Skewness as a test of non-Gaussian primordial density fluctuations

Peter Coles,1 Lauro Moscardini,2,3 Francesco Lucchin,2 Sabino Matarrese4 and Antonio Messina5,6

1Astronomy Unit, School of Mathematical Sciences, Queen Mary & Westfield College, Mile End Road, London E1 4NS
2Dipartimento di Astronomia, Università di Padova, vicolo dell’Osservatorio 5, I-35122 Padova, Italy
3Astronomy Centre, University of Sussex, Falmer, Brighton BN1 9QH
4Dipartimento di Fisica G. Galilei, Università di Padova, via Marzolo 8, I-35131 Padova, Italy
5Centre Européen de Calcul Atomique et Moleculaire, 91405 Orsay CEDEX, France
6Dipartimento di Fisica A. Righi, Università de Bologna, via Irnerio 46, I-40126 Bologna, Italy

Accepted 1993 April 20. Received 1993 March 8; in original form 1992 December 10

ABSTRACT
We investigate the evolution of the skewness of the distribution of density fluctuations in CDM models with both Gaussian and non-Gaussian initial fluctuations. We show that the skewness of galaxy counts is a potentially powerful test of the hypothesis of Gaussian primordial density fluctuations. We find, as expected, that the skewness of the mass distribution in models with initially non-Gaussian fluctuations shows systematic departures from the corresponding behaviour for Gaussian fluctuations on intermediate to large scales. We investigate the effect of peculiar velocity distortions and normalization upon the relationship between skewness and variance. These effects are generally small for the models we consider. Comparing our results to the QDOT measurements of the skewness, we find that our initially positive-skew models are clearly excluded by this analysis, but the available data do not rule out the negative-skew models.

Key words: galaxies: clustering – galaxies: formation – dark matter – early Universe – large-scale structure of Universe.

1 INTRODUCTION
The gravitational instability picture of galaxy formation is coming under increasing pressure from new observational data. In particular, models of structure formation involving dark matter – either hot (HDM) or cold (CDM) – have been found wanting in the light of new information on galaxy clustering and the cosmic microwave background radiation (CMBR). The ‘standard’ versions of both HDM and CDM incorporate the assumption that present-day structures grew by gravitational instability from small, primordial, Gaussian-distributed adiabatic perturbations with the scale-invariant Zel’dovich spectrum. Both these ‘standard’ cosmogonies seem, however, to be unable to account for all the observational constraints. As far as galaxy clustering data are concerned, HDM succeeds in reproducing the amount of clustering on very large scales but fails to account for the age of galaxies, and galaxies must be much less clustered than the mass on small scales to reproduce known clustering properties; CDM, on the other hand, reproduces small-scale structures better but suffers from general lack of power on large scales. In particular, the large-scale problem of CDM is indicated by a number of independent statistical tests, such as counts of galaxies in cells (Efstathiou et al. 1990; Saunders et al. 1991), the spatial two-point correlation function of rich clusters (Batuski, Melott & Burns 1987), the angular correlations of projected galaxy distributions (Maddox et al. 1990) and the power spectrum of the distribution of optical and radio galaxies (Peacock 1991).

In recent months, the COBE detection of large-scale temperature anisotropy in the CMBR has given independent information about the normalization of fluctuations in these models, and about the shape of the power spectrum on very large scales (Smoot et al. 1992). This poses a particular problem for CDM, since the normalization implied by COBE is rather high for the standard model. Normalized to match the COBE amplitude, CDM does quite well on large scales but has too high an amplitude on small scales, resulting in runaway clustering and very high peculiar velocities (Davis et al. 1985). Attempts to escape from these difficulties by a straightforward change in either the amplitude or the shape of the primordial power spectrum (e.g. Bardeen, Bond & Efstathiou 1987; Vittorio, Matarrese & Lucchin 1988) are strongly constrained, not only by COBE but also by other
observational limits on temperature anisotropies on the microwave background sky (Bond et al. 1991; Vittorio et al. 1991). Nevertheless, it is possible to overcome the COBE constraints by invoking a power-law inflationary model (Abbott & Wise 1984; Lucchin & Matarrese 1985a,b) that produces almost, but not quite, scale-invariant density fluctuations but also produces tensor perturbations (i.e. gravitational waves) with large amplitude (Davis et al. 1992; Liddle & Lyth 1992; Lidsey & Cole 1992; Lucchin, Matarrese & Mollerach 1992; Salopek 1992b). Such a model can allow a lower amplitude of density fluctuations to be compatible with the COBE anisotropy and can thus reconcile small-scale clustering with the CMBR anisotropy.

Another possible escape route from these constraints is to assume that the relation between galaxies and mass is different on different mass-scales: a scale-dependent bias. Given how little we know about galaxy formation this seems a reasonable choice, and specific models have been constructed that can alleviate the large-scale structure problem for CDM without damaging its small-scale successes (Babul & White 1991; Bower et al. 1993). The problem with these models is that they need to invoke collective physical processes acting on a very large scale (\(\sim 30~\text{h}^{-1}~\text{Mpc}\)), and feedback mechanisms acting on such large scales are hard to find. The need to introduce a physical scale into the problem is demonstrated by Cole (1993): any local biasing effect acting upon Gaussian density fluctuations cannot change the slope of the galaxy correlation function with respect to the mass autocorrelation function.

It is clear that many of the problems with large-scale structure theory can be traced back to the assumption of random-phase fluctuations within the gravitational instability model. Can we construct a self-consistent model for galaxy formation based upon non-Gaussian fluctuations? Within the inflatonary picture of the origin of perturbations, this issue has been discussed by a number of authors (e.g. Matarrese, Ortolan & Lucchin 1989; Barrow & Cole 1990; Kofman et al. 1991). The actual possibility of obtaining phase correlations on cosmologically relevant scales is restricted to multiple scalar field models (e.g. Allen, Grinstein & Wise 1987; Salopek, Bond & Bardeen 1989; Salopek & Bond 1991; Fan & Bardeen 1992; Salopek 1992a). Alternatively, non-Gaussian fluctuations can be produced by a discrete, random distribution of seed masses, such as topological defects like monopoles, cosmic strings or textures, provided by a phase transition in the early Universe (Turok 1989; Park, Spergel & Turok 1991; Scherrer & Bertschinger 1991; Scherrer 1992). An alternative way to obtain non-random phases is to invoke the cosmic explosion scenario (Ikeuchi 1981; Ostriker & Cowie 1981), where hydrodynamics rather than gravity plays the main role in the structure formation process.

A typical (although not mandatory) signature of non-Gaussian density fluctuations, \(\delta_{nG}\), is an initially non-vanishing skewness \(\langle \delta_{nG}^3 \rangle \neq 0\). Actually, according to the analysis of the QDOT IRAS-selected data by Saunders et al. (1991), a positive skewness of the distribution of IRAS galaxy counts is observed on quite large scales. A number of recent papers debate the issue of whether such a positive skewness has a primordial origin or is due simply to the non-linear, aggregating action of gravity on a primordial (i.e. unskewed) Gaussian field (e.g. Coles & Frenk 1991, hereafter CF91; Martel & F Preisler 1991; Park 1991; Silk & Juszkiewicz 1991; Bouchet & Hernquist 1992; Bouchet, Davis & Strauss 1992a; Bouchet et al. 1992b; Juszkiewicz & Bouchet 1992; Juszkiewicz, Bouchet & Colombi 1993; Lahav et al. 1993). N-body simulations (Matarrese et al. 1991; Moscardini et al. 1991, hereafter MMLM; Weinberg & Cole 1992) have shown that the primordial skewness is a strongly discriminating parameter in determining both the dynamics and the present clustering properties of the Universe. MMLM studied the origin of large-scale structures in skewed CDM models, while Weinberg & Cole (1992) considered non-Gaussian models obtained using a local non-linear transformation on scale-free Gaussian fields. These studies also showed that there are many similarities between skew-positive models and, e.g., the texture-seeded CDM model (Park, Spergel & Turok 1991; Cen et al. 1991). Actually, scenarios based on accreting HDM or CDM around seed masses always produce an excess of overdense regions. Cosmic explosions (e.g. Weinberg, Ostriker & Dekel 1989), and bubbles left over from a period of extended inflation in the early Universe (La & Steinhardt 1989a,b; Liddle & Wand 1991) give rise to an excess of low-density regions, resembling the initial conditions of primordial skew-negative models.

The initial skewness of non-Gaussian models is therefore a strong indicator of their clustering behaviour. As we have mentioned above, however, this primordial skewness is masked to some extent by the effect of gravitational evolution which generally tends to couple the skewness to the variance, which increases with time. Can we disentangle these two possible causes of skewness and use the skewness of the present-day distribution as a test of the hypothesis of Gaussian fluctuations? CF91 and, independently, Silk & Juszkiewicz (1991) have argued, on the basis of both analytical calculations and N-body experiments, that the answer to this question is yes, and they proposed a simple test for primordial non-Gaussianity based on the skewness of galaxy counts, which should be a robust test of initial conditions (see also Bouchet et al. 1992; Juszkiewicz & Bouchet 1992). It is the purpose of this paper to determine the power of this test relative to various non-Gaussian alternatives. To this end we have used the results of N-body simulations with both Gaussian and non-Gaussian initial conditions (Messina et al. 1992; Lucchin et al. 1993) that represent the Universe on a cube of side 260 \(\text{h}^{-1}~\text{Mpc}\). We have already used these simulations in a study of the large-scale topology of the Universe (Coles et al. 1993, hereafter CMPLMM), which is an alternative way of testing the Gaussian hypothesis (for a review, see Melott 1990).

2 N-BODY SIMULATIONS WITH NON-GAUSSIAN INITIAL CONDITIONS

The non-Gaussian statistics considered here are the same as adopted by MMLM, namely the lognormal (hereafter LN) and the chi-squared with one degree of freedom (hereafter \(\chi^2\)), which are chosen as distributions for the peculiar gravitational potential, \(\Phi\), before the modulation by the CDM.
transfer function. These distributions actually split into two
different types of model: the positive (LN, and $\chi^2$) and
negative (LN, and $\chi^2$) models, classified according to the sign
of the skewness for linear mass fluctuations, $\langle \delta M \rangle > 0$ for the
former and $\langle \delta M \rangle < 0$ for the latter.

We have to restrict the parameter space we analyse in
a way that $\Phi$ has the ‘standard’ CDM power spectrum

$$\mathcal{P}_\Phi(k)=\frac{9}{4}P_0 k^{-3}T^2(k),$$

(2.1)

where $\mathcal{P}_0$ is a normalization constant and $T(k)$ is the CDM
transfer function

$$T(k)=(1+6.8k^{72}+16.0k^{72})^{-1}$$

(2.2)

(e.g. Davis et al. 1985), assuming a flat (i.e. critical density)
universe with a Hubble constant $h=0.5$ in units of 100 km
s$^{-1}$ Mpc$^{-1}$. This choice for the spectrum allows us to make a
direct comparison with the Gaussian CDM (hereafter G)
model.

We use a particle-mesh code with $N_p=128^3$ particles on
$N_g=128^3$ grid-point (more details are given in Lucchin et al.
1993). Computations were performed at the CINECA
Center (Bologna) on a Cray YMP/432. The box size of our
simulations is $L=260 h^{-1}$ Mpc, and each particle has mass
$m=4.7\times10^{-12} M_\odot$. We run two realizations for each of
the five models considered. We evolve our models starting
from the same amplitude up to the ‘present time’ $t_0$. We define $t_0$ as
the time at which the galaxy two-point function is best fitted
by the power law $\xi(r)=(r/r_0)^{-r}$, with $\gamma=1.8$ in a suitable
interval. To obtain the galaxies in a given simulation we
proceed as follows. We filter the initial density field with a
Gaussian window function of radius $0.5 h^{-1}$ Mpc and pick
up as galaxies the particles closest to each peak, defined as
the grid-point with a positive density contrast larger than the
26 nearest grid-points: the result is a galaxy catalogue con-
taining ~ 60 000 galaxies per simulation. Due to the exceed-
ingly high mass of our particles, which is a result of the large
box size and low resolution, and to the rather simplified
Galaxy identification criterion, we can only assume that our
peak regions roughly trace the actual galaxy distribution.

Different epochs can be parametrized by the bias factor $b$
deﬁned by the rms linear mass ﬂuctuation on a sharp-edged
sphere of radius $R_s=8 h^{-1}$ Mpc, i.e.

$$\sigma^2(R_s) = \frac{P_0}{2\pi} \int_0^{\infty} \lambda B_2(k) W_{th}(kR_s) = \frac{1}{b^2},$$

(2.3)

where $W_{th}(x)=(3x)f_1(x)$ is a top-hat window function
and $f_1$ is the Bessel function of order 1. The present time $t_0$
corresponds to $b=1$ for the Gaussian model, $b=1.5$ for both
the positive models, $b=0.5$ for the negative $\chi^2$ and $b=0.4$
for the negative lognormal. Note that the method for deﬁning the
Galaxies used in this work is different from the ‘exclusion regions’
technique used in CMLMM, where a larger galaxy number density,
$3 \times 10^{-2} h^3$ Mpc$^{-3}$, was necessary in order to generate simulated Lick catalogues with ~ 530 000
galaxies in the whole simulation box. A consequence of this
change is, for example, that the present epoch, i.e. the slope
$\gamma=1.8$ for the correlation function, is reached slightly later
here than in CMLMM. Note that a fully consistent normali-
ization of mass ﬂuctuations should give CMBR ﬂuctuations in
agreement with those detected by COBE (Smoot et al. 1992),
which, for a standard CDM model, favour low values of $b$,
namely $b=0.8$. On the other hand, the statistical analysis of
CMBR anisotropies on large angular scales for non-
Gaussian models cannot be reduced to calculation of the rms
fluctuation.

The primordial gravitational potential is obtained by the
convolution of a real functional $\tau(x)$ with a random field $\varphi(x)$,

$$\Phi(x) = \int d^3y \tau(y-x) \varphi(y).$$

(2.4)

The field $\varphi$ is obtained by non-linear transformation on a
zero-mean Gaussian process $w$, with unit variance and
flicker-noise power spectrum; the function $\tau$ is ﬁxed by its
Fourier transform,

$$\tilde{\tau}(k) = \int d^3x e^{-ik \cdot x} \tau(x) = T(k) \tilde{F}(k),$$

(2.5)

where $T(k)$ is the CDM transfer function of equation (2.2)
and $\tilde{F}(k)$ is a positive correction factor which we applied to have
the exact CDM initial power spectrum of equation (2.1)
in all our models. The precise forms of the non-linear trans-
formations from $w$ to $\varphi$ are $\varphi(x) \propto e^{w/\sqrt{2}}$ and $\varphi(x) \propto w^2(x)$ for
LN and $\chi^2$ respectively (Coles & Barrow 1987; Coles &
Jones 1991; MMLM).

As MMLM have shown, both the clustering dynamics and
the present large-scale structure depend strongly upon the
sign of the primordial skewness: positive models rapidly
cluster to a lumpy structure with small coherence length,
while negative models build up a cellular structure by the
slow process of merging of shells around primordial under-
dense regions, with large coherence length. The general
conclusion of these previous studies is that, of the non-
Gaussian alternatives considered, the skew-negative models are
the more successful at reproducing the observed properties of
the large-scale structure. Indeed, CMLMM showed that
very strong constraints on non-Gaussian models of the types
considered here can be imposed by the topology test: only
Gaussian and skew-negative models survive the rigours of
such an analysis.

3 THE SKEWNESS TEST

CF91 described the physical motivation behind the use of
the skewness of cell counts as a diagnostic of large-scale
structure, so we just outline the basics here. Consider the
density contrast smoothed on a certain scale: $\delta_M(R)$. In terms
of the distribution of $\delta_M(R)$, called $f_R(\delta_M)$, we can define
moments as follows:

$$\langle |\delta_M(R)|^n \rangle = \int f_R(\delta_M) \delta_M(R)^n d\delta_M(R).$$

(3.1)

Clearly, $\langle \delta_M \rangle = 0$; the quantity $\langle \delta_M(R)^2 \rangle = \sigma_M(R)$ is the
variance of the smoothed mass density fluctuations and $\langle \delta_M(R) \rangle$ is the skewness, denoted $\gamma_M(R)$. A variety of
analytical approximation methods suggest that, for initially
Gaussian density perturbations, $\gamma_M$ grows according to

$$\gamma_M(R) = \pm \sigma_M(R),$$

(3.2)
where $S=3$ is roughly constant, i.e. almost independent of the scale upon which $\delta_M$ is smoothed, the background cosmology and the power spectrum of the primordial fluctuations (CF91 and references therein). Various authors have extended the arguments given by CF91 and have demonstrated that $S$ is actually a weak function of $R$ and the primordial power spectrum (Martel & Freudling 1991; Park 1991; Silk & Juszkiewicz 1991; Bouchet et al. 1992b; Juszkiewicz & Bouchet 1992; Juszkiewicz et al. 1993; Lahav et al. 1993). Because all our non-Gaussian models have the same initial power spectrum and we use the same kind of smoothing function throughout this work, we can ignore the dependence of $S$ upon these variables in this particular paper. Bouchet et al. (1992b) and Lahav et al. (1993) have also shown that the effects of redshift-space distortion – disregarded by CF91 – should be weak in the interesting regime. The test suggested by CF91, that one should plot $\gamma$ against $\sigma^2$ for different smoothing scales and look for departures from linearity, is therefore confirmed as being a potentially powerful test of non-Gaussian primordial density fluctuations (CF91 found the QDOT data of Saunders et al. (1991) to be consistent with Gaussian initial fluctuations) and may be sensitive enough to rule out viable non-Gaussian scenarios. Indeed, Silk & Juszkiewicz (1991) argued that $\gamma \propto \sigma^3$ for the cosmic textures model, which seems to be incompatible with the QDOT data. We shall see whether the non-Gaussian models described in Section 2 are also incompatible with the data.

There are two main problems when it comes to applying this test in practice. First, the discrete nature of number counts of galaxies itself introduces a skewness term into the cell-count distribution. Provided that one accepts that the galaxy counts correspond to a Poisson 'shot-noise' effect, then one can easily correct for the discreteness terms (see below for a discussion). Secondly, most models of galaxy formation involve some degree of bias in the ratio of luminous galaxies to mass. An arbitrary functional bias of the form discussed by Coles (1993) could seriously interfere with the skewness test. CF91 showed, using $N$-body simulations, that the standard 'high-peak' biasing scenario does in fact produce a galaxy distribution with second- and third-order moments scaling in the same way as (3.2). Nevertheless, different biasing models might produce different behaviours, since an arbitrary bias is in some senses equivalent to having non-Gaussian fluctuations.

To check the power of the skewness test in the light of these difficulties, we shall compare the effects of gravitational evolution on the skewness of the Gaussian and non-Gaussian models described in Section 2, for different clustering amplitudes and different levels of bias.

4 RESULTS

It is a relatively straightforward matter to extract estimates, $\hat{\Gamma}(R)$ and $\hat{\Sigma}(R)$, of the skewness and variance of cell counts in cells of different size $R$ from the simulations described in Section 2. For a large number, $N$, of cells we have

$$\hat{\Sigma}^2 = \frac{1}{N} \sum_{i=1}^{N} \frac{(n_i - \bar{n})^2}{\bar{n}^2} \quad (4.1a)$$

and

$$\hat{\Gamma} = \frac{1}{N} \sum_{i=1}^{N} \frac{(n_i - \bar{n})^3}{\bar{n}^3} \quad (4.1b)$$

where $n_i$ is the number of particles in the $i$th cell and $\bar{n}$ is the mean number per cell, i.e. $\bar{n} = \sum_{i=1}^{N} n_i / N$. Cell counts are inevitably skewed by virtue of their discrete (integer-valued) nature. To correct for the discreteness terms, one usually subtracts 'Poisson' terms from the estimates, to give estimates of $\sigma^2$ and $\gamma$ that refer to a continuously distributed variable:

$$\delta^2 = \hat{\Sigma}^2 - \frac{1}{\bar{n}} \quad (4.2a)$$

$$\hat{\gamma} = \hat{\Gamma} - 3 \delta^2 - \frac{1}{\bar{n}} \quad (4.2b)$$

(Peebles 1980; Saunders et al. (1991); CF91; Lahav et al. 1993). Possible reasons why this might not be an appropriate scheme, particularly if galaxy formation is in some sense cooperative, are discussed by CF91 (see also Bower et al. 1993). To check whether this correction preserves the shape of the $\gamma - \sigma^2$ relation (3.2), we look at both $\hat{\gamma} - \sigma^2$ (i.e. corrected) and $\hat{\Gamma} - \hat{\Sigma}^2$ (i.e. uncorrected) relationships. We need to assign confidence limits to our estimates in order to assess the significance of departures from the predicted behaviour. Approximate methods for placing error limits on the empirically determined estimates are discussed by Saunders et al. (1991), but these involve a complicated iterative procedure involving high-order moments. We can make a rough estimate of the error following Kendall & Stuart (1977). Suppose that each simulation consists of a random sample of $N$ taken from a Gaussian parent distribution with variance $\sigma^2$. The variances from sample to sample of estimates of $\gamma$ and $\sigma^2$ in such a case are $6\sigma^4/N$ and $2\sigma^4/N$ respectively. If we take $\sigma^2 = \hat{\sigma}^2$ then we can place approximate standard errors, $s$, on the estimates $\hat{\gamma}$ and $\hat{\sigma}^2$ as

$$s(\hat{\gamma}) = \frac{\sigma^2}{\sqrt{N}} \quad (4.3a)$$

$$s(\hat{\sigma}^2) = \frac{\sigma^4}{\sqrt{N}} \quad (4.3b)$$

Of course, our simulations are not random samples from a Gaussian parent; the cells are correlated and the distributions are non-Gaussian. We have also taken the sample variance and the parent population variance to be identical. The estimates (4.3) can be expected to give only a rough order-of-magnitude estimate of the probable confidence limits. We can also calculate error limits by using the two different simulations of each model to calculate an estimate of the ensemble variance analytically. This is still not completely satisfactory – ideally we would wish for many more simulations – but gives results in reasonable accord with the analytic estimates. We find that the estimated errors (4.3a,b) exceed the spread of the simulations on large scales by about 50 per cent, whereas the two estimates agree on intermediate scales. On small scales, our estimated errors (4.3a,b) are too small because of a systematic effect due to the resolution in our simulations (see below).
The simulations also allow us to investigate (i) the effect of the normalization of the model upon the skewness–variance relationship; (ii) whether the redshift–space relationship is significantly different from that in real space; (iii) whether the relationship for galaxies identified in the manner described in Section 2 is different from that for the dark matter particles; and (iv) whether the observed QDOT points rule out any of these models. Our results are displayed in Figs 1–3.

In Fig. 1(a) we show the $\Gamma - \Sigma^2$ (i.e. uncorrected) relationship for dark matter particles (left) and galaxies (right). The solid line in each panel is the theoretical relationship between skewness and variance. The downward-pointing arrows in the right panel indicate the corrected skewness, while the upward-pointing arrows indicate the uncorrected skewness. All models are normalized to the present time (see Section 2). The three crosses are the QDOT measurements with error bars. Error bars on the simulated results are estimated using equations (4.3a,b).

Figure 1. The relationship between skewness, $\gamma$, and variance, $\sigma^2$, for our five models. (a) shows the results without shot-noise correction; (b) has shot-noise corrections. The solid line is the theoretical relationship $\Gamma (\Sigma^2)$. The downward-pointing arrows in (b) indicate that the corrected skewness is negative; the arrows are plotted at the corrected variance value and originate at the uncorrected skewness value. All models are normalized to the present time (see Section 2). The three crosses are the QDOT measurements with error bars. Error bars on the simulated results are estimated using equations (4.3a,b).
our particle code does not describe the fluid nature of the matter very well and this results in large systematic discreteness effects which are not well described by the random error terms given in equations (4.3a,b). These systematic errors occur because the cell size for forming the cell counts

Figure 2. The effect of normalization upon the skewness-variance relationship. (a) shows all models normalized to \( b = 1 \) and (b) shows the normalization \( b = 2.5 \). The solid line is the theoretical relationship (3.2). All points are uncorrected for shot-noise. The small downward arrows on the theoretical line indicate that the uncorrected skewness is negative at that point. The three crosses are the QDOT measurements with error bars. Error bars on the simulated results are estimated using equations (4.3a,b).
gravity acts to increase the skewness further. The trend is somewhat less clear for the negative models. All such models develop positive skewness even to very large scales, which shows that even weakly non-linear gravitational effects can wipe out the initial negative skewness. These models, however, need to be evolved for a comparatively long time in order to reach the present time. The result seems to be a much smaller systematic departure from the Gaussian expectation than for the positive-skew models. Fig. 1(b) shows the effect of using the corrected values; we show the $\tilde{\gamma} - \delta^2$ version. The trends are the same and the quantitative agreement is good, particularly on large scales, where the shot-noise terms are small anyway. Note, however, that on small scales the shot-noise correction produces a negative skewness (indicated by the downward arrow plotted at the position of the uncorrected skewness). This confirms our argument that the failure of the relationship (3.2) in these simulations is due to a resolution effect.

To consider the effect of evolution we plot, in Figs 2(a) and (b), corresponding diagrams for models that are normalized at $b = 1$ (Fig. 1a) and $b = 2.5$ (Fig. 2b). Only the uncorrected skewness and variance are shown; discreteness effects have a similar influence on all our simulations. Note that the Gaussian model still follows the relationship (3.2) accurately regardless of the value of $b$; when $b = 2.5$ the skewness on the very largest scale comes out negative but is consistent with zero within the errors. The positive-skew models seem to be closer to the Gaussian relationship for smaller $b$. The discrepancies for the negative-skew models occur in a rather less predictable pattern, and the systematic shift is less apparent.

To check the effect of redshift distortions, we have plotted both the real-space and redshift-space skewnesses and variances for all the models at the present time. The results are shown in Fig. 3. It is clear that, with the exception of very small scales where resolution effects dominate, the effect of looking at redshift space rather than real space is minimal.

We can now look at the question of whether the QDOT points place any strong constraints on any of these models. These points are plotted in Figs 1 and 2 and can be seen to lie on the expected trajectory (3.2). Although these points are on large scales from an observational point of view, they are on quite small scales compared to these simulations. Looking only at the corrected results (Fig. 1b) – because the QDOT points are in corrected form – we see that the two negative-skew models are clearly excluded at $> 2\sigma$; the two negative-skew models are, however, consistent with the observed points (as in the Gaussian).

5 DISCUSSION AND CONCLUSIONS

We have investigated the behaviour of the skewness and variance of the distributions of both dark matter and galaxies in a number of models involving non-Gaussian initial conditions.

As expected, we find that non-linear gravitational evolution always acts in such a way that the skewness increases with time. This means that models with negative initial skewness display a positive skewness on cosmologically interesting scales even after very weak evolution. It is therefore difficult to see directly the sign of the initial skewness. Nevertheless, the fact that models with different
initial skewness evolve in different ways means that their systematic trends in the behaviour of skewness against variance do remain, even into the fairly strongly non-linear regime. In particular, initially positive-skew models seem to obey a similar scaling law to the initially Gaussian models (3.2), but with a higher value of $S$. For initially negative-skew models, the situation is somewhat less clear because systematic departures from the Gaussian are smaller. Part of the reason for this must be that, to obtain models in reasonable accord with observations on small scales, the negative-skew models must be highly evolved whereas the positive-skew models are less strongly evolved (MMLM). This difference in normalization to the present epoch tends to suppress differences compared to the Gaussian; the extra evolution required by the negative-skew models generates enough skewness to bring them roughly on to the Gaussian locus in the $\gamma-\sigma^2$ plane. Some systematic differences do remain, especially on large scales, but these are generally so small as to make discrimination between models difficult. Moreover, the typical errors for the negative-skew models are somewhat larger than those of the positive-skew models. The reason for this is probably that clustering evolves in the negative-skew models by forming a quasi-cellular network of bubbles characterized by a large coherence length. Since there are relatively few of these structures in the simulations, their presence can produce large fluctuations from simulation to simulation. Structures in the positive-skew models generally have a significantly smaller coherence length, and each simulation therefore contains more typical structures than the negative-skew case and fluctuations are correspondingly less.

We have confirmed the conclusion of CF91 that the relationship (3.2) seems to be reasonably well obeyed for Gaussian models by both the dark matter and the 'galaxy' distributions, at least for our particular scheme for identifying galaxies. Generally speaking, the behaviour of galaxy and mass fluctuations is similar for all our models; the most noticeable effect is that in the negative-skew models the galaxy skewness lies closer to the Gaussian locus than does the skewness of the mass fluctuations. The fact remains, however, that such analyses, which rely only on galaxy clustering to test primordial fluctuations, do rely on a particular relationship between galaxies and dark matter. Complicated non-linear and/or non-local biasing schemes could produce very different behaviour from that described here (Coles 1993; Bower et al. 1993).

By looking at the distribution of matter and galaxies in both redshift space and real space, we have confirmed that the effect of peculiar velocity distortions on the relationship (3.2) is small for all our models. This confirms the argument given by Bouchet et al. (1992b) and Lahav et al. (1993).

In comparison with the QDOT results for skewness discussed by Saunders et al. (1991), we find that initially positive-skew models fare rather poorly and are excluded by $S > 2$. Because the negative-skew models have rather larger errors associated with them and the systematic departures from the Gaussian form are rather smaller than for the positive-skew cases, these models cannot be ruled out by the available skewness measurements. To constrain these models more strongly, we would need measurements of skewness with much smaller errors (i.e. from catalogues containing more galaxies) and preferably out to larger scales. The

analysis of the IRAS 1.2-Jy survey by Bouchet et al. (1992a, c) fulfills the first of these requirements, but not the second. They do, however, find results consistent with Gaussian initial data.

Of course we must stress that we have considered only a very small subset of the space of possible non-Gaussian models. All our models have the CDM power spectrum. Models with more (or less) large- (or small-) scale power may well behave differently. We have also chosen models with a very specific form of statistical distribution, obtained by locally transforming a Gaussian field. Galaxy and structure formation involves a complicated interaction between primordial conditions and non-linear gravitational evolution, and it would be surprising if the effects of the primordial spectrum and statistics upon the final density distribution could be separated out completely. We suspect, however [and other work seems to confirm this idea (Lahav et al. 1993; Bouchet et al. 1992b)], that the dominant influence on the skewness-variance relationship at late times is indeed the primordial skewness of the density fluctuations. We have not proved this to be true, and to do so would require us to investigate all types of plausible initial power spectrum, background cosmology and so on. Although our work is thus, in a sense, limited, it does demonstrate that the skewness of observed galaxy fluctuations is a potentially powerful probe of the initial distribution of density fluctuations.

ACKNOWLEDGMENTS

PC acknowledges the SERC for support under the QMW rolling theory grant (GR/H09454) and thanks the Dipartimento di Astronomia at the Università di Padova for its hospitality during the visit that saw the initiation of this work. LM acknowledges the Astronomy Centre at the University of Sussex for hospitality during a visit when part of this work was carried out. FL, SM, AM and LM thank the Ministero Italiano dell'Università e della Ricerca Scientifica e Tecnologica for financial support. This work has been partially supported by Consiglio Nazionale delle Ricerche (Progetto Finalizzato: Sistemi Informatici e Calcolo Parallelo). The staff and the management of the CINECA Computer Center are warmly acknowledged for their assistance and for allowing the use of computational facilities.

REFERENCES

Barrow J. D., Coles P., 1990, MNRAS, 244, 188
Bouchet F. R., Davis M., Strauss M. A., 1992a, in Mamon G. A.
Gerbal D., eds. Proc. 2nd DAEC Meeting on The Distribution of Matter in the Universe. Meudon Observatory, Paris, p. 287

© Royal Astronomical Society • Provided by the NASA Astrophysics Data System
Skewness as a test of primordial density fluctuations 757


Salopek D. S., 1992a, Phys. Rev. D, 45, 1139


Saunders W. et al., 1991, Nat, 349, 32


© Royal Astronomical Society • Provided by the NASA Astrophysics Data System