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New Computations of the Tide-generating Potential

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Summary

A time-harmonic expansion of the gravitational tide potential is computed using an ephemeris of high precision for the Moon and the Sun and the latest I.A.U. astronomical constants. The results, which are computed for three different epochs and by novel methods, are compared with Doodson's classic expansion. The chief differences are due to secular trends in large terms and to revised constants which reduce all the solar terms. A new expansion is also given for the radiational tide potential.

Notation

t	Time (E.T. or U.T.) in mean solar days, usually from the epoch 1900 Jan 1·0
f	Frequency of general harmonic term in cycles per mean solar day
T	Time in Julian centuries of 36525 ephemeris days from the epoch 1900 January 0·5.
g	Gravitational acceleration at Earth's surface
θ, λ	Geocentric co-latitude (zero at North Pole) and east longitude of a place on the Earth
Θ, Λ	The same quantities for the Moon (′ for the Sun)
Π, Π'	Sine equatorial parallax of the Moon, Sun
ξ, ξ'	$= \Pi/\bar{\Pi}, \Pi'/\bar{\Pi}'$, where the bar denotes time-average.
L, L'	Mean longitude of the Moon, Sun
β, β'	Latitude of the Moon, Sun
ϖ, ϖ'	Mean longitude of the Moon's, Sun's perigee
Ω	Mean longitude of the Moon's ascending node
R'	Radius vector of the Sun in astronomical units
ε	Obliquity of the ecliptic
l, l', F, D	Principal arguments in Brown's development
a	Earth's equatorial radius
W_n^m	Complex spherical harmonic of order m, degree n (equation (10)).
c_n^m	Time dependent coefficient of W_n^m in gravitational potential
A_n	See equation (14)
H_s, θ_s	Amplitude and phase of general harmonic component (equation (13))
C_{j_1, j_2}	Filtered potential centred on tidal Group (j_1, j_2) (equation (15))
$(P, Q)_{j_1, j_2, j_3}$	Filtered potential at $\frac{1}{18}$ c yr^{-1} resolution (equation (17))
$F_1, (F_2, G_2)$	Filter characteristics associated with last two quantities (equations (16) and (18).

Introduction

A. T. Doodson's (1921)† harmonic expansion has for long been accepted as the most thorough development of the gravitational tidal potential ever carried out. It superseded G. H. Darwin's (1883) expansion, just as E. W. Brown's (1905) lunar theory, which Doodson used, superseded all earlier theories. However, while its principal features have been amply verified by analyses of tidal records as far as their lengths and geophysical noise levels permit, the finer details of Doodson's expansion have probably never been checked by independent calculation. In any case, the widespread revision of astronomical constants (Wilkins 1964, 1965), the introduction of Ephemeris Time (Sadler & Clemence 1954), and the re-calculation of Brown's coefficients (Eckert, Jones & Clark 1954), make the present time ripe for fresh calculations of the tidal potential. Such work has now been completed, and the results are presented in this paper.

Paradoxically, our motivation for this work arises not from the requirements of 'harmonic methods' of tidal analysis, but from those of a new method of analysing tidal data which is in principle non-harmonic. Standard 'harmonic methods' demand little accuracy in the harmonic amplitudes of the potential, since they use only the *frequencies* at which the larger amplitudes appear, and certain details on which to base 'nodal corrections'.‡ Indeed, recent efforts to extend such methods by nearly doubling the usual number of arbitrary terms (Zetler & Cummings 1967; Rossiter & Lennon 1968) have sought to identify compound frequencies arising from local effects of shallow water rather than neglected terms in the primary potential.

The non-harmonic method is the 'response method' of Munk & Cartwright (1966)—see also Cartwright (1968) and Cartwright, Munk & Zetler (1969). Here, the gravitational potential is computed *a priori* as a time-dependent series of *spherical* harmonics§,

$$V(\theta, \lambda, t)/g = \sum_m \sum_n c_n^{m*}(t)\, W_n^m(\theta, \lambda)$$

and the part of a given geophysical tidal variation $\zeta(t)$ which is linearly coherent with the harmonic of order m, degree n is expressed in the form§

$$\tilde{\zeta}_n^m(t) = \sum_{s=-S}^{S} R_n^{m*}(s)\, c_n^m(t-s\tau)$$

where the arbitrary time lag τ is usually taken as two days. Although direct reference to *time* harmonics is deliberately avoided, indirect reference is sometimes necessary, as when:

(a) it may be convenient to compute $c_n^m(t)$ itself, or a filtered part of it, directly from its harmonic expansion;

(b) one wants to generate a tidal prediction by the response method for a regime which is known only by its 'harmonic constants'; or

(c) one wants to compare the results of several 'response analyses' with each other, with existing 'harmonic analyses', and with dynamical theory, for which it is desirable to specify fixed frequencies.

† Reprinted as Doodson (1954); tables also in Neumann & Pierson (1966).

‡ The most thorough use of the potential for harmonic purposes is by Horn (1967).

§ In these two equations, the real part of complex products is understood, with * denoting the conjugate.

Cases (b) and (c), essentially matters of translation, are considered by Zetler, Cartwright & Munk (1970) and are implicit in Munk, Snodgrass & Wimbush (1970). At any specified frequency f', one defines the 'admittance' $Z_n^m(f')$ of the tidal motion to the spherical harmonic (m, n) of the potential,

$$Z_n^m(f') = \sum_{s=-S}^{S} R_n^m(s)\, e^{-2\pi i s f' \tau}.$$

The time harmonic of the motion corresponding to a 'line' H' with frequency f' in the potential is then simply $H' Z_n^m(f')$. Evidently, in any of these applications, the harmonic lines H', at least the larger ones, have to be known with some precision. Similar considerations also apply to the relationship between the precessional nutation of the Earth and the tidal potential, recently expounded by Melchior & Georis (1968).

Our method of computing the time harmonics of the potential differs considerably from that of Doodson, which was one of massive algebraic expansion from Brown's series. A suite of computer programs for tidal analysis by the 'response method' has been in use and well tested for some years (Cartwright 1967), and this was used to generate time-series of the coefficients for three spans, each a little more than 18 years. The harmonics were extracted from these series by carefully applied filtering techniques. In generating the time-series, special attention was paid to the accuracy of the ephemerides used for both Moon and Sun, which were made comparable with the most modern published ephemerides to six significant figures. To ensure this accuracy, the programs had to incorporate not only a fair length of the revised Brown series, but also various corrections such as those due to the nutation and to the planets, which were ignored by Doodson.

In what follows, we first outline the choice of terms for inclusion in the ephemeris calculations, then after defining the normalization used for the potentials, we describe the filtering processes, and tabulate the results, with comparisons with Doodson's tables. Finally, we add a harmonic expansion of the *radiational potential* (Munk & Cartwright 1966) which has not previously been calculated.

Calculation of the ephemeris

Eckert, Jones & Clark (1954)—hereafter referred to as EJC—re-worked Brown's (1905) theory from its fundamentals by automatic computer. Their resulting tables and corrections have now superseded Brown's (1919) tables, and represent the most precise expression of the Newtonian dynamics of the Earth–Moon–Sun system in existence. However, the accuracy of the EJC tables, about 10^{-7} (rad, or mean parallax), is far greater than is required for the present purpose. Munk & Cartwright (1966) obtained good tidal analyses using an ephemeris (essentially de Pontécoulant's to 3rd order), which contains errors of $0{\cdot}5 \times 10^{-2}$, as is to be expected from an expansion containing only 13 harmonic terms†. Longman (1959) and others have worked with a gravitational potential computed from only eight harmonic terms. Our aim has been to remove all doubts associated with such approximations, and in fact to maintain a level of precision rather better than Doodson's. Since a general property of the lunar series seems to be that total errors can amount to about ten times the largest neglected term, our computer program was arranged to include all terms from EJC in longitude and latitude ($\gamma_1 C$) greater than $0''{\cdot}190$, in latitude (S and N) greater than $1''{\cdot}85$, and in sine parallax greater than $0''{\cdot}0018$. These limits

† The printed formula for the Moon's longitude omitted the Annual Equation, included in the calculations. Error curves were calculated by Dr M. J. Krijger of the Hague (private communication).

entailed a total of 277 harmonic solar perturbations†, many of which of course shared common arguments, and also 15 very small planetary perturbations. Final errors were never found to exceed $1{\cdot}3\times10^{-5}$ rad or $0{\cdot}6\times10^{-5}$ mean parallax, and were usually much less (see Table 1).

The 'fundamental arguments', consisting of the mean longitudes of Moon, Sun and planets, of the Moon's and Sun's perigee and of the Moon's mean node, were computed in terms of ephemeris time T in Julian Centuries from formulae of type.

$$\theta(t) = A_0+A_1\,T+A_2\,T^2+A_3\,T^3+\sum_n c_n\cos(a_n+b_n\,T). \tag{1}$$

The secular arguments A_r are as printed in Meeus (1962), in EJC (with other units), and in modern editions of the *Astronomical Ephemeris*. We remark only that the constants of the Moon's mean longitude have been substantially altered to keep in line with the new (1954) revisions. The harmonic terms in (1) are long period perturbations to the Moon's elements, which we selected from Table II of EJC again only where c_n exceeds 0″·19. Twenty such terms were used, the largest by far being two terms in the Moon's node of amplitude 95″·96 and 15″·58 respectively with periods close to the nodal period, and the 'Great Venus Term' in longitude of amplitude 14″·27 and a period of 271 years.

The Moon's true longitude and sine parallax are then computed by adding the high frequency perturbations in terms of Brown's four arguments:

$$\begin{aligned} l &= L-\varpi && = \text{Moon's mean anomaly}\\ l' &= L'-\varpi' && = \text{Sun's mean anomaly}\\ F &= L-\Omega && = \text{Moon's mean elongation from the node}\\ D &= L-L' && = \text{Moon's mean elongation from the mean Sun,}\end{aligned}$$

by formulae of type:

$$\delta\theta(t) = \sum_n \mu_n r_n \,{}^{\sin}_{\cos}(i_n\,l+j_n\,l'+k_n\,F+m_n\,D) + \sum_n \rho_n \,{}^{\sin}_{\cos}(\text{lunar and planetary arguments}). \tag{2}$$

In (2), r_n and ρ_n are the coefficients of solar and planetary perturbations respectively, chosen as previously described from Table III of EJC, each being associated with a set of integers (i_n, j_n, k_n, m_n), in our case all between ±6. Sines of arguments are used for longitude, cosines for parallax. The μ_n are multipliers close to unity of the form

$$\mu_n = e^{|i_n|}e'^{|j_n|}\gamma^{|k_n|}$$

as detailed on p. 344 of EJC. They allow for small differences between actual and nominal orbital parameters, chiefly solar eccentricity e', corresponding to $e'(T)/e'(0)$ in formula (5).

The final increment used to obtain the Moon's true longitude, (referred to the true equinox of date) is that due to the Earth's nutation. Woolard's expressions for the nutation are tabulated in Sadler & Clemence (1954), from which for the present accuracy we have extracted the following increments to longitude L and obliquity ε (seconds of arc):

$$\begin{matrix}\delta L\\ \delta\varepsilon\end{matrix} = \begin{matrix}-17{\cdot}23\sin\\ +\;9{\cdot}21\cos\end{matrix}\,\Omega\;\begin{matrix}-1{\cdot}27\sin\\ +0{\cdot}55\cos\end{matrix}\,2L'\;\begin{matrix}+0{\cdot}21\sin\\ -0{\cdot}09\cos\end{matrix}\,2\Omega\;\begin{matrix}-0{\cdot}20\sin\\ +0{\cdot}09\cos\end{matrix}\,2L \tag{3}$$

† A few terms with amplitude a little lower than the stated limits were also included where their arguments were inevitably used in the longitude, viz. Serial Nos. 676, 753, 872, 912.

The sine parallax is converted to its normalized value ξ by dividing by 3422″·70, which is the nominal mean value of the tables. ξ is precisely the quantity occurring in tidal potential theory, whereas for the construction of astronomical tables it is converted to the arc† by adding a cubic correction of order 10^{-4}. Where we require the numerical value of mean sine equatorial parallax, we use the 1964 I.A.U. value 3422″·451, (Wilkins 1965), which again differs from the value 3422″·54 at present adopted in the *Astronomical Ephemeris*.

We compute the Moon's latitude in the formalism adopted by EJC:

$$\beta = (1+C)(\gamma_1 \sin S+\gamma_2 \sin 3S+N), \quad S = F+\delta F+\delta S$$

where $F+\delta F$ is the true elongation from the node, already described, and δS and N (sines) and $\gamma_1\, C$ (cosines) are obtained by summing harmonic terms similar to (2), though without planetary terms, which are negligible here. We also use $\gamma_1 = 18519''{\cdot}70$, $\gamma_2 = -6''{\cdot}24$, and ignore a very small term γ_3. This was the formalism used by Brown in his final tables (1919), although Doodson (1921) and Meeus (1962) refer to a more explicit form for latitude given in Brown (1905).

Maintaining the same accuracy in the Sun's ephemeris, we have used Newcomb's formulae as in all official work, for convenience as tabulated in Meeus (1962). In brief, the 'apparent' longitude L_a' and radius vector R_a' (in this case equal to $1/\xi'$) are compounded of the following terms:

$$\left.\begin{aligned} L_a' &= L'+\zeta L'_{add}+\delta L'_{ellipse}+\delta L'_{planet}+\delta L'_{lunar}+\delta L'_{nut} \\ R_a' &= 1+\delta R'_{ellipse}+\delta R'_{planet}+\delta R'_{lunar} \end{aligned}\right\}. \tag{4}$$

Here, δL_{add} consists of the 'additive' terms of long period, already referred to in formula (1), although considerably smaller than the corresponding lunar terms. The next terms in (4) are the classical variations of elliptic motion, with eccentricity given by:

$$e' = 0{\cdot}01675104-0{\cdot}00004180T-0{\cdot}000000126T^2. \tag{5}$$

These are the only terms considered by Doodson, who took e' as a constant at $T=0$.

The planetary terms in (4) are similar in form to those in (2), but are relatively more important than in the lunar motion and can amount to as much as 10^{-4}. 45 harmonic terms (23 arguments) are included in the computation, principally due to Venus and Jupiter, but with some non-negligible amplitudes due to Mars and Saturn‡.

The lunar terms in (4) express the changes in apparent position of the Sun due to the Earth's reflection of the Moon's orbit about their joint centre of gravity. Following Meeus (1962, p. 31), we use the geometrical formula:

$$\begin{matrix}\delta L'_{lunar} \\ \delta R'_{lunar}\end{matrix} = 3{\cdot}12\times 10^{-5}(\xi'/\xi)\cos\beta \begin{matrix}\sin \\ \cos\end{matrix}(L_a-L_a') \tag{6}$$

to this we finally add the nutational increment to longitude δL from formula (3). These two increments are interesting as being the only means whereby lunar frequencies, (principally modulations of one synodic month and the nodal period) enter the solar tide.

† Meeus and others make the approximation that 3422″·70 is in fact the mean arc, although strictly incorrect according to EJC.

‡ Strictly, the planetary effects on tides, though minute, are incomplete, because we have not included the direct tidal potential of the planets. The present object is merely to establish an accurate ephemeris.

Normally, the Sun's apparent longitude is allowed a further increment $(-20''\cdot47/R')$ due to the aberration of light, but this is omitted here as inappropriate to calculations of gravity. For consistency in precision, two small planetary terms and a lunar term related to (6) are combined to make a non-zero solar latitude β'.

As an overall test of the above procedures, and of the computer logistics, the six lunar and solar elements were compared with corresponding values in the *Astronomical Ephemeris* every 10 days from 1959 Jan 0 to 1967 Dec 24, and the mean, standard, and maximum errors are given in Table 1. In the comparison due allowance was made for solar aberration and the difference between arc and sine of lunar parallax. Errors in the lunar values are similar to those described by Meeus (1962, pp 47–51) from a much shorter comparison with his tables. Our errors in lunar parallax are significantly smaller; in fact deliberately so, since the tidal potential involves the cube. Meeus's solar elements are nearly perfect, since he includes an extensive range of planetary and nutational terms. Our's have errors comparable with but smaller than our lunar errors as befits the present work. It is difficult to compare with Doodson's level of accuracy, but his errors must certainly be greater in every case.

At this stage, the reader may wonder why we bother to compute the ephemeris at all when it is already available to higher precision in published form. The main reasons are that modern computers can compute faster and more efficiently than they can read data (the calculations above take about 45 s for a year's ephemeris), and that tidal analyses are sometimes required for rather ancient epochs. (As an extreme example, the senior author has recently used this program to analyse tidal observations made by Maskelyne (1762) before he published the first *Nautical Almanac*.)

Table 1

Statistics of differences between present computations and published ephemerides, 1959–1967

		Units	Mean	S.D.	Maximum	Dates of maximum
Moon	Normalised sine parallax	10^{-5}	0·20	0·18	−0·56	1966 Aug. 21
Sun	,,	10^{-5}	0·03	0·08	+0·29	1959 June 9, 1962 Nov. 10
Moon	Longitude	10^{-5} × radians (i.e. 2″)	0·16	0·31	+1·21	1963 Nov. 5
Sun	,,		0·01	0·20	+0·58	1965 Nov. 24
Moon	Latitude		0·04	0·19	−0·50	1959 March 31†
Sun	,,		−0·02	0·03	−0·07	1962 July 3

† +0·50 in Moon's latitude also occurred on 1961 Aug. 17 and 1965 July 27.

The final steps taken to produce quantities directly usable for calculations of the gravitational potential are as follows. The ecliptic latitudes and longitudes are converted to cosines and sines of *co-declination* Θ (polar angle) and *right ascension*, and the latter transferred to terrestial east longitude Λ from the Greenwich ephemeris meridian by effectively subtracting the ephemeris sidereal time. This involves some well-known trigonometrical formulae; also the obliquity of the ecliptic, for which we take

$$\varepsilon = 84428''\cdot26 - 46''\cdot85\,T + \delta\varepsilon \tag{7}$$

and the sidereal time angle (in revolutions) reckoned from the true equinox, namely

$$t + 0\cdot27691940 + 100\cdot00213590\,T + 0\cdot00000108\,T^2 + (129600)^{-1}\,\delta L\,\cos\varepsilon$$

where $\delta\varepsilon$ and δL are the nutational increments in (3). The lunar parameters ξ, $\frac{\cos}{\sin}(\Theta, \Lambda)$ are at first computed at *Oh* and 12*h* E.T., and the solar parameters ξ', $\frac{\cos}{\sin}(\Theta', \Lambda')$ at *Oh* E.T. only. At a later stage of the computation, these elements are interpolated by Everett formulae to a shorter time interval (3 hourly for the present purpose) in *Universal Time*, while Λ and Λ' are adjusted from the ephemeris meridian to the *geographical meridian* of Greenwich. These last adjustments use the series of measured time differences

$$\Delta T = \text{E.T.} - \text{U.T.}$$

published in the *Astronomical Ephemeris*, and thus involve the known vagaries of the Earth's rotation, to produce as realistic values as possible.

Calculation of the potential

We consider the gravitational potential on a sphere with the Earth's equatorial radius. The adjustment to the actual radius of the geoid is a secondary matter which need not concern us here. We have then

$$V/g = \sum_{n=2}^{\infty} K_n \xi^{n+1} P_n(\cos\alpha), \quad K_n = a(M/M_\oplus)\,\Pi^{n+1} \tag{8}$$

where M (or M') is the Moon's (or Sun's) mass, Π its mean sine parallax, and α its zenith angle relative to the place on the sphere with co-ordinates (θ, λ).

The P_n are Legendre Polynomials, which can be expanded in terms of the ephemeris elements Θ, Λ described in the last section as follows†:

$$P_n(\cos\alpha) = \frac{4\pi}{2n+1} \cdot \text{Re}\left[W_n^{0*}(\Theta, \Lambda)\, W_n^{0}(\theta, \lambda) + 2\sum_{m=1}^{n} W_n^{m*}(\Theta, \Lambda)\, W_n^{m}(\theta, \lambda)\right] \tag{9}$$

where $W_n^m(\theta, \lambda)$ denotes the spherical harmonic

$$(-1)^m \left[\frac{2n+1}{4\pi} \cdot \frac{(n-m)!}{(n+m)!}\right]^{\frac{1}{2}} P_n^m(\cos\theta)\, e^{im\lambda} \tag{10}$$

and

$$P_n^m(\mu) = \frac{(1-\mu^2)^{\frac{1}{2}m}}{2^n \,.\, n!} \frac{d^{m+n}}{d\mu^{m+n}} [(\mu^2-1)^n]\,. \tag{11}$$

Using the 1964 I.A.U. constants (Wilkins 1965):

$$M/M_\oplus = 1/81{\cdot}30, \qquad M'/M_\oplus = 332958,$$

$$\Pi = 3422''{\cdot}451, \qquad \Pi' = 8''{\cdot}794,$$

$$a = 6378160 \text{ metres, so that}$$

$$K_2 = 0{\cdot}358378\text{ m}, \qquad K_2' = 0{\cdot}164577\text{ m},$$

$$K_3 = 0{\cdot}005946\text{ m}, \qquad K_3' = 0{\cdot}000007\text{ m},$$

† We here follow the procedure and notation of Munk & Cartwright (1966), except that our ζ, Θ, Λ are their $\bar{R}/R$, Z, L, respectively.

equations (8)–(11) and the computed ephemeris are used quite simply to compute the series of time-dependent coefficients $c_n^m(t)$ in the relation

$$V/g = \sum_{n=2,\,3} \sum_{m=0}^{n} c_n^{m*}(t)\, W_n^m(\theta, \lambda) \text{ metres} \tag{12}$$

mentioned in the Introduction. We compute only for $n = 2$ and 3 (Moon) and for $n = 2$ (Sun) because of the ordering of magnitude due to the factor Π^{n+1}. Corresponding lunar and solar series are added to define the total potential.

Doodson's development differs from ours in normalization. His G (in which ρ is a misprint for ρ^2) corresponds to our $\frac{3}{4}gK_2$ and is taken out as an arbitrary factor, so that most of his numerical coefficients are hardly affected by changes in basic astronomical constants, but only by the small differences in the ephemeris calculations. However, his solar terms, denoted by G_m, all contain a factor K_2'/K_2 which he took to be 0·46040, whereas the modern constants give 0·45923. His third degree terms denoted by G_m' (our $n = 3$) also contain the factor Π which he took to be 3422″·70, but since these terms never involve more than four significant figures this particular error is negligible. Apart from such discrepancies, Table 2 details our normalization (equations (10) and (11)), and the resulting ratio ρ of Doodson's coefficients to corresponding terms in c_n^m.

Table 2

Normalization and ratio ρ = (Doodson : C_n^m)

m	n	$e^{-im\lambda}\, W_n^m(\theta, \lambda)$	$1/\rho$	ρ
0	2	$\sqrt{(5/4\pi)}(\frac{3}{2}\cos^2\theta - \frac{1}{2})$	$-\sqrt{(9\pi/20)}\,K_2$	−2·34681
1	2	$-\sqrt{(5/24\pi)}\,3\sin\theta\cos\theta$	$-\sqrt{(6\pi/5)}\,K_2$	−1·43712
2	2	$\sqrt{(5/96\pi)}\,3\sin^2\theta$	$\sqrt{(6\pi/5)}\,K_2$	1·43712
0	3	$\sqrt{(7/4\pi)}(\frac{5}{2}\cos^3\theta - \frac{3}{2}\cos\theta)$	$-1{\cdot}11803\sqrt{(9\pi/7)}\,K_2$	−1·24182
1	3	$-\sqrt{(7/48\pi)}\,\frac{3}{2}\sin\theta(5\cos^2\theta - 1)$	$0{\cdot}72618\sqrt{(12\pi/7)}\,K_2$	1·65576
2	3	$\sqrt{(7/480\pi)}\,15\sin^2\theta\cos\theta$	$2{\cdot}59808\sqrt{(6\pi/35)}\,K_2$	1·46349
3	3	$-\sqrt{(7/2880\pi)}\,15\sin^3\theta$	$-6\sqrt{(\pi/35)}\,K_2$	−1·55227

Harmonic development and filtering

With t in Universal Time measured in mean solar days from 1900 Jan 1·0, we wish to express the time series $c_n^m(t)$ as closely as possible in the form

$$c_n^m(t) = \sum_s H_s \begin{matrix}\cos\\ \sin\end{matrix}\, \theta_s,$$

$$\theta_s = 2\pi f_s t + \phi_s = \sum_{r=1}^{6} k_r^{(s)}\,(2\pi f_r t + \phi_r) \tag{13}$$

where, for each s, $k_1 \dots k_6$ is an array of small integers, and the bracketed arguments (defined precisely in Table 3) correspond in a reasonable manner with the following concepts in descending order of frequency:

	Doodson	Brown	
1	τ	$360°\,t - D + 180°$	Time angle in lunar days ($f_1 = 1 - f_2 + f_3$)
2	s	L	Moon's mean longitude
3	h	L'	Sun's mean longitude
4	p	ϖ	Longitude of Moon's mean perigee
5	N'	$-\Omega$	Negative longitude of Moon's mean node
6	p_1	ϖ'	Longitude of Sun's mean perigee

Classical analysis shows that the cosines in (13) are appropriate to $(m+n)$ even, the sines to $(m+n)$ odd.

It is of course accepted that f_r and ϕ_r will vary on a very long time scale, (as they do in Doodson's model), but we also have to make some compromise for the fact that many of the amplitudes in the ephemeris calculation were allowed slight secular variations. This, together with the planetary terms, the irregular time scale introduced by the conversion from E.T. to U.T., and 'numerical noise' due to imperfections in computing, make the problem better suited to least-squares estimation than to precise algebraic expansion. In fact, we analyse $c_n^m(t)$ by methods similar to those suitable for real geophysical time series of tidal nature with very low background noise.

We first note that the real and imaginary parts of $c_n^m(t)$ are orthogonal in time, (any term $H\cos\omega t$ in the real part occurs as $-H\sin\omega t$ in the imaginary part), so we shall consider only the former in what follows. Secondly, since the order m separates the spectra into tidal 'Species' with frequencies centred on m cycles per lunar day ($k_1 = m$), and the spectral analyses of Munk, Zetler & Groves (1965) show that the spectral energy is reduced by at least 10^{10} (amplitude reduced by 10^5) at a separation of $1c/ld$, therefore we worked (as is very convenient) with the summed series

$$A_n(t) = \mathrm{Re}\sum_{m=0}^{n} c_n^m(t), \quad n = 2, 3, \tag{14}$$

and left the filtering process to separate the component parts.

The next procedure was to apply orthogonal pairs of filters, each designed to pass only one tidal 'Group' (k_1, k_2) with little amplitude reduction. This operation is defined by

$$C_{0,0}(t-t_0) = N^{-1}\sum_{r=-\frac{1}{2}N}^{\frac{1}{2}N} A_n(t+r\Delta t)(1+\cos\pi r/N),$$

$$C_{j_1,j_2}(t-t_0) = \exp\{2\pi i(j_1 f_1+j_2 f_2-j_2 f_3)(t-t_0)\}\,.$$

$$\left[2N^{-1}\sum_{r=-\frac{1}{2}N}^{\frac{1}{2}N} A_n(t+r\Delta t)(1+\cos\pi r/N)\exp(2\pi ipr/N)\right], \tag{15}$$

where

$$N = 472,\ \Delta t = \tfrac{1}{8},\ (N\Delta t = 59\ \text{days})$$

and

$$p = 57j_1+2j_2,$$

with the following combinations:

$$j_1 = 0, j_2 = 1(1)4;$$

$$j_1 = 1, 2, j_2 = -4(1)4;$$

$$j_1 = 3, j_2 = -2(1)2,\ \text{for } n = 3 \text{ only.}$$

The general effect of (15) is to multiply the amplitude H_s of a term with frequency f_s by the filter characteristic:

$$F_1(f_s) = \frac{\sin^2\nu\cos\nu\delta}{\sin(\nu+\nu\delta)\sin(\nu-\nu\delta)}\cdot\frac{S(\pi\delta)}{S(\nu\delta)} \tag{16}$$

where $\nu = \pi/N$, $S(x) = \sin x/x$, $\delta = 59f_s - p$. The form of $F_1(f)$ is plotted in Fig. 1. It is near unity for all relevant frequencies in the Group $(k_1, k_2) = (j_1, j_2)$, centred fairly close to $k_3 = -j_2$. It greatly attenuates neighbouring Groups and virtually eliminates neighbouring Species (different k_1). The small interference from neighbouring Groups will be removed by the next filter characteristic F_2, (18), whose envelope is also shown in Fig. 1.

The effect of the first exponential factor in (15) is to 'heterodyne' by the central frequency of the Group, that is to subtract $j_1 f_1 + j_2(f_2 - f_3)$ from the frequency of all harmonic components. The complex series $C_{j_1, j_2}(t)$ referred to an arbitrary time origin t_0 (defined later), thus contains only very low frequency variations from its own Group, and small variations of up to a few cycles per month from the attenuated neighbouring Groups. The only precaution needed is to ensure that none of the latter frequencies is 'aliassed', that is made indistinguishable from very low frequencies, by too long a sampling interval in t. A sampling interval of 5 days was chosen as satisfactory. As shown in Fig. 1, this produces low frequencies by 'aliassing' Groups $(j_1, j_2 \pm 6)$, but the value of F_1 at $\delta \sim 13$ is so small that the effect is well below numerical noise level, and in any case the frequencies of the aliassed lines do not tally with those of Group (j_1, j_2). The redundant operations inherent in applying the 59-day filter (15) at 5-day intervals were avoided by efficient computer logistics.

The next operation was to apply direct Fourier transforms to an 18-year span

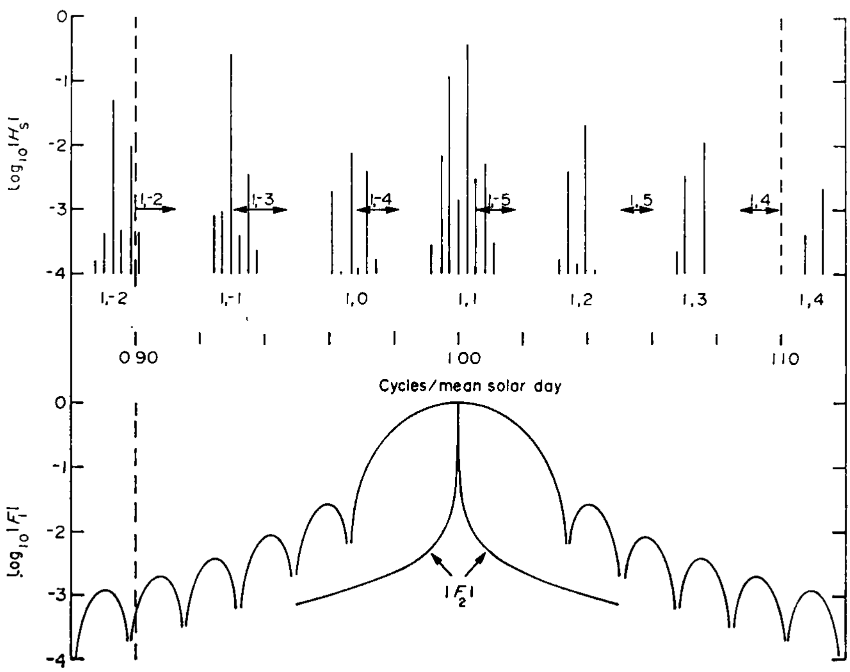

Fig. 1. The top panel shows the main constituents of the W_2^1 diurnal tide, with Group numbers (k_1, k_2). The vertical pecked lines show the 'Nyquist' frequencies of the filtered series $C_{1,1}(t)$ when computed at 5-day intervals, and the horizontal lines are the positions of 'aliassed' Groups. Amplitudes of the aliassed Groups are greatly reduced by the filter $F_1(f)$ acting at its proper (non-aliassed) frequency. (Group (1, −5), reduced by more than 6000, is well below the threshold level.) The central portion of F_1 appropriate to $C_{1,1}$, is in the lower panel, as well as the envelope of the Fourier filter F_2 appropriate to $(P, Q)_{1,1,0}$.

of the Group series $C_{j_1, j_2}(t')$, $(t' = t - t_0)$:

$$P_{0,0,0} = M^{-1} \sum_{r=0}^{M}{}'' C_{0,0}(t' + r\Delta t')$$

$$P_{0,0\ j_3} + iQ_{0,0,j_3} = (-1)^{j_t} 2M^{-1} \sum_{r=0}^{M}{}'' C_{0,0}(t' + r\Delta t') \exp(-2\pi i j_3 r/M)$$

$$P_{j_1, j_2, j_3} + iQ_{j_1, j_2, j_3} = (-1)^{j_3} M^{-1} \sum_{r=0}^{M}{}'' C_{j_1, j_2}(t' + r\Delta t') \exp(-2ij_3 r/M) \tag{17}$$

where

$$M = 1315, \ \Delta t' = 5, \ (M\Delta t' = 6575 \text{ days})$$

and $\sum''$ represents a summation whose first and last terms are halved. For Group (0, 0), $j_3 = 1(1)80$; otherwise $j_3 = -80(1)80$. It is now appropriate to state that t_0 was chosen as the central time of the 18-year span, (see Table 3), so that all 'phases' θ_s in (13) refer to this time.

The filter characteristic of (17) is such that for Group (0, 0)

$$(P, Q)_{0,0,j_3} = \sum_s F_1 H_s(F_2 \cos\theta_s, G_2 \sin\theta_s),$$

$$(F_2, G_2) = \left\{ \frac{\sin 2\mu(|j_3| + \varepsilon)}{\sin \mu(2|j_3| + \varepsilon)}, \frac{\sin 2\mu|j_3|}{\sin \mu(2|j_3| + \varepsilon)} \right\} \frac{S(\pi\varepsilon)}{S(\mu\varepsilon)}, \tag{18}$$

where

$$\mu = \pi/M, \ \varepsilon = 6575|f_s - j_1 f_1 - j_2 f_2 + j_2 f_3| - |j_3|.$$

For all other Groups, (F_2, G_2) is replaced by

$$\{\tfrac{1}{2}(F_2 \pm G_2), \tfrac{1}{2}(F_2 \pm F_2)\}, \tag{19}$$

the (+) signs being taken when $f_s - j_1 f_1 - j_2 f_2 + j_2 f_3$ has the same sign as j_3, the (−) sign when different. The function is always rather similar to its dominant factor $S(\pi\varepsilon)$, and only its envelope for the case $j_3 = 0$ is shown in Fig. 1.

The Fourier harmonics $(P, Q)_{j_1, j_2, j_3}$ already give a good first approximation to the lines

$$F_1 H_s(\cos\theta_s, \sin\theta_s),$$

as the typical examples in Fig. 2 clearly show. 6575 days being within 16h of 18 tropical years, unit incıements in k_3 correspond fairly precisely with 18 increments in j_3. Unit increments in k_4 (8·85 yr) and k_5 (18·61 yr) give increments of 2 and 1 to j_3 with somewhat less precision. Non-zero k_6 is recognizable from the phase change of some 282° in ϕ_6. However, it is possible for two or more distinct lines H_s, closely spaced in frequency, to be unresolved without further analysis. Careful algebraic study shows that close terms from the same spherical harmonic can differ in frequency only by

$$2f_6, \ (1 \text{ cycle}/10470 \text{ y})$$

or

$$\delta f_7 = f_4 - 2f_5 \pm f_6, \ (1 \text{ c}/180 \text{ y}). \tag{20}$$

Doodson's tables show six such pairs, all in the solar Groups, differing by $2f_6$†, but some others involving amplitudes below the threshold of 10^{-4} may have been omitted. Another difficulty we have to resolve is that all terms (P, Q) contain small

† The difference $f_4 - 2f_5$ also appears, but only between terms from $W_2{}^m$ and terms from $W_3{}^m$

contributions from lines at more than $\frac{1}{18}$ c y^{-1} separation, through the 'sidebands' of the filter (F_2, G_2).

Our final steps for extracting reasonably accurate values from (P, Q) were as follows:

1. For reasons irrelevant to this paper, it was convenient to compute 18-year time series of $A_2(t)$ and $A_3(t)$, (14) for a recent epoch with central date in 1960. In order to search unambiguously for frequency differences δf_7 (20), a similar span was also computed about 90 years earlier, with central date in 1870. A third convenient span, with central date in 1924, was also used. For each span, mean values of $f_2 \ldots f_6$ and $\phi_2 \ldots \phi_6$ were computed from values at the start and end times of $L, L', \varpi, -\Omega, \varpi'$ respectively, using the long period 'additive' terms (equation (1)), and also the appropriate adjustments from Ephemeris Time to Universal Time. The precise dates and arguments are listed in Table 3.

Table 3

Times (U.T.) and mean arguments for the three 6575 day spans.

Span No.	Start time (ΔT)	End time (ΔT)	Central time	t_0 (from 1900·0)
1	1861 Sep 21·0 (3·1)	1879 Sep 22·0 (−7·7)	1870 Sep 19·5	−10693·5
2	1915 May 16·0 (16·4)	1933 May 22·0 (23·6)	1924 May 21·5	8906·5
3	1951 May 23·0 (29·7)	1969 May 23·0 (40·0)	1960 May 22·5	22056·5

	$r = 2$	$r = 3$	$r = 4$	$r = 5$	$r = 6$
f_r 1	0·03660 11013	0·00273 79093	0·00030 94562	0·00014 70943	0·00000 01307
f_r 2	25	92	54	41	08
f_r 3	23	92	48	40	08
ϕ_r 1	135°·22275	180°·16879	223°·08434	254°·58011	280°·71758
ϕ_r 2	272°·60245	058°·85684	246°·60455	212°·47704	281°·64011
ϕ_r 3	022°·22101	060°·11923	271°·56503	188°·82048	282°·25919

'Start' and 'End' correspond to the terms $r = 0$, M, in equation (17)
Figures in brackets at ΔT = ET–UT in seconds
For each period, $f_1 = 1 - f_2 + f_3$, $\phi_1 = 180° - \phi_2 + \phi_3$

2. The 'sideband' noise level for each Group j_1, j_2, (see Fig. 2), was greatly reduced by assuming the indisputable k_r values for the frequencies of the major lines in the Group, (and in some cases for adjacent Groups $j_1, j_2 \pm 1$ also) and subtracting their sidebands according to the filter functions 17, 18 and 19.

3. Each $(P, Q)_{j_1, j_2, j_3}$ whose amplitude stood well clear of the reduced noise level was tested for all possible combinations of three lines H_s with frequencies f_s determined by the scheme:

$$k_1 = j_1,\ k_2 = j_2,\ k_3 = k_3' - j_2;$$

$$(k_4, k_5) = (k_4', k_5'),\ \text{or}\ (k_4' + 1,\ k'_5 - 2),\ \text{or}\ (k_4' - 1,\ k_5' + 2);$$

$$k_6 = 0\ \text{or}\ \pm 1,\ \text{or in certain cases}\ \pm 2;$$

where

$$k_3'\ \text{is the nearest integer to}\ j_3/18,$$

$$k_4'\ \text{is the integral part of}\ (j_3 - k_3')/2,$$

and

$$k_5' = j_3 - k_3' - 2k_4'.$$

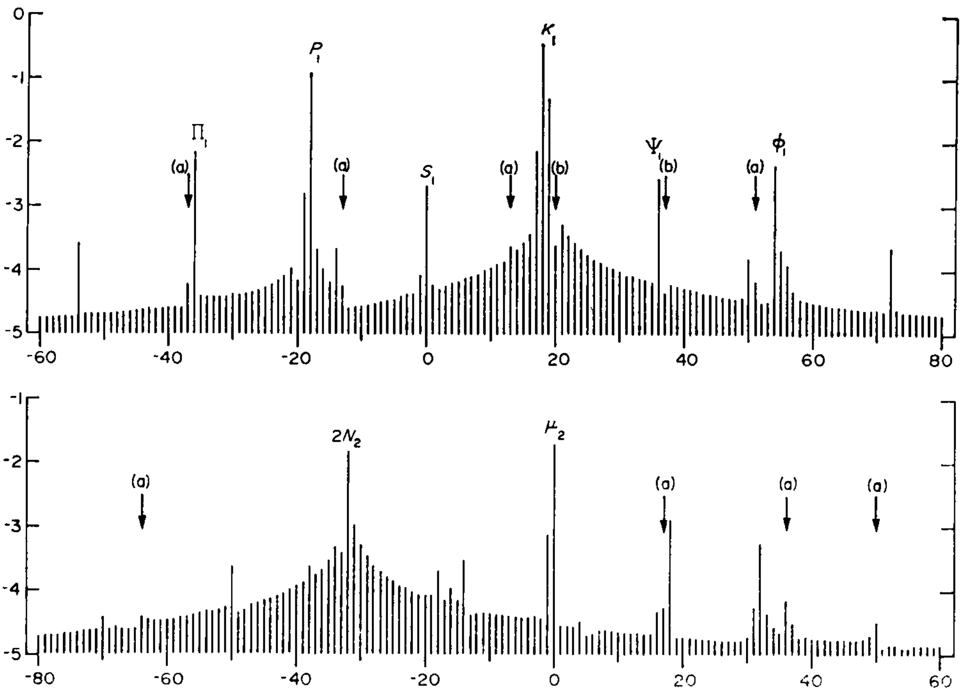

FIG. 2. Two groups, (1, 1) and (2, −2) of 'untreated' Fourier harmonics, $\log_{10}|P+iQ|$, plotted against j_3. Suffixed letters above strong lines are the conventional Darwin symbols. Harmonics marked (a) correspond to lines H_s below Doodson's threshold level, but included in present tables. Harmonics marked (b) are negative anomalies which become positive when the 'sidebands' are subtracted. The sideband subtraction process reduces the background level to below −5 on the above scale.

The test consisted in determining a triplet H_s to minimize

$$v = <[(\sum_s F_2 H_s \cos\phi_s - P)^2 + (\sum_s G_2 H_s \sin\phi_s - Q)^2]> \tag{21}$$

where < > denotes an ensemble average over the harmonics from the three 18-year periods. The appropriate combination was then easily picked out by the smallness of its v_{min} (independently of the choice made at step 2), and in most cases indicated a single large H_s and two other negligibly small amplitudes. Where two comparable amplitudes appeared, their frequencies were always separated by the 'permissible' values $2f_6$ or δf_7, (20).

4. The solutions from step 3 were used to subtract sidebands of higher accuracy from the original (P, Q) values and thus to iterate step 2. The sequence 2–3 was repeated until stable values of H_s and a generally low amplitude level ($< 10^{-6}$) at non-contributing (P, Q) was obtained. Three iterations were usually sufficient.

The solutions from (21), converted to true amplitudes H_s by dividing by the broad filter function F_1, (16), agreed roughly with Doodson's values, (with some differences discussed in the next section) and included several reliable amplitudes below Doodson's threshold of 10^{-4}. However, we noticed that the residual variances v_{min} associated with the largest lines such as M_2, K_1, and the constant term, were substantially greater than with small lines. Examination showed this to be due to discernible secular trends in the amplitudes themselves, resulting from the relative changes of 5×10^{-4} per century in mean obliquity ε, (7) and 25×10^{-4} per century in solar eccentricity e' (5). Since the above procedure established that there were never more than two lines contributing significantly to any (P, Q) after removal of side-bands, it was possible to evaluate H_s separately from each 18-year period, so we thought it wiser to present the amplitudes from all three epochs, rather than the ensemble averages derived from (21). These show the magnitude of the secular trends, allowing interpolation or extrapolation to other epochs, as well as confirming the stability of our method of evaluation.

Finally, for direct comparison with Doodson's coefficients, a fourth value was calculated specifically for the epoch 1900·0 by the least-squares interpolation:

$$H_s(o) = 0{\cdot}5504H_s(-10693{\cdot}5)+0{\cdot}3066H_s(8906{\cdot}5)+0{\cdot}1430H_s(22056{\cdot}5) \quad (22)$$

and converted to Doodson's scaling by the factors ρ given in Table 2. All values above a threshold of $4{\cdot}5 \times 10^{-5}$ in Doodson's scale are tabulated in Tables 4 and 5.

Comments on Tables 4 and 5

Table 4(a), (b) and (c) list the terms derived from the spherical harmonics of 2nd degree, contributing to tides of Species 0 (low frequency), 1 (diurnal), and 2 (semi-diurnal), respectively. We have headed these 'principal terms', because they include the largest amplitudes, although many of their terms are less than the largest terms in the 3rd degree harmonics. Table 5(a), (b), (c) and (d) list the terms from the spherical harmonics of 3rd degree (Doodson's G') ,which contribute to the same tidal species as in Table 4, and also to Species 3 (ter-diurnal).

In each table, the first columns contain the six integers k_r defining the argument (equation (13)), and the amplitudes H_s derived from the three epochs t_0 defined in Table 3. The six integers separated by a central dot repeat the k_r in Doodson's notation, whereby all except k_1 are increased by 5 to avoid minus signs, and the number 10, where it appears, is denoted by X. The columns headed 1900·0 contain the amplitudes interpolated between the three given amplitudes by equation (22) and converted to Doodson's scaling, and the last columns contain Doodson's coefficients for comparison. Doodson (1921, 1954) also lists some coefficients $> 10^{-4}$ in Groups for which $k_2 = \pm 5$ and 6. We have not computed these because experience has shown that their contributions to tidal records are invariably below noise level.

Secular trends, mentioned in the last section, are seen clearly only in amplitudes greater than 0·01. Below this level, variations of 1 or 2 in the last digit may be taken as a measure of the extent of inaccuracy, possibly due to the omission of a small line here and there.

Comparisons with Doodson's values are generally very good, with a few minor exceptions, discussed below. They certainly confirm that he omitted no major term and made no mistakes in sign. The most consistent differences occur in the larger solar terms, because of the inaccuracy in Doodson's conversion factor K_2'/K_2, mentioned earlier. If, for example, one re-adjusts his coefficient for S_2 (2 2 − 2 0 0 0) to the modern constants, one gets 0·42250, which is much closer to our figure. However, differences up to seven in the last decimal occur in purely lunar terms, and these must be attributable to our improved ephemeris and possibly more accurate method of calculation. This also explains why we obtain several lines with amplitude just above Doodson's threshold of 0·00010; they were probably just below it in his calculations.

On the other hand, the effects of some of our more obvious improvements in the ephemeris are hardly detectable to the present accuracy. The largest planetary terms in the Sun's orbit should produce anomalous lines modulating the strong solar lines at harmonic separations of $j_3 = 11{\cdot}3$, $16{\cdot}5$, $22{\cdot}5$ and $33{\cdot}0$ c/18y, but these were not identifiable. Similarly, the effect of the Earth's lunar motion on the Sun's apparent position modulates the strong solar lines by one cycle per synodic month (0 1 − 1 0 0 0), producing differences from Doodson's figures at that frequency and at (1 0 1 0 0 0), (1 2 − 1 0 0 0), (2 1 − 1 0 0 0) and (2 3 − 3 0 0 0). In fact, the differences at these lines are mostly about 2 units, which is not remarkable, and the last is below both threshold levels. However, such small terms, of which there is a considerable number, can accumulate in the time domain to give occasionally much larger increments.

Four terms in Table 4 deserve some comment. Our amplitude at (2 2 2 0 0 0) agrees with the corrected figure in Doodson (1954), but not with that printed in 1921. The two small lines at (0 0 2 0 0 − 2) and (1 1 − 2 0 0 2) differ from Doodson's by more than usual. He lists them as pure solar terms, and these can be checked to have in his scale the respective amplitudes:

$$0{\cdot}46\, e'^2\, (3 - \tfrac{9}{2} \sin^2 \varepsilon) = 0{\cdot}00030$$

and

$$-0{\cdot}46\, e'^2\, (\tfrac{9}{4} \sin \varepsilon \cos \varepsilon) = -0{\cdot}00011$$

as in his table. We had to derive both terms by separation from considerably larger terms at a frequency interval of $2f_6$, but this procedure does not appear to incur any special errors, and there are similar cases which give the expected results. We can only suggest that there may be lunar terms at the same frequencies which were overlooked or did not appear in Doodson's expansion.

Our line at (2 − 2 0 0 0 1) is the only one in Table 4 which is well above Doodson's threshold but is not included in his tables. In fact, this set of k_r can arise by expansion only from rather obscure combinations of arguments. However, a term of the given amplitude is undoubtedly present, and it cannot be accounted for any any other combination, aliassed or otherwise. (Fig. 2, lower panel, $j_3 = -36$, gives no indication of its presence, but it becomes obvious after the first removal of sidebands). Its constancy over the three epochs adds confidence.

The largest differences from Doodson occur in the 3rd degree term of Group (1, 2), Table 5(b). He shows an amplitude of −0·00089 at (1 2 − 2 2 1 0) where we have nothing, while we obtain −0·00098 at (1 2 0 0 1 0) where he shows nothing. Our results here are indisputable, and it seems probable that Doodson made a slip in adding some of his argument-numbers.

Table (4a)

Low-Frequency tides—Principal terms

GROUP 0,0

	1	2	3	1900·0		
0 0 0 0 0 0	-0.31447	-0.31452	-0.31456	055.555	0.73807	0.73869
0 0 0 0 1 0	0.02794	0.02793	0.02793	055.565	-0.06556	-0.06552
0 0 0 0 2 0	-0.00027	-0.00028	-0.00027	055.575	0.00064	0.00064
0 0 0 2 1 0	0.00004	0.00004	0.00004	055.765	-0.00009	
0 0 1 0-1-1	-0.00004	-0.00004	-0.00004	056.544	0.00009	
0 0 1 0 0-1	-0.00493	-0.00493	-0.00492	056.554	0.01156	0.01160
0 0 1 0 0 1	0.00027	0.00026	0.00026	056.556	-0.00063	-0.00061
0 0 1 0 1-1	0.00004	0.00004	0.00005	056.564	-0.00010	
0 0 2-2-1 0	0.00002	0.00002	0.00002	057.345	-0.00005	
0 0 2-2 0 0	-0.00031	-0.00031	-0.00031	057.355	0.00073	0.00073
0 0 2 0 0 0	-0.03097	-0.03095	-0.03095	057.555	0.07266	0.07299
0 0 2 0 0-2	-0.00006	-0.00006	-0.00008	057.553	0.00015	0.00030†
0 0 2 0 1 0	0.00075	0.00077	0.00077	057.565	-0.00178	-0.00181
0 0 2 0 2 0	0.00019	0.00017	0.00017	057.575	-0.00042	-0.00040
0 0 3 0 0-1	-0.00182	-0.00181	-0.00181	058.554	0.00426	0.00427
0 0 3 0 1-1	0.00004	0.00003	0.00003	058.564	-0.00008	
0 0 4 0 0-2	-0.00007	-0.00007	-0.00007	059.553	0.00017	0.00017

Table (4a) *continued*

GROUP 0,1

0 1-3 1-1 1	0.00002	0.00003	0.00002	062.646	-0.00005	
0 1-3 1 0 1	-0.00029	-0.00028	-0.00029	062.656	0.00067	0.00067
0 1-3 1 1 1	0.00002	0.00002	0.00002	062.666	-0.00005	
0 1-2-1-2 0	0.00003	0.00003	0.00003	063.435	-0.00006	
0 1-2-1-1 0	0.00007	0.00007	0.00007	063.445	-0.00016	-0.00016
0 1-2 1-1 0	0.00048	0.00048	0.00048	063.645	-0.00113	-0.00113
0 1-2 1 0 0	-0.00673	-0.00673	-0.00673	063.655	0.01579	0.01578
0 1-2 1 1 0	0.00043	0.00043	0.00043	063.665	-0.00101	-0.00103
0 1-1-1-1 1	0.00002	0.00002	0.00002	064.446	-0.00005	
0 1-1-1 0 1	-0.00022	-0.00021	-0.00021	064.456	0.00050	0.00051
0 1-1-1 1 1	0.00003	0.00002	0.00000	064.466	-0.00005	
0 1-1 0 0 0	0.00019	0.00020	0.00020	064.555	-0.00046	-0.00044
0 1-1 1 0-1	0.00005	0.00005	0.00005	064.654	-0.00011	-0.00010
0 1 0-1-2 0	-0.00003	-0.00003	-0.00003	065.435	0.00007	
0 1 0-1-1 0	0.00231	0.00231	0.00231	065.445	-0.00542	-0.00542
0 1 0-1 0 0	-0.03517	-0.03518	-0.03518	065.455	0.08255	0.08254
0 1 0-1 1 0	0.00228	0.00228	0.00228	065.465	-0.00535	-0.00535
0 1 0 1 0 0	0.00188	0.00188	0.00189	065.655	-0.00441	-0.00442
0 1 0 1 1 0	0.00076	0.00077	0.00077	065.665	-0.00180	-0.00179
0 1 0 1 2 0	0.00021	0.00021	0.00021	065.675	-0.00049	-0.00047
0 1 1-1 0-1	0.00018	0.00018	0.00018	066.454	-0.00043	-0.00043
0 1 2-1 0 0	0.00050	0.00049	0.00049	067.455	-0.00116	-0.00116
0 1 2-1 1 0	0.00026	0.00025	0.00024	067.465	-0.00059	-0.00058
0 1 2-1 2 0	0.00005	0.00005	0.00004	067.475	-0.00011	
0 1 3-1 0-1	0.00002	0.00003	0.00003	068.454	-0.00006	

GROUP 0,2

0 2-4 2 0 0	-0.00011	-0.00011	-0.00011	071.755	0.00026	0.00026
0 2-3 0 0 1	-0.00038	-0.00038	-0.00038	072.556	0.00090	0.00091
0 2-3 0 1 1	0.00003	0.00002	0.00002	072.566	-0.00006	
0 2-2 0-1 0	-0.00042	-0.00042	-0.00042	073.545	0.00098	0.00098
0 2-2 0 0 0	-0.00582	-0.00582	-0.00582	073.555	0.01366	0.01370
0 2-2 0 1 0	0.00037	0.00037	0.00037	073.565	-0.00087	-0.00088
0 2-2 2 0 0	0.00004	0.00004	0.00004	073.755	-0.00009	
0 2-1-2 0 1	-0.00004	-0.00004	-0.00004	074.356	0.00009	
0 2-1-1 0 0	0.00003	0.00003	0.00003	074.455	-0.00007	
0 2-1 0 0-1	0.00007	0.00007	0.00007	074.554	-0.00016	-0.00017
0 2-1 0 0 1	-0.00020	-0.00020	-0.00020	074.556	0.00046	0.00048
0 2-1 0 1 1	-0.00004	-0.00004	-0.00004	074.566	0.00010	0.00012
0 2 0-2-1 0	0.00015	0.00015	0.00015	075.345	-0.00036	-0.00036
0 2 0-2 0 0	-0.00288	-0.00288	-0.00288	075.355	0.00676	0.00677
0 2 0-2 1 0	0.00018	0.00019	0.00019	075.365	-0.00044	-0.00044
0 2 0 0 0 0	-0.06669	-0.06664	-0.06662	075.555	0.15645	0.15642
0 2 0 0 1 0	-0.02763	-0.02762	-0.02762	075.565	0.06482	0.06481
0 2 0 0 2 0	-0.00258	-0.00258	-0.00258	075.575	0.00605	0.00607
0 2 0 0 3 0	0.00007	0.00005	0.00007	075.585	-0.00014	-0.00013
0 2 1-2 0-1	0.00003	0.00003	0.00003	076.354	-0.00007	
0 2 1 0 0-1	0.00023	0.00023	0.00023	076.554	-0.00054	-0.00054
0 2 1 0 1-1	0.00006	0.00006	0.00006	076.564	-0.00014	-0.00014
0 2 2-2 0 0	0.00020	0.00020	0.00020	077.355	-0.00047	-0.00047
0 2 2-2 1 0	0.00008	0.00008	0.00008	077.365	-0.00018	-0.00019
0 2 2 0 2 0	0.00003	0.00003	0.00003	077.575	-0.00006	

Table (4a) *continued*

GROUP 0,3

0 3-5 1 0 1	-0.00002	-0.00002	-0.00002	080.656	0.00005	
0 3-4 1 0 0	-0.00017	-0.00017	-0.00017	081.655	0.00041	0.00042
0 3-3-1 0 1	-0.00007	-0.00007	-0.00007	082.456	0.00016	0.00016
0 3-3 1 0 1	-0.00012	-0.00011	-0.00012	082.656	0.00027	0.00026
0 3-3 1 1 1	-0.00005	-0.00004	-0.00004	082.666	0.00011	0.00011
0 3-2-1-1 0	-0.00009	-0.00010	-0.00010	083.445	0.00022	0.00022
0 3-2-1 0 0	-0.00091	-0.00091	-0.00091	083.455	0.00213	0.00217
0 3-2-1 1 0	0.00006	0.00006	0.00006	083.465	-0.00014	-0.00014
0 3-2 1 0 0	-0.00242	-0.00242	-0.00242	083.655	0.00569	0.00569
0 3-2 1 1 0	-0.00100	-0.00100	-0.00100	083.665	0.00235	0.00236
0 3-2 1 2 0	-0.00009	-0.00009	-0.00009	083.675	0.00021	0.00021
0 3-1-1 0 1	-0.00013	-0.00013	-0.00013	084.456	0.00031	0.00028
0 3-1-1 1 1	-0.00004	-0.00004	-0.00004	084.466	0.00010	0.00010
0 3-1 0 0 0	0.00007	0.00007	0.00006	084.555	-0.00016	-0.00016
0 3-1 0 1 0	0.00003	0.00003	0.00003	084.565	-0.00007	
0 3-1 1 0-1	0.00002	0.00002	0.00003	084.654	-0.00005	
0 3 0-3 0 0	-0.00023	-0.00023	-0.00023	085.255	0.00054	0.00054
0 3 0-3 1-1	0.00004	0.00004	0.00004	085.264	-0.00009	
0 3 0-3 1 1	0.00004	0.00004	0.00004	085.266	-0.00008	
0 3 0-1 0 0	-0.01277	-0.01275	-0.01275	085.455	0.02995	0.02995
0 3 0-1 1 0	-0.00528	-0.00528	-0.00528	085.465	0.01240	0.01241
0 3 0-1 2 0	-0.00048	-0.00049	-0.00051	085.475	0.00114	0.00117
0 3 0 1 2 0	0.00005	0.00005	0.00005	085.675	-0.00011	-0.00012
0 3 0 1 3 0	0.00002	0.00002	0.00002	085.685	-0.00005	
0 3 1-1 0-1	0.00011	0.00011	0.00011	086.454	-0.00025	-0.00026
0 3 1-1 1-1	0.00004	0.00004	0.00004	086.464	-0.00009	

GROUP 0,4

0 4-4 0 0 0	-0.00008	-0.00008	-0.00008	091.555	0.00018	0.00020
0 4-4 2 0 0	-0.00006	-0.00006	-0.00006	091.755	0.00015	0.00014
0 4-4 2 1 0	-0.00002	-0.00003	-0.00002	091.765	0.00006	
0 4-3 0 0 1	-0.00014	-0.00014	-0.00014	092.556	0.00033	0.00032
0 4-3 0 1 1	-0.00005	-0.00006	-0.00006	092.566	0.00013	0.00013
0 4-2-2 0 0	-0.00010	-0.00010	-0.00011	093.355	0.00024	0.00025
0 4-2 0 0 0	-0.00206	-0.00206	-0.00205	093.555	0.00483	0.00478
0 4-2 0 1 0	-0.00085	-0.00085	-0.00085	093.565	0.00200	0.00200
0 4-2 0 2 0	-0.00008	-0.00008	-0.00008	093.575	0.00018	0.00019
0 4-1-2 0 1	-0.00003	-0.00003	-0.00003	094.356	0.00007	
0 4-1 0 0-1	0.00003	0.00003	0.00003	094.554	-0.00007	
0 4 0-2 0 0	-0.00169	-0.00169	-0.00169	095.355	0.00396	0.00396
0 4 0-2 1 0	-0.00070	-0.00070	-0.00070	095.365	0.00164	0.00165
0 4 0-2 2 0	-0.00006	-0.00006	-0.00006	095.375	0.00016	0.00016

Table (4b)

Diurnal tides—Principal terms

	1	2	3	1900·0		
			GROUP 1,-4			
1-4 0 3-1 0	-0.00014	-0.00014	-0.00014	115.845	0.00021	0.00021
1-4 0 3 0 0	-0.00074	-0.00075	-0.00075	115.855	0.00107	0.00108
1-4 1 1 0 1	0.00004	0.00004	0.00003	116.656	-0.00005	
1-4 2 1-1 0	-0.00036	-0.00037	-0.00036	117.645	0.00052	0.00053
1-4 2 1 0 0	-0.00193	-0.00193	-0.00193	117.655	0.00278	0.00278
1-4 3 1 0-1	-0.00015	-0.00015	-0.00015	118.654	0.00021	0.00021
1-4 4-1-1 0	-0.00007	-0.00007	-0.00007	119.445	0.00010	0.00010
1-4 4-1 0 0	-0.00037	-0.00037	-0.00037	119.455	0.00054	0.00054
1-4 5-1 0-1	-0.00004	-0.00004	-0.00004	119.454	0.00006	
			GROUP 1,-3			
1-3-1 2 0 1	0.00009	0.00009	0.00009	124.756	-0.00013	-0.00013
1-3 0 0-2 0	0.00004	0.00004	0.00003	125.535	-0.00006	
1-3 0 2-2 0	0.00005	0.00004	0.00003	125.735	-0.00006	
1-3 0 2-1 0	-0.00125	-0.00125	-0.00125	125.745	0.00180	0.00180
1-3 0 2 0 0	-0.00664	-0.00664	-0.00663	125.755	0.00954	0.00955
1-3 1 0 0 1	0.00011	0.00012	0.00011	126.556	-0.00016	-0.00016
1-3 1 1 0 0	0.00007	0.00007	0.00006	126.655	-0.00010	-0.00011
1-3 1 2 0-1	-0.00011	-0.00010	-0.00011	126.754	0.00015	0.00015
1-3 2 0-2 0	0.00005	0.00005	0.00005	127.535	-0.00007	
1-3 2 0-1 0	-0.00151	-0.00151	-0.00150	127.545	0.00217	0.00218
1-3 2 0 0 0	-0.00801	-0.00801	-0.00800	127.555	0.01151	0.01153
1-3 2 2 0 0	0.00007	0.00007	0.00006	127.755	-0.00009	
1-3 3 0-1-1	-0.00009	-0.00010	-0.00010	128.544	0.00014	0.00014
1-3 3 0 0-1	-0.00054	-0.00054	-0.00055	128.554	0.00078	0.00079
1-3 4-2-1 0	-0.00004	-0.00005	-0.00004	129.345	0.00006	
1-3 4-2 0 0	-0.00025	-0.00025	-0.00024	129.355	0.00035	0.00035
1-3 4 0 0 0	0.00007	0.00008	0.00007	129.555	-0.00010	
1-3 4 0 1 0	-0.00003	-0.00003	-0.00004	129.565	0.00005	
			GROUP 1,-2			
1-2-2 1-2 0	0.00004	0.00004	0.00004	133.635	-0.00006	
1-2-2 3 0 0	0.00016	0.00016	0.00016	133.855	-0.00023	-0.00023
1-2-1 1-1 1	0.00007	0.00007	0.00007	134.646	-0.00010	
1-2-1 1 0 1	0.00042	0.00042	0.00042	134.656	-0.00061	-0.00061
1-2 0-1-3 0	0.00004	0.00004	0.00004	135.425	-0.00005	
1-2 0-1-2 0	0.00019	0.00019	0.00019	135.435	-0.00028	-0.00028
1-2 0 1-2 0	0.00029	0.00029	0.00029	135.635	-0.00041	-0.00042
1-2 0 0 0 1	-0.00005	-0.00004	-0.00004	135.556	0.00006	
1-2 0 1-1 0	-0.00946	-0.00946	-0.00946	135.645	0.01359	0.01360
1-2 0 1 0 0	-0.05020	-0.05019	-0.05018	135.655	0.07214	0.07216
1-2 0 3 0 0	0.00014	0.00014	0.00014	135.855	-0.00020	-0.00019
1-2 1-1 0 1	0.00010	0.00009	0.00009	136.456	-0.00014	-0.00013
1-2 1 0-1 0	0.00005	0.00005	0.00005	136.545	-0.00007	
1-2 1 0 0 0	0.00027	0.00027	0.00027	136.555	-0.00039	-0.00039
1-2 1 1-1-1	-0.00008	-0.00008	-0.00007	136.644	0.00011	0.00011
1-2 1 1 0-1	-0.00046	-0.00046	-0.00046	136.654	0.00066	0.00068
1-2 2-1-2 0	0.00006	0.00005	0.00005	137.435	-0.00008	
1-2 2-1-1 0	-0.00180	-0.00180	-0.00180	137.445	0.00258	0.00258
1-2 2-1 0 0	-0.00954	-0.00953	-0.00953	137.455	0.01370	0.01371
1-2 2 1 0 0	0.00055	0.00055	0.00055	137.655	-0.00079	-0.00078
1-2 2 1 1 0	-0.00017	-0.00017	-0.00017	137.665	0.00024	0.00024
1-2 3-1-1-1	-0.00008	-0.00008	-0.00008	138.444	0.00012	0.00011
1-2 3-1 0-1	-0.00044	-0.00044	-0.00044	138.454	0.00063	0.00064
1-2 3 1 0-1	0.00004	0.00004	0.00004	138.654	-0.00006	
1-2 4-1 0 0	0.00012	0.00012	0.00012	139.455	-0.00017	-0.00014
1-2 4-1 1 0	-0.00003	-0.00003	-0.00003	139.465	0.00005	

Table (4b) *continued*

GROUP 1,-1

1-1-2 0-2 0	0.00011	0.00011	0.00011	143.535	-0.00016	-0.00017
1-1-2 2-1 0	0.00014	0.00014	0.00014	143.745	-0.00020	-0.00020
1-1-2 2 0 0	0.00079	0.00079	0.00079	143.755	-0.00113	-0.00113
1-1-1 0-1 1	0.00011	0.00011	0.00011	144.546	-0.00016	-0.00015
1-1-1 0 0 1	0.00091	0.00090	0.00090	144.556	-0.00130	-0.00130
1-1-1 1 0 0	-0.00004	-0.00004	-0.00004	144.655	0.00006	
1-1 0 0-2 0	0.00152	0.00153	0.00153	145.535	-0.00220	-0.00218
1-1 0 0-1 0	-0.04943	-0.04944	-0.04944	145.545	0.07105	0.07105
1-1 0 0 0 0	-0.26229	-0.26223	-0.26219	145.555	0.37690	0.37689
1-1 0 2 0 0	0.00169	0.00169	0.00169	145.755	-0.00243	-0.00243
1-1 0 2 1 0	0.00027	0.00028	0.00028	145.765	-0.00039	-0.00040
1-1 1 0-1-1	-0.00008	-0.00008	-0.00008	146.544	0.00012	0.00012
1-1 1 0 0-1	-0.00076	-0.00076	-0.00076	146.554	0.00109	0.00115
1-1 2-2 0 0	0.00015	0.00015	0.00015	147.355	-0.00021	-0.00021
1-1 2 0-1 0	-0.00010	-0.00010	-0.00010	147.545	0.00014	0.00014
1-1 2 0 0 0	0.00343	0.00342	0.00342	147.555	-0.00492	-0.00491
1-1 2 0 1 0	-0.00074	-0.00075	-0.00075	147.565	0.00107	0.00107
1-1 2 0 2 0	-0.00005	-0.00005	-0.00005	147.575	0.00007	
1-1 3 0 0-1	0.00022	0.00023	0.00023	148.554	-0.00032	-0.00033
1-1 4-2 0 0	0.00006	0.00006	0.00006	149.355	-0.00009	

GROUP 1,0

1 0-3 1 0 1	0.00009	0.00009	0.00009	152.656	-0.00013	-0.00014
1 0-2 1-1 0	0.00044	0.00044	0.00044	153.645	-0.00063	-0.00063
1 0-2 1 0 0	0.00193	0.00193	0.00193	153.655	-0.00278	-0.00278
1 0-1 0 0 0	-0.00004	-0.00004	-0.00004	154.555	0.00006	
1 0-1 1 0 1	-0.00010	-0.00010	-0.00010	154.656	0.00015	0.00015
1 0 0-1-2 0	-0.00012	-0.00012	-0.00012	155.435	0.00018	0.00017
1 0 0-1-1 0	0.00137	0.00137	0.00137	155.445	-0.00197	-0.00197
1 0 0-1 0 0	0.00742	0.00742	0.00742	155.455	-0.01066	-0.01065
1 0 0 1-1 0	-0.00060	-0.00060	-0.00060	155.645	0.00086	0.00085
1 0 0 1 0 0	0.02062	0.02062	0.02061	155.655	-0.02963	-0.02964
1 0 0 1 1 0	0.00413	0.00414	0.00414	155.665	-0.00594	-0.00594
1 0 0 1 2 0	-0.00011	-0.00012	-0.00011	155.675	0.00016	0.00017
1 0 1 0 0 0	-0.00011	-0.00011	-0.00011	156.555	0.00016	0.00016
1 0 1 1 0-1	0.00013	0.00013	0.00013	156.654	-0.00018	-0.00018
1 0 2-1-1 0	-0.00011	-0.00011	-0.00011	157.445	0.00016	0.00016
1 0 2-1 0 0	0.00394	0.00394	0.00394	157.455	-0.00567	-0.00566
1 0 2-1 1 0	0.00087	0.00087	0.00087	157.465	-0.00125	-0.00124
1 0 3-1 0-1	0.00017	0.00017	0.00017	158.454	-0.00024	-0.00024
1 0 3-1 1-1	0.00004	0.00004	0.00004	158.464	-0.00006	

GROUP 1,1

1 1-4 0 0 2	-0.00029	-0.00029	-0.00029	161.557	0.00042	0.00042
1 1-3 0-1 1	0.00006	0.00006	0.00006	162.546	-0.00008	
1 1-3 0 0 1	-0.00716	-0.00715	-0.00714	162.556	0.01028	0.01029
1 1-2 0-2 0	-0.00010	-0.00010	-0.00010	163.535	0.00014	0.00014
1 1-2 0-1 0	0.00137	0.00137	0.00137	163.545	-0.00197	-0.00199
1 1-2 0 0 0	-0.12211	-0.12207	-0.12205	163.555	0.17546	0.17584
1 1-2 0 0 2	0.00002	0.00003	0.00003	163.557	-0.00004	-0.00011†
1 1-2 2 0 0	0.00019	0.00018	0.00018	163.755	-0.00027	-0.00026
1 1-2 2 1 0	0.00004	0.00004	0.00004	163.765	-0.00005	
1 1-1 0 0-1	0.00103	0.00102	0.00103	164.554	-0.00147	-0.00147
1 1-1 0 0 1	0.00290	0.00289	0.00289	164.556	-0.00416	-0.00423
1 1-1 0 1 1	-0.00007	-0.00008	-0.00008	164.566	0.00011	
1 1 0-2-1 0	0.00007	0.00007	0.00007	165.345	-0.00010	
1 1 0 0-2 0	0.00005	0.00005	0.00005	165.535	-0.00007	
1 1 0 0-1 0	-0.00732	-0.00730	-0.00730	165.545	0.01051	0.01050
1 1 0 0 0 0	0.36890	0.36882	0.36876	165.555	-0.53009	-0.53050
1 1 0 0 1 0	0.05000	0.05001	0.05001	165.565	-0.07186	-0.07182
1 1 0 0 2 0	-0.00108	-0.00108	-0.00108	165.575	0.00156	0.00154
1 1 1 0 0-1	0.00294	0.00293	0.00293	166.554	-0.00422	-0.00423
1 1 1 0 1-1	0.00005	0.00005	0.00005	166.564	-0.00008	
1 1 2-2 0 0	0.00018	0.00018	0.00018	167.355	-0.00026	-0.00026
1 1 2-2 1 0	0.00006	0.00006	0.00006	167.365	-0.00008	
1 1 2 0 0-2	0.00006	0.00007	0.00008	167.553	-0.00010	-0.00011
1 1 2 0 0 0	0.00525	0.00525	0.00525	167.555	-0.00755	-0.00756
1 1 2 0 1 0	-0.00020	-0.00020	-0.00020	167.565	0.00029	0.00029
1 1 2 0 2 0	-0.00010	-0.00010	-0.00010	167.575	0.00014	0.00014
1 1 3 0 0-1	0.00031	0.00031	0.00031	168.554	-0.00044	-0.00044

Table (4b) *continued*

GROUP 1,2

1 2-3 1 0 1	0.00017	0.00017	0.00017	172.656	-0.00024	-0.00024
1 2-3 1 1 1	0.00003	0.00003	0.00003	172.666	-0.00005	
1 2-2-1-1 0	0.00012	0.00012	0.00012	173.445	-0.00017	-0.00017
1 2-2 1-1 0	-0.00013	-0.00013	-0.00013	173.645	0.00018	0.00018
1 2-2 1 0 0	0.00394	0.00394	0.00394	173.655	-0.00567	-0.00566
1 2-2 1 1 0	0.00078	0.00078	0.00078	173.665	-0.00112	-0.00112
1 2-1-1 0 1	0.00013	0.00013	0.00012	174.456	-0.00018	-0.00018
1 2-1 0 0 0	-0.00012	-0.00011	-0.00011	174.555	0.00017	0.00016
1 2 0-1-1 0	-0.00061	-0.00060	-0.00060	175.445	0.00087	0.00087
1 2 0-1 0 0	0.02062	0.02062	0.02061	175.455	-0.02963	-0.02964
1 2 0-1 1 0	0.00409	0.00409	0.00409	175.465	-0.00587	-0.00587
1 2 0-1 2 0	-0.00010	-0.00009	-0.00007	175.475	0.00014	0.00013
1 2 0 1 0 0	-0.00032	-0.00032	-0.00032	175.655	0.00046	0.00046
1 2 0 1 1 0	-0.00020	-0.00020	-0.00020	175.665	0.00029	0.00029
1 2 0 1 2 0	-0.00012	-0.00012	-0.00012	175.675	0.00017	0.00017
1 2 1-1 0-1	-0.00010	-0.00010	-0.00010	176.454	0.00015	0.00015
1 2 2-1 0 0	-0.00008	-0.00008	-0.00008	177.455	0.00012	0.00012
1 2 2-1 1 0	-0.00007	-0.00006	-0.00006	177.465	0.00009	

GROUP 1,3

1 3-4 2 0 0	0.00006	0.00007	0.00006	181.755	-0.00009	
1 3-3 0 0 1	0.00023	0.00023	0.00023	182.556	-0.00033	-0.00032
1 3-3 0 1 1	0.00004	0.00004	0.00005	182.566	-0.00006	
1 3-2 0-1 0	0.00011	0.00011	0.00011	183.545	-0.00016	-0.00016
1 3-2 0 0 0	0.00343	0.00343	0.00342	183.555	-0.00493	-0.00492
1 3-2 0 1 0	0.00067	0.00067	0.00067	183.565	-0.00097	-0.00096
1 3-1 0 0-1	-0.00007	-0.00007	-0.00007	184.554	0.00010	
1 3 0-2-1 0	-0.00004	-0.00004	-0.00004	185.345	0.00006	
1 3 0-2 0 0	0.00169	0.00169	0.00169	185.355	-0.00243	-0.00240
1 3 0-2 1 0	0.00033	0.00033	0.00033	185.365	-0.00048	-0.00048
1 3 0 0 0 0	0.01130	0.01129	0.01129	185.555	-0.01623	-0.01623
1 3 0 0 1 0	0.00723	0.00723	0.00723	185.565	-0.01039	-0.01039
1 3 0 0 2 0	0.00151	0.00151	0.00152	185.575	-0.00217	-0.00218
1 3 0 0 3 0	0.00010	0.00010	0.00010	185.585	-0.00014	-0.00014
1 3 1 0 0-1	-0.00004	-0.00004	-0.00004	186.554	0.00006	

GROUP 1,4

1 4-4 1 0 0	0.00011	0.00011	0.00011	191.655	-0.00015	-0.00015
1 4-3-1 0 1	0.00004	0.00004	0.00004	192.456	-0.00006	
1 4-2-1 0 0	0.00055	0.00055	0.00055	193.455	-0.00079	-0.00078
1 4-2-1 1 0	0.00011	0.00011	0.00011	193.465	-0.00016	-0.00015
1 4-2 1 0 0	0.00041	0.00041	0.00041	193.655	-0.00059	-0.00059
1 4-2 1 1 0	0.00026	0.00026	0.00026	193.665	-0.00038	-0.00038
1 4-2 1 2 0	0.00005	0.00005	0.00005	193.675	-0.00007	
1 4 0-3 0 0	0.00013	0.00013	0.00014	195.255	-0.00019	-0.00019
1 4 0-1 0 0	0.00216	0.00216	0.00216	195.455	-0.00311	-0.00311
1 4 0-1 1 0	0.00139	0.00138	0.00138	195.465	-0.00199	-0.00199
1 4 0-1 2 0	0.00029	0.00029	0.00029	195.475	-0.00042	-0.00042

Table 4(c)

Semi-diurnal tides—Principal terms

	1	2	3	1900·0		
		GROUP 2,-4				
2-4 0 4 0 0	0.00018	0.00019	0.00019	215.955	0.00027	0.00027
2-4 2 2 0 0	0.00077	0.00077	0.00077	217.755	0.00111	0.00111
2-4 3 2 0-1	0.00006	0.00006	0.00006	218.754	0.00009	
2-4 4 0 0 0	0.00048	0.00048	0.00048	219.555	0.00069	0.00069
2-4 5 0 0-1	0.00006	0.00006	0.00006	21X.554	0.00009	
		GROUP 2,-3				
2-3 0 1 0 1	0.00006	0.00006	0.00006	225.656	0.00009	
2-3 0 3-1 0	-0.00007	-0.00007	-0.00007	225.845	-0.00010	
2-3 0 3 0 0	0.00180	0.00180	0.00180	225.855	0.00258	0.00259
2-3 1 1 0 1	-0.00009	-0.00009	-0.00009	226.656	-0.00013	-0.00012
2-3 1 3 0-1	0.00004	0.00004	0.00004	226.854	0.00006	
2-3 2 1-1 0	-0.00017	-0.00017	-0.00018	227.645	-0.00025	-0.00025
2-3 2 1 0 0	0.00466	0.00465	0.00465	227.655	0.00669	0.00671
2-3 3 1 0-1	0.00035	0.00035	0.00036	228.654	0.00051	0.00054
2-3 4-1-1 0	-0.00003	-0.00003	-0.00003	229.445	-0.00005	
2-3 4-1 0 0	0.00090	0.00090	0.00090	229.455	0.00129	0.00130
2-3 5-1 0-1	0.00010	0.00010	0.00010	22X.454	0.00015	0.00015
		GROUP 2,-2				
2-2-2 4 0 0	-0.00006	-0.00006	-0.00006	233.955	-0.00009	
2-2-1 2 0 1	-0.00022	-0.00022	-0.00022	234.756	-0.00032	-0.00031
2-2 0 0-2 0	-0.00010	-0.00010	-0.00009	235.535	-0.00014	-0.00014
2-2 0 0-1 1	0.00004	0.00005	0.00005	235.546	0.00007	
2-2 0 0 0 1	0.00012	0.00012	0.00012	235.556	0.00017	+
2-2 0 2-1 0	-0.00059	-0.00060	-0.00060	235.745	-0.00086	-0.00086
2-2 0 2 0 0	0.01599	0.01599	0.01599	235.755	0.02298	0.02301
2-2 1 0 0 1	-0.00027	-0.00028	-0.00027	236.556	-0.00039	-0.00040
2-2 1 1 0 0	-0.00017	-0.00017	-0.00017	236.655	-0.00024	-0.00025
2-2 1 2 0-1	0.00025	0.00025	0.00025	236.754	0.00036	0.00036
2-2 2 0-1 0	-0.00072	-0.00072	-0.00072	237.545	-0.00104	-0.00104
2-2 2 0 0 0	0.01930	0.01930	0.01929	237.555	0.02774	0.02774
2-2 3-1 0 0	-0.00004	-0.00005	-0.00004	238.455	-0.00006	
2-2 3 0-1-1	-0.00005	-0.00005	-0.00005	238.544	-0.00007	
2-2 3 0 0-1	0.00131	0.00130	0.00131	238.554	0.00188	0.00189
2-2 4-2 0 0	0.00059	0.00059	0.00059	239.355	0.00085	0.00085
2-2 4 0 0-2	0.00005	0.00005	0.00005	239.553	0.00007	
2-2 5-2 0-1	0.00005	0.00005	0.00005	23X.354	0.00007	
		GROUP 2,-1				
2-1-2 1-2 0	-0.00010	-0.00010	-0.00010	243.635	-0.00015	-0.00015
2-1-2 3 0 0	-0.00039	-0.00039	-0.00039	243.855	-0.00056	-0.00056
2-1-1 1-1 1	0.00003	0.00003	0.00003	244.646	0.00005	
2-1-1 1 0 1	-0.00102	-0.00102	-0.00102	244.656	-0.00147	-0.00147
2-1 0-1-2 0	-0.00047	-0.00046	-0.00047	245.435	-0.00067	-0.00063
2-1 0 1-2 0	0.00006	0.00006	0.00007	245.635	0.00009	
2-1 0 0 0 1	0.00010	0.00009	0.00010	245.556	0.00014	0.00014
2-1 0 1-1 0	-0.00452	-0.00451	-0.00451	245.645	-0.00649	-0.00648
2-1 0 1 0 0	0.12094	0.12094	0.12095	245.655	0.17380	0.17387
2-1 1-1 0 1	-0.00023	-0.00022	-0.00023	246.456	-0.00032	-0.00033
2-1 1 0 0 0	-0.00065	-0.00065	-0.00066	246.555	-0.00094	-0.00094
2-1 1 1-1-1	-0.00004	-0.00004	-0.00004	246.644	-0.00005	
2-1 1 1 0-1	0.00113	0.00113	0.00113	246.654	0.00163	0.00163
2-1 2-1-1 0	-0.00086	-0.00086	-0.00086	247.445	-0.00123	-0.00123
2-1 2-1 0 0	0.02297	0.02297	0.02297	247.455	0.03301	0.03303
2-1 2 1 0 0	0.00010	0.00010	0.00010	247.655	0.00014	0.00017
2-1 2 1 1 0	-0.00008	-0.00008	-0.00008	247.665	-0.00012	-0.00012
2-1 3-1-1-1	-0.00004	-0.00004	-0.00004	248.444	-0.00006	
2-1 3-1 0-1	0.00106	0.00106	0.00106	248.454	0.00153	0.00153

Table (4c) *continued*

GROUP 2,0

2 0-3 2 0 1	-0.00008	-0.00008	-0.00008	252.756	-0.00011	-0.00011
2 0-2 0-2 0	-0.00027	-0.00027	-0.00028	253.535	-0.00039	-0.00040
2 0-2 2-1 0	0.00007	0.00007	0.00007	253.745	0.00010	
2 0-2 2 0 0	-0.00190	-0.00190	-0.00190	253.755	-0.00273	-0.00273
2 0-1 0-1 1	0.00005	0.00005	0.00005	254.546	0.00007	
2 0-1 0 0 1	-0.00218	-0.00218	-0.00218	254.556	-0.00313	-0.00314
2 0-1 1 0 0	0.00010	0.00009	0.00009	254.655	0.00014	0.00014
2 0 0 0-2 0	0.00033	0.00033	0.00034	255.535	0.00047	0.00047
2 0 0 0-1 0	-0.02361	-0.02356	-0.02357	255.545	-0.03390	-0.03386
2 0 0 0 0 0	0.63184	0.63187	0.63189	255.555	0.90805	0.90812
2 0 0 2 0 0	0.00036	0.00037	0.00037	255.755	0.00052	0.00053
2 0 0 2 1 0	0.00013	0.00014	0.00013	255.765	0.00019	0.00019
2 0 1 0-1-1	-0.00004	-0.00004	-0.00004	256.544	-0.00006	
2 0 1 0 0-1	0.00193	0.00192	0.00192	256.554	0.00277	0.00276
2 0 2-2 0 0	-0.00036	-0.00036	-0.00036	257.355	-0.00052	-0.00052
2 0 2 0 0 0	0.00072	0.00072	0.00072	257.555	0.00104	0.00107
2 0 2 0 1 0	-0.00036	-0.00035	-0.00035	257.565	-0.00051	-0.00051
2 0 2 0 2 0	0.00012	0.00012	0.00012	257.575	0.00017	0.00018
2 0 3 0 0-1	0.00005	0.00005	0.00005	258.554	0.00007	

GROUP 2,1

2 1-3 1 0 1	-0.00022	-0.00022	-0.00023	262.656	-0.00032	-0.00013
2 1-2 1-1 0	0.00021	0.00021	0.00021	263.645	0.00030	0.00024
2 1-2 1 0 0	-0.00466	-0.00466	-0.00466	263.655	-0.00669	-0.00670
2 1-1-1 0 1	-0.00007	-0.00007	-0.00007	264.456	-0.00010	-0.00010
2 1-1 0 0 0	0.00011	0.00011	0.00011	264.555	0.00015	0.00017
2 1 0-1-1 0	0.00065	0.00066	0.00065	265.445	0.00094	0.00095
2 1 0-1 0 0	-0.01787	-0.01786	-0.01787	265.455	-0.02567	-0.02567
2 1 0 1-1 0	-0.00009	-0.00009	-0.00008	265.645	-0.00012	-0.00012
2 1 0 1 0 0	0.00447	0.00446	0.00446	265.655	0.00642	0.00643
2 1 0 1 1 0	0.00197	0.00197	0.00197	265.665	0.00283	0.00283
2 1 0 1 2 0	0.00028	0.00027	0.00028	265.675	0.00040	0.00040
2 1 2-1 0 0	0.00085	0.00085	0.00086	267.455	0.00122	0.00123
2 1 2-1 1 0	0.00041	0.00041	0.00042	267.465	0.00059	0.00059
2 1 2-1 2 0	0.00003	0.00004	0.00005	267.475	0.00006	

GROUP 2,2

2 2-4 0 0 2	0.00070	0.00070	0.00070	271.557	0.00101	0.00101
2 2-3 0 0 1	0.01724	0.01722	0.01720	272.556	0.02476	0.02479
2 2-2 0-1 0	0.00067	0.00066	0.00066	273.545	0.00095	0.00094
2 2-2 0 0 0	0.29397	0.29399	0.29400	273.555	0.42248	0.42358
2 2-2 2 0 0	0.00004	0.00004	0.00004	273.755	0.00006	
2 2-1 0 0-1	-0.00247	-0.00247	-0.00246	274.554	-0.00355	-0.00354
2 2-1 0 0 1	0.00063	0.00062	0.00062	274.556	0.00090	0.00092
2 2-1 0 1 1	-0.00004	-0.00004	-0.00004	274.566	-0.00005	
2 2 0 0-1 0	-0.00103	-0.00102	-0.00103	275.545	-0.00147	-0.00147
2 2 0 0 0 0	0.08001	0.07997	0.07993	275.555	0.11495	0.11506
2 2 0 0 1 0	0.02383	0.02383	0.02382	275.565	0.03424	0.03423
2 2 0 0 2 0	0.00259	0.00259	0.00259	275.575	0.00372	0.00372
2 2 1 0 0-1	0.00063	0.00063	0.00063	276.554	0.00091	0.00092
2 2 2 0 0 0	0.00053	0.00053	0.00053	277.555	0.00076	0.00078†

GROUP 2,3

2 3-3 1 0 1	0.00004	0.00004	0.00004	282.656	0.00005	
2 3-2-1-1 0	0.00006	0.00006	0.00006	283.445	0.00008	
2 3-2-1 0 0	0.00005	0.00004	0.00004	283.455	0.00006	
2 3-2 1 0 0	0.00085	0.00085	0.00085	283.655	0.00123	0.00123
2 3-2 1 1 0	0.00037	0.00037	0.00037	283.665	0.00053	0.00054
2 3-2 1 2 0	0.00004	0.00004	0.00004	283.675	0.00006	
2 3 0-1-1 0	-0.00009	-0.00008	-0.00008	285.445	-0.00012	-0.00012
2 3 0-1 0 0	0.00446	0.00447	0.00446	285.455	0.00642	0.00643
2 3 0-1 1 0	0.00194	0.00194	0.00194	285.465	0.00279	0.00280
2 3 0-1 2 0	0.00021	0.00022	0.00021	285.475	0.00031	0.00030
2 3 0 1 0 0	-0.00003	-0.00003	-0.00003	285.655	-0.00005	

Table (4c) *continued*

GROUP 2,4

2 4-4 0 0-1	0.00006	0.00006	0.00005	291.554	0.00008	
2 4-3 0 0 1	0.00005	0.00005	0.00005	292.556	0.00007	
2 4-2 0 0 0	0.00074	0.00074	0.00073	293.555	0.00106	0.00107
2 4-2 0 1 0	0.00032	0.00032	0.00031	293.565	0.00046	0.00046
2 4-2 0 2 0	0.00003	0.00003	0.00003	293.575	0.00005	
2 4 0-2 0 0	0.00036	0.00036	0.00036	295.355	0.00052	0.00053
2 4 0-2 1 0	0.00016	0.00016	0.00016	295.365	0.00023	0.00023
2 4 0 0 0 0	0.00118	0.00117	0.00117	295.555	0.00169	0.00168
2 4 0 0 1 0	0.00102	0.00102	0.00102	295.565	0.00146	0.00146
2 4 0 0 2 0	0.00033	0.00033	0.00033	295.575	0.00047	0.00047
2 4 0 0 3 0	0.00005	0.00005	0.00005	295.585	0.00007	

† See comment in text

Table 5(a)

Low-frequency tides—3rd-degree terms

	1	2	3	1900·0		
GROUP 0,0						
0 0 0 1 0 0	-0.00020	-0.00020	-0.00021	055.655	0.00025	0.00026
0 0 2-1 0 0	-0.00004	-0.00004	-0.00004	057.455	0.00005	
GROUP 0,1						
0 1-2 0 0 0	0.00004	0.00004	0.00004	063.555	-0.00005	
0 1 0 0-1 0	0.00019	0.00020	0.00019	065.545	-0.00024	-0.00024
0 1 0 0 0 0	-0.00375	-0.00375	-0.00375	065.555	0.00466	0.00466
0 1 0 0 1 0	-0.00059	-0.00059	-0.00059	065.565	0.00074	0.00073
0 1 0 0 2 0	0.00005	0.00005	0.00005	065.575	-0.00006	
GROUP 0,2						
0 2-2 1 0 0	-0.00012	-0.00012	-0.00012	073.655	0.00015	0.00015
0 2 0-1 0 0	-0.00061	-0.00061	-0.00061	075.455	0.00076	0.00076
0 2 0-1 1 0	-0.00010	-0.00010	-0.00010	075.465	0.00012	0.00012
GROUP 0,3						
0 3-2 0 0 0	-0.00010	-0.00010	-0.00010	083.555	0.00013	0.00013
0 3 0-2 0 0	-0.00007	-0.00007	-0.00007	085.355	0.00009	
0 3 0 0 0 0	-0.00031†	-0.00030	-0.00030	085.555	0.00038	0.00038
0 3 0 0 1 0	-0.00019	-0.00019	-0.00019	085.565	0.00023	0.00024
0 3 0 0 2 0	-0.00004	-0.00004	-0.00004	085.575	0.00005	
GROUP 0,4						
0 4 0-1 0 0	-0.00008	-0.00008	-0.00008	095.455	0.00010	0.00011
0 4 0-1 1 0	-0.00005	-0.00005	-0.00005	095.465	0.00006	

Table (5b)

Diurnal tides—3rd-degree terms

	1	2	3	1900·0		
			GROUP 1,-4			
1-4 0 2 0 0	-0.00006	-0.00006	-0.00006	115.755	-0.00010	-0.00010
1-4 2 0 0 0	-0.00006	-0.00006	-0.00006	117.555	-0.00010	-0.00010
			GROUP 1,-3			
1-3 0 1-1 0	-0.00014	-0.00014	-0.00014	125.645	-0.00023	-0.00023
1-3 0 1 0 0	-0.00035	-0.00035	-0.00035	125.655	-0.00058	-0.00058
1-3 2-1 0 0	-0.00007	-0.00007	-0.00007	127.455	-0.00011	-0.00011
			GROUP 1,-2			
1-2 0 0-2 0	-0.00004	-0.00004	-0.00004	135.535	-0.00007	
1-2 0 0-1 0	-0.00051	-0.00050	-0.00050	135.545	-0.00083	-0.00084
1-2 0 0 0 0	-0.00128	-0.00128	-0.00128	135.555	-0.00211	-0.00211
1-2 0 2 0 0	-0.00008	-0.00008	-0.00008	135.755	-0.00013	-0.00013
1-2 2 0 0 0	-0.00011	-0.00011	-0.00011	137.555	-0.00018	-0.00018
			GROUP 1,-1			
1-1 0-1 0 0	0.00007	0.00007	0.00007	145.455	0.00012	0.00012
1-1 0 1-1 0	0.00010	0.00010	0.00010	145.645	0.00016	0.00016
1-1 0 1 0 0	-0.00065	-0.00065	-0.00065	145.655	-0.00108	-0.00108
1-1 0 1 1 0	0.00009	0.00008	0.00009	145.665	0.00014	0.00014
1-1 2-1 0 0	-0.00013	-0.00013	-0.00013	147.455	-0.00021	-0.00021
			GROUP 1,0			
1 0 0 0-1 0	0.00059	0.00059	0.00059	155.545	0.00098	0.00098
1 0 0 0 0 0	-0.00399	-0.00399	-0.00399	155.555	-0.00660	-0.00661
1 0 0 0 1 0	0.00052	0.00052	0.00052	155.565	0.00086	0.00086
			GROUP 1,1			
1 1-2 1 0 0	-0.00004	-0.00004	-0.00004	163.655	-0.00007	
1 1 0-1-1 0	0.00003	0.00003	0.00003	165.445	0.00005	
1 1 0-1 0 0	-0.00022	-0.00022	-0.00022	165.455	-0.00036	-0.00036
1 1 0-1 1 0	0.00003	0.00003	0.00003	165.465	0.00005	
1 1 0 1 0 0	-0.00008	-0.00008	-0.00008	165.655	-0.00013	-0.00013
1 1 0 1 1 0	-0.00003	-0.00003	-0.00003	165.665	-0.00005	
			GROUP 1,2			
1 2-2 0 0 0	-0.00005	-0.00005	-0.00005	173.555	-0.00008	
1 2 0 0-1 0	0.00005	0.00005	0.00005	175.545	0.00008	
1 2 0 0 0 0	-0.00146	-0.00146	-0.00146	175.555	-0.00242	-0.00241
1 2 0 0 1 0	-0.00059	-0.00059	-0.00059	175.565	-0.00098	(-0.00089)†
1 2 0 0 2 0	-0.00005	-0.00005	-0.00005	175.575	-0.00008	
			GROUP 1,3			
1 3-2 1 0 0	-0.00005	-0.00005	-0.00005	183.655	-0.00008	
1 3 0-1 0 0	-0.00024	-0.00024	-0.00024	185.455	-0.00039	-0.00040
1 3 0-1 1 0	-0.00010	-0.00010	-0.00010	185.465	-0.00016	-0.00016
			GROUP 1,4			
1 4-2 0 0 0	-0.00004	-0.00004	-0.00004	193.555	-0.00007	
1 4 0 0 0 0	-0.00006	-0.00005	-0.00005	195.555	-0.00009	
1 4 0 0 1 0	-0.00005	-0.00005	-0.00005	195.565	-0.00008	

Table 5(c)

Semi-diurnal tides—3rd-degree terms

	1	2	3	1900·0		
			GROUP 2,-4			
2-4 2 1 0 0	-0.00006	-0.00006	-0.00006	217.655	-0.00008	
			GROUP 2,-3			
2-3 0 2 0 0	-0.00018	-0.00018	-0.00018	225.755	-0.00027	-0.00027
2-3 2 0-1 0	-0.00003	-0.00003	-0.00003	227.545	-0.00005	
2-3 2 0 0 0	-0.00019	-0.00018	-0.00018	227.555	-0.00027	-0.00027
			GROUP 2,-2			
2-2 0 1-1 0	-0.00018	-0.00018	-0.00018	235.645	-0.00027	-0.00027
2-2 0 1 0 0	-0.00107	-0.00107	-0.00107	235.655	-0.00156	-0.00156
2-2 2-1-1 0	-0.00003	-0.00003	-0.00003	237.445	-0.00005	
2-2 2-1 0 0	-0.00020	-0.00020	-0.00020	237.455	-0.00029	-0.00029
			GROUP 2,-1			
2-1 0 0-2 0	0.00003	0.00004	0.00003	245.535	0.00005	
2-1 0 0-1 0	-0.00066	-0.00066	-0.00066	245.545	-0.00097	-0.00097
2-1 0 0 0 0	-0.00389	-0.00389	-0.00389	245.555	-0.00569	-0.00569
2-1 0 2 0 0	0.00007	0.00007	0.00007	245.755	0.00010	0.00011
2-1 2 0 0 0	0.00010	0.00010	0.00010	247.555	0.00014	0.00015
			GROUP 2,0			
2 0-2 1 0 0	0.00005	0.00005	0.00005	253.655	0.00008	
2 0 0-1-1 0	0.00004	0.00004	0.00004	255.445	0.00005	
2 0 0-1 0 0	0.00022	0.00022	0.00022	255.455	0.00032	0.00032
2 0 0 1-1 0	-0.00003	-0.00003	-0.00003	255.645	-0.00005	
2 0 0 1 0 0	0.00059	0.00059	0.00059	255.655	0.00086	0.00086
2 0 0 1 1 0	0.00011	0.00011	0.00011	255.665	0.00016	0.00016
2 0 2-1 0 0	0.00011	0.00011	0.00011	257.455	0.00017	0.00017
			GROUP 2,1			
2 1 0 0-1 0	-0.00021	-0.00021	-0.00021	265.545	-0.00031	-0.00031
2 1 0 0 0 0	0.00359	0.00359	0.00359	265.555	0.00525	0.00525
2 1 0 0 1 0	0.00068	0.00068	0.00068	265.565	0.00099	0.00099
			GROUP 2,2			
2 2-2 1 0 0	0.00004	0.00004	0.00004	273.655	0.00005	
2 2 0-1 0 0	0.00019	0.00019	0.00019	275.455	0.00028	0.00029
2 2 0-1 1 0	0.00004	0.00004	0.00004	275.465	0.00005	
			GROUP 2,3			
2 3-2 0 0 0	0.00004	0.00004	0.00004	283.555	0.00006	
2 3 0 0 0 0	0.00033	0.00033	0.00033	285.555	0.00048	0.00048
2 3 0 0 1 0	0.00021	0.00021	0.00021	285.565	0.00031	0.00031
2 3 0 0 2 0	0.00004	0.00004	0.00004	285.575	0.00006	
			GROUP 2,4			
2 4 0-1 0 0	0.00005	0.00005	0.00005	295.455	0.00008	

Table 5(d)

Ter-diurnal tides—3rd-degree terms

	1	2	3	1900·0		
		GROUP 3,−2				
3−2 0 2 0 0	0.00036	0.00037	0.00037	335.755	−0.00057	−0.00056
3−2 2 0 0 0	0.00037	0.00037	0.00037	337.555	−0.00057	−0.00057
		GROUP 3,−1				
3−1 0 1−1 0	−0.00012	−0.00012	−0.00012	345.645	0.00018	0.00018
3−1 0 1 0 0	0.00210	0.00210	0.00210	345.655	−0.00326	−0.00326
3−1 2−1 0 0	0.00039	0.00039	0.00039	347.455	−0.00061	−0.00061
		GROUP 3,0				
3 0−2 2 0 0	−0.00005	−0.00005	−0.00005	353.755	0.00007	
3 0 0 0−1 0	−0.00043	−0.00043	−0.00043	355.545	0.00067	0.00066
3 0 0 0 0 0	0.00765	0.00765	0.00765	355.555	−0.01188	−0.01188
		GROUP 3,1				
3 1−2 1 0 0	−0.00011	−0.00011	−0.00011	363.655	0.00017	0.00017
3 1 0−1 0 0	−0.00043	−0.00043	−0.00043	365.455	0.00067	0.00067
3 1 0 1 0 0	0.00016	0.00016	0.00016	365.655	−0.00025	−0.00025
3 1 0 1 1 0	0.00007	0.00007	0.00007	365.665	−0.00011	−0.00011
		GROUP 3,2				
3 2 0 0−1 0	−0.00004	−0.00004	−0.00004	375.545	0.00006	
3 2 0 0 0 0	0.00100	0.00100	0.00100	375.555	−0.00155	−0.00155
3 2 0 0 1 0	0.00044	0.00044	0.00043	375.565	−0.00068	−0.00068
3 2 0 0 2 0	0.00005	0.00005	0.00005	375.575	−0.00007	

† See comment in text

Expansion of the radiational potential

The radiational potential was introduced by W. H. Munk to account for motions of tidal nature which are caused directly or indirectly by the Sun's radiation. Such motions dominate the atmospheric tides, and they are also detectable in the ocean. Since response-type analyses often include coefficients of the radiational potential, it is desirable to know their harmonic amplitudes to add to the gravitational tides.

If α is the Sun's zenith angle at the place (θ, λ) the potential is defined in the present notation as

$$\Psi = S\xi \cos\alpha \text{ for } 0 \leqslant \alpha \leqslant \tfrac{1}{2}\pi \text{ (day)},$$

or

$$0 \text{ otherwise (night).} \tag{23}$$

where S is the solar constant, taken as the unit. Expansion in Legendre polynomials, ignoring the parallax Π' in comparison with unity, gives

$$\Psi = S\xi(\tfrac{1}{4}+\tfrac{1}{2}P_1(\cos\alpha)+\tfrac{5}{16}P_2(\cos\alpha)-\tfrac{3}{32}P_4(\cos\alpha)+\ldots). \tag{24}$$

P_3 does not appear because odd order terms other than P_1 contain the factor Π'. The series (24) differs from the gravitational formula (8) mainly in the appearance of P_1, which is due to the day–night asymmetry of (23), and in the different powers of ξ†, which alters the fine structure in the tidal Groups.

The harmonics of 1st degree arising from P_1 contain strong lines at the seasonal annual Sa and daily S_1 frequencies, which do not strictly appear in the gravitational expansion, although it has some close minor terms depending on the solar anomaly (non zero k_6). The harmonics of 2nd degree occupy the same frequencies as the corresponding solar gravitational terms but can be distinguished in long quiet records by the absence of lunar effects. Cartwright (1966) found the radiational content of S_2 of several records of sea level to average 18 per cent of the gravitational content.

The time harmonics from P_1 and P_2, listed in Table 6, were derived from (24) by algebraic expansion, which is fairly easy in the case of the Sun, using equations (9), (10) and (11), and the relations (for $\beta' = 0$):

$$\cos\Theta' = \sin(L'+\delta L')\sin\varepsilon,$$

$$\cos\Lambda' \sin\Theta' = \cos\tau' \cos(L'+\delta L')+\sin\tau' \sin(L'+\delta L')\cos\varepsilon$$

$$\tau' = (f_1+f_2)\,t+\pi,$$

$$\delta L' = 2e' \sin l' + 0(e'^2),$$

$$\xi' = 1+e' \cos l' + 0(e'^2).$$

Since there is no call for great accuracy here, only the first power of e' was retained in the expansions, and the numerical values of e' and ε were taken at the epoch 1950·0 (equations (5) and (7) with $T = 0{\cdot}5$). Omission of terms in e'^2 limits the accuracy to about $\pm 0{\cdot}0020$. All coefficients in Table 6 were confirmed to this accuracy by comparison with spectral analyses of 3-year time series.

The possible relevance of P_4 in (24) to the radiational tide has not been ascertained.

† G. W. Groves and H. G. Loomis (unpublished MS) have experimented with a radiational function $\propto \xi^2$.

Table 6

Radiational potential

1	2	3	1900·0
1ST DEGREE	GROUP 0,0	2ND DEGREE	
		0 0 0 0 0 0	-0.18894
0 0 0 0 0 1	-0.00341	0 0 1 0 0-1	-0.00316
0 0 1 0 0 0	0.40694	0 0 1 0 0 1	0.00148
0 0 2 0 0-1	0.01022	0 0 2 0 0 0	-0.05879
0 0 3 0 0-2	0.00024	0 0 3 0 0-1	-0.00246
1ST DEGREE	GROUP 1,1	2ND DEGREE	
		1 1-3 0 0 1	0.00967
		1 1-2 0 0 0	0.23140
1 1-2 0 0 1	-0.03482	1 1-1 0 0-1	-0.00581
1 1-1 0 0 0	-1.38710	1 1-1 0 0 1	-0.00185
1 1 0 0 0-1	0.01161	1 1 0 0 0 0	-0.22144
1 1 0 0 0 1	0.00050	1 1 1 0 0-1	-0.00185
1 1 1 0 0 0	-0.05969	1 1 1 0 0 1	0.00025
1 1 2 0 0-1	-0.00150	1 1 2 0 0 0	-0.00996
		1 1 3 0 0-1	-0.00042
	GROUP 2,2	2ND DEGREE	
		2 2-3 0 0 1	0.02333
		2 2-2 0 0 0	0.55741
		2 2-1 0 0-1	-0.01400
		2 2-1 0 0 1	0.00040
		2 2 0 0 0 0	0.04800
		2 2 1 0 0-1	0.00040
		2 2 2 0 0 0	0.00103

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