the results. Therefore, over 40 images need to be included in each group under test for this analysis to provide worthwhile information. The software available at http://faith.med.jhmi.edu/edma.html alerts the investigator to the existence of possible outliers, and calculates marginal confidence intervals for the elements of the FDM. The visualization and interpretation of EDMA

**Table 1** Form difference matrix (FDM).

| Euclidean distance | | Ratio |
|---|---|---|
| Figure 2a | B–Gn | 0.569 |
| | S–Ar | 0.583 |
| | Gn–Go | 0.589 |
| | N–ANS | 0.591 |
| Figure 2b | S–Go | 0.605 |
| | N–A | 0.614 |
| | Ar–Go | 0.619 |
| | S–PNS | 0.619 |
| | Ar–Gn | 0.620 |
| | S–Gn | 0.629 |
| | PNS–Gn | 0.631 |
| | B–Go | 0.632 |
| | PNS–Go | 0.637 |
| Figure 2c | N–Gn | 0.652 |
| | Ar–PNS | 0.653 |
| | Ar–B | 0.657 |
| | N–Go | 0.659 |
| | **S–B** | **0.663** |
| | ANS–A | 0.666 |
| | N–B | 0.670 |
| | ANS–Go | 0.670 |
| | N–PNS | 0.678 |
| | PNS–B | 0.679 |
| | Ar–ANS | 0.680 |
| | S–ANS | 0.682 |
| | A–Go | 0.683 |
| | PNS–ANS | 0.694 |
| | Ar–A | 0.695 |
| | S–A | 0.695 |
| Figure 2d | N–Ar | 0.702 |
| | A–Gn | 0.703 |
| | ANS–Gn | 0.712 |
| | PNS–A | 0.723 |
| Figure 2e | S–N | 0.762 |
| | ANS–B | 0.770 |
| | A–B | 0.770 |

Landmark abbreviations as in Figure 1. The ratios are graphically displayed in Figure 2a–e.
*T* statistic (max/min), 1.354; median ratio in bold.
(S–B = 0.663).



**Figure 2** EDMA ratios.

results is complex. At present, the software available does not produce a graphical display of the results. However, this can be produced indirectly. Clinically important ratios can be depicted by lines representing the relevant interlandmark distances (Figure 2). Thus, EDMA can be used to identify the regions of shape change between cephalograms due to growth or orthodontic treatment.

EDMA has been used to study craniofacial growth in both normal individuals and in those with Crouzon syndrome (Richtsmeier and Lele, 1990), the craniofacial morphology in subjects with Class III malocclusions (Singh *et al.*, 1998a,b,d), and the evaluation of the stability of osteotomies (Ayoub *et al.*, 1993, 1994, 1995, 2000), and of surgical changes in craniofacial microsomia patients treated with an inverted 'L' osteotomy (Hay *et al.*, 2000; Cerajewska and Singh, 2001).
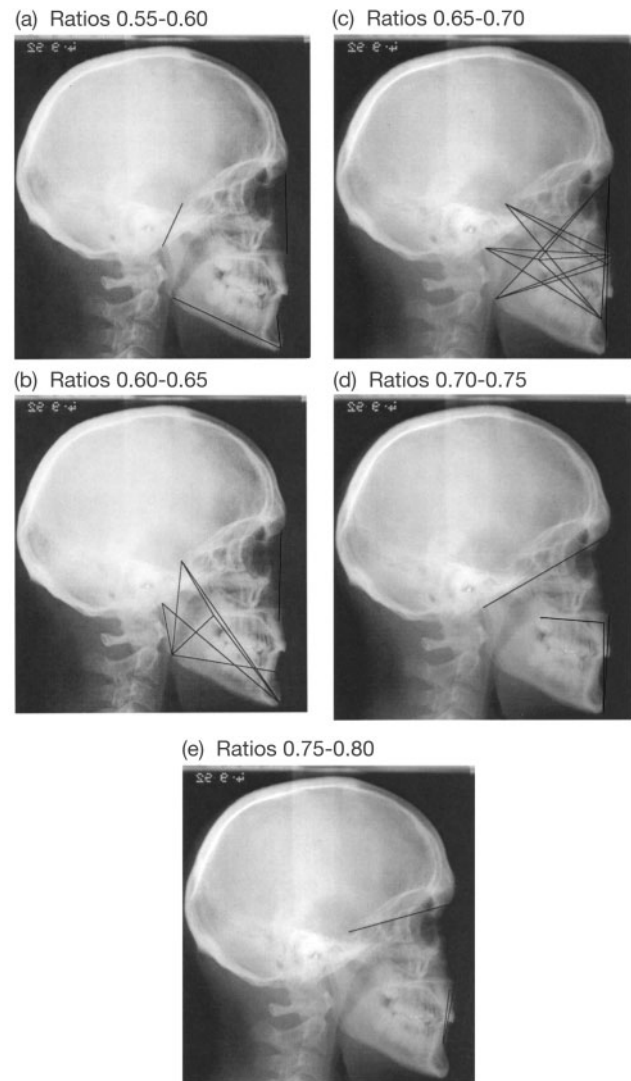
*Thin-plate spline analysis (TPS)*

TPS quantitatively analyses shape change (Bookstein, 1989) using the theory of surface spline interpolations (Bookstein, 1991) to express the differences between two landmark configurations as a continuous deformation. 'Spline' is a smooth piecewise polynomial function, named after the draughtsman's instrument used to draw curves (Segner, 1986). TPS uses an interpolation function representing a mapping. This models the 'biological homology' of landmark pairs. The interpolant is basically a smooth function fitted to the landmark set. The TPS function, colloquially known as the 'bending energy' is visualized as an infinitely thin metal sheet draped over a set of landmarks, extending to infinity in all directions. The surface of the metal sheet demonstrates pairwise displacements of each landmark as a deformation (Bookstein, 1989). The height over each landmark is

equal to the differences between the forms. TPS software is available at ftp://life.bio.sunysb.edu/morphmet/tpssplnw.exe (tpssplin.exe programme).

The configurations of the two forms are matched exactly to minimize the bending energy (Richtsmeier *et al*., 1992). If the two forms are identical, then the bending energy is zero, and the plate is flat. The magnitude and location of bending energy can be identified depending on the size and position of the deformation of the plate. The total spline depicts the vectors of deformation in registration-free morphospace. This deformation can be decomposed into affine and non-affine transformations. The affine transformation delineates changes due to size differences, rotation, and uniform shape change. It should be noted that there are no size-related affine transformations when the forms are uniformly scaled beforehand. The affine change has been described as 'the parallel lines remain parallel' (Slice *et al.*, 1998a). The bending energy of an affine transformation is zero and only tilting of the plate may occur (Figure 3a). Non-affine transformations (Figure 3b) delineate non-uniform or local deformations. These can be further decomposed into localized components, represented by partial warps (Slice *et al*., 1998b), corresponding to deformations at differing geometric scales. The systematic comparison of individual partial warps towards the total spline determines the contribution of the partial warp to the morphology under test. The contribution of each partial warp to the non-affine component is determined by its eigenvalue, magnitude, and bending energy. High eigenvalues relate to localized transformations, whereas a high magnitude relates to a shape difference affecting the entire landmark configuration. The bending energy quantifies the 'transformation', representing the amount of bending of the metal plate. This bending energy is greater for localized deformations than for generalized changes. Shape changes can be statistically analysed using multivariate statistical techniques based on the matrices of partial-warp scores (Lux *et al*., 2001).

TPS produces a visually appealing representation of the morphological change between the forms—an excellent tool to localize shape differences due to growth or orthodontic treatment between two series of cephalograms. Notwithstanding, critics suggest that as TPS is a mathematically based product, the choice of the spline function is dependent on mathematical properties rather than the potentially more relevant biological model (as in EDMA) when dealing with biological data (Lele and Richtsmeier, 1991). Furthermore, the interlandmark data generated in TPS transformations should be ignored, as it is potentially inaccurate.

TPS has been utilized to analyse the facial and tongue morphology in obstructive sleep apnoea (Pae *et al*., 1997a). Singh *et al*. (1997a,c,d,f, 1998e, 1999c)
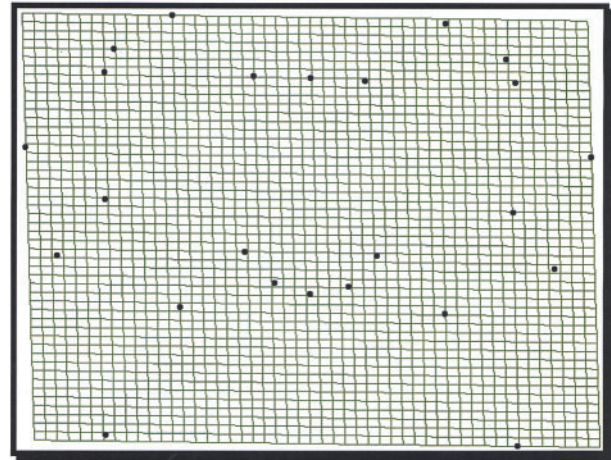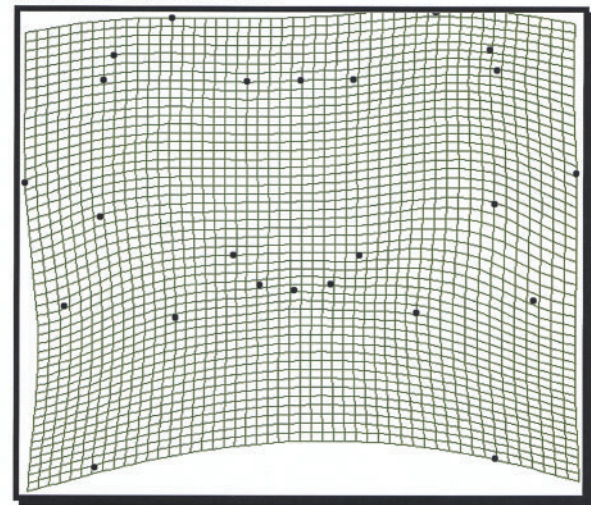


**Figure 3** Thin-plate spline transformation of postero-anterior cephalometric landmarks.

comprehensively evaluated the craniofacial morphology in subjects with Class III malocclusions with TPS. Hay and Singh (2000) and Cerajewska and Singh (2001) used TPS to analyse the effect of an inverted 'L' osteotomy in craniofacial microsomia patients. Baccetti *et al.* (1999) employed TPS to evaluate the effects of rapid maxillary expansion and face mask therapy in Class III malocclusions, whilst Franchi *et al.* (2001) used TPS to analyse mandibular growth.
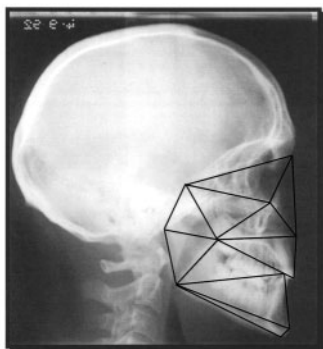
*Finite element morphometry/scaling analysis (FEM/FESA)*

FEM/FESA was developed from an engineering model for use in biological morphometrics. Finite element analysis (without 'scaling') is used in continuum mechanics to estimate the deformation resulting from a pattern of forces acting on a mechanical system. In biological morphometrics, FEM is used inversely to

calculate the strains that represent the hypothetical forces required to distort one form to the other (Slice *et al.*, 1998a). FESA software can be found at ftp://life.bio.sunysb.edu/morphmet/chev.exe. The forms of the two averaged landmark configurations are divided into triangles (and where appropriate tetrahedra, hexahedra, octahedra). These are the *finite elements* (Figure 4), each consisting of a boundary with the landmarks or 'nodes' at each apex and an internal continuum of particles or points. Because the finite elements in cephalometry are not of uniform size (Figure 4), their relative importance can be weighted. The quantitative expression of the deformation of the finite elements of the reference and target forms provides a numerical representation of form change (Lozanoff, 1999). This output can be expressed as a size ratio, shape ratio, and the angle of maximum strain value for each element (Moss *et al.*, 1995). There are no established statistical procedures to test these data (Sameshima *et al.*, 1997); however, an analysis of variance is one appropriate method.

The assumption is that the interiors of the finite elements deform uniformly in relation to their defining landmarks. FEM overcomes this problem by allowing deformations to be assessed at each point (Cheverud *et al.*, 1983). Thus, FEM is a sensitive morphometric technique. Nonetheless, the validity of interlandmark information obtained by any interpolation method is dubious. Consequently, where interpolation is not used and only triangles based on specific landmarks are utilized, the relevance of the analysis is improved. The magnitude of local size and shape changes and their contribution to the overall morphological differences can be visualized by a colour spectrum and a calibration axis (Singh and Clark, 2001).

FEM uses the triangle as its basic unit for form measurement, and the inherent limitations of triangles in biological morphometrics have been described earlier. The algebraic limitations are overcome because FEM is co-ordinate invariant, measuring only the resultant strain required to deform one object into the other—not comparing a series of individual measurements obtained from each form (as in CCA). This means that FEM can estimate the shape change of the structure under examination, in all directions, and at each and every landmark. This is not possible with CCA. Differences in craniofacial form due to growth or skeletal discrepancy are anisotropic (involving size and shape) and non-linear (changes in linear distance measurements and angles are not insignificant) and thus the use of the principles of non-linear continuum mechanics in FEM is mathematically advantageous in comparison with the geometric simplicity of CCA.

FEM was developed for use where each element relates to an inanimate homogeneous structure. This is not the case in cephalometry. There is also a major difference between measuring mechanical strains and the craniofacial complex, where the only physiological forces present are gravity and muscle pull (Melsen and Baumrind, 1995), exerting minimal influence in craniofacial morphology. Whilst most applications of FEM in cephalometry have been research-based, clinically useful software is available (Sameshima and Melnick, 1994). Because FEM is a sensitive technique, the level of residual measurement error adversely influences the reliability of the method (Ayoub and Stirrups, 1993). Thus, FEM is not suitable for the individual case. However, the effect of measurement error is reduced with inter-group comparisons, and the reliability of the analysis is satisfactory.

FEM/FESA has been widely used with cephalometry. The craniofacial morphology and growth in normal subjects and those with Crouzon and Apert syndromes has been investigated by Richtsmeier and Cheverud (1986), Richtsmeier (1987, 1988), and Richtsmeier and Lele (1990). Changes in craniofacial morphology with two modes of orthodontic treatment were evaluated using FEM by Book and Lavelle (1988). Hammond *et al.* (1993) characterized shape and size differences between unilateral cleft lip and palate subjects and controls using FEM. Sameshima *et al.* (1997) assessed the ethnic differences in craniofacial growth in response to orthodontic treatment using FEM. FESA has been extensively used in the analysis of the craniofacial morphology of subjects with Class III malocclusions (Singh *et al.*, 1997a,b,e, 1998c, 1999a,b,d, 2000a,b). Similarly, FESA has been applied in evaluating the stability of osteotomies (Ayoub *et al.*, 1993, 1994), and the skeletal changes produced by genioplasties (Ayoub and Stirrups, 1993). Singh and Hay (1999) reported on the use of FEM of the mandible in prepubertal craniofacial microsomia patients following an inverted 'L' osteotomy, whilst Singh and Clark (2001) used FEM to analyse the mandibular changes associated with Twin Block therapy.



**Figure 4** Finite element discretization for FEM.

## Elliptical Fourier functions (EFF)

EFF software can be found at ftp://life.bio.sunysb. edu/morphmet/efaz.exe. The EFF technique was developed originally for military aircraft identification (Lestrel *et al.*, 1999), and like conventional Fourier functions is a curve-fitting procedure. The basic principle involves embedding a set of closely spaced observed measurements on an object's boundary into a mathematical function. EFF is a parametric solution to shape description, deriving a pair of equations as functions of a third variable (Lestrel, 1989a). The advantage of EFF over landmark-based techniques is that EFF does not require landmarks for the analysis to operate, although these can be included. Multiple points are digitized along the outline of the structure under consideration (*observed form*) and EFF computes the *predicted form* using a stepwise procedure based on harmonic coefficients (Chen *et al.*, 2000). As the number of harmonics is calculated to be half the number of data points, the closer the points, the more accurate the fit of the polygon. The first harmonic represents an ellipse, with higher harmonics detecting increasingly localized shape differences. The accuracy of the procedure can be determined by calculating a residual value—the difference between the observed data and the predicted values derived from the EFF. Chen *et al.* (2000) noted that residual values less than 0.3 mm are desirable. Like EDMA, FEM, and TPS, EFF is co-ordinate invariant. Although EFF represents shape, shape change can be calculated following size standardization. To investigate shape change, the size of all the specimens under consideration can be standardized and superimposed. The distances between the centroid and predicted landmark points on the boundary can then be calculated and tested for statistical significance using multivariate statistical techniques (Hotelling's $T^2$ test, MANOVA, cluster analysis, principal co-ordinate analysis). In contemporary craniofacial morphometrics the greatest limitation of EFF is that it can only be used with two-dimensional images. This prevents potentially valuable three-dimensional EFF-derived information from being compared with cephalometric EFF data. EFF has been used in several previous lateral cephalometric studies: investigating skeletal jaw relationships (Lowe *et al.*, 1994), quantification of function regulator therapy (Lestrel and Kerr, 1993), and evaluating the shape changes in the cleft palate maxilla (Lestrel *et al.*, 1999). EFF has also been used in the evaluation of mandibular form from lateral cephalograms (Ferrario *et al.*, 1999; Chen *et al.*, 2000), and from the Bolton templates (Ferrario *et al.*, 1996).

## Medial axis analysis (or transformation) (MAA)

*Median axes* are a geometric transformation of an outline identifying a branching set of points constituting the middle of a form (Straney, 1990). MAA software can be found at ftp://life.bio.sunysb.edu/morphmet/ stran1.exe. The medial axis can be considered as conjoined centres of circles maximally contacting the shape boundary. Where a circle contacts more than two points on the shape boundary, a branch point is identified for the medial axis. The medial axes begin and end where anatomical structures of the bilateral sides converge on the image, such as at the coronoid processes. This axis, in addition to the expression of its distance from the peripheral boundary, provides shape information, independent of size. A series of measurements can also be derived from the medial axes and statistically tested using univariate and multivariate techniques (Pae *et al.*, 1997b). MAA has not been widely used for the examination of craniofacial morphology. However, Lavelle (1984) found that the results of mandibular shape produced using MAA differed considerably from published results using CCA, whilst Lavelle (1985, 1987) found that MAA of the mandible and the basicranial axis form on lateral cephalograms differed between micro-, macro-, and normo-cephalic individuals. Grayson *et al.* (1986) also used MAA to investigate the mandible in mandibulofacial dysostosis. The complexities of medial axes and the measurements derived from them mean that MAA is not useful for the clinical management of individual patients, but more so for intergroup comparisons. Moreover, MAA is only suitable for relatively simplistic shapes such as the mandibular or soft palate outlines. The application of MAA to the craniofacial complex would produce a myriad of medial axes. These would be confusing and difficult to interpret.

## Comment

There is no universal agreement between mathematicians, statisticians, researchers, and clinicians as to the most appropriate method of analysing cephalograms (Table 2). Many of the arguments regarding the interpretation of shape from size-based computations have been resolved, and it is recognized that the erudite geometric morphometric techniques are the most appropriate methods of deriving shape information from cephalograms. Nevertheless, CCA provides predominantly size-based data and limited morphological information. Despite the well-known drawbacks associated with multiple univariate tests, few cephalometric studies employ multivariate techniques. It remains that information derived from CCA is useful for analysis of the individual case and CCA has a significant role in routine clinical practice, where morphometric analyses of a single cephalogram are not possible. Moreover, there is no consensus about the suitability of morphometric techniques in differing circumstances (O'Higgins and Jones, 1998). Procrustes superimposition techniques scale a

**Table 2** Summary of analytical techniques used in cephalometry.

| Technique | Landmarks required? | Size data | Shape data | Statistical treatment of data | Analysis of an individual case? | Analysis of groups of cephalograms? | Visual output |
|---|---|---|---|---|---|---|---|
| CCA | Yes | Yes | No | Various univariate/ multivariate methods | Yes | Yes | Poor, must be produced indirectly |
| Procrustes superimposition | Yes | No | Yes | Principal component analysis | No | Yes | Good |
| EDMA | Yes | Yes | Yes | Compare observed $T$ to distribution of $T$ values (non-parametric bootstrap) | No | Yes | Must be produced indirectly |
| TPS | Yes | No | Yes | Multivariate analysis of partial warp scores | No | Yes | Good |
| FEM | Yes | Yes | Yes | Various univariate/ multivariate methods | See text | Yes | Good |
| EFF | No, can be included | Yes | Yes | Various univariate/ multivariate methods | No | Yes | Good |
| MAA | No | No | Yes | Various univariate/ multivariate methods | Possible | Yes | Difficult to interpret |

landmark configuration to a uniform size and facilitate the quantification and visualization of shape variance around landmarks. Although the Procrustes residuals can be analysed statistically, perhaps to differentiate the shape changes that occur with growth, Procrustes superimposition should precede other morphometric techniques if *purely* shape information is to be derived. EDMA provides a comprehensive numerical output of the morphological differences between two forms. The statistical significance of the test statistic ($T$) can determine whether there is a shape difference between the forms under comparison. The ratios determined to be of clinical importance, perhaps those representing greater than a 10 per cent shape difference, can be graphically displayed to demonstrate the shape difference. EDMA is specifically indicated where the detection of the influential landmarks in the form difference is desirable, such as the comparison of two types of appliance therapy. TPS analysis deforms one landmark configuration into another, illustrating this shape change as the deformation of a grid. TPS has specific cephalometric indications for displaying shape differences due to different orthodontic treatment techniques or growth-related changes. FEM quantifies the differences between two forms, deforming one landmark configuration into another by calculating the required strain. The vivid display that can be generated is one means of comparing two orthodontic treatments, accurately localizing and quantifying the shape difference between them. EFF and MAA are techniques that are particularly useful for analysing the shape of outlines of structures, especially where viable landmarks do not fully represent the curving biological form, such as the lateral cephalometric mandibular outline. Because they do not

rely on individual landmarks, they are not limited by the inherent error of landmark identification.

The choice of the individual morphometric technique used can be likened to the holistic principle (Anekàntvàda) of Jain logic: if six blind men each touch a different part of an elephant, they come to a differing opinion. In consequence, the elephant should be looked at from all sides (Mardia, 1999). Thus, the use of only one morphometric technique in the evaluation of cephalometric craniofacial form may only, in part, describe overall form. The particular technique selected will depend on the type of information that is required to be derived, be that size, shape, or overall morphology. Moreover, where any doubt exists as to the best analytical method to use, be it CCA, Procrustes, EDMA, TPS, FEM, EFF, or MAA, it may be preferable to use more than one technique. With such an approach to the evaluation of craniofacial form, the corroboration of results from different techniques would be ideal. Non-corroborative results (including contradictory results) could be explained by the limitations of the individual techniques. The computer age continues to provide tremendous opportunities for the development of morphometric techniques. Nevertheless, because of the practical difficulties in interpreting morphometric data and the graphical display of results, it is likely that the morphometric toolkit will remain within the realm of orthodontic research. Although CCA will continue to be widely utilized by clinical orthodontists, univariate statistics are overused and future clinical research should instead make greater use of more appropriate multivariate techniques. Furthermore, the opportunity exists for future cephalometric studies to utilize the symbiosis of CCA and sophisticated

morphometric techniques. This is of particular relevance where shape and size changes characterize a form difference such as that which occurs with growth. Moreover, shape changes that occur during growth may not be detected if CCA is used in isolation to measure growth-related changes.

## Further information

For information, software for download, and links to other morphometrics websites see: http://life.bio.sunysb.edu/morph/ and http://www.cwru.edu/dental/orth/ortho/morphmet/mmresc.html.

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