Secular trends in mean tidal range around the British Isles and along the adjacent European coastline

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SUMMARY
Time series of annual Mean Tidal Range (MTR) have been assembled from 13 ports around the British Isles and secular trends in MTR computed at each site. Trends vary between −1.8 and 1.3 mm yr⁻¹ depending on location. At many sites the values are significantly non-zero implying different trends in Mean High Waters (MHWs) and Mean Low Waters (MLWs). Such tidal behaviour has also been observed previously at stations along the adjacent European coastline, but is not well understood. At several places, the trends are sufficiently large that they should be taken into account in investigations of impacts of sea level change and in extreme level engineering studies. They also suggest that, in general, time series of MWH should not be used as proxies for series of Mean Sea Level (MSL). For most of the British Isles data, MTR secular trend is larger (more positive) for larger trend in local MSL, or water depth. Lerwick and Newlyn hourly heights have been used to show that the observed MTR trends at most locations must be due primarily to changes in the dominant M2 tidal constituent. A comparison is given of the British Isles findings to those from neighbouring countries; British, Irish, French, Belgian and perhaps southern Dutch MTR trends are found to be considerably less than those reported from the northern Netherlands and the German Bight.

Key words: coastal impacts of level changes, high and low waters, mean sea level trends, nodal tidal cycles, tidal constituent changes.

INTRODUCTION
Many researchers will be familiar with the geographical pattern of secular trends in Mean Sea Level (MSL) around the British Isles and along the adjacent European coastline. The rate of change of MSL, determined from tide gauge data and averaged over the last century, is of order 1–2 mm yr⁻¹ at most stations with smaller, or even negative, trends observed in Scotland and northern and eastern Ireland, and larger than average trends found in southeast England and at mid-Channel ports of England and France (Rossiter 1967; Carter 1982; Emery & Aubrey 1985; Woodworth 1987). The spatial pattern is similar to that determined from geological data averaged over several thousand years, modulo an overall trend reflecting different regional eustatic changes over the different averaging periods (Flemming 1982; Shennan 1987, 1989). Altogether, the trend data are usually interpreted as spatial variations in vertical land movements due to isostatic effects and local tectonics, superimposed upon which are regionally coherent eustatic MSL trends due to slow climatic change.

Less well-known, and less well-understood, is the fact that at many British Isles and European locations the secular trends in Mean High Waters (MHWs) and Mean Low Waters (MLWs), which taken together define the trend in MSL [or, specifically, the trend in the almost identical quantity Mean Tide Level (MTL)], are significantly different, resulting in apparent long-term changes in Mean Tidal Range (MTR) where $\text{MTR} = \text{MHW} - \text{MLW}$ (de Ronde 1983, 1989; Führbörter & Jensen 1985; Jensen et al. 1988; Führbörter, Dette & Töppe 1990; J. Jensen et al., private communication).

There are four main reasons for the present study. Firstly, the intention is to provide estimates of MTR trends at a representative number of stations around the British Isles to provide a comparison data set to those from the adjacent European coastline and to extend previous work at this laboratory on MTR trends in the River Thames (Rossiter 1969; Bowen 1972; Amin 1983). Secondly, there exist important long MHW records at several UK tide gauge stations for which there are no corresponding MLW records due to gauges bottoming-out at low tide. Therefore, if it could be demonstrated that MTR trends in British and Irish waters are generally small, then, in principle, MHW records could be employed in future MSL studies as proxies for MSL records. Thirdly, for those interested in the impacts
of sea level change, then MHW (or alternatively MSL and MTR in combination) is the variable of interest, rather than MSL alone. Therefore, an intention of this study is to demonstrate how MHWs are changing differently from MSLs. Knowledge of such differences is also of importance to coastal engineers for extreme level studies. Finally, there is continued interest in understanding the temporal changes in the tides of the deep ocean and the changes in tidal dynamics of the northwest European continental shelf, particularly from the point of view of understanding the palaeotides of the region and the future tidal changes which might result from significant increases in depth due to ‘greenhouse warming’. Such studies will not be possible without the most complete historical tidal data set.

This paper is the first of two investigations into long-term tidal changes around the British Isles which have been undertaken as part of a sea level research programme commissioned by the UK Ministry of Agriculture, Fisheries and Food. It will concentrate on a presentation of the historical MTR data set which has been assembled over the past two years. A second paper will consist of a numerical modelling study of changes in shelf tides with changes in water depth and with changing meteorology and will address aspects of the data presented in this study.

DATA SOURCES AND ANALYSIS METHODS

Table 1 lists the British Isles tide gauge stations which have been used in this analysis; station locations are shown in Fig. 1. In order to establish a MTR trend, at least 15 years of data were required spanning at least 28 years, approximately one and a half lunar nodal cycles. From the size of the standard errors of the trends determined below it will be seen that such short records are the minimum necessary to provide adequate estimates. Data are available in addition from another 14 stations with even shorter records. These are not used in this report but have been included in our data set with the purpose of adding future data to them in order to maintain an ongoing British Isles-wide monitor of MTR change.

Table 1 specifies for each station-year of data the method by which the annual MTR was computed or the reference from which it was obtained. The two methods employed were ‘turning points’ and ‘tabulations’. In the turning point method, hourly heights of sea level from the data bank of the Tidal Computation and Statistics Section (TCSS) of the Proudman Oceanographic Laboratory (POL) were interpolated, using a cubic spline fitted to four consecutive hourly heights as interpolation function, in order to provide the heights and times of high and low waters. All the ports we have studied are in areas of predominantly semi-diurnal tide and there are few cases of double high or low water. Those which do occur are due to strong storm surge events and in such cases the data set was manually inspected and adjusted for correct choice of higher high water or lower low water. Turning point values were subsequently added together to form annual MHW, MLW and MTR with the proviso that an acceptable annual mean had to include data from at least 90 per cent of the year, in order to avoid significant distortion to the mean from missing data and from the seasonal cycle of MTR.

Table 1. British Isles tide gauge stations together with the periods of MTR data available (ignoring gaps) and the method by which MTR has been computed. Some years of data have been subsequently dropped from the analysis following selection criteria described in the text.

<table>
<thead>
<tr>
<th>Station</th>
<th>Period</th>
<th>Method of Data Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lerwick</td>
<td>1959-1987</td>
<td>Turning points</td>
</tr>
<tr>
<td>Aberdeen</td>
<td>1862-1913</td>
<td>From Thompson (1914), see text</td>
</tr>
<tr>
<td></td>
<td>1931-1962</td>
<td>HWs and LWs from Harbour Board</td>
</tr>
<tr>
<td></td>
<td>1968-1986</td>
<td>Turning points</td>
</tr>
<tr>
<td>Blyth</td>
<td>1954-1987</td>
<td>HWs and LWs from Harbour Board</td>
</tr>
<tr>
<td>Immingham</td>
<td>1956-1986</td>
<td>Turning points</td>
</tr>
<tr>
<td>Southend</td>
<td>1934-1996</td>
<td>Trends have been taken from Rossiter (1989) and Bowen (1972)</td>
</tr>
<tr>
<td>Dover</td>
<td>1925-1986</td>
<td>Turning points</td>
</tr>
<tr>
<td>Newlyn</td>
<td>1915-1985</td>
<td>Turning points</td>
</tr>
<tr>
<td>Avonmouth</td>
<td>1925-1987</td>
<td>HWs and LWs from Harbour Board</td>
</tr>
<tr>
<td>Holyhead</td>
<td>1936-1967</td>
<td>From annual tabulations by Harbour Board archived at the FOMSL</td>
</tr>
<tr>
<td></td>
<td>1964-1986</td>
<td>Turning points</td>
</tr>
<tr>
<td>Liverpool</td>
<td>1858-1911</td>
<td>HWs and LWs from Harbour Board (George’s Pier)</td>
</tr>
<tr>
<td></td>
<td>1963-1985</td>
<td>Turning points (Princes Pier)</td>
</tr>
<tr>
<td>Douglas</td>
<td>1938-1970</td>
<td>From annual tabulations by Harbour Board archived at the FOMSL 1977</td>
</tr>
<tr>
<td>Belfast</td>
<td>1917-1963</td>
<td>HWs and LWs from Harbour Board</td>
</tr>
<tr>
<td>Dublin</td>
<td>1938-1988</td>
<td>From annual tabulations by Harbour Board archived at the FOMSL</td>
</tr>
</tbody>
</table>

‘Tabulations’ took two forms. The first form comprised compilations of annual MHWs and MLWs made available by harbour authorities or taken from previous analyses. The complete details of such calculations are not always reported and the quoted values cannot always be checked. A second form consisted of lists of historical daily high and low waters supplied to us for this analysis by UK and Irish harbour authorities. These sea levels were entered into a computer database and annual mean values of high and low waters computed using the same criteria as for the turning point method. In the case of data from Liverpool George’s Pier, annual MTR values were derived from on average every third year, in view of there being many incomplete or missing years; this is sufficient to sample adequately the nodal cycle and any secular trend. Emphasis was given to data from after 1880 as they were more copious and evidently of better quality. Annual MTR values from Avonmouth are, in principle, available up to the present. However, after the mid-1970s progressive siting took place at the tide gauge with subsequent loss of low water values and, therefore, rejection by the selection criteria for an acceptable annual mean.

All new data acquired as a result of this work can be obtained for further analysis through the POL–TCSS. We estimate that approximately the same amount again of UK data could be made available for further research, although considerable effort would be required. For example, much historical UK sea level data are preserved as hand-written or typed three-hourly heights derived from digitizations of tidal charts made many years ago. In principle, these values could be copied straightforwardly to a computer database,
although that in itself would be a major task. However, the cubic spline turning point method applied to one-hourly heights would not be of adequate accuracy for the three-hourly data and it would be necessary to employ a different method. One possibility would be to determine MHWs and MLWs by performing a harmonic tidal analysis for each year of data, with a suitable number of constituents, and by computing turning points from the one-hourly heights of the reconstructed tidal time series.

**ERROR SOURCES**

Determinations of annual MTR will naturally contain errors of both statistical and systematic character although it is only the latter which are of importance to this study. Even if each individual high and low water were to be measured to relatively poor accuracy (e.g. to many centimetres), the large number of measurements obtained during the required minimum of 90 per cent of a full year would ensure that
different systematic biases to MTR measurements at different periods of recording, without any such evidence coming up-river from the mouth to Eastham, as shown by tidal surveys. For example, recording at Liverpool switched several times, and the turning points on the authority tidal charts. Liverpool George's Pier levels were documented at 15 min intervals for approximately 60 yr during the nineteenth and early twentieth centuries, although some years are incomplete or absent. The largest documented level around a maximum defined the 'high tide' itself in the harbour authority tabulations. Liverpool has a predominantly semi-diurnal tide, so the 15 min sampling implies a maximum 7.5 min timing error or 3.75 degrees phase error from the high (or low) tide to the closest sampling. This corresponds to a maximum 0.21 per cent and an average 0.07 per cent systematic underestimate of MTR.

Movement of the tide gauge location in a large harbour with strong spatial tidal gradients is another way of introducing systematic biases into the MTR measurements. For example, recording at Liverpool switched several hundred metres down-river from George's Pier to Princes Pier around 1918. In the River Mersey the amplitude of the dominant M2 constituent increases by approximately 3 per cent up-river from the mouth to Eastham, as shown by tidal constants obtained from the TCSS data bank and by a numerical ocean tide model of Liverpool Bay and the River Mersey (D. Prandle, private communication). From inspection of the model in the vicinity of the tide gauges, we conclude that the shift in recording location could well have resulted in an approximately 0.4 per cent apparent reduction in MTR. We have searched the documentation available for all other stations for evidence of similar gauge movements or operational changes which might have introduced different systematic biases to MTR measurements at different periods of recording, without any such evidence being discovered.

Data gaps can introduce systematic errors if they tend to occur at particular times of the year. However, although MTR varies by approximately 2 per cent seasonally (Corkan 1934; Pugh & Vassie 1976; Baker & Alcock 1983), the criterion for an annual mean to contain data from at least 90 per cent of the year should result in biases of order only 0.2 per cent, an amount which is less than determined standard errors in MTR trends over a century at all stations.

Consideration of the major systematic error which besets MSL measurements, that of datum accuracy, is not relevant to MTR studies as the same datum errors will apply both to high and low waters and should cancel in the determination of MTR. The absence of reliable datum information is the reason for there being at some ports longer records of MTR than of MSL or MTL. Their absence also precludes the presentation of all time series of MHWs and MLWs individually.

Systematic errors become important to trend studies only if annual MTR values with errors from different sources in different years are employed in the same time series analysis. Table 1 specifies which station records contain MTR computations by more than one technique; in the present analysis, Liverpool and Aberdeen are the stations primarily concerned.

MEAN TIDAL RANGE TIME SERIES

Figures 2(a–l) show the time series of MTR for each port listed in Table 1, with the exception of Southend for which we have taken trend values published in Rossiter (1969) and Bowen (1972). Superimposed on each one is the result of a linear regression fit to all years of data of the form

$$MTR(t) = a + bt + c \cos(wt - d)$$

where MTR at time 't' is parametrized in terms of a linear trend 'bt', a nodal term in which the angular frequency 'w' = (2\pi/18.61) radians per year, and fit parameters 'a, b, c and d'. Also superimposed on each time series is the linear trend part of the fit (a + bt) alone. Beneath each MTR plot is shown the fit residual time series in order to demonstrate fit quality.

Table 2(a) lists the secular trends in MTR determined from the linear regression fits. For each station are given the computed linear trend in MTR and its standard error, the percentage change in MTR per century (PMTRC) computed over the available record length and using epoch 1950 MTR as the reference MTR value. Trends can be seen to be mostly positive, of order a millimetre per year, less than but comparable to the size of typical MSL trends in the British Isles (Woodworth 1987). Table 2(a) also presents those corresponding fit parameters for Liverpool (1859–1911) and for Aberdeen (1862–1913) using the MTR data determined by a common method for these subsets. In both cases the computed trend values are consistent with those obtained from the total time series. In Liverpool's case, the MTR trend from the last to the present century is positive, that is opposite to the possible small systematic negative shift of MTR expected from the change of tide gauge location mentioned above.

Inspection of the residuals is important in forming an
Figure 2. Time series of annual MTR (large dots) with values relative to the left-hand vertical scale (mm). Each series has superimposed the result of a linear plus nodal regression fit (small dots) and its linear part alone as described in the text. Fit values are shown connected together for consecutive years of data. Beneath each MTR series is the corresponding fit residual series (crosses) with values relative to the right-hand vertical scale (mm).
Figure 2. (continued)
Secular trends in British Isles mean tidal range

Figure 2. (continued)
Figure 2. (continued)
impression of fit quality. In particular, this means a search for irregular episodic changes in MTR, as might be expected from major dredging works, for example, and a search for long-term departures from the linear trend, or 'accelerations', in MTR. No evidence for either of these can be obtained from the residuals to any great extent, although it is clear that certain individual fits, such as those for Belfast and Dublin, do not represent the nodal variations well overall. Belfast Lough has had a long history of land reclamation (Carter 1982) which may have distorted its tidal characteristics over a long period. The apparent '50 year wave' in Aberdeen MTR data from the last century, quite evident in the residuals, was commented on by Thompson (1914) without explanation; the behaviour does not manifest itself in later data. Liverpool residuals prior to 1880 tend to be much larger than later ones and many years of data have been deleted from the time series before that date due to incompleteness. However, restriction of the record to later than 1880 hardly changes the obtained MTR trend. No obvious correlation exists between residuals from neighbouring stations.

It is to be expected that the residual time series will contain variability arising from noise from non-tidal energy at tidal frequencies, for example from meteorological effects or from ocean modulation of tidal behaviour, and from any measurement errors of statistical character (Pugh 1987, section 4.6). However, it is interesting that several of the records show the residual variance to contain energy over several years or decades which is difficult to explain (Thompson 1914), and which could result in significantly different trend values if subsets of the time series are employed. It is possible that meteorological or ocean influences might also contribute in some way to MTR decadal or secular changes. Local processes, whether natural or anthropogenic, might also take place over many years.

Table 2(a) shows trends in MTR to be small, or even negative, in Scotland (Aberdeen and Lerwick) and in the central and western Irish Sea (Holyhead, Douglas and Dublin), while in most of England they are generally of order 0.5-1.0 \( \text{mm yr}^{-1} \). Expressed as a percentage change in MTR (PMTRC), trends vary within approximately \( \pm 0.5 \) per cent per century. There are, therefore, large differences in MTR trend at different stations. From knowledge of the shallow water tides of the northwest European continental shelf, MTR changes might be expected to depend on changes in MSL, or water depth. Table 2(b) includes a compilation of trends in MSL at the same ports using, as far as possible, the same epochs of measurements as for the

### Table 2. (continued)

#### (c) Amplitude of the nodal variation in MTR expressed in mm as a percentage change in MTR at 1950, and time of nodal minimum MTR closest to 1950. Southend values have been computed from information in Bowen (1972).

<table>
<thead>
<tr>
<th>Station</th>
<th>Amplitude</th>
<th>Percentage</th>
<th>Minimum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lerwick</td>
<td>46</td>
<td>3.5</td>
<td>1950.6</td>
</tr>
<tr>
<td>Aberdeen</td>
<td>84</td>
<td>3.0</td>
<td>1950.6</td>
</tr>
<tr>
<td>Blyth</td>
<td>89</td>
<td>2.7</td>
<td>1950.6</td>
</tr>
<tr>
<td>Immingham</td>
<td>136</td>
<td>2.8</td>
<td>1951.1</td>
</tr>
<tr>
<td>Southend</td>
<td>93</td>
<td>2.3</td>
<td>1950.6</td>
</tr>
<tr>
<td>Dover</td>
<td>127</td>
<td>2.7</td>
<td>1950.7</td>
</tr>
<tr>
<td>Newlyn</td>
<td>102</td>
<td>2.8</td>
<td>1950.6</td>
</tr>
<tr>
<td>Avonmouth</td>
<td>201</td>
<td>2.1</td>
<td>1951.1</td>
</tr>
<tr>
<td>Holyhead</td>
<td>66</td>
<td>1.8</td>
<td>1953.4</td>
</tr>
<tr>
<td>Liverpool</td>
<td>183</td>
<td>2.8</td>
<td>1950.5</td>
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<tr>
<td>Douglas</td>
<td>110</td>
<td>2.3</td>
<td>1950.8</td>
</tr>
<tr>
<td>Belfast</td>
<td>68</td>
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</tr>
<tr>
<td>Dublin</td>
<td>72</td>
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<table>
<thead>
<tr>
<th>Station</th>
<th>Years</th>
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<th>MTR Trend</th>
<th>MTR 1950</th>
<th>PMTRC</th>
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<tbody>
<tr>
<td>Lerwick</td>
<td>17</td>
<td>1959-1987</td>
<td>-0.71 +/- 0.14</td>
<td>1268</td>
<td>-5.63 +/- 1.10</td>
</tr>
<tr>
<td>Aberdeen</td>
<td>93</td>
<td>1862-1988</td>
<td>-0.28 +/- 0.07</td>
<td>2767</td>
<td>-1.00 +/- 0.24</td>
</tr>
<tr>
<td>Blyth</td>
<td>34</td>
<td>1954-1987</td>
<td>0.00 +/- 0.39</td>
<td>3290</td>
<td>2.72 +/- 1.16</td>
</tr>
<tr>
<td>Immingham</td>
<td>28</td>
<td>1958-1988</td>
<td>0.10 +/- 0.54</td>
<td>4869</td>
<td>0.21 +/- 1.11</td>
</tr>
<tr>
<td>Southend</td>
<td>34</td>
<td>1934-1966</td>
<td>1.14 +/- 0.42</td>
<td>1922</td>
<td>3.25 +/- 2.99</td>
</tr>
<tr>
<td>Dover</td>
<td>23</td>
<td>1926-1987</td>
<td>0.81 +/- 0.51</td>
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<td>1.92 +/- 0.76</td>
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<tr>
<td>Newlyn</td>
<td>64</td>
<td>1916-1984</td>
<td>0.36 +/- 0.13</td>
<td>3820</td>
<td>1.08 +/- 0.25</td>
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<tr>
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<td>1925-1980</td>
<td>1.12 +/- 0.62</td>
<td>9722</td>
<td>1.16 +/- 0.65</td>
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<tr>
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<td>1938-1988</td>
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<td>-6.46 +/- 2.12</td>
</tr>
<tr>
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<td>1856-1893</td>
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<td>6653</td>
<td>0.95 +/- 0.27</td>
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<tr>
<td>Douglas</td>
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<td>-0.59 +/- 0.44</td>
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<td>50</td>
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<td>0.03 +/- 0.27</td>
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<td>0.17 +/- 0.94</td>
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</table>

<table>
<thead>
<tr>
<th>Station</th>
<th>Years</th>
<th>Period</th>
<th>MTR Trend</th>
<th>MTR 1950</th>
<th>PMTRC</th>
</tr>
</thead>
<tbody>
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<td>26</td>
<td>1959-1987</td>
<td>-1.61 +/- 0.61</td>
<td>1288</td>
<td>-6.63 +/- 1.10</td>
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<tr>
<td>Aberdeen</td>
<td>120</td>
<td>1862-1988</td>
<td>0.54 +/- 0.08</td>
<td>2767</td>
<td>-1.00 +/- 0.24</td>
</tr>
<tr>
<td>Blyth</td>
<td>34</td>
<td>1954-1987</td>
<td>5.59 +/- 0.01</td>
<td>3290</td>
<td>2.72 +/- 1.16</td>
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<tr>
<td>Immingham</td>
<td>26</td>
<td>1960-1988</td>
<td>1.47 +/- 0.76</td>
<td>4869</td>
<td>0.21 +/- 1.11</td>
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<tr>
<td>Southend</td>
<td>34</td>
<td>1934-1966</td>
<td>3.11 +/- 0.46</td>
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<td>Dover</td>
<td>23</td>
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<td>Holyhead</td>
<td>40</td>
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<td>0.95 +/- 0.27</td>
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<td>0.95 +/- 0.27</td>
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<td>Douglas</td>
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<td>Belfast</td>
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<td>0.17 +/- 0.94</td>
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<tr>
<td>Dublin</td>
<td>51</td>
<td>1938-1988</td>
<td>0.17 +/- 0.35</td>
<td>2839</td>
<td>0.17 +/- 0.94</td>
</tr>
</tbody>
</table>
MTR data. Secular trends in MSL for Lerwick, Aberdeen, Immingham, Newlyn and Douglas were discussed in Woodworth (1987) and were derived from data obtained from the Permanent Service for Mean Sea Level (PSMSL). Those in Table 2(b) are consistent with the previously reported values with any differences arising as a result of different epochs chosen for trend computation. Values for Dover, Holyhead, Liverpool, Belfast and Dublin were derived from PSMSL data available since Woodworth (1987). There are two trend values for Liverpool corresponding to the two epochs of recording; available datum information from Liverpool does not allow the two records to be connected. The Southend trend was taken from Rossiter (1969) and Bowen (1972). Avonmouth and Blyth trends were obtained from the MTL = (MHW + MLW)/2 values compiled in this analysis.

The MSL trend for Holyhead may be overestimated owing to possible datum errors in the record around 1960 (Rossiter & Lennon 1967); restriction to data from the period 1960–1988 yields a trend of 1.62 ± 0.71 mm yr⁻¹ from 20 yr of data. The MSL trend value for Blyth also has to be regarded with considerable caution in view of the large amount of mining subsidence reported in the area. The nearby North Shields MSL trend for a similar epoch (1954–1985) is 1.1 ± 0.8 mm yr⁻¹ although the long-term North Shields trend is approximately 2.6 mm yr⁻¹ (Woodworth 1987). MTR values are available from North Shields for only 19 yr spanning the 27 yr period 1962–1988 and have, therefore, not been included in Tables 1 and 2. The computed MTR trend is 0.67 ± 0.60 mm yr⁻¹ (2.01 ± 1.81 per cent per century).

Figures 3(a) and (b) show the overall dependence of British Isles MTR trends and PMTRC values respectively upon the corresponding trends in MSL. Fig. 3(a) shows an apparent dependence of MTR trend on MSL trend with the possible exception of Holyhead, while Fig. 3(b) suggests a relationship between PMTRC and trend in MSL, with the possible exceptions of Holyhead and Belfast. PMTRC might be expected to be a better function of MSL (water depth) than MTR if non-linear tide-depth interactions are responsible for the tidal changes. MTR trends or PMTRC values have no apparent dependence upon MTR itself alone, as might have been anticipated if the largest effects were to occur in the higher range (more resonant) areas.

The seasonal dependence of MTR trends was investigated using the longer records from Avonmouth, Blyth and Aberdeen, without any evidence for significant dependence being obtained. Larger (more positive) than average trends were obtained at the end of the year at Aberdeen, while larger than average trends were obtained at Avonmouth in summer months, although for each station the fluctuations are not statistically significant. Blyth MTR trends showed no clear seasonal dependence. Different secular trends for MTR through the year, in so far as MTR depends primarily on M2, would correspond to trends in the harmonic constituents MA2 and MB2 (sometimes called H1 and H2) separated in frequency from M2 by ± one cycle per year. However, these constituents are understood to arise partly from an astronomical component and partly, either from global or regional seasonal ocean modulations which affect the M2 tide, or possibly from interaction between M2 and the MSL seasonal cycle. All of these are quasi-stationary quantities, therefore real long-term changes in the MTR seasonal cycle would appear unlikely (see Discussion section below).

The extent to which MTR trends depend simply upon trends in M2 amplitude was tested using hourly height data

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**Figure 3.** (a) Values of secular trend in MTR (mm yr⁻¹) for British Isles stations plotted as a function of trend in MSL (mm yr⁻¹) at the same site; (b) values of percentage trend in MTR per century (PMTRC) for British Isles stations plotted as a function of trend in MSL (mm yr⁻¹) at the same site. Notation is Lerwick (L), Aberdeen (A), Blyth (B), Immingham (I), Southend (S), Dover (D), Newlyn (N), Avonmouth (AV), Holyhead (H), Liverpool (LI), Douglas (DO), Belfast (BE) and Dublin (DU).
from Lerwick and Newlyn, at opposite ends of the British Isles. Each year of data was subjected to a separate tidal analysis with 63 harmonic constituents and with each determined harmonic constant corrected for lunar nodal variations using equilibrium tide relationships. At Lerwick, twice the secular trend in M2 amplitude was found to be $-0.64 \pm 0.12$ mm yr$^{-1}$ over the period 1959–1987, consistent with the MTR trend in Table 2(a). At Newlyn the corresponding figure was $0.41 \pm 0.10$ mm yr$^{-1}$ for the period 1916–1984, again consistent with the observed MTR trend. These results imply that at most locations the observed MTR changes must be primarily the result of M2 changes, rather than of a conspiracy involving changes amongst shallow water constituents (Appendix A). This is important for the discussion of the following sections in that most previously reported results on secular trends in tidal parameters have been concerned with changes in M2. Of the other constituents, secular trends of M4 amplitude at Lerwick and Newlyn were found to be $0.060 \pm 0.019$ and $0.098 \pm 0.013$ mm yr$^{-1}$ respectively; MA2 amplitude trends were found to be $-0.015 \pm 0.046$ and $-0.153 \pm 0.033$ mm respectively; and MB2 amplitude trends were found to be $-0.012 \pm 0.067$ and $-0.088 \pm 0.038$ mm yr$^{-1}$ respectively.

The apparent significant trend in the annual variations in the M2 amplitude at Newlyn are not real, however, in that MA2 and MB2 have to be added together to give the overall annual modulation (Appendix B). When this is done, the secular trend in the annual modulation amplitude of M2 (i.e. the trend in 'S' in Appendix B) was found to be $-0.021 \pm 0.022$ mm yr$^{-1}$. The reason for the apparent trends in MA2 and MB2 individually are not well understood but may originate from the character of meteorological and instrumental noise in particular years of the record.

The magnitude of the nodal signals in the MTR time series is considerably spatially dependent (Table 2c). If the tide were to be composed of the M2 component alone, a nodal signal of amplitude 3.7 per cent of mean MTR would be obtained (Doodson & Warburg 1941), whereas the observed amplitude varies from 3.5 per cent of epoch 1950 MTR at Lerwick, dropping to approximately 2.7 per cent along most of the east and south coasts of Britain, to approximately 2.0 per cent in the Bristol Channel, Wales and Isle of Man areas, and to approximately 2.6 per cent in NW England and Ireland. This behaviour is consistent with previous research and is a result of non-linear frictional damping (Amin 1983, 1985). Only in the shortest records will uncertainties in the parametrization of the nodal term affect the determination of the MTR trend. Table 2(c) shows the cycle minima to occur at most places at approximately 1950.6 $\pm 18.61$ (integer $n$) as expected from the dominant M2 constituent. The Holyhead data again appear anomalous with the nodal cycle displaced from expectation by about 2 yr and with a very low percentage value for the anticipated nodal amplitude (Amin 1987).

**COMPARISONS TO PREVIOUSLY REPORTED MEASUREMENTS**

A short review of previous studies of historical tides is given in Pugh (1987, section 10.5). Cartwright (1971) showed from analysis of eighteenth and twentieth century measurements at Saint Helena that, as far as can be determined, the deep ocean tides have probably changed little over the past several centuries. The deep ocean tides, of course, provide the input boundary conditions to the tides observed on the northwest European continental shelf. More relevant to the present study, Cartwright (1972) demonstrated an apparent secular trend in the diurnal and semi-diurnal tides at Brest, France with the amplitude of the dominant M2 tide decreasing by approximately 1 per cent per century since 1711. If one makes the reasonable assumption that M2 changes primarily determine MTR changes (Appendix A and see above), then one can infer an order 1 per cent per century secular trend reduction in MTR. This independent evidence for long-term changes in shelf tides, commencing well before major port development in Europe, provides support for a regional secular trend explanation of the British Isles MTR trends presented in this report.

Cartwright (1972) had available for analysis data sets from 1711–1716, 1864–1884, 1898–1914, 1916–1936 and 1960 from which M2 parameters were extracted by means of the Response Method (Munk & Cartwright 1966). Simon (1982) also found Brest M2 amplitude to have decreased by approximately 1 per cent over the past century, although the two analyses differ in their use of recent data in that it is clear from Simon (1982) that the 1960 data used by Cartwright (1972) will have resulted in smaller M2 amplitude than is reasonable on average for recent years, and that M2 has been approximately constant at Brest since the 1916–1936 period. Over the period 1807–1987 MSL rose at Brest by an average of 0.92 mm yr$^{-1}$, although the MSL secular trend was greater in the twentieth century (Woodworth 1990). Fig. 4(a) includes a point for Brest representing these long-term trends for which we have assigned a PMTRC value of $-1$ per cent per century and an MSL trend of 0.92 mm yr$^{-1}$. This can be seen to be a similar PMTRC trend for a given MSL trend as for the British Isles data in Fig. 3(b), although slightly more negative.

Decreasing M2 amplitude has also been observed elsewhere along the French coast although the data have not so far been published. Simon (private communication) found an 8 mm reduction in M2 amplitude at Cherbourg between 1903–1908 and 1978–1988, or a PMTRC of $-0.35$ per cent per century, and a 9 cm reduction at St Malo between 1880–1898 and 1941–1964 (odd years of data only) which is a PMTRC of approximately $-4$ per cent per century. The corresponding MSL trends are approximately 2 mm yr$^{-1}$ at each site. These have also been included in Fig. 4(a). Like that for Brest, the Cherbourg value for PMTRC appears a little negative for given MSL trend compared to the British Isles data, while the St Malo M2 or MTR changes seem anomalously negative; the values for the latter were obtained prior to the construction of the tidal power station at La Rance. Overall, the three French PMTRC values have more negative PMTRC than corresponding British Isles data for the same MSL trend.

Preliminary information is available from the Belgian port of Ostend where secular trends of both MHW and MLW over the combined periods 1835–1852 and 1927–1988 have been found to be approximately 1 mm yr$^{-1}$ (Baeteman et al. 1990; C. Van Cauwenberge, private communication). From these it can be concluded that Ostend had had an MSL trend during this time of order 1 mm yr$^{-1}$ and a MTR trend.
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Figure 4. (a) Values of PMTRC for French, Belgian and Irish stations plotted as a function of trend in MSL (mm yr\(^{-1}\)) at the same site. Notation is Brest (B), Cherbourg (CH), St Malo (St. M), Ostend (O), Ballycastle (BA), Courtown (CO) and Castletownsend (CA); (b) values of PMTRC for previously published Dutch stations plotted as a function of trend in MSL (mm yr\(^{-1}\)) at the same site. Notation is Flushing (F), IJmuiden (IJ), Den Helder (DH), Harlingen (H), Terschelling (T), Hook of Holland (1900-1980) and Delfzijl (1900-1977).

Table 3. Secular trends in MTR in the Netherlands together with their corresponding PMTRC values and MSL trends. The trends for MTR and MSL (in mm yr\(^{-1}\)) for the first five stations were computed over the period 1901-1986 except for those MTR trends marked (**) which were computed over 1933-1986 (de Ronde 1989). Values for the last two stations were taken from de Ronde (1983).

<table>
<thead>
<tr>
<th>Station</th>
<th>MTR Trend</th>
<th>PMTRC</th>
<th>MSL Trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flushing</td>
<td>1.5</td>
<td>4.3</td>
<td>2.1</td>
</tr>
<tr>
<td>IJmuiden</td>
<td>0.9</td>
<td>6.6</td>
<td>2.2</td>
</tr>
<tr>
<td>Den Helder</td>
<td>1.1 **</td>
<td>8.5</td>
<td>1.4</td>
</tr>
<tr>
<td>Harlingen</td>
<td>2.1 **</td>
<td>12.7</td>
<td>1.1</td>
</tr>
<tr>
<td>Terschelling</td>
<td>3.4 **</td>
<td>21.5</td>
<td>1.0</td>
</tr>
<tr>
<td>Hook of Holland (1900-1980)</td>
<td>0.6</td>
<td>3.8</td>
<td>1.9</td>
</tr>
<tr>
<td>Delfzijl (1900-1977)</td>
<td>1.4</td>
<td>5.2</td>
<td>1.6</td>
</tr>
</tbody>
</table>

Table 4. Values for the last two stations were taken from de Ronde (1983).

We now turn to a comparison to the Dutch MTR information, the need for a better understanding of which was one of the main motivations for the present study. Table 3 summarizes MSL and MTR trends observed at Dutch tide gauges from de Ronde (1982, 1989). We have converted published MTR trends to PMTRC values by dividing by twice the M2 amplitude given in the Admiralty Tide Tables (1990). The stations of Flushing (Vlissingen) and Hook of Holland are the nearest to Ostend and the UK. Fig. 4(b) and Table 3 show that these have PMTRC-MSL relationships more similar to those of Fig. 3(b) than do those from the northern Netherlands. Even so, they have an approximately 2-3 times larger MTR secular trend, for a given MSL trend, than do those from the northern Netherlands. Even so, they have an approximately 2-3 times larger MTR secular trend, for a given MSL trend, than one would expect from most of the British Isles data. The largest values of PMTRC are found at Terschelling, Harlingen and Den Helder, all of which
border the Wadden Sea. We conclude, therefore, that, with the possible exception of southern Dutch stations, the observed MTR behaviour shown in Dutch tide gauge records has no parallel in data from the British Isles, Belgium or France.

Comparison to European data would not be complete without mention of the very large changes in MTR observed in records from the German Bight (Führböter & Jensen 1985; Jensen et al. 1988; Führböter et al. 1990). For example, at Dagebüll on the mainland coast south of the island of Sylt, MTR increased at the rate of 89 cm per century between 1954 and 1986, and a rate of 34 cm per century was found at List on Sylt with similar large values observed throughout the German Bight. However, the authors (J. Jensen et al., private communication) conclude that there is no uniform secular trend in the tidal regime of the German Bight, with most of the very large changes occurring since 1950 primarily occurring in low waters (Führböter et al. 1990). Significant departures from the long-term trend have also been observed at some Dutch stations since 1950 (J. de Ronde, private communication). It is almost certain that the tidal response of the extensive coastal flats of the area coupled to major coastal engineering from the River Elbe in the north to the Dutch coast in the south has resulted in a unique tidal development.

**DISCUSSION**

Table 2(a) immediately provides a fulfilment of two of the objectives of the present study. Firstly, non-zero trends in MTR are observed at many locations around the British Isles. This is consistent with, but not proof of, the suspicions of Amin (1983) and other authors that the tidal changes observed at Southend and in the River Thames have a common origin with sea level and tidal changes in the North Sea and elsewhere. It also confirms a possibility suggested by Dutch and German studies that long-term tidal changes might extend to the British Isles, although much smaller secular trends are observed on average in the present study than were found in the previous ones, and may not have a common origin. Secondly, the trends are in general large enough to suggest that MSL analysts cannot, without great caution, employ time series of MHWs as proxies for MSL records. This is still a useful result, if a somewhat negative one.

In order to understand properly the determined spatial pattern of MTR trends, one has to examine different features of the tidal response of the continental shelf derived from previous experience of the tides of the region. As explained in the Introduction, numerical modelling exercises to investigate tidal changes with changing water depth and in the presence of differing meteorological environments will form the basis of a future report. Nevertheless, it is still useful to consider different aspects of tides to gain some pre-modelling insight into which factors may or may not be responsible for the long-term MTR changes.

Long-term changes in the tidal regime of the northwest European continental shelf could result from two general causes. The first could be changes in the deep ocean tides bordering the shelf. These can stem from long-term changes in the tidal potential arising from variations in the orbital elements of the Sun and Moon, or from long-term changes in the shape or depth of the major ocean basins or in the rate of global tidal dissipation. However, as the period of interest in this study is of the order to several decades or one century, we can ignore all of these factors. The second general cause concerns the shelf itself with changes in its own shape or depth or in the major river estuaries and inland seas (e.g. Wadden Sea) which connect to it. These may in turn be consequences of either of the following.

1. Anthropogenic effects, including dredging of rivers and other shallow areas through which navigation is required. Such work causes a reduction in tidal friction and an increased MTR and is a well understood phenomenon in rivers although less well studied in off-shore shallow areas. Other anthropogenic effects include modification to the bathymetry or coastlines, for example through extraction of oil and gas from the several major fields situated in and around the North Sea, or through large land reclamation schemes (e.g. the closure of the Zuiderzee in 1932).

2. Natural processes, including long-term changes in regional eustatic sea levels and in vertical land movements, together resulting in natural long-term changes in MSL, or water depth, relative to the local land. Natural accretion and erosion in river deltas and shallow inland seas, in sandbank areas and along coastlines also contribute.

A possibility which immediately presents itself for an explanation of the data in Table 2(a) comes from (1). As most tide gauges are located at or near to large coastal towns and ports, the observed changes in MTR may be the results of anthropogenic modifications in general and especially of the local history of port development and dredging. If this explanation were to pertain to data from all British Isles stations, then it would be impossible to explain the available data in any consistent way and Table 2(a) would only be of further application to local studies (e.g. to studies of local extreme levels). However, we consider that this possibility can be discounted for the present for the following reasons. Firstly, meaningful histories of port and river development do not exist for all stations, although such histories could probably be assembled for the larger ports. In principle, one might then expect to correlate the histories with MTR trends and fit residuals, although any comparisons would necessarily be qualitative only. In fact, the MTR fit residuals do not appear in general to contain large irregular events which one might associate with dredging. Secondly, dredging will increase MTR primarily through lowering of low waters, rather than raising high waters, which would lead to a negative correlation between observed MTR and MSL trends, other factors aside. Figs 3(a) and (b) could not support such a conclusion and in fact suggest a positive correlation. Thirdly, the episodic fluctuations in the MTR fit residuals do not appear to be well correlated either negatively or positively with the episodic interannual variability of MSL which would point to MLWs or MHWs respectively being responsible for the fluctuations. For example, the MTR residuals of Fig. 2 are correlated with the residuals of a linear regression fit to the annual MSL values at Newlyn, Aberdeen, Avonmouth and Dublin with coefficients $-0.19$, $-0.23$, $0.04$ and $0.00$ respectively. Finally, we have the evidence from Brest (Cartwright 1972) that real secular trends in regional ocean...
tides could well exist, irrespective of whatever local processes are at work. Consequently, while local changes in tides certainly cannot be ruled out as contributing to the MTR trends, it is certainly valid to search for a coherent regional interpretation of the observations.

Another possible explanation for the MTR changes could come from (2): studies of variations in the shelf tides due to local depth changes, subject to constant deep ocean tide boundary conditions. In these investigations a numerical ocean tide model of the shelf could be employed with bottom friction and other parameters correctly adjusted to simulate the well the observed present-day regime of diurnal and semi-diurnal shelf tides and shallow water constituents. The shelf water depth would then be varied to test how each major and minor constituent would change. Such exercises have been performed several times from the point of view of palaeotides with average sea levels lower by several metres (Austin 1988), and of future tides to be expected if sea levels rise by several metres due to greenhouse warming (Rijkswaterstaat 1986; de Ronde 1989). Each of these investigators have to add or subtract several metres (typically 5 m) of water before significant changes to M2, and therefore to MTR, become apparent and it is clear that the typical 10–20 cm changes in MSL along most of the British Isles and European Atlantic coastlines over the past century, if common to most of the shelf, cannot explain the MTR changes within the context of such numerical tidal models. For example, Austin (1988) showed that a 5 m depth change would result in M2 range changes less than or of the order of 20 cm around most of the British Isles coast; that is less than a 1 cm range change for a 20 cm depth change. However, Table 2(a) demonstrates MTR changes of about 10 cm per century at many locations, or an order of magnitude larger than expected from the model. Similar conclusions can also be obtained for North Sea ports (Rijkswaterstaat 1986; de Ronde 1989). Consequently, it seems that, while numerical tide models have never been developed with water depth changing non-uniformly throughout the shelf, as would be a realistic scenario, simple depth change effects on tides are unlikely to reproduce the observations without other parameters, such as bottom friction, also being varied.

A third possible explanation, also implied in (2), is to include some kind of high-order, non-linear dependence of M2 (and MTR) upon depth into the numerical models. A well-known example of non-linear interaction is the tide-surge interaction in the southern North Sea and River Thames where positive surge events are statistically more likely to occur on the rising tide (Prandle & Wolf 1978). Non-linear tide-depth interactions are also thought to be partly responsible for an annual modulation of M2 amplitude around the British Isles (Corkan 1934; Cartwright 1968; Pugh & Vasse 1976; Amin 1982) which results in the harmonic constituents MA2 and MB2 separated in frequency from M2 by ± one cycle per year. When MA2 and MB2 are added together, a typical annual modulation of 1 per cent of M2 amplitude results with peak M2 amplitude in the first half of the year. There is also in most areas a semi-annual modulation of M2 due to increased non-linear interactions of the semi-diurnal tides at the equinoxes (Baker & Alcock 1983). It is of interest that annual modulations of order 1 per cent of M2 or MTR are possible, alongside annual MSL amplitudes of approximately 7 cm (Woodworth 1984), as that is the order of magnitude response of MTR to MSL change required in Fig. 3(b), if the apparent PMTR-MSL relationship is real. Corkan (1934) showed that annual modulations of M2 occur at many ports around the world and, if the long-term and annual changes are caused by the same processes, one concludes that non-zero secular MTR trends might exist worldwide. In the 55 years since Corkan’s work, no fundamental new insight has been obtained into the causes of the annual modulations in M2, aside from recognizing that tides will have to respond in some way to the seasonal cycle in other ocean processes (Cartwright 1968), and apart from observing that similar modulations also take place in other tidal constituents (Amin 1982; Baker & Alcock 1983). However, Amin (1983) showed that there are certainly also interannual changes in tidal constituents at Southend which appear to depend upon local MSL. Therefore, given the new results presented in this paper, we believe there is considerable scope for further modelling studies of seasonal and secular changes in shelf tides and surges with small changes in water depth.

CONCLUSIONS

We have compiled time series of MTR from 13 stations around the British Isles and have computed secular trends of MTR at each station. Together the data show the following results.

(a) MTR cannot in general be considered a constant quantity over time-scales of several decades or a century but varies at rates between −1.8 and 1.3 mm yr−1 depending on location. These trends are in addition to the lunar nodal (18.61 yr) modulations of MTR.

(b) The non-uniform geographical distribution and the quantitative range of MTR trends is such that, at a station where only MHWs are available due to gauges bottoming-out at low tide, MHW trends should not be used without caution in studies of trends in MSL.

(c) Conversely, MTR trends are usually large enough that they should be included in geographical studies of impacts of sea level change, should only MSL trend estimates be currently available, in order to reconstruct MHWs. They should also be incorporated into extreme level engineering studies.

(d) Analysis of hourly heights from Lerwick and Newlyn demonstrates that the MTR changes are due primarily to changes in the dominant M2 constituent.

(e) The amplitude of the nodal signal in MTR records varies from 3.5 per cent of epoch 1950 MTR at Lerwick, dropping to approximately 2.7 per cent along most of the east and south coasts of Britain, to approximately 2.0 per cent in the Bristol Channel, Wales and Isle of Man areas, and to approximately 2.6 per cent in NW England and Ireland.

(f) MTR and PMTR trends presented in this report for the British Isles appear consistent with previous estimates of long-term tidal changes in Ireland the Belgium, and perhaps the southern Netherlands. Northern French MTR trends are more negative than those on neighbouring coastlines. All of these are significantly smaller (for a given MSL trend) than
those observed in the northern Netherlands and along the German Bight coast. Given the fact that there have been extensive coastal and river changes in the latter areas, one might conclude that their MTR changes are of different origin to those observed in the British Isles, Belgium and France.

(g) The suspicions of Amin (1983), that the MTR changes observed in the River Thames cannot be explained entirely by changes to the river alone, are confirmed in the sense that long-term MTR changes are observed to be taking place throughout the continental shelf. However, common but local factors could in principle also be responsible.

(h) An apparent relationship exists between MTR and PMTRC secular trends observed around the British Isles and trends in MSL observed at the same locations. Exceptions to this general relationship might be found in Northern Ireland and at Holyhead.

(i) The relationship described in (h) may be accidental but, more likely, may point to a MTR dependence on depth not so far included in most numerical ocean tide models of the area. The order of magnitude of the PMTRC–MSL relationship is comparable to that observed between the annual modulation of M2 and MSL around the British Isles.

(j) Extensive numerical modelling of the tides of the continental shelf is required in order to fully understand the historical secular trends in MTR and their apparent relationship to depth changes. Also, firmer information is required, particularly from estuaries (Prandle 1989), on how tides may change in the future if sea levels rise significantly due to global warming. It is intended that this modelling activity will form the second half of this study.

ACKNOWLEDGMENTS

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REFERENCES


angular frequency

appendix a: terms contributing to

ANNUAL MTR MODULATION

constituent with amplitude R

frequency \( \omega \). We can write the total height \( h \) at any

where

following doodson & warburg (1941) sections 11.7 and

11.9, we consider adding a secondary tidal constituent with

angular frequency \( n \) to a dominant M2 signal with angular

frequency \( \omega \). We can write the total height \( h \) at any

instant \( t \) as

\[ h(t) = M \cos(\omega t) + A \cos(nt) + B \sin(nt) \]  

(A1)

where \( M \) is the amplitude of the M2 component, the time

origin \( t = 0 \) is defined at M2 maximum, and \( A \) and \( B \)
define the in-phase and quadrature parts of the secondary

constituent with amplitude \( R = \sqrt{A^2 + B^2} \). Differentiation

of equation (A1) with respect to time shows that the combined

high water level \( H \) will occur approximately when

\[ t = (Bn)/(Mw^2) \]

and with magnitude

\[ H = M + A + (nB)^2 \]

\[ 2Mw^2 \]

A similar calculation, defining the \( t = 0 \) at M2 minimum,

shows that low water will occur approximately when

\[ t = -(Bn)/(Mw^2) \]

and with magnitude

\[ L = -M + A - (nB)^2 \]

\[ 2Mw^2 \]

Now, over many high (or low) tides, each with a different

phase of the secondary constituent with respect to the \( t = 0 \)
of each particular M2 maximum (or minimum), the time

average of \( A \) will be zero, while the time average of \( B^2 \)

will be \( R^2/2 \). The Mean Tidal Range \( 'MTR' \) will, therefore, be

\[ MTR = \langle H \rangle - \langle L \rangle = 2M + \frac{(Rn)^2}{2Mw^2}, \]  

(A2)

or slightly more than twice the M2 amplitude. In a practical

situation with many secondary constituents, one has to sum

the additional term over all constituents other than M2 (with

the exception of those with angular frequencies which are

multiples of the M2 angular frequency, see below).

For example, at Newlyn the MTR is approximately

3620 mm whereas twice the M2 amplitude is 3424 mm

(amin 1985). Approximately 135 of the 196 mm discrepancy

can be accounted for by \( S2, N2 \) and \( K2 \) for which

\( (Rn)^2/2Mw^2 = 96, 31 \) and \( 8 \) mm respectively. Other

semi-diurnal constituents together account for less than

10 mm. The \( n^2 \) term in the numerator ensures that diurnal

and longer period constituents have only small contributions

to make; for example, those of O1 and K1 at Newlyn are

only 0.2 and 0.3 mm respectively.

The above summation should include only those tidal

constituents with angular frequencies which are not

multiples of that for M2. That is to say, M4, M6 etc. have to

be treated separately. Section 11.7 of doodson & warburg

(1941) states that it is only the even multiples (M4, M8 etc.)

which will contribute to MTL and implies that it is only

the odd multiples (M6, M10 etc.) which will contribute to MTR.

Pugh (1987, p. 304) makes a similar statement. However,

this is only true if the M2 maximum (or minimum) coincides

precisely with the combined high (or low) water i.e. if M2 is

not just much larger than M4 etc. but is completely

dominant. In fact, Pugh [1987, fig. 7.2(a)] shows

schematically an example in which finite M4 can contribute

to MTR. If one restricts attention to M4 alone, as most of

the other shallow water constituents will be very small, then

it can be shown that M4 will contribute a fixed amount to

MTR of order

\[ 2m \sin(F) \sin(2\sin F) \]  

(A3)

where \( m \) is the M4 amplitude, \( r = 2m/M \), and \( F = 2G - g \),

and where \( 'G' \) and \( 'g' \) are Greenwich phase lags for M2 and

M4 respectively.

It is important to observe that, whatever the phase

relationship between M2 and M4, the two sine terms in (A3)

will ensure that M4 will always contribute positively to MTR. In the example of Newlyn, M2 has a phase lag of
135.3° and M4 an amplitude of 112 mm and phase lag of 169.63° (Amin 1985), which adds 56 mm to MTR in this way.

For areas in which the tidal regime is not predominately semi-diurnal (e.g. for ports such as Southampton at which double high or low waters occur) then this simplified description of the component terms of MTR will not apply.

**APPENDIX B: COMBINATION OF MA2 AND MB2 INTO ONE M2 ANNUAL MODULATION**

Consider the combination of three tidal harmonic constituents with speed number 's' and astronomical argument 'V':

(i) MA2 (or $H_1$), $s = s_1 = 28.94304° \text{hr}^{-1}$, $V = V_{M2} - 280.19°$,

(ii) M2, $s_{M2} = 28.98410° \text{hr}^{-1}$, $V = V_{M2}$,

(iii) MB2 (or $H_2$), $s_2 = 29.02517° \text{hr}^{-1}$, $V = V_{M2} + 280.19°$,

where $V_{M2}$ is the astronomical argument for M2 and the time origin ($t = 0$) defined at the start of the year results in the 280.19° offset in $V$ for MA2 and MB2 (Doodson & Warburg 1941). The total tide 'h' at any instant 't' can then be written as an amplitude modulation of M2 as follows (ignoring nodal factors):

\[
h(t) = H_{M2} \Re \{e^{i \arg_{M2}}\}
+ H_1 \Re \{e^{i \arg_{M2}} e^{-(et - G_1)}
+ G_{M2} - 280.19\}
+ H_2 \Re \{e^{i \arg_{M2}} e^{(et - G_2)
+ G_{M2} + 280.19}\},
\]

where

\[
\arg_{M2} = s_{M2}t - G_{M2} + V_{M2}
\]

and

\[
e = s_2 - s_{M2} = s_{M2} - s_1,
\]

and where 'H_{M2}', 'H_1', and 'H_2' are the amplitudes of M2, MA2 and MB2 respectively, 'G_{M2}', 'G_1', and 'G_2' are the Greenwich phase lags for M2, MA2 and MB2 respectively, and where 'Re' implies real part, 'exp' implies exponentiation, and 'i' = $\sqrt{-1}$. The expression for 'h' can be interpreted as an M2 carrier wave with an annual modulation 'S' in the amplitude where

\[
S(t) = H_1 \cos (-et - G_1 + G_{M2} - 280.19)
+ H_2 \cos (et - G_2 + G_{M2} + 280.19).
\]

If we define the two quantities

\[
D_1 = G_1 - G_{M2} - 280.19, \quad D_2 = -G_2 + G_{M2} + 280.19,
\]

then the modulation will be a maximum, positively or negatively, when

\[
H_1 \sin (-et - D_1) - H_2 \sin (et + D_2) = 0,
\]

or

\[
\tan (et) = -\frac{(H_1 \sin D_1 + H_2 \sin D_2)}{(H_1 \cos D_1 + H_2 \cos D_2)}.
\]

Of course, the two solutions for 'et' correspond to maximum positive and negative 'S'. In practice, the values of the MA2 and MB2 constituents quoted in Pugh & Vassie (1976) and Amin (1982) lead to peak M2 amplitudes in the second quarter of the year and maximum $S$ values of about 1 per cent of the M2 amplitude.