Dynamic stress variations due to shear faults in a plane-layered medium

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SUMMARY
A complete set of expressions is presented for the computation of elastic dynamic stress in plane-layered media. We use a discrete-wavenumber reflectivity method to compute the stress field radiated by arbitrary moment-tensor sources. The expressions derived here represent an interesting tool for both the observational and theoretical analysis of dynamic stress changes associated with earthquake phenomena. Dynamic stress changes associated with a strike-slip fault having unilateral rupture are shown. This modelling, which is similar to the 1992 Landers California earthquake, illustrates the effects of distance, directivity and depth on transient stress changes.

Key words: dynamic stress, Landers earthquake, reflectivity, triggered seismicity, wavenumber.

INTRODUCTION
Earthquake rupture complexity can be partly due to triggered slip phenomena. In this process, a slip on one fault can be triggered by an earthquake on another fault, or by a distant segment rupturing on the same fault. If the triggered slip comes soon after faulting, it is usually considered to be part of a complex dynamic event. Such heterogeneous slip distributions have been revealed by numerous waveform inversions in the last decade (e.g. Hartzell & Heaton 1983; Fukuyama & Irikura 1986; Beroza & Spudich, 1988). All these inversions are based on kinematic fault models. Their results show slip values as a function of space and time, but they do not describe the associated stress changes on and around the fault. Nevertheless, it is of great interest to quantify the radiated transient stress waves. These stress waves can probably reach a high level and trigger events on other sections of the fault which were not themselves ready to self-nucleate.

If, on the other hand, the slip triggering is delayed compared to the time of the main event rupture, it is most probably related to changes in the static-stress field. It has been recognized by several authors that the increase in static stress produced by an earthquake can trigger, with a certain time delay, earthquakes on nearby faults (e.g. Smith & Van de Lindt 1969; Yamashina 1978; Das & Scholz 1981). Using analytical expressions to quantify static-stress changes (e.g. Chinnery 1963; Okada 1985, 1992; Pan 1989), numerous papers in the last 10 years have shown that aftershocks are more likely to occur in locations where static stresses due to the main shock have increased (e.g. Reasenberg & Simpson 1992; King, Stein & Lin 1994).

At large distances (more than tens of kilometres), the computed static-stress changes appear to be too low to cause triggering (Hill et al. 1993). The M = 7.4 Landers earthquake, however, triggered widespread remote seismicity in the western United States (Hill et al. 1993; Anderson et al. 1994). These distant triggered slips could be related to the dynamic-stress field propagating from the main rupture event. For this reason, several studies have tried to quantify dynamic transient stress by estimating peak dynamic strain from surface ground-motion records (Hill et al. 1993). Since frequency dependence and variability of stress with depth and geometric structure could be important to the understanding of dynamic triggering of earthquakes, there have also been attempts to quantify the full dynamic stress or strain tensor (e.g. Gomberg & Bodin 1994; Anderson et al. 1994, Spudich et al. 1995). Spudich et al. (1995) have used the displacements recorded by the UPSAR (USGS Parkfield small aperture array) array to determine the surface strain and stress tensor as a function of time. Gomberg & Bodin (1994) have developed an approach to quantify the theoretical dynamic strain tensor at remote distances from the main event. Synthetic seismograms including the fundamental and first higher modes of surface waves are computed. Differentiation of the particle displacement expressions then yields the dynamic strains. Closer to the fault, however, particle motion is not dominated by surface waves, and the quantification of theoretical dynamic stress tensor changes is not possible with this method.

Both the phenomena of distant triggered seismicity and the complex multi-event nature of earthquakes suggest that the quantification of transient stresses is important. This article...
Dynamic stress variations

This study attempts to give theoretical expressions of stress tensor changes as functions of time, associated with shear rupture on a fault plane. We present a complete set of expressions for the calculation of dynamic stress variations resulting from a shear point source in a plane-layered elastic medium. These expressions are valid for all source-station distances. The main goals of this paper are (1) to give the expressions of the stress tensor field radiated by a shear rupture in a plane-layered medium; and (2) to show numerical results of dynamical stress changes due to a strike-slip rupture.

METHOD OF COMPUTATION

We compute the stress field radiated by the six elements of the moment tensor using the reflectivity method of Kennett & Kerry (1979) with the discrete wavenumber decomposition of the Green's functions given by Bouchon (1981) for an axisymmetric medium. These two methods are well known and have been extensively used to compute displacement fields in plane-layered media. The stress field is computed by carrying out one more differentiation step from the displacement field, once displacements have been derived from potentials. We have used as a starting point an extensively used and tested code (Coutant 1990, private communication), which computes the displacements radiated by moment sources. In this paper, we present the analytic formulae giving the stress Green's functions for six independent moment sources (three dipoles and three double couples) in a homogeneous space, using the discrete wavenumber representation with cylindrical coordinates. These formulae have been arranged to be included in a reflectivity propagation method. The resulting program allows one to compute stress at several receivers, due to several sources, in any geometry except when the source and the receiver are at the same depth.

Let us briefly describe the method of computation. We adopt the formalism described by Müller (1985), where seismic

![Figure 1](https://example.com/figure1.png)

Figure 1. Comparison of stress tensor static changes computed by our method with those derived from the analytical expression of Okada (1985). These comparisons are done for a source located at a depth of 5000 m and 21 stations located at the surface of a half-space. Two different source mechanisms are tested: (a) strike = 0°, rake = 70°, rake = 90°; (b) strike = 0°, rake = 70°, rake = 0°.
wavefields are expressed using the three scalar potentials \((\phi, \psi, \chi)\) representing, respectively, \(P\), \(SV\) and \(SH\) waves in the cylindrical coordinate \((r, \theta, z)\) system. The displacement field is given by

\[
\mathbf{u}(r, \theta, z) = \nabla \phi + \mathbf{V} \times (\mathbf{e}_z \psi) + \nabla \times (\mathbf{e}_r \chi).
\]  

(1)

We distinguish three major steps in the computation:

1. From the expressions giving the potentials radiated by single forces (see e.g. Müller 1985; Bouchon 1981), we derived the upward and downward potentials in the source layer(s) radiated by six independent moment tensor sources. The radiated fields can be expressed as the linear combination of the six elementary potential sources given by (see appendix for detail)

\[
\begin{align*}
S_1(z) &= e^{-ik_0 z - z_0} \\
S_2(z) &= \text{sign}(z-z_0)e^{ik_0 z - z_0} \\
S_3(z) &= e^{-i\ell(z-z_0)} \\
S_4(z) &= \text{sign}(z-z_0) e^{i\ell(z-z_0)} \\
S_5(z) &= e^{-i\ell(z-z_0)} \\
S_6(z) &= \text{sign}(z-z_0) e^{i\ell(z-z_0)}
\end{align*}
\]

P–SV conversions

(2)

In these expressions, \(z_0\) is the source depth.

2. The potential fields for these six elementary sources are propagated through the layers. In the receiver layer(s), each of them yields upgoing and downgoing potentials, including possible conversions.

3. Previous potentials are differentiated to obtain displacements using eq. (1), and then the stress \(\tau_{ij}\) is computed using time differentiations and relations derived from Hooke's law:

\[
\begin{align*}
\tau_{rr} &= (\lambda + 2\mu) u_{r,r} + \frac{\lambda}{r} u_{\theta,\theta} + \frac{\lambda}{r} u_{z,z} + \frac{\lambda}{r} u_r, \\
\tau_{\theta\theta} &= \lambda u_{r,r} + \frac{\lambda + 2\mu}{r} u_{\theta,\theta} + \frac{\lambda}{r} u_{z,z} + \frac{\lambda + 2\mu}{r} u_r, \\
\tau_{zz} &= \lambda u_{r,r} + \frac{\lambda}{r} u_{\theta,\theta} + (\lambda + 2\mu) u_{z,z} + \frac{\lambda}{r} u_r, \\
\tau_{\theta r} &= \mu(u_{r,\theta} + u_{\theta,r}), \\
\tau_{r \theta} &= \mu(u_{r,\theta} + u_{\theta,r}), \\
\tau_{r 0} &= \mu(u_{0,r} + u_r).
\end{align*}
\]  

(3)

Details of stress and displacement expressions may be found in the appendix. Using a discrete wavenumber decomposition with cylindrical coordinates is known to produce small artefacts. A plane wave propagating vertically is sometimes observed in all expressions, including the term \(J_0(k_r r)\), where \(J_0\) is the zero-order Bessel function and \(k_r\) is the radial wavenumber. To reduce this artefact, we use the solution proposed by Herrmann & Mandal (1986).

**TESTS**

To check the validity of our expressions, we have performed two tests: (1) comparison with static stresses obtained analytically, and (2) comparison with dynamic stresses obtained, using a finite-difference approximation of the derivative operator, from computed displacements.

**Comparison with static results**

In the last decade, several authors have proposed analytical expressions for static displacement, strain and stress due to a point source in an elastic medium (e.g. Chinnery 1963; Okada, 1985, 1992). The expressions described in this paper allow the computation of both the dynamic stress associated with propagating elastic waves and the static stress variations. The purpose of this test is to check the static values obtained with our computation against the values obtained from the analytical expressions given by Okada (1985). In these tests (Fig. 1), the source is a point source with a moment of \(1.0 \times 10^{10}\) Nm. The coordinates of source are \(x = 0, y = 0\) and \(z = 5000\) m, and the stations are at the free surface (\(x = 5000, y\) between \(-5000\) and \(5000\) m). The static stress changes are compared for two different source mechanisms and 21 stations in an elastic half-space \((v_p = 4000\) m \(s^{-1}, v_s = 2700\) m \(s^{-1}\) and \(\rho = 2500\) kg \(m^{-3}\)). Static changes obtained from our calculations match perfectly the values calculated using Okada's (1985) analytic expressions.

**Comparison with finite-difference results**

We now check the accuracy of the expressions for stress Green's functions by comparing our results with those obtained by computing numerically the stress from the
displacement values. We first compute the ground displacement at four closely located points whose coordinates are given in m by: \( P_1 = (1000, 4000, 5000) \), \( P_2 = (1000.05, 4000, 5000) \), \( P_3 = (1000, 4000.05, 5000) \), and \( P_4 = (1000, 4000, 5000.05) \). The source is a point source located at the origin with a moment of 1 Nm, a strike of 10°, a dip of 20° and a rake of 30°. The source function is a ramp with a rise time of 0.1 s. The characteristics of the medium are \( v_p = 4000 \text{ m s}^{-1} \), \( v_s = 2390 \text{ m s}^{-1} \) and \( \rho = 2500 \text{ kg m}^{-3} \). Spatial derivatives of the displacement are approximated using first-order finite differences. From these, we calculate the strain variations and, using Hooke's law, the stress variations near the station \( P_1 \). These stress variations compare perfectly (Fig. 2) with the results obtained directly using the expressions above.

Figure 3. Top: map view of the fault (solid line) and position of the stations (circles) used in the simulations of Figs 4, 5, 6 and 7. Triangles indicate the location of the three observation points: S1 is located at \((x, y) = (-63 188 \text{ m}, -12 497 \text{ m})\), S2 at \((94 311 \text{ m}, 47 502 \text{ m})\) and S3 at \((184 311 \text{ m}, -32 497 \text{ m})\). Bottom: the fault plane is 12.5 km high and discretized into 130 point sources (black squares). The rupture front has a velocity of \(2.7 \text{ km s}^{-1} \) and is propagating from the epicentre to the north. Its position is represented every 5 s by grey lines on the figure.

Figure 4. Graphical presentation of eq. (4) and illustration of the static Coulomb stress changes. The panels show static Coulomb stress changes calculated for right-lateral faults parallel to the master fault. These maps are obtained from the Coulomb failure criterion changes calculated at each of the 156 stations shown in Fig. 3. In these calculations, the friction coefficient is equal to 0.4. In this and subsequent plots, the maximum and minimum stress changes exceed the plotted colour bar range. Stress is sampled at a depth of 5500 m.
EXAMPLE OF COMPUTATION FOR A FINITE FAULT

We present results of dynamical stress changes due to a shear rupture on a finite fault and illustrate the different effects that control transient stress variations.

In these simulations, the characteristics of the medium are \( v_p = 4000 \text{ m s}^{-1} \), \( v_s = 2700 \text{ m s}^{-1} \) and \( \rho = 2500 \text{ kg m}^{-3} \). A vertical fault (70 km long and 12.5 km wide) is assumed to be located at the centre of a grid containing 156 stations (Fig. 3).

The fault is represented by 130 point sources, whose focal mechanism is a pure right-lateral strike-slip (strike = 0). The tapered slip on the fault is set to 4.2 m. These parameters represent an earthquake with a moment of \( 6.2 \times 10^{19} \text{ Nm} \) and a mean slip value of 3.89 m. The rupture initiates at the southern end of the fault and the rupture front velocity is 2.7 km s\(^{-1}\). The source function at each point source is a ramp with a rise time of 5 s. We compute for each station 180 s of stress time history with a frequency band of 0–0.18 Hz. This means that the distance between two point sources is less than one-sixth of the minimal wavelength of \( P \) waves.

Laboratory studies of rock friction and failure suggest that tangential and normal forces are important in determining resistance to sliding or failure. To characterize the conditions under which failure or sliding occurs, the Coulomb failure criterion is usually used. In the Coulomb failure criterion, failure occurs on a plane when the Coulomb stress \( \sigma_c \) exceeds a specific value. The Coulomb stress is

\[
\sigma_c = \tau + \mu \sigma, \tag{4}
\]

where \( \tau \) is the shear stress in the slip direction, \( \sigma \) is the normal stress (extension positive) on the failure plane, and \( \mu \) is the friction coefficient. In our simulations, time histories of the stress tensor are computed with the expressions described above. Shear and normal stress changes on observation planes that are arbitrarily oriented are also calculated. Assuming a given coefficient of friction, we finally compute the spatio-temporal variations of the Coulomb stress. Fig. 4 illustrates Coulomb stress calculations obtained using eq. 4. The contributions of the shear and normal static components to the failure conditions, and the resulting Coulomb stresses for infinitesimal right-lateral slipping faults parallel to the master
fault, are shown in separate panels. This figure can successfully be compared qualitatively with the static results of King et al. (1994, Fig. 2a): the shear stress change field is distributed symmetrically across the fault; the normal stress change field is distributed antisymmetrically.

**Variation of transient stress with depth**

To illustrate the dependence of transient stress variations on depth, stress tensor histories are calculated beneath the observation point S2 at different depths between 1500 m and 13 500 m. The results (Fig. 5) show that dynamic stress changes at depth are not correctly represented by estimations from surface seismograms, and Fig. 5 clearly shows that the xz, yz and zz components of both the static and dynamic stresses are increasing with depth. This dependence has also been noted by Gomberg & Bodin (1994) and Spudich et al. (1995). These numerical tests confirm the analysis obtained by Spudich et al. (1995) in their study of dynamical stress variations detected at Parkfield after the Landers strike-slip earthquake: in the upper crust, transient stresses associated with horizontally oriented tractions on vertical planes (xx, yy and xy components) dominate. As already mentioned by Spudich et al. (1995), this means that the transient stresses due to a major strike-slip event tend to favour faulting on vertical planes in the upper crust, and other orientations below. Should these dynamic stresses trigger seismicity by direct working and weakening of material, the triggered seismicity would exhibit different mechanisms at different depths, something that has not yet been observed.

**Distance and directivity effects**

To illustrate distance and directivity effects, normal and horizontal in-plane stresses are computed on right-lateral slipping vertical fault planes striking N at a depth of 5500 m for each

![Fig. 7. The panels show the minima (a) and maxima (b) of dynamic Coulomb stress changes. These maps are obtained from the Coulomb failure criterion minimum and maximum change calculated at each of the 156 stations shown in Fig. 3. In these calculations, the friction coefficient is equal to 0.4.](image-url)
compared with the functions 

\[ \frac{1}{r} \]

and 

\[ \frac{1}{r^2} \]

distance from the fault. This is clearly a directivity effect: the changes (crosses) as a function of the distance distances from the fault. The Coulomb stress is calculated with at the observation points. Dynamic peak stress changes (circles) and static stress changes associated with propagating waves have a fall-off of less than \( r^{-1} \). In this figure, \( r \) is the distance between one of the 130 stations (top panel, Fig. 3) and the closest point source used to model the fault (bottom panel, Fig. 3). Static Coulomb stress changes have a fall-off of between \( r^{-1} \) and \( r^{-2} \).

**Figure 8.** Dynamic peak stress changes (circles) and static stress changes (crosses) as a function of the distance from the fault, calculated for each of the 156 stations of Fig. 3. These results are compared with the functions \( 1/r \) and \( 1/r^2 \).

of our 156 stations. Fig. 6 shows the results of these calculations at the observation points S1, S2 and S3 located at different distances from the fault. The Coulomb stress is calculated with a friction coefficient of 0.4. One can see that static stresses are significant only at S2 in the neighborhood of the fault (static stress changes observed at locations S1 and S3 are null). On the other hand, dynamic stresses are still important at observation points S1 and S3. The difference in amplitude between S1 and S3 is striking because these two points are at the same distance from the fault. This is clearly a directivity effect: the fault rupture propagates northwards and waves have focused towards the north. Fig. 7 shows that the directivity effect is strongly controlling the spatial distribution of stress-change extrema; furthermore, static stress changes rapidly decrease with increasing distance (Fig. 4), and dynamic-stress maxima changes can be still greater than 1 bar at a distance of 70 km N of the fault. Maximum and minimum dynamic normal stress changes are pattern-mirrored at the N-S axis. The fact that this is not exactly the case is a result of an accuracy effect due to the coarse gridding. We show (Fig. 8) that dynamic Coulomb stress changes associated with propagating waves have a fall-off of less than \( r^{-1} \). In this figure, \( r \) is the distance between one of the 130 stations (top panel, Fig. 3) and the closest point source used to model the fault (bottom panel, Fig. 3). Static Coulomb stress changes have a fall-off of between \( r^{-1} \) and \( r^{-2} \).

**DISCUSSIONS AND CONCLUSIONS**

Our modelling can be considered as a simplified model of the 1992 Landers, California, earthquake (similar seismic moment, length and mechanism). It provides some elements of the physical processes that caused aftershocks of this earthquake. Correlating the spatial distribution of aftershocks with characteristics of the stress fields associated with mainshocks is important in order to understand the physics of the generation of instability on a fault. At first inspection, we might reasonably expect aftershocks to be generated by dynamic stress changes, since they may be several orders larger than static changes (Fig. 8). However, the spatial distribution of dynamic stress-change peaks and static stress changes are very different. Dynamic stress peaks are mainly distributed N of the fault (Fig. 7), while static changes are distributed symmetrically around the centre of the fault (Fig. 4). In this 80–100 km radius around the fault (a distance of one rupture dimension, the typical rule-of-thumb for defining aftershocks), seismicity studies (Hauksson 1994; King et al. 1994) have not pointed out that aftershocks of the Landers earthquake are mainly distributed N of the fault in a region where we show that peak dynamic stresses are at a maximum (Figs 7a and b). However, within this 80–100 km radius, the seismicity distribution and changes in static stresses appear to correlate statistically (King et al. 1994). Hence, even if dynamic stresses are larger than static stresses, static changes seem to correlate with the distribution of the aftershocks, whereas dynamic stress changes do not. One possible interpretation of this observation is that earthquakes are not triggered by transient stresses. This means that failure does not occur when the Coulomb stress change reaches a critical value. Theoreticalmodels of fault stability offer an explanation for this phenomenon. Earthquakes are frictional instabilities, and an instability does not occur at the peak value of stress; instead, it occurs in the post-peak region, when the frictional resistance force is falling faster than the system can respond (Scholz 1990, Fig. 2.16). There is a mechanical breakdown associated with this resistance-weakening, and this requires a finite displacement for it to occur (and because this breakdown, or nucleation, as it is also called, is quasi-static, it also requires a finite time to occur). This frictional behaviour in which strength falls with slip is known as slip-weakening. As a result, this nucleation is insensitive to a short-term transient. It follows that this kind of system can always support higher loads if they are of short rather than of long duration. The intrinsic time dependence of this phenomenon is responsible for the time delay of the triggered events.

The drawback of this interpretation is that it does not explain the remote triggering of earthquakes, at distances where static stress changes appear to be too low to cause triggering (Hill et al. 1993; Anderson et al. 1994; Gomberg & Bodin 1994). Could the rupture-initiating mechanism for remotely triggered earthquakes be different from that for nearby aftershocks? Could another triggering mechanism be invoked that explains the triggering of aftershocks at all distances?

Answering these questions would imply observational and theoretical analysis of dynamic stress changes associated with earthquake phenomena. Our method offers the possibility to compute theoretical stressgrams in order to estimate the dynamic stress tensor associated with fault rupture; it represents a powerful tool for this purpose. We have shown that the quantification of dynamic stress variations is important in understanding the dynamic triggering of earthquakes, and our numerical results illustrate directivity effects and the depth dependence of dynamic stress tensor variations. The authors are ready to provide the source code to interested researchers.

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REFERENCES


APPENDIX A: STRESS GREEN’S FUNCTION

The following notation is used in expressions below:

\[ J_i = J_i(k, r), \text{ where } J_i() \text{ is the Bessel function of order } i; \]

\[ k_r = \frac{2\pi n}{L} \] is the radial wavenumber, \( r \) is the distance between source and receiver, and \( L \) is the spatial period (Bouchon 1981);

\[ k_s = \frac{\omega}{a}, k_p = \frac{\omega}{\beta} \] are the \( P, S \)-wave wavenumbers;

\[ \gamma, \nu = \sqrt{k_s^2 - k_t^2}, \sqrt{k_p^2 - k_t^2} \] are the \( P, S \)-wave vertical wavenumbers with \( \text{Im}(\gamma) < 0 \) and \( \text{Im}(\nu) < 0 \).

We also define the following variables.

\[ C_f = \frac{1}{2\pi n L} \]

\[ K_0 = k_n J_0 \]

\[ K_1 = k_r J_0 - 2 J_1 / r \]

\[ K_2 = k_r J_0 - J_1 / r \]

\[ K_3 = \frac{3 k_r J_0}{r} - \frac{3}{2} K_1 - k_s^2 J_1 \]

\[ K_4 = \frac{3 k_r J_0}{r} - k_s J_1 \]

\[ K_5 = k_r J_0 - J_1 / r \]

\[ K_6 = \frac{3 k_r J_0}{r} - k_s^2 K_5 + \frac{3}{2} K_1 - \frac{3}{2} K_3 = -k_s^2 (K_1 + K_2) - 2 K_7 \]

\[ \cos^2 = \frac{k_s^2}{r^2}; \quad \cos^2 = \frac{k_s^2}{r^2}; \quad \cos^2 = \frac{k_s^2 + 2 k_s^2}{r^2} \]

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A1 Potential Green's functions for a homogeneous space

Couples and dipoles are derived from the three elementary unidirectional forces, \( F_x, F_y \) and \( F_z \). Dependences on space coordinates \((r, z, \phi)\) are implicit.

**Single couples** (e.g. \( M_{xy} = \partial F_x/\partial z_0 \) and \( M_{xy} = \partial F_y/\partial x_0 \))

\[
\begin{align*}
M_{xy} : \quad \phi &= C_f F_z \cos \theta \sin \theta (k_0^2 + i\nu) K_1 e^{-ir|z-z_0|} \\
\psi &= C_f F_z \cos \theta \sin \theta \text{sign}(z-z_0) K_1 e^{-ir|z-z_0|} \\
\chi &= C_f F_z \left( k_0^2 + i\nu \right) \left( \sin^2 \theta K_1 + K_2 \right) e^{-ir|z-z_0|}
\end{align*}
\]

\[
\begin{align*}
M_{xz} : \quad \phi &= C_f F_z \cos \theta \text{sign}(z-z_0) (-k_0^2) J_1 e^{-ir|z-z_0|} \\
\psi &= C_f F_z \cos \left( -i\frac{k_0^2}{i\nu} \right) e^{-ir|z-z_0|} \\
\chi &= 0
\end{align*}
\]

**Double couples and dipoles** (e.g. \( M_{xy} = M_{xy} + M_{xz} \) with \( F_x = F_x, F_y \))

\[
\begin{align*}
M_{XY} : \quad \phi &= C_f F \sin \theta \left( k_0^2 + i\nu \right) \cos \theta K_1 e^{-ir|z-z_0|} \\
\psi &= C_f F \sin \theta \left( k_0^2 + i\nu \right) \sin \theta K_1 e^{-ir|z-z_0|} \\
\chi &= C_f F \left( k_0^2 + i\nu \right) \cos \left( k_0^2 + i\nu \right) K_1 e^{-ir|z-z_0|}
\end{align*}
\]

\[
\begin{align*}
M_{XY} : \quad \phi &= C_f F \left( k_0^2 + i\nu \right) \cos \theta \text{sign}(z-z_0) \left( k_0^2 + i\nu \right) K_1 e^{-ir|z-z_0|} \\
\psi &= C_f F \cos \theta \text{sign}(z-z_0) \left( k_0^2 + i\nu \right) K_1 e^{-ir|z-z_0|} \\
\chi &= C_f F \cos \theta \left( k_0^2 + i\nu \right) K_1 e^{-ir|z-z_0|}
\end{align*}
\]

**A2 Potential Green's functions for a layered medium, expressions in the source layer**

Replacing the elementary sources, defined by Eq. (2) (see text), in the previous expressions gives us the following.

\[
\begin{align*}
M_{XY} : \quad \phi &= C_f F \sin \theta \left( k_0^2 + i\nu \right) \cos \theta K_1 e^{-ir|z-z_0|} \\
\psi &= C_f F \sin \theta \left( k_0^2 + i\nu \right) \sin \theta K_1 e^{-ir|z-z_0|} \\
\chi &= C_f F \left( k_0^2 + i\nu \right) \cos \left( k_0^2 + i\nu \right) K_1 e^{-ir|z-z_0|}
\end{align*}
\]

\[
\begin{align*}
M_{XY} : \quad \phi &= C_f F \left( k_0^2 + i\nu \right) \cos \theta \text{sign}(z-z_0) \left( k_0^2 + i\nu \right) K_1 e^{-ir|z-z_0|} \\
\psi &= C_f F \cos \theta \text{sign}(z-z_0) \left( k_0^2 + i\nu \right) K_1 e^{-ir|z-z_0|} \\
\chi &= C_f F \cos \theta \left( k_0^2 + i\nu \right) K_1 e^{-ir|z-z_0|}
\end{align*}
\]

\[
\begin{align*}
M_{XY} : \quad \phi &= C_f F \sin \theta \left( k_0^2 + i\nu \right) \cos \theta K_1 e^{-ir|z-z_0|} \\
\psi &= C_f F \sin \theta \left( k_0^2 + i\nu \right) \sin \theta K_1 e^{-ir|z-z_0|} \\
\chi &= C_f F \left( k_0^2 + i\nu \right) \cos \left( k_0^2 + i\nu \right) K_1 e^{-ir|z-z_0|}
\end{align*}
\]

\[
\begin{align*}
M_{XY} : \quad \phi &= C_f F \left( k_0^2 + i\nu \right) \cos \theta \text{sign}(z-z_0) \left( k_0^2 + i\nu \right) K_1 e^{-ir|z-z_0|} \\
\psi &= C_f F \cos \theta \text{sign}(z-z_0) \left( k_0^2 + i\nu \right) K_1 e^{-ir|z-z_0|} \\
\chi &= C_f F \cos \theta \left( k_0^2 + i\nu \right) K_1 e^{-ir|z-z_0|}
\end{align*}
\]

**A3 Displacement Green's functions in receiver layer**

We evaluate here the displacements in the \( i \)th layer at depth \( z_i \), where \( z_i < z < z_i + h_i \) \((h_i = \text{ith layer thickness})\). When we start to propagate the potentials with the reflectivity algorithm, we set the upgoing/downgoing source potentials in the source(s)/layer(s). At the end, we know the upgoing/downgoing potentials in any layer and we deduce the corresponding displacements by
differentiating:

\[ u_r = \frac{\partial \phi}{\partial r} + \frac{\partial^2 \psi}{\partial r \partial z} + \frac{1}{r} \frac{\partial \chi}{\partial \theta}, \]

\[ u_\theta = \frac{1}{r} \frac{\partial \phi}{\partial \theta} + \frac{1}{r} \frac{\partial^2 \psi}{\partial \theta \partial z} - \frac{1}{r^2} \frac{\partial \chi}{\partial \theta}, \]

\[ u_z = \frac{\partial \phi}{\partial z} - \frac{\partial^2 \psi}{\partial r^2} - \frac{1}{r} \frac{\partial \psi}{\partial r} - \frac{1}{r^2} \frac{\partial^2 \psi}{\partial \theta^2}. \]

The last expression can be rewritten using the equation of propagation for \( \psi \):

\[ u_z = -\frac{\partial \phi}{\partial z} + k^2 \psi. \]

**Propagating the six elementary sources**

Each of the six elementary sources located in the source layer radiates in any layer the following \( \phi, \psi \) or \( \chi \) potentials with an upgoing and a downgoing component:

\[ S_1 \rightarrow S_{1\uparrow}^\phi, S_{1\downarrow}^\phi, S_{1\uparrow}^\psi, S_{1\downarrow}^\psi, \]

\[ S_2 \rightarrow S_{2\uparrow}^\phi, S_{2\downarrow}^\phi, S_{2\uparrow}^\psi, S_{2\downarrow}^\psi, \]

\[ S_3 \rightarrow S_{3\uparrow}^\phi, S_{3\downarrow}^\phi, S_{3\uparrow}^\psi, S_{3\downarrow}^\psi, \]

\[ S_4 \rightarrow S_{4\uparrow}^\phi, S_{4\downarrow}^\phi, S_{4\uparrow}^\psi, S_{4\downarrow}^\psi, \]

\[ S_5 \rightarrow S_{5\uparrow}^\phi, S_{5\downarrow}^\phi, S_{5\uparrow}^\psi, S_{5\downarrow}^\psi, \]

\[ S_6 \rightarrow S_{6\uparrow}^\phi, S_{6\downarrow}^\phi. \]

The \( \rightarrow \) symbol represents all the reflections/transmissions computed with the reflectivity algorithm. We then have

\[ S_{n\uparrow}^\phi = P_n^\phi e^{i(z-z_n)}, \]

\[ S_{n\downarrow}^\phi = P_n^\phi e^{-i(z-z_n)}, \]

\[ S_{n\uparrow}^\psi = P_n^\psi e^{i(z-z_n)}, \]

\[ S_{n\downarrow}^\psi = P_n^\psi e^{-i(z-z_n)}, \]

\[ \ldots, \]

where \( P_n^\phi \) are the reflectivity coefficients.

**Propagating a general source**

A general seismic source radiating \( \phi \) and/or \( \psi \) and/or \( \chi \) potential(s) in the source layer is a linear combination of the six elementary sources with \( r \)-dependent coefficients. Let us take a simple example where the seismic source is expressed as \( f(r)S_z(z_0) \) in the source layer. The radial displacement observed at depth \( z \) in any layer is

\[ u_r = \frac{\partial}{\partial r} f(r)(S_{\uparrow}^\phi(z) + S_{\downarrow}^\phi(z)) + \frac{\partial^2}{\partial r^2} f(r)(S_{\uparrow}^\psi(z) + S_{\downarrow}^\psi(z)). \]

The upgoing and downgoing derivatives only differ by the sign due to the \( \partial/\partial z \) derivative. We thus obtain

\[ \frac{\partial}{\partial z} f(r)(S_{\uparrow}^\phi(z) + S_{\downarrow}^\phi(z)) = f(r)(-i\gamma S_{\downarrow}^\phi(z) + i\gamma S_{\uparrow}^\phi(z)). \]

Using the following notation:

\[ S_f(z) = S_{\uparrow}^\phi(z) + S_{\downarrow}^\phi(z) \]

and

\[ e^{i\gamma} S_f(z) = e^{i\gamma} S_{\uparrow}^\phi(z) + e^{i\gamma} S_{\downarrow}^\phi(z), \]

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where $\epsilon = 1$ for the upcoming wave and $\epsilon = -1$ for the downgoing, we obtain a simpler expression:
\[ u_r = S_1^2(2) \psi' + \epsilon i \gamma S_1^2 \psi' \].

**Expressions of displacement for double couples, dipoles**

Using the previous notation, we obtain the displacements due to a unit moment function.

<table>
<thead>
<tr>
<th>Contribution</th>
<th>$\phi$</th>
<th>$\psi$</th>
<th>$\chi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{XY}$</td>
<td>$u_r = C_f \sin 2\theta \cos 2 \theta K_3 (S_1^2 + i \gamma S_1^4)$</td>
<td>$+ C_f \sin 2\theta K_3 (S_1^2 + i \gamma S_1^4)$</td>
<td>$+ C_f 2 \sin 2\theta \cos 3 K_4 S_6^5$</td>
</tr>
<tr>
<td></td>
<td>$u_\theta = C_f \cos 2\theta \cos 2 \theta K_4 (S_1^2 + i \gamma S_1^4)$</td>
<td>$+ C_f \cos 2 \theta K_4 (S_1^2 + i \gamma S_1^4)$</td>
<td>$+ C_f \cos 2 \theta \cos 3 K_3 S_6^5$</td>
</tr>
<tr>
<td></td>
<td>$u_z = C_f \sin 2\theta \cos 2 \theta K_1 (e^{i \phi} S_1^2 + k_2 S_1^4)$</td>
<td>$+ C_f \sin 2\theta K_1 (e^{i \phi} S_1^2 + k_2 S_1^4)$</td>
<td>$+ C_f \cos 2 \theta \cos 3 K_3 S_6^5$</td>
</tr>
<tr>
<td>$M_{XZ}$</td>
<td>$u_r = C_f 2 \cos \theta \cos (k_2^2) K_3 (S_1^2 + i \gamma S_1^4)$</td>
<td>$+ C_f \cos \theta \cos 9 K_5 (S_1^2 + i \gamma S_1^4)$</td>
<td>$- C_f \cos \theta (k_2^2) K_2 S_6^5$</td>
</tr>
<tr>
<td></td>
<td>$u_\theta = C_f 2 \sin \theta \cos (k_2^2) K_2 (S_1^2 + i \gamma S_1^4)$</td>
<td>$- C_f \sin \theta \cos 9 K_2 (S_1^2 + i \gamma S_1^4)$</td>
<td>$- C_f \cos \theta (k_2^2) K_2 S_6^5$</td>
</tr>
<tr>
<td></td>
<td>$u_z = C_f 2 \cos \theta \cos (k_2^2) J_1 (e^{i \phi} S_1^2 + k_2 S_1^4)$</td>
<td>$+ C_f \sin \theta \cos 9 J_1 (e^{i \phi} S_1^2 + k_2 S_1^4)$</td>
<td>$+ C_f \sin \theta (k_2^2) K_3 S_6^5$</td>
</tr>
<tr>
<td>$M_{YZ}$</td>
<td>$u_r = C_f 2 \sin \theta (k_2^2) K_3 (S_1^2 + i \gamma S_1^4)$</td>
<td>$+ C_f \sin \theta \cos 9 K_5 (S_1^2 + i \gamma S_1^4)$</td>
<td>$- C_f \sin \theta (k_2^2) K_2 S_6^5$</td>
</tr>
<tr>
<td></td>
<td>$u_\theta = C_f 2 \cos \theta (k_2^2) K_2 (S_1^2 + i \gamma S_1^4)$</td>
<td>$- C_f \cos \theta \cos 9 K_2 (S_1^2 + i \gamma S_1^4)$</td>
<td>$- C_f \cos \theta (k_2^2) K_2 S_6^5$</td>
</tr>
<tr>
<td></td>
<td>$u_z = C_f 2 \sin \theta (k_2^2) J_1 (e^{i \phi} S_1^2 + k_2 S_1^4)$</td>
<td>$+ C_f \sin \theta \cos 9 J_1 (e^{i \phi} S_1^2 + k_2 S_1^4)$</td>
<td>$+ C_f \sin \theta (k_2^2) K_3 S_6^5$</td>
</tr>
<tr>
<td>$M_{XX}$</td>
<td>$u_r = C_f (K_3 \cos^2 \theta + K_4) \cos 2 \phi (S_4^2 + i \gamma S_4^4)$</td>
<td>$+ C_f (K_3 \cos^2 \theta + K_4) (S_4^2 + i \gamma S_4^4)$</td>
<td>$+ C_f \cos 2 \theta \cos 3 K_4 S_6^5$</td>
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<td>$u_\theta = C_f \sin 2 \phi \cos 2 \theta K_4 (S_4^2 + i \gamma S_4^4)$</td>
<td>$- C_f \sin 2 \theta K_4 (S_4^2 + i \gamma S_4^4)$</td>
<td>$- C_f \cos \theta \sin \theta \cos 3 K_3 S_6^5$</td>
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<tr>
<td></td>
<td>$u_z = C_f \cos 2 \theta \cos \phi \cos 3 K_3 S_6^5$</td>
<td>$+ C_f \cos 2 \theta \cos \phi \cos 3 K_3 S_6^5$</td>
<td>$+ C_f \cos 2 \theta \cos \phi \cos 3 K_3 S_6^5$</td>
</tr>
<tr>
<td>$M_{YY}$</td>
<td>$u_r = C_f (K_3 \sin^2 \theta + K_4) \cos 2 \phi (S_4^2 + i \gamma S_4^4)$</td>
<td>$+ C_f (K_3 \sin^2 \theta + K_4) (S_4^2 + i \gamma S_4^4)$</td>
<td>$- C_f \cos 2 \theta \cos 3 K_4 S_6^5$</td>
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<tr>
<td></td>
<td>$u_\theta = C_f \sin 2 \phi \cos 2 \theta K_3 (S_4^2 + i \gamma S_4^4)$</td>
<td>$- C_f \sin 2 \theta K_3 (S_4^2 + i \gamma S_4^4)$</td>
<td>$- C_f \cos \theta \sin \theta \cos 3 K_3 S_6^5$</td>
</tr>
<tr>
<td></td>
<td>$u_z = C_f \cos 2 \theta \cos \phi \cos 3 K_3 S_6^5$</td>
<td>$+ C_f \cos 2 \theta \cos \phi \cos 3 K_3 S_6^5$</td>
<td>$+ C_f \cos 2 \theta \cos \phi \cos 3 K_3 S_6^5$</td>
</tr>
<tr>
<td>$M_{ZZ}$</td>
<td>$u_r = C_f k_4 \sin \phi J_1 (S_4^2 + i \gamma S_4^4)$</td>
<td>$+ C_f k_2^3 J_1 (S_4^2 + i \gamma S_4^4)$</td>
<td>$- C_f k_4 \cos \phi J_0 (e^{i \phi} S_4^2 + k_2 S_4^4)$</td>
</tr>
<tr>
<td></td>
<td>$u_\theta = 0$</td>
<td>$+ C_f k_2^3 J_1 (S_4^2 + i \gamma S_4^4)$</td>
<td>$- C_f k_4 \cos \phi J_0 (e^{i \phi} S_4^2 + k_2 S_4^4)$</td>
</tr>
<tr>
<td></td>
<td>$u_z = -C_f k_4 \cos \phi J_0 (e^{i \phi} S_4^2 + k_2 S_4^4)$</td>
<td>$- C_f k_4 \cos \phi J_0 (e^{i \phi} S_4^2 + k_2 S_4^4)$</td>
<td></td>
</tr>
</tbody>
</table>

These expressions can be rearranged into the following.

<table>
<thead>
<tr>
<th>Contribution</th>
<th>$M_{XY}$</th>
<th>$M_{XZ}$</th>
<th>$M_{YZ}$</th>
<th>$M_{XX}$</th>
<th>$M_{YY}$</th>
<th>$M_{ZZ}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_r$</td>
<td>$C_f \sin 2 \theta (K_3 \cos 2 \theta + K_4) [e^{i \phi} S_1^2 + k_2 S_1^4]$</td>
<td>$C_f \cos \theta (K_3 \cos 2 \theta + K_4) [e^{i \phi} S_1^2 + k_2 S_1^4]$</td>
<td>$C_f \sin \theta J_1 [e^{i \phi} S_1^2 + k_2 S_1^4]$</td>
<td>$C_f (K_3 \cos^2 \theta + K_4) [e^{i \phi} S_1^2 + k_2 S_1^4]$</td>
<td>$C_f (K_3 \sin^2 \theta + K_4) [e^{i \phi} S_1^2 + k_2 S_1^4]$</td>
<td>$C_f k_4 \cos \phi J_0 (e^{i \phi} S_1^2 + k_2 S_1^4)$</td>
</tr>
<tr>
<td>$u_\theta$</td>
<td>$C_f \cos 2 \theta (K_3 \cos 2 \theta + K_4) [e^{i \phi} S_1^2 + k_2 S_1^4]$</td>
<td>$C_f \sin 2 \theta (K_3 \cos 2 \theta + K_4) [e^{i \phi} S_1^2 + k_2 S_1^4]$</td>
<td>$C_f \cos \theta J_1 [e^{i \phi} S_1^2 + k_2 S_1^4]$</td>
<td>$C_f (K_3 \cos^2 \theta + K_4) [e^{i \phi} S_1^2 + k_2 S_1^4]$</td>
<td>$C_f (K_3 \sin^2 \theta + K_4) [e^{i \phi} S_1^2 + k_2 S_1^4]$</td>
<td>$C_f k_4 \cos \phi J_0 (e^{i \phi} S_1^2 + k_2 S_1^4)$</td>
</tr>
<tr>
<td>$u_z$</td>
<td>$C_f \sin 2 \theta (K_3 \cos 2 \theta + K_4) [e^{i \phi} S_1^2 + k_2 S_1^4]$</td>
<td>$C_f \cos 2 \theta (K_3 \cos 2 \theta + K_4) [e^{i \phi} S_1^2 + k_2 S_1^4]$</td>
<td>$C_f \sin \theta J_1 [e^{i \phi} S_1^2 + k_2 S_1^4]$</td>
<td>$C_f (K_3 \cos^2 \theta + K_4) [e^{i \phi} S_1^2 + k_2 S_1^4]$</td>
<td>$C_f (K_3 \sin^2 \theta + K_4) [e^{i \phi} S_1^2 + k_2 S_1^4]$</td>
<td>$C_f k_4 \cos \phi J_0 (e^{i \phi} S_1^2 + k_2 S_1^4)$</td>
</tr>
</tbody>
</table>

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Dynamic stress variations

Expressions of stresses for double couples and dipoles

We use Hooke's law in cylindrical coordinates (see eq. 4) and we note:

\[ A_j = S_i^j + \epsilon \gamma S_i^j, \]
\[ B_j = \epsilon \nu S_i^j + k^2 S_i^j, \]
\[ C_i = S_i^j, \]

and their derivatives with respect to \( z \):
\[ A'_j = \epsilon \nu S_i^j - \gamma^2 S_i^j, \]
\[ B'_j = -\nu^2 S_i^j + \epsilon \nu k^2 S_i^j, \]
\[ C'_j = \epsilon \nu S_i^j. \]

Using the previous expression of the displacements, the Lamé parameters \((\lambda, \mu)\) and setting \( \eta = \lambda + 2\mu \), we finally obtain the following for the stress tensor.

\[
\begin{align*}
M_{XY} & : \\
T_{rr} & = C_f \sin \theta \left( [\eta K_4 + \lambda K_0](A_1 c_2 + A_4) + \lambda K_1(B_1 c_2 + B_4) \right) + 2K_1(\eta - \lambda) c_3 c_5 \\
T_{\theta \theta} & = C_f \sin \theta \left( [\lambda K_0 + \eta K_4](A_1 c_2 + A_4) + \lambda K_1(B_1 c_2 + B_4) \right) + 2K_1(\eta - \lambda) c_3 c_5 \\
T_{zz} & = C_f \sin \theta \left( [\lambda K_0 + \lambda K_4](A_1 c_2 + A_4) + \eta K_1(B_1 c_2 + B_4) \right) + 2K_1(\eta - \lambda) c_3 c_5 \\
T_{r \theta} & = \mu C_f \sin \theta \left( K_3(A_1 c_2 + A_4) + K_3(B_1 c_2 + B_4) \right) + 2K_4 c_3 c_5' \\
T_{r z} & = \mu C_f \sin \theta \left( 2K_4(A_1 c_2 + A_4) + 2K_4(B_1 c_2 + B_4) - K_3 c_3 c_5' \right) + (K_{10}) c_3 c_5 \\
M_{XZ} & : \\
T_{rr} & = C_f \cos \theta \left( [\eta K_4 + \lambda K_0](-2k^2 A_2 + A_3 c_9) + \lambda J_1(-2k^2 B_2 + B_3 c_9) \right) - 2\mu K_4 k_3^2 c_6 \\
T_{\theta \theta} & = C_f \cos \theta \left( [\lambda K_0 + \eta K_4](-2k^2 A_2 + A_3 c_9) + \lambda J_1(-2k^2 B_2 + B_3 c_9) \right) + 2\mu K_4 k_3^2 c_6 \\
T_{zz} & = C_f \cos \theta \left( [\lambda K_0 + \lambda K_4](-2k^2 A_2 + A_3 c_9) + \eta J_1(-2k^2 B_2 + B_3 c_9) \right) + 2\mu K_4 k_3^2 c_6 \\
T_{r \theta} & = K_5(-2k^2 A_2 + A_3 c_9) + K_5(-2k^2 B_2 + B_3 c_9) - K_2 k_3^2 c_6' \\
T_{r z} & = \mu C_f \cos \theta \left( -K_2(-2k^2 A_2 + A_3 c_9) - K_2(-2k^2 B_2 + B_3 c_9) + K_5 k_3^2 c_6' \right) + K_3 k_3^2 c_6 \\
M_{YZ} & : \\
T_{rr} & = C_f \sin \theta \left( [\eta K_4 + \lambda K_0](-2k^2 A_2 + A_3 c_9) + \lambda J_1(-2k^2 B_2 + B_3 c_9) \right) - 2\mu K_4 k_3^2 c_6 \\
T_{\theta \theta} & = C_f \sin \theta \left( [\lambda K_0 + \eta K_4](-2k^2 A_2 + A_3 c_9) + \lambda J_1(-2k^2 B_2 + B_3 c_9) \right) + 2\mu K_4 k_3^2 c_6 \\
T_{zz} & = C_f \sin \theta \left( [\lambda K_0 + \lambda K_4](-2k^2 A_2 + A_3 c_9) + \eta J_1(-2k^2 B_2 + B_3 c_9) \right) + 2\mu K_4 k_3^2 c_6 \\
T_{r \theta} & = \mu C_f \cos \theta \left( K_5(-2k^2 A_2 + A_3 c_9) + K_5(-2k^2 B_2 + B_3 c_9) - K_2 k_3^2 c_6' \right) + K_3 k_3^2 c_6 \\
T_{r z} & = \mu C_f \cos \theta \left( 2K_4(-2k^2 A_2 + A_3 c_9) + 2K_4(-2k^2 B_2 + B_3 c_9) - K_3 k_3^2 c_6 \right) + (K_{10}) c_3 c_5 \\
M_{XX} & : \\
T_{rr} & = C_f \left( [\eta K_4 + \lambda K_1] - \cos 2\theta(2\lambda K_1) + \cos^2 \theta(\eta K_4 + \lambda K_1) \right)(A_1 c_2 + A_4) + \lambda (\lambda K_2 + \cos^2 \theta \lambda K_1)(B_1 c_2 + B_4) \\
& + 2\mu K_4 \cos 2\theta c_3 c_5 \\
T_{\theta \theta} & = C_f \left( [\lambda K_4 + \eta K_1] - \cos 2\theta(2\lambda K_1) + \cos^2 \theta(\eta K_4 + \lambda K_1) \right)(A_1 c_2 + A_4) + \lambda (\lambda K_2 + \cos^2 \theta \lambda K_1)(B_1 c_2 + B_4) \\
& - 2\mu K_4 \cos 2\theta c_3 c_5 \\
T_{zz} & = C_f \left( [\lambda K_4 + \lambda K_1] - \cos 2\theta(2\lambda K_1) + \cos^2 \theta(\lambda K_4 + \lambda K_1) \right)(A_1 c_2 + A_4) + \lambda (\eta K_2 + \cos^2 \theta \eta K_1)(B_1 c_2 + B_4) \\
& + \cos 2\theta K_4 \mu c_3 c_5' \\
T_{r \theta} & = C_f \mu \sin 2\theta \left( -K_4(A_1 c_2 + A_4) - K_4(B_1 c_2 + B_4) \right) - k_3^2 c_6' \\
T_{r z} & = C_f \mu \sin 2\theta \left( -2K_7(A_1 c_2 + A_4) \right) - \frac{k_3^2}{2} c_6' \\
\end{align*}

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A5 Wavenumber integration

To obtain the stress Green's function in the space/frequency domain, we use the modification proposed by Herrmann & Mandal (1986). The summation is performed as:

\[
T(x, \omega) = \sum_{n=0}^{N} T(k_n, \omega),
\]

where the radial wavenumber is given a small offset: \(k_n = (2\pi n/L) + 0.258(2\pi/L)\). The summation is truncated at step \(N\), when for each moment all stress tensor elements have reached the convergence criterion.