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GRACE's spatial aliasing error

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SUMMARY

The GRACE satellite mission provides a near-continuous sequence of approximately 30-d gravity field solutions in the form of spherical harmonics (SH). Because SH functions are global while GRACE measurements are sensitive mainly to variations along the ground-track, undersampling (alias contamination) occurs. Here we investigate how geophysical signals are likely to cause alias error in GRACE gravity fields. We use actual GRACE orbits and systematically sample several types of time-varying signals that might represent either errors in geophysical models such as tide models, or unmodelled geophysical signals. We show how error in semi-diurnal tides like S_2 can alias into long period variations in particular harmonics, particularly as a possible error source in the degree 2, order 0 term (C_{20}) of GRACE fields. We also show that aliasing associated with non-tidal geophysical model errors is significant at order 15 or multiples of 15, due to the GRACE ground track spacing in longitude. This can be predicted from Kaula's resonance formula and might be reduced by suppressing amplitudes of affected harmonics.

Key words: Satellite geodesy; Time variable gravity.

1 INTRODUCTION

The NASA/DLR Gravity Recovery and Climate Experiment (GRACE) satellite mission (Tapley *et al.* 2004) has been providing a nearly continuous sequence of monthly gravity field solutions in the form of spherical harmonic (SH) coefficients over the period from April 2002 to the present. Temporal gravity changes at time scales of a few years and less are mainly due to mass redistribution from tides, postglacial rebound, ocean bottom pressure, atmospheric surface pressure and the water cycle (NRC 1997). Tides and oceanic and atmospheric mass redistribution are removed by subtracting model predictions during processing, but these predictions are imperfect. Although GRACE solutions are monthly averages, error in model predictions with periods of hours to days will contaminate monthly solutions, an effect known as aliasing in sampling theory. Aliasing in the case of GRACE is complex because mass variations occur over the entire Earth, in both space and time, but are sampled mainly along orbital ground tracks and then represented in terms of global SH functions.

Previous studies of aliasing by Thompson *et al.* (2004) and Han *et al.* (2004) used numerical oceanic and atmospheric models to show that aliasing is a significant problem relative to GRACE measurement noise. Ray *et al.* (2003) calculated aliasing periods due to tides, finding for K_1, K_2, S_1, S_2 and P_1 constituents alias periods of 7.48 yr, 3.74 yr, 322 d, 161 d and 171 d, respectively. Schrama & Visser (2007) estimated aliasing error via simulated GRACE data and showed that signals at periods shorter than 3 months were not well retrieved due to errors in geophysical background models, also a possible source of alias contamination. Seo *et al.* (2007) estimated aliasing error associated with ocean tides using actual GRACE ground tracks, showing that orbit decay and monthly averages of GRACE sampling cause aliasing periods to change. The purpose of this study is to estimate GRACE spatial aliasing error in the SH domain. Aliasing error is generated by a combination of GRACE along-track measurements and variations of geophysical signals in time and space, and therefore, will likely contaminate specific SH degrees and orders. We examine aliasing error associated with single SH terms using synthetic GRACE data as well as geophysical models, and investigate error in monthly GRACE products.

2 METHODS TO SIMULATE ALIASING ERROR

2.1 Aliasing error from single SH terms

Numerical experiments to simulate aliasing error from single SH terms are useful to understand how alias contamination occurs. Experiments consist of three steps. First, we take GRACE ground tracks (GNV1B) for 2005 from the Physical Oceanography Distributed Active Archive Center (PO.DAAC) (NASA Jet Propulsion Laboratory, Pasadena, CA, http://podaac.jpl.nasa.gov.). GNV1B provides 3-D positions of the two satellites as a function of GPS time (at 60 s intervals) past 12:00:00 on January 1, 2000 along with orbit

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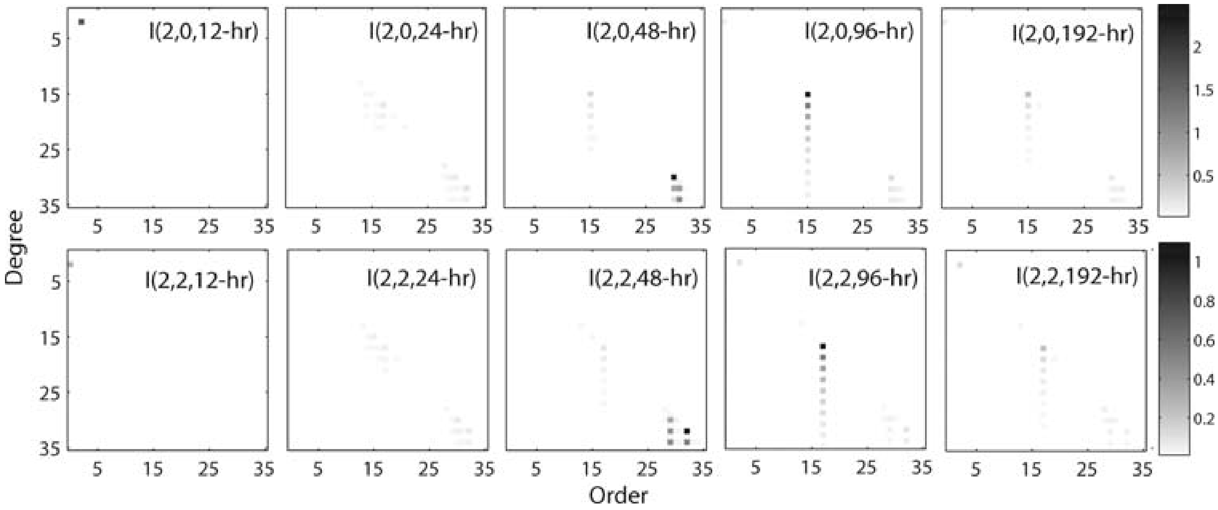

Figure 1. Normalized rms aliasing error from single SH components and single sinusoidal periods. Horizontal axis is for order and vertical is for degree. Top and bottom panels show aliasing errors from SH (2,0) and SH (2,2) as inputs, respectively. From left to right, periods of inputs vary from 12 to 192 hr. This shows that the SH (2,2) component in semi-diurnal tides aliases to SH (2,0). The aliasing error from non-tidal input error is sensitive to the spacing of successive GRACE ground tracks in longitude.

error, which is less than a few centimetres. Second, we obtain amplitudes of residuals of potential differences due to time-varying geophysical model error, such as atmospheric pressure fields or ocean tides. Then, a single SH component is used as the source of gravity field perturbations. Residuals of potential differences at GRACE satellites orbits are calculated as follow:

$$\mathrm{d}V^t(r_1,\theta_1,\lambda_1,r_2,\theta_2,\lambda_2) = \delta V_1^t(r_1,\theta_1,\lambda_1) - \delta V_2^t(r_2,\theta_2,\lambda_2), \qquad (1)$$

where δV_1 and δV_2 are resulting perturbed gravity potentials at the positions of the two GRACE satellites $(r_i, \theta_i, \lambda_i)$, and t is sampling time. The gravity potential due to a single SH component is derived as follows:

$$\delta V_i^t(r_i,\theta_i,\lambda_i) = \frac{GM}{R}\left(\frac{R}{r_i}\right)^{l+1}\tilde{P}_{lm}(\cos\theta_i) \times [\delta C^t\cos(m\lambda_i) + \delta S^t\sin(m\lambda_i)], \qquad (2)$$

where i is 1 or 2, G is the gravity constant, M and R are the mass and mean radius of Earth, respectively, $\tilde{P}_{lm}$ are normalized associated Legendre polynomials, and l and m are degree and order of a single SH component. We vary δC^t and δS^t sinusoidally via

$$\begin{aligned}&\delta C^t = A\cos(2\pi t/P),\ \delta S^t = \mathrm{d}A\sin(2\pi t/P)\\ &d = -1,\ 0,\ \text{ or } 1\end{aligned} \qquad (3)$$

in which P is period, A is amplitude and d is a factor to control direction of propagation of the single harmonic. Third, we sample $\mathrm{d}V^t$ at 60 s intervals, as in GNV1B, over intervals of 10 d and estimate Stokes coefficients to degree and order 35 from a least square fit of SH functions to sampled potential differences (Han *et al.* 2004). Because this aliasing simulation is sensitive to $\mathrm{d}V^t$ between the two satellites and we estimate SH coefficients up to degree and order 35, the orbit error (<2–3 cm) should be insignificant in this study. This is a simplified approach without full consideration of the real GRACE data processing, and a 60 s sampling is sparse compared to GRACE data. Actual GRACE data provides K-band ranging every 5 s (Bettadpur 2004). However, this scheme using actual GRACE ground tracks should replicate the main features of aliasing error, which is likely to occur due to undersampling along longitude direction. Seo *et al.* (2007) simulated the alias of ocean tides using the same approach as this study, and compared the simulated aliasing error with the results in Ray & Luthcke (2006) to show that this simplified method achieves about the same results, and therefore, is a good proxy for the alias simulation

2.2 Aliasing error from geophysical models

We simulate aliasing associated with geophysical model error or omissions for September 2005 using an approach similar to that in the previous section. We assume that the Global Land Data Assimilation Scheme (GLDAS, Rodell *et al.* 2004), Estimating the Circulation and Climate of the Ocean (ECCO, Fukumori *et al.* 2000) and National Center for Environmental Prediction (NCEP/NCAR, Kalnay *et al.* 1996) represent true geophysical signals for terrestrial water, ocean bottom pressure and atmospheric surface pressure, respectively. The GRACE Atmosphere and Ocean Dealiasing (AOD) time varying mass fields (which differ from these three models) are used in processing to remove ocean and atmosphere signals from GRACE observations (Flechtner 2005). Therefore, potential difference residuals ($\mathrm{d}V^t$) are calculated from GLDAS + ECCO + NCEP/NCAR − AOD. Temporal sampling of these geophysical models is 3 hr or longer, requiring linear interpolation to sample $\mathrm{d}V^t$ every 60 s. To estimate ocean tide model error, we use the difference between two tide models, GOT00.2 (an update of Ray 1999) and TPXO6.2 (an update of Egbert *et al.* 1994) with 8 major diurnal and semi-diurnal constituents (Q_1, O_1, P_1, K_1, N_2, M_2, S_2 and K_2).

3 ALIASING ERROR ESTIMATES

3.1 Aliasing error from single SH terms

We examine effects of errors in SH (2,0) and SH (2,2), as examples of individual harmonics. Both are prominent elements of tidal and other signals. The error in single SH terms will be useful to explore the cause and the effect of the alias. Six sinusoidal time variations are examined with 12-, 24-, 48-, 96-, 192- and 384-hr periods. The

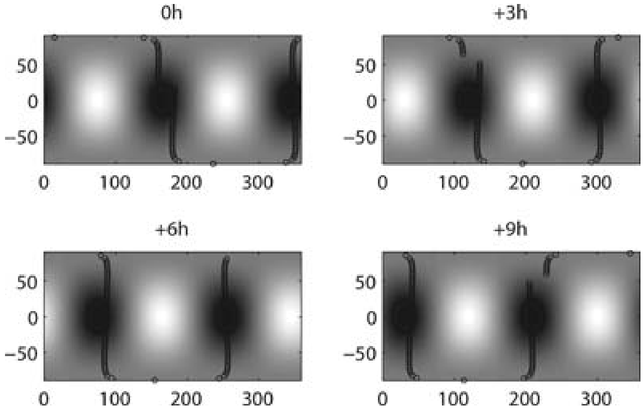

Figure 2. Grey scaled maps show snap shots of SH (2,2) components of the S_2 tide. GRACE ground tracks are plotted over those maps. The ground tracks overpass almost the same phase of the S_2 tide. This sampling produce an apparent SH (2,0) component from the SH (2,2) component.

12- and 24-hr periods simulate semi-diurnal and diurnal tides, while 48-, 96-, 192- and 384-hr periods are intended to represent errors in synoptic variations of geophysical signals at periods longer than the tides. The longer chosen periods are selected only to be representative of changes in aliasing as periods of input error vary. To describe each simulation we use the notation I (SH degree, SH order, period). For example, a spatial variation of SH (2,0) and temporal period of 12 hr is I (2,0,12-hr). Aliasing error is the difference between estimated Stokes coefficients from the average of simulated GRACE observations and the true average over 10-d. We vary the propagation direction, and examine the three cases (westward, eastward and stationary) for the SH (2,2) term.

Fig. 1 shows rms amplitudes of aliasing error normalized by amplitudes of the sampled harmonic. In this case the single harmonic error is propagating westward, with other propagation cases shown in later figures. The left two panels show aliasing error from semi-diurnal and diurnal tidal periods. For I (2, 0, 12 hr), the strongest alias contamination is at SH (2,2), because the sign of the input harmonic changes twice every 12 hr while GRACE surveys the globe about every 12 hr. However, for the case I (2, 2, 12 hr), the strongest aliasing error is at SH (2,0). Fig. 2 shows how GRACE observes an apparent SH (2,0) from I (2, 2, 12 hr). I (2, 2, 12 hr) and GRACE orbit ground tracks are plotted every 3 hr. The orbit drifts westward almost at the same speed as the semi-diurnal signal. As a result, GRACE observes an apparent SH (2,0) signal. This spatial aliasing error of semi-diurnal tides should apply to any harmonics with order 0 or 2, which are the dominant components of semi-diurnal tides. For example, a numerical test (not shown) shows that I (6, 2, 12 hr) leads to aliasing error at SH (6,0), SH (4,0) and SH (2,0). The small period difference between S_2 (12 hr) and half a sidereal day (11.9664 hr) makes the orbit ground track shift slowly relative to SH (2,2) of the S_2 tidal wave, and this produces long period aliasing error from residual semi-diurnal tides. The period of aliasing error associated with I(2, 0, 12 hr) and I (2, 2, 12 hr) is about 160 d, which confirms the estimate by Ray *et al.* (2003). Aliasing error due to diurnal tides, I (2, 0, 24 hr) and I (2, 2, 24 hr), is small, apparently because diurnal tide errors mostly average to zero as GRACE surveys the globe about every 12 hr.

Fig. 1 also shows that aliasing error from the longer period single harmonics is significant at specific SH orders. Variations at 48 and 96 hr produce large errors, but errors diminish at longer periods. Aliases from I (2, 0, 384 hr) and I (2, 2, 384 hr) are negligible (not shown). SH degrees and orders with significant aliasing error are predicted by Kaula's resonance formula (Kaula 1966),

$$(l-2p)\dot{\omega}+(l-2p+q)\dot{M}+m(\dot{\Omega}-\dot{\theta})=0, \tag{4}$$

where l and m are SH degree and order, p and q are coefficients of inclination and eccentricity functions (Kaula 1966), and $\dot{\omega}$, $\dot{M}$, $\dot{\Omega}$ and $\dot{\theta}$ are the first derivative with time for the argument of perigee, the mean anomaly, the longitude of the ascending node and Greenwich sidereal time. Since $\dot{\omega}$ and $\dot{\Omega}$ are very small and $\dot{M}$ is the mean motion of the satellite (N), eq. (4) can be rewritten (Lambeck 1988):

$$N \approx \frac{m\dot{\theta}}{l-2p+q}, \tag{5}$$

where N is about 15 for GRACE, $\dot{\theta}$ is 1 for Earth's rotation rate, and $q = 0$ since the eccentricity of GRACE is nearly zero, so eq. (5) is satisfied when, $m = 15$ when $l = 15, 17, 19, 21, \ldots$, $m = 30$ when $l = 30, 32, 34, \ldots$ and so on. The predicted SH order 15 from the eq. (5) is approximately equivalent to the longitudinal spacing of GRACE's successive ground track.

Fig. 3 shows I (2,2,96-hr), propagating westward, eastward and stationary. The left-hand panel is the same as the I (2,2,96-hr) case in Fig. 1. For the westward case, aliasing error is dominant at $m = 15 + 2$ when $l = 17, 19, 21, \ldots$ For the eastward case, aliasing error is significant at $m = 15 - 2$ when $l = 13, 15, 17, 19, \ldots$. The stationary case shows that SH orders of dominant aliasing error are both $m = 15 + 2$ and $m = 15 - 2$. Additional aliasing simulations, using stationary single SHs with degree 4 and a given order X with a 96-hr period, show that SH orders of dominant aliasing error are $m = 15 - X$ and $m = 15 + X$. Consequently, if the input error order (m) is not zero, SH orders are linear combinations of SH orders of input error and Kaula-resonant orders.

To examine periods of aliasing error, error time-series are presented in Fig. 4. Left, middle and right panels in the figure are aliasing error associated with inputs at 96-, 192- and 384-hr periods, respectively. Aliasing error due to different SH terms are shown

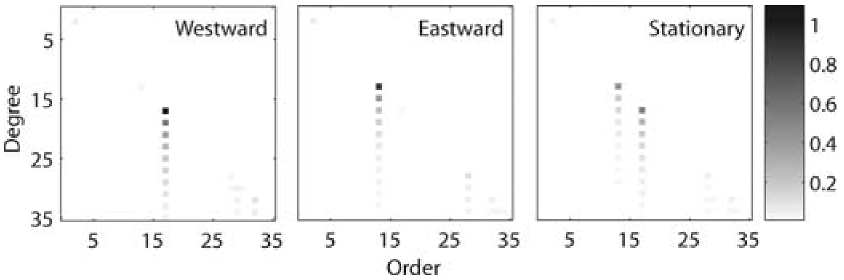

Figure 3. Normalized rms aliasing error from input of SH (2,2) term with a 96 hr period. Three different cases, are SH (2,2) propagates westward, eastward and stationary, show aliasing error at different orders.

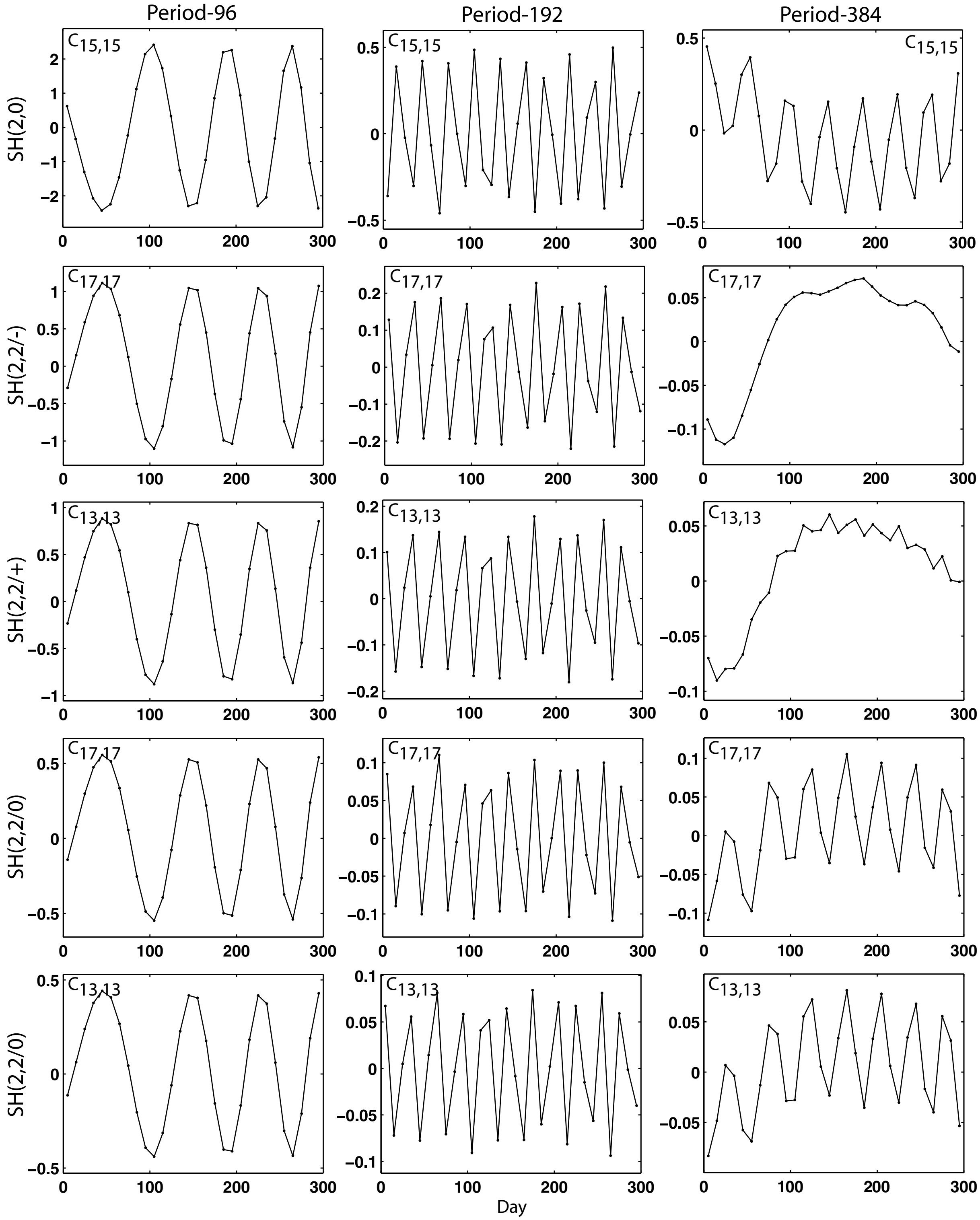

Figure 4. Time-series of aliasing error normalized by amplitudes of input SH. The three columns represent aliasing errors caused by three different periods of input in hours, and the five rows show the aliases seen in different harmonics. +, − and 0 in parenthesis indicate directions of propagation (eastward, westward and stationary, respectively) of SH terms.

from top to bottom panels. Symbols, +, − and 0 in parenthesis represent eastward, westward and stationary propagation of SH terms. For example, the left panel in the third row shows time-series of aliasing error caused by I (2,2,96-hr) that propagates eastward. The choice of SH degrees and orders for time-series of aliasing errors is based on results in Fig. 3, showing aliasing error is dominant at SH orders 13, 15 and 17. Amplitudes of time-series in aliasing error are normalized by amplitudes of the sampled harmonic. Aliasing error from a single harmonic with a 96-hr period (left-hand panels) shows non-stationary time-series. This is because the GRACE orbit

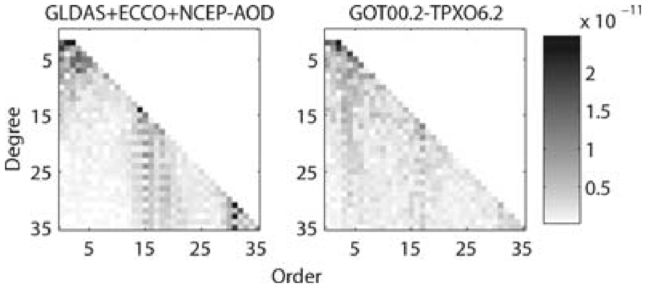

Figure 5. Spectra of synthetic data from geophysical model error (left-hand panel) and ocean tidal model error (right-hand panel). Terrestrial water and unmodelled ocean bottom pressure and atmospheric surface pressure signal induces aliasing error at predicted resonant orders. Units are mm in geoid.

decays over time so that GRACE sampling in longitude is not regular (Seo *et al.* 2007). Periods of aliasing errors are dependent mainly on periods of the input harmonic. Different SH terms and directions of propagation in input are relatively less important in determining aliasing periods.

In summary, experiments sampling single SH variations using GRACE ground tracks show two types of aliasing error. The first is associated with variations at tidal periods. Similar westward speeds of tides and GRACE ground tracks produce aliasing error at long periods. The second is related to the spatial scale of GRACE's successive ground track, equivalent to approximately order 15 and multiples of 15. Non-tidal geophysical error would predominantly produce this type of aliasing.

3.2 Aliasing error from geophysical models

To explore the spatial alias examined with single SH terms for more realistic cases, here we incorporate geophysical models. We examine aliasing errors of non-tidal and tidal geophysical models separately, because characteristics of the aliasing problem are very different as shown in Fig. 1. Fig. 5 shows spectra of synthetic GRACE data for September 2005. The left-hand panel of the figure shows spectra of gravity fields resulting from non-tidal sources (GLDAS + ECCO + NCEP/NCAR − AOD), and the right-hand panel represents spectra of gravity fields resulting from ocean tide error (GOT00.2 − TPXO6.2). Aliasing due to geophysical model error (left-hand panel) is evident around SH orders 15 and 30. Large amplitudes in the left panel at low SH degrees and orders are due mainly to the hydrologic model (GLDAS). Because diurnal and semi-diurnal ocean tides tend to average to zero over monthly intervals, amplitudes of sampled tidal errors (right-hand panel) are mostly due to the longer period aliases. Aliasing error caused by ocean tide model error is also significant at low SH degrees and orders as discussed above, and in a previous study (Seo *et al.* 2007). There is insignificant aliasing error associated with ocean tides at the resonant orders. This implies that the alias due to ocean tides is not resonance free, but its impact is small compared to that from non-tidal geophysical error. Therefore, Fig. 5 shows the two different aliasing effects as discussed in the previous section based on analysis of a single SH source.

Fig. 1 shows that alias contamination at particular SH orders occurs over a range of SH degrees. To examine this, we plot SH cosine terms at orders 15, 16 and 30 versus SH degree in Fig. 6. Orders 15, 16 and 30 are associated with resonant orders seen in Fig. 1 and Fig. 5. Black lines are SH terms in synthetic GRACE data from ECCO + NCEP − AOD, and red lines are SH terms in synthetic GRACE data from GLDAS + ECCO + NCEP − AOD plus GOT00.2 − TPXO6.2. There is a correlated pattern, alternating with even and odd degrees. Swenson & Wahr (2006) found that the correlated pattern causes north–south stripes in GRACE data. The alias due to unmodelled atmospheric and oceanic pressure field accounts for much of this pattern. Therefore, the well-recognized north–south noise stripes in GRACE fields are likely caused by aliasing of mismodelled non-tidal geophysical signal, particularly from atmospheric and oceanic sources.

4 ALIASING ERROR IN GRACE MONTHLY SOLUTIONS

4.1 Aliasing error from tides

To examine aliasing associated with ocean tides, we use time-series of zonal harmonics from the CSR RL04 product (Bettadpur 2007) As illustrated in Figs 1 and 2 zonal harmonics tend to be contaminated by SH order 2 terms of ocean tide model error. CSR RL04 employs FES2004 (an update of Lefevre *et al.* 2002) to remove ocean tide effect.

Seo *et al.* (2007) simulated aliasing error from ocean tides using actual GRACE ground tracks, finding that K_1, K_2, P_1, S_2 and M_2 are particularly problematic for GRACE because the alias periods (7.48 yr, 3.74 yr, 171 d, 161 dand 140 d, respectively) exceed 1 month. Because the aliases will not tend to average to zero in monthly sampling, GRACE solutions may include aliasing error at these five periods. To examine this in more detail, we first remove known geophysical signal and error other than the alias, including a seasonal cycle and the unmodelled ocean pole tide. We use periods of 365.25 and 433 d for seasonal cycle and ocean pole tides and

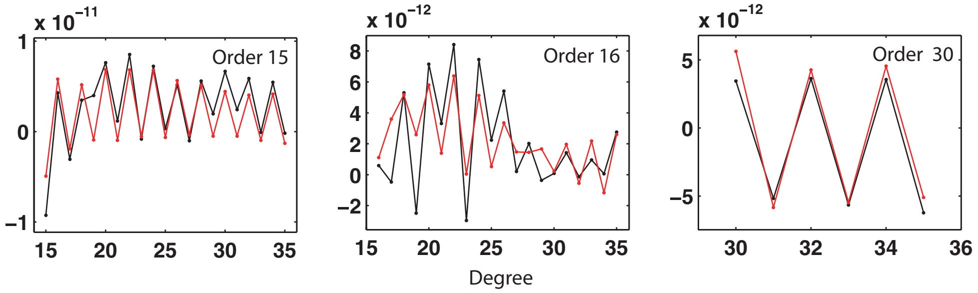

Figure 6. Cosine terms of SH orders 15, 16 and 30 versus degree from synthetic data associated with ECCO + NCEP − AOD (red) and GLDAS + ECCO + NCEP − AOD + GOT00.2 − TPXO6.2 (black). Correlations at even and odd degrees are evident, as shown by Swenson and Wahr (2006). Units are mm in geoid.

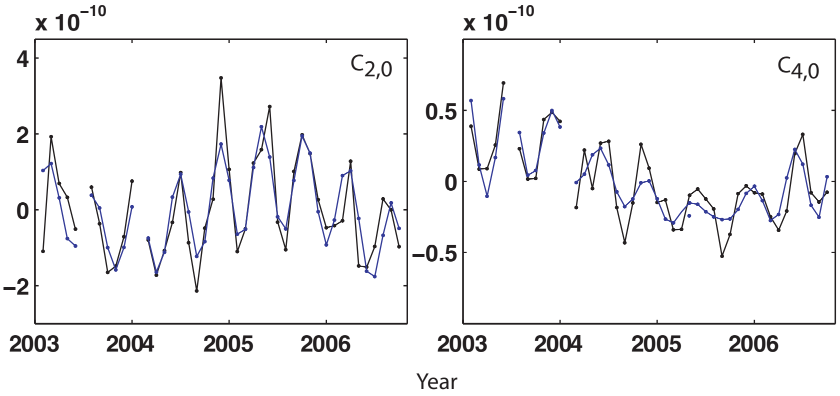

Figure 7. Black lines are time-series of $C_{2,0}$ and $C_{4,0}$ in CSR RL04 after removing a seasonal cycle and ocean pole tide. The blue lines are the best fits to black lines with five sinusoids at aliasing periods (7.48 yr, 3.74 yr, 171 d, 161 d and 140 d). The blue lines agree with black lines fairly well, suggesting these variations are due to tidal aliases. Units are mm in geoid.

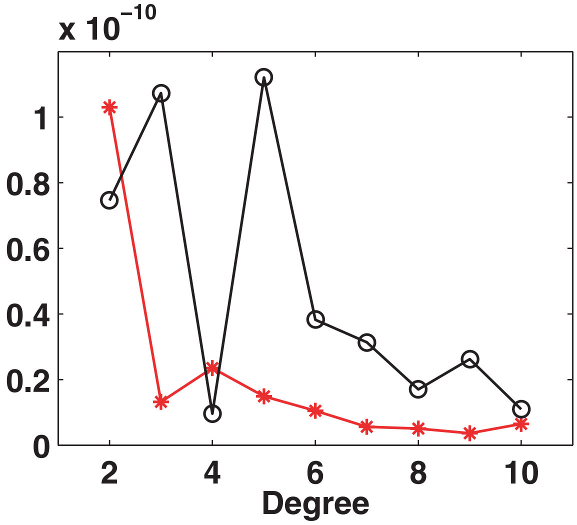

Figure 8. Red line is rms aliasing error, and black line is amplitude in seasonal cycle (365.25 d period) from time-series of zonal harmonic in CSR RL04. The aliasing error due to oceans tides are bigger than seasonal cycle at $C_{2,0}$ and $C_{4,0}$. Large error in $C_{2,0}$ implies that this SH coefficient is vulnerable to aliasing error. Units are mm in geoid.

remove these by a least square fit. Black lines in Fig. 7 show resulting time-series of $C_{2,0}$ and $C_{4,0}$ from CSR RL04. The blue lines are least square fits to black lines using sinusoids at the five aliasing periods. The five aliasing periods account quite well for variability seen in the GRACE measurements (CSR RL04). This is true for other zonal harmonics, as well.

Fig. 8 summarizes rms amplitudes of aliasing error associated with ocean tides (red) and those of the seasonal cycle (black) in low degree zonal harmonics. The $C_{2,0}$ and $C_{4,0}$ coefficients have larger error than seasonal cycle, and in particularly, the $C_{2,0}$ coefficient has significant error over other coefficients. Relative amounts of aliasing error in $C_{2,0}$ from K_1, K_2, P_1, S_2 and M_2 are about 23, 32, 7, 32 and 6 per cent, respectively. The estimation for K_1 and K_2 aliasing error may be inaccurate because (1) the GRACE measurement period is not long enough to separate those long period errors (7.48 yr for K_1 and 3.74 yr for K_2) and (2) real geophysical signals having interannual variation, possibly affecting the estimate. However, this indicates that SH order 2 with even degrees, the largest components in semi-diurnal tides, aliases to SH (2,0).

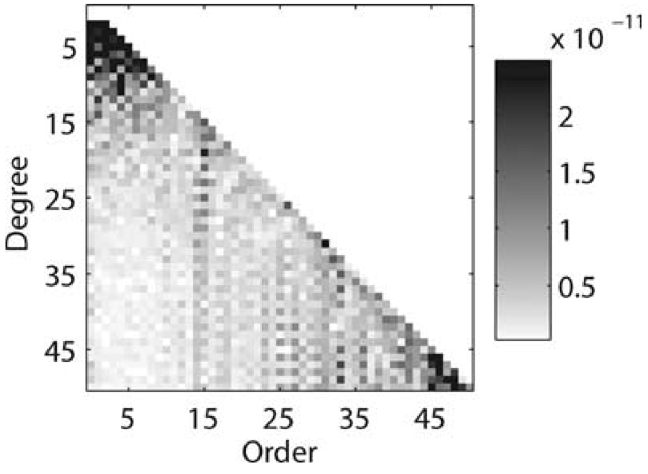

Figure 9. Spectrum of GRACE for September 2005 relative to a mean field. Significant amplitudes are present around orders 15, 30 and 45, as shown in Fig 5. Unit is mm in geoid.

4.2 Aliasing error from non-tidal signal

Fig. 9 shows a degree-order spectrum of GRACE data for September 2005 minus the average field of 2003. The spectrum shows large amplitudes around orders 15, 30 and 45. However, relative to Fig. 5, amplitudes here are larger than the synthetic data. One possible explanation is that the synthetic data (Fig. 5) are noise free, but actual GRACE data (Fig. 9) are contaminated by measurement noise. In any case, it appears that the synthetic data underestimate aliasing error.

Swenson & Wahr (2006) presented a new filter to remove longitudinal noise stripes by removing correlation of GRACE SH at even and odd degrees at a given order. Here we explore a different approach to remove these stripes, noting that particular SH degrees and orders contaminated by aliasing error are predicted around multiples of order 15. Fig. 10(a) shows a mass field (in units of equivalent water layer thickness, relative to a mean field) from September 2005 after 400 km Gaussian smoothing. Longitudinal stripes contaminate the map. Fig. 10(b) shows the same field after 1000 km Gaussian smoothing, with the result that the stripes are suppressed, but the undesirable effect is that apparent signal amplitudes are reduced relative to Fig. 10(a). As an alternative method, we apply 400 km Gaussian smoothing, and then replace particular SH coefficients

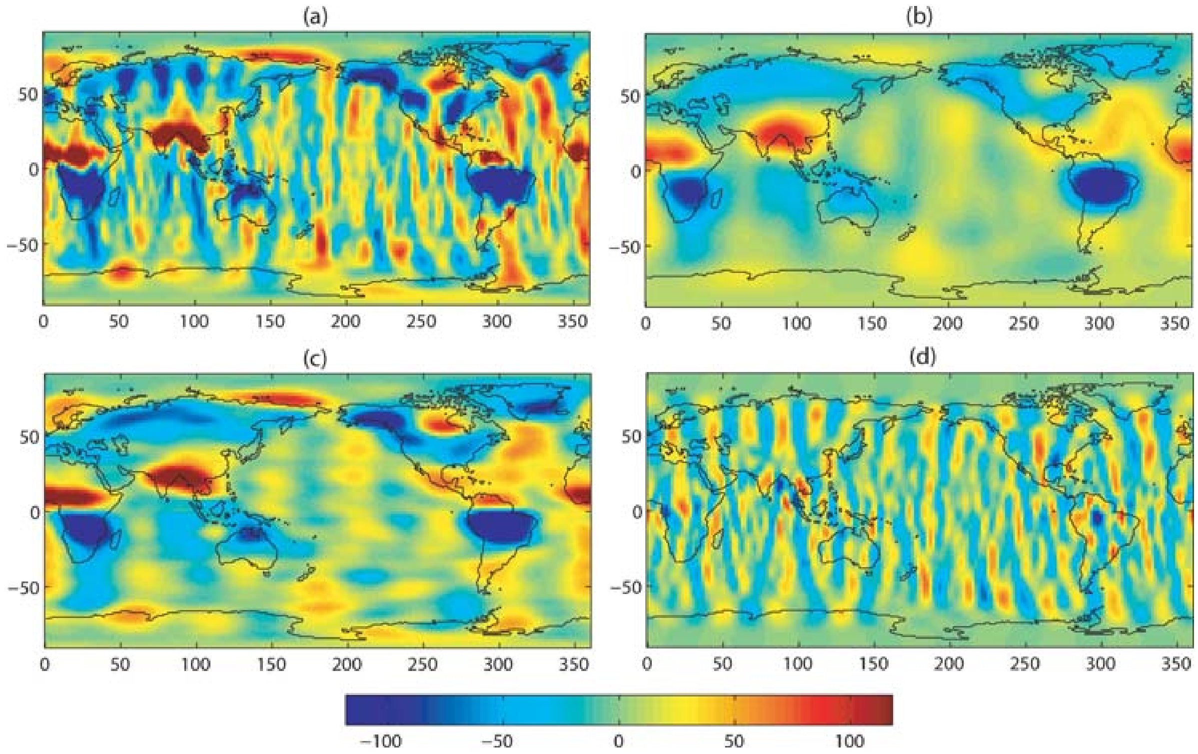

Figure 10. Equivalent water thickness map from GRACE for September 2005 after 400 km Gaussian filtering (a) and after 1000 km Gaussian filtering (b), (c) shows a similar map after combining 400 and 1000 km Gaussian filtering as described in the text to suppress specific SH orders, (d) is the difference between (a) and (c),

with those degrees and order from a 1000 km Gaussian smoothed field. The selected SH coefficients are SH orders from $15 - 7$ to $15 + 7$, from $30 - 7$ to $30 + 7$ and $45 - 7$ to $45 + 7$ because the aliasing simulation with single SH components show that the alias due to the resonance is present at SH orders in the linear combinations of the Kaula's resonant orders (15, 30, 45...) and input error orders. The range ±7 is chosen through trial and error, and implies that un-modelled geophysical signal (error in the AOD model) is significant to SH order 7. SH orders 15, 30 and 45 are predicted as the resonant order in the previous section. Panel (c) shows an improved map by combing 400 and 1000 km filtered fields in this way. Amplitudes are comparable to (a), but the north–south stripes are not present. Panel (d) is the difference between (a) and (c), which show the difference is dominantly the noise stripes. This clearly shows that their origin is aliasing error related to the GRACE ground track spacing in longitude.

5 SUMMARY AND DISCUSSION

We show that two different types of aliasing error contaminate gravity fields of GRACE. The aliasing problems caused by errors in geophysical model synoptic variations (∼5–10 d) can be explained by Kaula's resonance formula in eq. (4). This error is sensitive to ground track spacing in longitude. This type of aliasing error is associated with multiples of SH order 15 and produces longitudinal striped patterns in mass anomaly maps from GRACE. Aliasing of ocean tide model error can explain partly the poor estimate of $C_{2,0}$ coefficient in the CSR RL04 solutions, which uses the FES2004 tide model. The $C_{2,0}$ coefficient is particularly vulnerable to aliasing error associated with ocean tides because error in SH order 2 with even degrees, dominant in semi-diurnal tides, aliases to SH (2,0).

The two types of aliasing error cause tides to corrupt low SH degrees and orders, and non-tidal variations to contaminate at orders around 15. Different strategies to remove the two types of error are necessary. For tide-related aliases, the error may be separated if the GRACE time-series is long enough to resolve the alias error periods. On the other hand, error from non-tidal geophysical model or other errors over broad ranges of periods will contaminate particular SH orders. The varied filtering as a function of SH order is effective, as shown in Fig. 10, Alternatively, one can remove the correlated pattern in GRACE data (Swenson & Wahr 2006) to suppress the error.

As shown in Fig. 1, amplitudes of aliasing error may be larger than the improperly sampled source of the error. For example, we found that aliasing error at SH (15,15) was more than double the amplitude of the undersampled source at SH (2,0). This implies that the benefit from better geophysical models in the de-aliasing process may exceed the accuracy of the models. Aliasing error in measuring Earth's time-varying gravity, as in the GRACE mission, is inevitable because background models of tides, ocean bottom pressure and atmospheric surface pressure will never be perfect. For a given precision of geophysical models, improvements will come from additional sampling, for example, new satellite configurations. For example, two pairs of GRACE satellite separated significantly in longitude, might remove the problem at lower SH orders. Further study regarding this issue is appropriate because the recent United States National Research Council Decadal survey (NRC 2007) calls for planning a GRACE-2 mission.

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