Feedback and the fundamental line of low-luminosity low-surface-brightness/dwarf galaxies

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ABSTRACT
We study in simple terms the role of feedback in establishing the scaling relations of low-surface-brightness (LSB) and dwarf galaxies with stellar masses in the range $6 \times 10^5 \leq M_\ast \leq 3 \times 10^{10} M_\odot$. These galaxies – as measured, for example, from the Sloan Digital Sky Survey (SDSS) and in the Local Group – show tight correlations of internal velocity, metallicity and surface brightness (or radius) with $M_\ast$. They define a \textit{fundamental line} which distinguishes them from the brighter galaxies of high surface brightness and metallicity. The idealized model assumes spherical collapse of cold dark matter (CDM) haloes to virial equilibrium and angular momentum conservation. The relations for bright galaxies are reproduced by assuming that $M_\ast$ is a constant fraction of the halo mass $M$. The upper bound to the low-luminosity LSBs coincides with the virial velocity of haloes in which supernova feedback could significantly suppress star formation, $V < 100$ km s$^{-1}$. We argue that the energy fed to the gas obeys $E_{SN} \propto M_\ast$ despite the radiative losses, and equate it with the binding energy of the gas to obtain $M_\ast/M \propto V^2$. This idealized model provides surprisingly good fits to the scaling relations of low-luminosity LSBs and dwarfs, which indicates that supernova feedback had a primary role in determining the fundamental line. The apparent lower bound for galaxies at $V \sim 10$ km s$^{-1}$ may be caused by the cooling barrier at $T \sim 10^4$ K. Some fraction of the dark haloes may show no stars as a result of complete gas removal, either by supernova winds from neighbouring galaxies or by radiative feedback after cosmological reionization at $z_{ion}$. Radiative feedback may also explain the distinction between dwarf spheroidals (dE) and irregulars (dI), where the dEs, typically of $V \leq 30$ km s$^{-1}$, form stars before $z_{ion}$ and are then cleaned out of gas, whereas the dIs, with $V > 30$ km s$^{-1}$, retain gas-rich discs with feedback-regulated star formation.

Key words: stars: winds, outflows – supernova remnants – galaxies: dwarf – galaxies: formation – galaxies: fundamental parameters – Local Group.

1 INTRODUCTION
The galaxies can be crudely divided into two main classes based on their location in the plane of surface brightness versus luminosity or stellar mass $M_\ast$ [see, for example, fig. 1 in Dekel & Silk (1986, hereafter DS), fig. 7a of Kauffmann et al. (2003b, hereafter K03) and other references cited below]. The bright galaxies, dominated at the bright end by ellipticals and early-type spirals, have relatively high surface brightnesses which are only weakly correlated with stellar mass. The fainter galaxies, spanning the broad range from relatively bright late-type spirals all the way down to the Local Group dwarf galaxies, have their conditional average surface brightness at a given $M_\ast$ decrease with decreasing $M_\ast$. The \textit{transition} occurs near $M_\ast \sim 3 \times 10^{10} M_\odot$ (corresponding to absolute magnitudes of about $-20.8$ and $-19.0$ in the r and b bands, respectively). Other global galaxy properties, such as the mean metallicity, behave in a similar manner as a function of stellar mass, with the transition seen at a similar characteristic scale (see references below). We refer to these two general classes hereafter as HH versus LL galaxies, standing for high-luminosity high-surface-brightness (HSB) galaxies versus low-luminosity low-surface-brightness (LSB) and dwarf galaxies. For the purpose of our current idealized theoretical modelling, we distinguish simply between these two coarse types based on the $M_\ast$ transition scale.\textsuperscript{1} This kind of classification can be traced

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\textsuperscript{1}LSBs are sometimes defined in the literature by having central blue surface brightness $> 22$ mag arcsec$^{-2}$, 1σ from Freeman's (1970) mean value for bright HSB spirals ($21.65 \pm 0.35$), and sometimes by being $> 23$ mag
back, for example to Binggeli, Sandage & Tarenghi (1984), Wirth & Gallagher (1984), Kormendy (1985) and Hoffman et al. (1985). Dekel & Silk (1986) have highlighted this classification scheme (their fig.1) in the context of their early theoretical modelling (their fig. 6).

1.1 HH and LL galaxies

The analysis by K03 of 80 000 galaxies from the Sloan Digital Sky Survey (SDSS) highlight the bivariate distribution of relatively bright galaxies in the plane of surface brightness and stellar mass, above their claimed completeness limits of absolute r magnitude of −17 and effective surface brightness of 23 mag arcsec−2. With the spectral information available for SDSS galaxies, their stellar masses can be evaluated more reliably than before using population-synthesis models (Kauffmann et al. 2003a). The transition scale shows very clearly at $M_*=3 \times 10^{10} M_\odot$ (K03, fig. 7a). The bright galaxies in the range $3 \times 10^{10} < M_* < 10^{12} M_\odot$ have their effective surface brightnesses scattered about a mean value of $\mu_* \sim 10^{2} M_\odot$ kpc−2 (referring to the mean surface brightness within the half-light radius; the central surface brightness is typically larger by a factor of ~3), with only a weak systematic trend of roughly $\mu_* \propto M_*^{0.6}$. (When viewed as a function of luminosity, the surface brightness is actually decreasing slowly with luminosity in this range, because the stellar mass-to-light ratio is increasing — see Blanton et al. 2003.)

On the other hand, the correlation at the top part of the LL regime, $10^{6} < M_* < 3 \times 10^{10} M_\odot$, is well fit by $\mu_* \propto M_*^{0.6}$ (or even slightly steeper). A similar correlation, with a slope of $0.6-0.7$, is found in the top LL regime between surface brightness and i absolute magnitude based on 144 609 SDSS galaxies (Blanton et al. 2003), indicating that the translation to stellar mass by K03 makes a negligible difference in this regime. A consistent correlation is measured from other samples of galaxies as well: for example, Cross et al. (2001) find a slope of $0.42$ in the 2dF survey over the whole range of galaxies brighter than $M_0 \sim -16$, de Jong & Lacey (2000) measure a slope of 0.5 for Sdm galaxies, Driver (1999) finds a slope of 0.67 for a sample of fainter galaxies in the Hubble Deep Field, and Ferguson & Binggeli (1994) reported a slope of 0.7 for Virgo dwarf galaxies. The same kind of correlation can even be seen in fig. 4 of Bothun, Impey & McCaugh (1997) based on their samples of LSB galaxies, independent of the fact that they measure a larger scatter. When one translates this figure into a distribution of surface brightness at a given luminosity (lines parallel to the dashed line), one finds that the conditional average surface brightness at a given luminosity below a certain characteristic luminosity seems to be decreasing with decreasing luminosity (by about 1.5 magn across this sample, though it is hard to quantify this trend given the possible selection effects).

The spread in surface brightness at a given luminosity, which is not directly relevant for our theoretical analysis in the current paper, is a matter of debate among the observers. The uniformly selected SDSS data show a relatively tight distribution about the mean relation in the $\mu_*-M_*$ plane, both above and below the transition scale (see K03, figs 7 and 10). We learn, for example, that low-luminosity galaxies with high surface brightness, such as M32, seem to be rare.

If the photometric completeness limit of the SDSS data in r is indeed below 23.0 mag arcsec−2, then, for a given $M_*$ near $M_* \sim 10^{9} M_\odot$, these data indicate a significant drop in the galaxy count as a function of decreasing surface brightness.2 A similar conclusion, of a dearth of high-luminosity galaxies with low surface brightness, is obtained from the 2dF survey by Cross et al. (2001). On the other hand, there are claims in the literature for a significant population of low surface brightness galaxies with high luminosities (e.g. Disney 1976; Phillips, Davies & Disney 1988, 1990; Davies et al. 1994; Impey et al. 1996; Bothun et al. 1997; Sprayberry et al. 1997; Davies, Impey & Phillips 1999; O’Neil & Bothun 2000).3 The SDSS and the 2dF results argue that this is either a smaller population, or a separate population that occupies a different locus in the $\mu_*-M_*$ plane, below the surface-brightness limits of these surveys. Independently of this ongoing dispute, we seek in the current paper theoretical understanding for the general correlation between average surface brightness and stellar mass, as indicated by the SDSS and the other data sets across five decades in $M_*$, from the transition scale to the smallest dwarfs. As mentioned in Section 8, the scatter in surface brightness at a given luminosity may be affected by other physical processes not studied here in detail – the obvious one being angular momentum (e.g. Dalcanton, Spergel & Summers 1997).

We notice in passing that, given the observed Schechter luminosity function and the transition at $M_* \sim 3 \times 10^{10} M_\odot$ (about a factor or 2 below the mass corresponding to Schechter’s characteristic $L_*$), more than 95 per cent of the galaxies are below the transition scale, whereas most of the light still comes from the bright galaxies above this scale. In terms of mass, if the mass function of haloes is similar to that predicted for the cold dark matter (ΛCDM) cosmology by simulations or Press–Schechter-like approximations, then the vast majority of the virialized mass is in haloes of LL galaxies. Clearly, this motivates a major theoretical effort aimed at understanding the origin of the low-luminosity LSBs.

At least two additional independent relations, beyond the $\mu_*-M_*$ relation, are apparent in the SDSS data of the relatively bright galaxies. The second, based on preliminary reports (Tremonti et al., in preparation), is a reconfirmation of a scaling relation involving the metallicity, Z, of roughly $Z \propto M_*^{0.6}$ at the high end of the LL regime, turning into no significant correlation between $Z$ and $M_*$ in the HH regime, $Z \approx$ const. Similar gradients in gas metallicity have been seen before in other samples of LSBs (e.g. McGaugh 1994; de Blok & McGaugh 1997).

The third, which is typically the tightest correlation obeyed by galaxies, is between their luminosity and the characteristic velocity

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1 Blanton et al. (in preparation) provide further evidence for their completeness down to below 23.0 by showing that when the galaxies are binned according to Galactic extinction $E$, the peak of the apparent surface brightness distribution shifts accordingly, from about 21.2 at $E=0$ to about 23.0 at $E=1.8$ (a test first applied by Davies et al. 1993 to the European Southern Observatory’s ESO-Uppsala catalogue).

2 O’Neil & Bothun argue for a population of luminous LSBs based on their finding that the surface brightness distribution function is flat down to below a central blue value of 24 − down to the survey completeness limit. They infer that many of the LSBs are luminous and extended, based on the moderate differences in the distributions of sizes and velocities between HSBs and LSBs in these surveys. With the correlation $\mu_* \propto M_*^{0.6-0.7}$, the expected variation in radius across this decade of $\mu_*$ is only a factor of ~2, which may be consistent with the data. The corresponding variation in velocity, $V \propto M_*^{0.2}$, is similarly weak. The flat surface-brightness distribution can therefore be consistent with the correlation detected in the SDSS, the 2dF and other surveys between surface brightness and luminosity.

V measuring the depth of the gravitational potential well, corresponding to roughly $V \propto M^{1/4}$ (e.g. Bernardi et al. 2003). This is the Tully–Fisher relation for the rotation velocity in discs and the Faber–Jackson relation (or a projection of the generalized Fundamental Plane relation) for the dispersion velocity in spheroids. We term this kind of relation between $M_*$ and $V$ a ‘TF’ relation. A similar TF relation seems to extend down at least to the top part of the LL regime, with no obvious change at the transition between HH and LL galaxies (e.g. Sprayberry et al. 1995; Zwaan et al. 1995; Dale et al. 1999).

K03 (figs 9 and 11) also find that the luminosity ‘concentration’ within the galaxies correlates with stellar mass, which they interpret as a measure of bulge-to-disc ratio. The bright galaxies are dominated by elliptical galaxies, the LLs near $M_* \sim 10^6 M_\odot$ are dominated by discs, and in between, the bulge-to-disc ratio is gradually decreasing with decreasing $M_*$. Associated with this trend is an increasing gas-to-star ratio (e.g. Longmore et al. 1982; McGaugh & de Blok 1997) and a younger, bluer stellar population in galaxies of decreasing $M_*$ down to $\sim 10^5 M_\odot$ (e.g. de Blok, van der Hulst & Bothun 1995; McGaugh, Schombert & Bothun 1995; Sprayberry et al. 1995; another interpretation in van den Hoek et al. 2000). At a given $M_*$, the galaxies with lower bulge-to-disc ratio and younger stellar ages tend to be of lower surface brightness (K03, fig. 14). Preliminary results from the SDSS data (Brinchmann et al., in preparation) indicate that while the current star formation rate (SFR) in HHs shows no clear correlation with stellar mass, there is a correlation of the sort $M_* \propto M_*$ in the top LL regime.

1.2 Local Group dwarfs

The dwarf galaxies of the Local Group (LG, see Section 6), as well as those observed in the Virgo cluster and the Local Supercluster (e.g. Binggeli & Cameron 1991; Ferguson & Binggeli 1994), seem to be in a crude sense an extension of the LL population of the SDSS, stretching its range to almost $10^7 M_\odot$ in comparison with the SDSS results for the bright end of the LLs. The stellar mass $M_*$ of each galaxy has been derived from the observed magnitudes using the mean age and metallicity of the stellar population and a simple population-synthesis model (kindly provided by G. Kauffmann); the results were found to be quite insensitive to the details of this derivation. For the central surface brightness, WD find a tight correlation for the scaling relation $\mu_* \propto M_*^{0.55\pm0.03}$, extending down to $\mu_* \sim 3 \times 10^6 M_\odot$ kpc$^{-2}$ at the faint end. The best-fitting slope is determined by WD via a linear regression of the log variables, taking into account the errors in both (i.e. minimizing the two-dimensional $\chi^2$ as in section 15.3 of Press et al. 1992). The Pearson correlation coefficient (equation 14.5.1 of NR) is $r = 0.88$. The slope is quite similar to the slope obtained for the effective surface brightness of LLs sampled by the SDSS, although the amplitude for the effective $\mu_*$ is lower by a factor of $\sim 3$.

For the metallicity $Z$, WD took the stellar [Fe/H] (mostly for dE) and/or a constant factor times the oxygen abundance of the gas (mostly for dI), where the constant factor had been chosen to minimize the scatter in the $Z$–$M_*$ relation. They find a tight correlation, with the best-fitting scaling relation being $\propto M_*^{0.50\pm0.02}$ and $r = 0.92$.

For the internal velocity $V$, WD adopted the observed maximum circular velocity for the dIs and $\sqrt{3} \sigma_r$ for dEs, where $\sigma_r$ is the observed projected central dispersion velocity. When the fit is performed across the whole dwarf range, the TF scaling relation is $V \propto M_*^{0.24\pm0.01}$ with $r = 0.89$. When inspected more carefully, the dIs at the bright end show a slight steepening which merges smoothly into the known TF relation for bright galaxies. At the faint end, $M_* < 3 \times 10^5 M_\odot$, there is an indication that the velocities of the dEs are bound from below by $V \geq 10$ km s$^{-1}$, and can actually be fit by $V \propto$ const. (see Section 7). Nevertheless, the tight scaling relations over the whole range indicate that the LL/dwarf galaxies constitute basically a one-parameter family, which calls for a simple physical explanation.

1.3 Supernova feedback

The dwarf galaxies are central players in one of the main problems facing galaxy formation theory in the context of CDM cosmology – the so called ‘missing dwarf problem’. This refers to the apparent discrepancy between the predicted abundances of halo masses in the CDM cosmology, especially subhaloes within larger haloes, and the relatively few, faint dwarf galaxies observed – in the Local Group, for example (Klypin et al. 1999; Moore et al. 1999; Springel et al. 2001). This problem is related to the fact that for galaxies fainter than $L_*$, the luminosity function is observed to be flatter than the mass function predicted for haloes in the $\Lambda$CDM cosmology. This implies that the stellar-to-virial mass ratio $M_*/M$ must be decreasing with decreasing $M$ – namely, fewer stars were formed per unit total mass in fainter LSB galaxies. The systematic variation in $M_*/M$ can serve as a clue for understanding the LL–LSB phenomenon. Following preliminary ideas by Larson (1974), Dekel & Silk (1986) studied the general scenario where the key physical process governing this phenomenon is the supernova feedback from a first generation of stars, which either drives out a significant fraction of the original halo gas or suppresses star formation in the retained and added gas. DS showed that the observed scaling properties of dwarf galaxies
can be qualitatively consistent with this picture, provided that the potential wells are dominated by non-gaseous dark haloes with a structure that crudely resembles the predictions of the CDM scenario. They studied the amount of energy fed into the interstellar gas by supernova ejecta subject to radiative losses and found that haloes with virial velocities significantly lower than a critical value of the order of $\sim 100 \, \text{km} \, \text{s}^{-1}$ can lose significant amounts of their gas and/or effectively suppress further star formation.

Recent observations provide cumulative evidence for massive outflows from galaxies, consistent with being generated by supernovae. For example, H$_I$ maps of large disc galaxies show empty bubbles associated with outflowing clouds, which indicate an energy equivalent to $\sim 100$ normal supernovae (Boomsma et al. 2002). Chandra X-ray measurements show extended winds of soft X-ray gas about galaxies such as M82, associated with hard X-ray sources in the galaxy, indicative of young massive stars and supernovae (Roy et al. 2000; Martin, Kobulnicky & Heckman 2002). Outflows are detected directly in local starburst galaxies, and are seen to be driven by SNIa activity (e.g. Legrand et al. 1997; Martin 1999b; Heckman et al. 2001). In some of these galaxies, outflows of a few hundred km s$^{-1}$ are inferred based on the blueshifted metal absorption lines from the approaching foreground compared to the redshifted Ly-$\alpha$ photons back-scattered from the receding background. Spectroscopy of the brighter lensed galaxies at high redshifts reveals using a similar effect typical outflows of 200–800 km s$^{-1}$ (Frax et al. 1997; Frye & Broadhurst 1998; Pettini et al. 2001; Frye, Broadhurst & Benitez 2002). New evidence for strong outflows at $z \sim 3$ is provided by Adelberger et al. (2003), who interpret their measurements of absorption systems in quasi-stellar object (QSO) spectra as bubbles of radius $\sim 1 \, \text{h}^{-1} \, \text{Mpc}$ (comoving) around Lyman-break galaxies, almost empty of neutral hydrogen. Combined with the measured outflow velocities of a few hundred km s$^{-1}$, this is an indication for energetic winds that persist for a few hundred Myr and drive away nearby intergalactic gas. A strong observed correlation of galaxies with intergalactic metals supports the idea that the IGM has been enriched by the outflows from Lyman-break galaxies. In several cases, the outflow rate is observed to be proportional to the SFR (Martin 1999a), consistent with a stellar feedback origin for the outflows. This body of evidence indicates that supernova-driven winds actually exist, which helps in motivating our theoretical modelling of the relevant features of galaxy formation.

In this paper, we improve the DS scenario for the formation of dwarf galaxies in view of the developments in galaxy formation theory and the refined observed scaling relations across the LL/dwarf family. Using a simple energetics criterion and standard assumptions regarding the origin of galaxy sizes, we now show that the observed scaling relations naturally emerge from the simplest possible supernova-feedback scenario, even before one tries to model and simulate in detail the complex physics of the feedback mechanism, and before one worries about the different types of dwarf and LSB galaxies and the scatter in their properties. We then address the possible role of radiative feedback in distinguishing between dEs and dIls and in preventing star formation altogether in some halos.

In Section 2, we address the role of standard assumptions in determining the scaling relations for galaxies in general. The assumptions include spherical collapse to virial equilibrium in CDM haloes and angular momentum conservation. In Section 3, we apply the analysis to bright galaxies where we assume that $M_*/M \approx \text{const.}$. In Section 4, we summarize the DS derivation of the velocity characterizing the supernova-feedback scale. In Section 5, we use simple theoretical considerations regarding supernova feedback to derive the scaling relations of the LL/dwarf family. In Section 6, we compare the model predictions to the observed relations shown by the Local Group dwarfs and comment on the comparison with the SDSS data. In Section 7, we discuss the possible role of radiative feedback in distinguishing between dEs and dIls. In Section 8, we discuss our results and related issues.

2 SCALING RELATIONS: GENERAL

We show that the basic observed scaling relations for galaxies in the two regimes can be reproduced to a surprising accuracy based on the simplest possible physical assumptions. These include the virial theorem for spherical cold-dark-matter haloes, and the notion that a fraction $\eta$ of the original gas makes stars in a disc such that the size of the stellar system is determined by angular momentum. For HH galaxies, we recover the scaling relations by taking $\eta$ to be independent of halo mass, assuming that feedback is not too effective there. For LL galaxies, where feedback is a key factor, we use in Section 5 below a simple energy constraint for the efficiency of supernova feedback to predict how $\eta$ should vary with the halo virial velocity. Together with the constraints from the virial theorem and from angular momentum, this leads to the characteristic scaling relations in the LL regime. In this section we derive the scaling relations in general terms without specifying the behaviour of $\eta$.

Assume a dark-matter halo of mass $M$ reaching virial equilibrium at a time corresponding to cosmological expansion factor $a = (1 + z)^{-1}$. The virial radius $R$ is defined in the spirit of the spherical collapse model by a given density contrast $\Delta$ relative to the mean universal density at that time, namely by $M/R^3 \propto \Delta^{1/3}$. At early times, when $\Omega_\text{m} \approx 1$, the relevant density contrast is $\Delta \approx 180$, whereas for the standard $\Lambda$CDM cosmology (with $\Omega_\Lambda = 0.7$ and $\Omega_\text{m} = 0.3$ today), it rises to $\Delta \approx 340$ today. In the following, we ignore the weak redshift dependence of the $\Delta$ factor. The virial velocity is defined by the virial theorem, $V^2 \propto M/R^4$, such that the three virial quantities at $a$ define a one-parameter family:

$$M \propto a^{3/2} V^3 \propto a^{-3} R^3.$$  \hfill (1)

In the simplest analysis, we ignore the possible systematic increase of $a$ as a function of halo mass (but see below). This dependence is relatively weak already, as predicted by cosmological spherical collapse in the $\Lambda$CDM cosmology; it gets weaker for smaller haloes as the rms density fluctuations approach a constant on small scales and it is weakened further by effects like the merging of early-forming small haloes into bigger ones (see Wechsler et al. 2002). The virial relations for typical haloes thus take in this approximation the simple form

$$M \propto V^3 \propto R^3.$$  \hfill (2)

We may keep tracing the $a$ dependence in the general expressions below in order to allow small corrections owing to its possible weak dependence on $M$, when desired.

Considering next the baryonic component, we assume that the halo is initially filled with gas of mass $M_b \sim f_b M$, where $f_b (\sim 0.13)$ is the universal baryonic fraction. For large galaxies, the gas is assumed to be shock-heated to the halo virial temperature, but as long as

\footnote{4 The maximum change is obtained at low redshifts. For example, in the range $z = 0$–2, the change is roughly $\Delta \propto a^{1/3}$, which implies that $a$ in the following expressions should be replaced by $a^{1/3} \propto a^{1/2}$. This is a weak effect, which becomes even weaker at higher redshifts.}

\begin{equation}
\frac{\Delta_{\text{const}}}{\Delta_{\text{mass}}} = \frac{M_{\text{const}}}{M_{\text{mass}}} = \frac{a^{3/2}}{a^{1/3}} = a^{1/2}.
\end{equation}
as \( M < 10^{12} - 10^{13} \, M_\odot \), the gas in the halo can cool in a dynamical
time (shorter than the Hubble time) and contract to form stars (Rees & Ostriker 1977; Silk 1977; White & Rees 1978; Blumenthal et al.
1984, in the context of dark haloes). We denote the ratio of luminous
stellar mass \( M_* \) to initial gas mass \( M_\gamma \) by \( \eta \),

\[
M_* = \eta M_\gamma \propto \eta M,
\]

without yet specifying how \( \eta \) may depend on \( M \). Substituting
equation (3) in the virial relations, equation (1), we obtain straightforwardly a general TF relation between \( V \) and \( M_* \):

\[
V \propto \eta^{-1/2} \eta^{-1/3} M_*^{2/3}.
\]

As long as the halo rotation curves are roughly flat at large radii, we
game the difference between the virial velocity and the observed velocity \( V_{\text{max}} \).

If the baryons within the halo virial radius \( R_v \) cool and contract to a
centrifugally supported disc of radius \( R_v \), while preserving their
specific angular momentum \( j \), then, following Fall & Efstathiou (1980) and Mo, Mao & White (1998), we write \( R_v \approx \lambda R \), where
\( \lambda = j / (RV) \) is the initial baryonic spin parameter [according to the
revised, practical definition of Bullock et al. (2001b)]. Then, from the
virial relations above,

\[
R_v \propto \lambda M^{1/3}.
\]

With \( M_* \propto \eta M \), this implies that, for the surface brightness,

\[
\mu_* \propto M_* R_v^{-2} \propto \lambda^{-2} \eta^{-2/3} M_*^{1/3}.
\]

The characteristic radii and surface brightnesses for disc
that formation to be observed may be obtained from the straightforward prediction based on spherical collapse, ignoring

the fact that many early-forming small haloes eventually merge
together ones and thus weaken the a(M) relation. For a power spectrum
of linear density fluctuations that resembles the power law \( P_k \propto k^n \)
at the vicinity of the scales relevant for galactic haloes, the typical
mean density fluctuation within a protohalo is \( \delta \propto M^{-(a+3)/6} \),
where \( D(t) \) is the linear growth rate and \( D(t) \propto a \) for the Einstein–de
Sitter cosmology relevant at high redshifts. The formation time in the
spherical collapse model can be approximated by \( \delta \approx 1.7 \)
for the linearly extrapolated mean density fluctuation, so one obtains

\[
a \propto M^{(a+3)/6}.
\]

The virial relations for typical haloes, equation (1), thus become

\[
M \propto V^{12/(1-a)} \propto R^{6(5+a)}.
\]

The TF relation, equation (4), is now

\[
V \propto (\eta^{-1} M_\gamma)^{1-a/12}.
\]

The stellar radius is now given by \( R_* \propto \lambda M^{(5+a)/6} \) such that the
surface brightness, equation (6), is replaced by

\[
\mu_* \propto \lambda^{-2} \eta^{-3/3} M_*^{2(3+a)/3}.
\]

The expressions so far should be valid in general, both for HH
and for LL galaxies. The differences between the two classes enter mainly via the behaviour of \( \eta \), with an additional weak effect owing to the difference in the effective \( n \) in the maximum \( M \) dependence of \( a \).

3 HH GALAXIES

For HH galaxies, we take \( \eta \) to be roughly independent of halo mass.

This is based on the assumption that feedback effects do not signifi-
cantly heat or remove most of the gas from these galaxies such that
most of the gas, or a constant fraction of it, eventually forms stars
(see Section 4). With \( \eta \) and \( a \) independent of mass in equations (4),
(6), and (7), the scaling relations for HH galaxies become

\[
V \propto M_*^{1/3}, \quad \mu_* \propto M_*^{1/3}, \quad Z \propto \text{const}.
\]

These are already in qualitative agreement with the observed rela-
tions for HH galaxies.

When considering the limit of maximum \( M \) dependence of \( a \), we
recall that big galactic haloes in a \( \Lambda CDM \) cosmology correspond to
the part of the power spectrum where \( n \lesssim -2 \). For example, with
\( n = -2 \) at the bright end, one has a \( a \propto M^{1/6} \). Then the virial relations
become \( M \propto V^4 \propto R^2 \). With \( \eta \) assumed to be independent of \( V 
\)
for HH galaxies, and with the maximum \( M \) dependence of \( a \) computed
above, the predicted scaling relations become

\[
V_{\text{max}} \propto M_*^{1/4}, \quad \mu_* \propto \text{const}, \quad Z \propto \text{const}.
\]

In order to compare with observations in terms of luminosity rather
than stellar mass, one can assume that for HH galaxies, the stellar
mass-to-light ratio varies as \( M_*/L \propto L^{0.3} \) (e.g. Courteau et al., in prep-
ation). [This is based, for example, on the reading of fig. 7
of Courteau et al. (2001) that \( (V - I) \approx -0.09M_\gamma \), combined with the
result from table 1 of Bell & de Jong (2001) that \( (M_*/L) \approx 1.35 \)
\( (V - I) \). A similar result is obtained for elliptical galaxies
(Bender, Burstein & Faber 1992)]. Thus with \( V_{\text{max}} \propto V \), we
recover the observed TF relation roughly: \( L \propto V_{\text{max}}^{3/2} \). The surface
brightness as measured in terms of luminosity is predicted to be
slowly decreasing with luminosity, \( I \propto L/R_*^{2} \propto L^{-0.3} \), in qualitative
agreement with observations. We note that in the TF relation,
the correction due to the correlation of \( a \) and \( M \) balances roughly
the correction due to the correlation of \( M_* \) and \( M \). However, the corresponding corrections to the relation of surface brightness and
luminosity add up. In any case, the corrections to the simple predictions of equation (12) are small and we expect the predicted scaling relations for HH galaxies to lie somewhere between the relations in equations (12) and (13). This range is in general agreement with the observed scaling relations for HH galaxies.

Recall that, beyond the standard assumptions of virial equilibrium and spherical collapse, the key assumption for HHs was that most of the original gas, or a constant fraction of it, turns into stars – namely, $\eta \simeq \text{const.}$

4 SUPERNova FEEDBACK: THE CRITICAL SCALE

Dekel & Silk (1986) evaluated the maximum total energy fed into the interstellar gas by a collection of supernova explosions as a result of a period of star formation at a constant rate $M_*$, taking into account the radiative losses (based on the standard evolution of a supernova remnant in a uniform interstellar medium, e.g. Spitzer 1978; Ostriker & McKee 1988). They found that at time $t$, this energy can be approximated by

$$E_{SN}(t) \simeq \epsilon v M_{\text{rad}} f(t),$$

where $\epsilon$ is the initial energy released by a typical supernova ($\epsilon \sim 10^{51}$ erg) and $v$ is the number of supernovae per unit mass of forming stars (which for a typical IMF is $v \sim 1$ per 50 $M_\odot$ of stars). The characteristic time $t_{\text{rad}}$ marks the end of the ‘adiabatic’ phase and the onset of the ‘radiative’ phase of a typical supernova remnant, by which it has radiated away a significant fraction of its energy. The dimensionless factor $f(t)$ turns out to be of order unity when $t \sim t_{\text{rad}}$; it grows roughly proportional to $t$ for $t < t_{\text{rad}}$ and proportional to $t^{-0.4}$ for $t > t_{\text{rad}}$.

We assume here that the stellar population of mass $M_*$ has formed over some constant multiple $\tau$ of the free-fall time $t_{\text{ff}}$, namely

$$M_* = M_*/t_{\text{ff}}.$$  

Substituting in equation (14), we obtain that the total energy fed into the gas is

$$E_{SN} \propto M_* t_{\text{rad}}/t_{\text{ff}}.$$  

DS noticed that in the temperature range $6 \times 10^4 < T < 6 \times 10^5$ K, the cooling rate scales approximately like $\Lambda \propto T^{-1}$, which implies that the ratio $t_{\text{rad}}/t_{\text{ff}}$ is roughly a constant, of the order of $10^{-2}$, independent of the gas density or the halo parameters. This leads to

$E_{SN} \propto M_*$, which we show below as being a key for deriving the scaling relations of LL galaxies (Section 5). Note that DS originally assumed that $M_* \propto M_*/t_{\text{ff}}$ rather than the $M_* \propto M_*/t_{\text{ff}}$ of equation (15). The two assumptions are roughly equivalent in the case of bright galaxies and when trying to estimate the transition scale between HH and LL galaxies where $M_* \sim M_*$.

DS also showed that if star formation is rapid, $\tau \sim 1$, then the filling factor of the expanding supernova shells within the halo is of order unity when the typical shell is at the end of its adiabatic phase, at $t_{\text{rad}}$. This coincidence indicates that the supernova energy (minus the radiative losses) can be fed quite evenly and efficiently into most of the gas via the expanding shells that reach a significant mutual overlap roughly at the time after which they become ineffective. It also justifies the adoption of $f \sim 1$ in equation (14).

A necessary condition for heating or unbinding most of the initial gas of mass $M_*$ is obtained by requiring that the energy fed by supernovae is comparable to the binding energy of the gas in the halo potential well,

$$E_{SN} = (1/2)M_* V^2.$$  

Here, $V$ is the virial velocity of the halo, which we assume for simplicity to be isothermal and to dominate the potential. DS then pushed this approximate relation to the limit where a large fraction of the gas turns into stars and obtained the critical velocity

$$V_{SN} = \left(2 \epsilon v f(t_{\text{rad}}/t_{\text{ff}})\right)^{1/2} \simeq 100 \text{ km s}^{-1}.$$  

This critical velocity is evaluated using the typical values of $\epsilon$ and $v$ with $f \simeq \tau \sim 1$ and is independent of the gas density because of the robustness of $t_{\text{rad}}/t_{\text{ff}}$. The interpretation of this critical velocity is that gas removal becomes possible in haloes with virial velocities smaller than $V_{SN}$. We note that the corresponding virial mass is

$$M_{SN} \simeq 2.2 \times 10^{11} M_\odot \left(\frac{V_{SN}}{100 \text{ km s}^{-1}}\right)^3 a^{3/2}.$$  

With $M_* \simeq M_*/f_{\text{b}}$ and the universal baryonic fraction $f_{\text{b}} \simeq 0.13$, the corresponding characteristic stellar mass today is

$$M_{SN} \simeq 3 \times 10^{10} M_\odot,$$

in excellent agreement with the transition scale seen in the SDSS data.

The haloes of bright galaxies, which have retained most of their gas, are therefore limited to the regime of deep potential wells, $V > V_{SN}$. We associate the galaxies that form in haloes below the critical supernova scale with LL or dwarf galaxies. The importance of feedback effects in the history of these galaxies implies that their scaling relations can be very different from those shown by galaxies that live in haloes of virial velocities larger than $V_{SN}$.

5 LL GALAXIES

We now use the feedback energetics constraints, equations (16) and (17), to determine the behaviour of $\eta$ in the LL regime, where we expect feedback effects to allow only a fraction of the gas to turn into stars,

$$\eta \equiv \frac{M_*}{M_\ast} < 1.$$  

The supernovae resulting from the first burst of stars either blow out the rest of the gas or at least provide enough feedback energy to regulate the subsequent SFR and keep it low. We assume that $\eta \simeq 1$ at $V = V_{SN} \sim 100 \text{ km s}^{-1}$ and that $\eta$ becomes gradually smaller for haloes of smaller velocities. The following simple analysis is actually carried out in the limit of strong feedback, $\eta \ll 1$.

Our key starting point is equation (16) with $t_{\text{rad}}/t_{\text{ff}} = \text{const.}$: namely, the energy fed into the interstellar gas by supernovae is proportional to the final stellar mass,

$$E_{SN} \propto M_*.$$  

Without the radiative losses of the supernova energy, this would have been anybody’s first intuitive guess for a relation between these quantities. We argue here, in the spirit of the DS analysis, that the actual energy fed into the gas after significant radiative losses is still a constant fraction of the original supernova energy. This makes equation (22) a valid approximation in the realistic case.

In order to allow significant heating or total blowout of the initial gas, the total input by supernovae should be at least comparable to
the binding energy of the gas, equation (17). With equation (22), the energy condition becomes $M_\ast \propto M_\mathrm{f} V^2$, namely

$$\eta \propto V^2.$$

(23)

The scaling relations for LLs all follow from this basic relation, which measures the strength of the feedback effects along the halo sequence characterized by the parameter $V$ in the range $V < V_{\mathrm{SN}}$.

Equation (23), combined with the virial relations for the halo, equation (1), and then with $M_\ast = \eta M_\ast$, yields

$$\eta \propto a^{-1} M_\ast^{2/3} \propto a^{-3/5} M_\ast^{1/5}.$$  

(24)

Recall that in the instantaneous-recycling approximation, for $\eta \ll 1$, the metallicity is simply $Z \propto \eta$, so the mean scaling relation involving metallicity is given by equation (24).

Substituting $\eta$ in equation (4), we obtain for the TF relation in the LL regime

$$V \propto a^{-3/10} M_\ast^{1/5}.$$  

(26)

Then substituting $\eta$ in equations (5) and (6), we obtain for the radius $R_\ast \propto \lambda^{-1/5} M_\ast^{1/5}$ and for the surface brightness $\mu_\ast \propto \lambda^{-2/5} M_\ast^{3/5}$.

(27)

In summary, when ignoring possible weak systematic dependences of $a$ and $\lambda$ on $M$, the scaling relations for LL/dwarf galaxies are predicted to be

$$V \propto M_\ast^{1/5}, \quad Z \propto M_\ast^{2/5}, \quad \mu_\ast \propto M_\ast^{3/5}.$$  

(28)

In order to evaluate the maximum correction due to the possible dependence of $a$ on $M$, we use equation (8) in equation (24) and obtain

$$\eta \propto M_\ast^{(1-n)/6} \propto M_\ast^{(1-n)/(7-n)}.$$  

(29)

Then the TF relation, equation (26), becomes

$$V \propto M_\ast^{(1-n)/14-2n}.$$  

(30)

and the surface brightness, equation (27), becomes

$$\mu_\ast \propto \lambda^{-2/5} M_\ast^{3-1+n/(7-n)}.$$  

(31)

Very small galaxies in the $\Lambda$CDM cosmology correspond to the part of the power spectrum where $n$ is not much larger than the lower limit of $n = -3$, implying a similar formation time for dwarf galaxies of all masses and therefore a constant $a$ in the above relations, thus leading to equation (28). For LL galaxies not much below $V_{\mathrm{SN}}$, we may try a typical $n = -2.5$, for example, for which $a \propto M_\ast^{(6+3)/(7-n)} \propto M_\ast^{1/3}$. This implies negligible effects on the TF relation and the metallicity relation, but the weak correction to the surface brightness relation may be marginally detectable: $\mu_\ast \propto \lambda^{-2} M_\ast^{9/10}$ compared to $\mu_\ast \propto \lambda^{-2} M_\ast^{3/5}$.

In this case, however, we may also wish to incorporate the possible mass dependence of $\lambda$. To a first approximation – as said above, based on cosmological simulations – the distribution of halo spin parameter is independent of the halo virial properties and its formation time. As long as the baryons trace the spatial distribution and kinematics of the halo initially, their $\lambda$ distribution can be assumed to be independent of $M$ and $a$. However, whereas the baryons in bright disc galaxies seem to have spin parameters similar to those of their host haloes, LSB disc galaxies may tend to be associated with a higher spin parameter. For example, van den Bosch, Burkert & Swaters (2001, hereafter BBS) studied the spin in a sample of 14 LSB discs with an estimated average of $V \approx 60$ km s$^{-1}$. They found an average spin parameter of about 50% cent larger than that of the dark haloes (see Maller & Dekel (2002), fig. 8). At the same time, BBS estimated in these galaxies an average baryonic fraction of only $f_\mathrm{a} \approx 0.035$, which translates in our terminology to $\eta(V = 60) = f_\mathrm{a}/f_\mathrm{b} \approx 0.27$. Following Maller & Dekel (2002), we model these systematic trends based on preferential blowout of low-spin material in dwarf galaxies. In order to obtain the 50 per cent change in spin parameter over the same $\eta$ range (between 1 and 0.27), the effect of blowout would roughly scale as $\lambda \propto \eta^{-0.3} \propto a^{0.18} M_\ast^{-0.12}$. Plugged into equation (27), using $n = -2.5$, we now obtain $\mu_\ast \propto M_\ast^{0.7}$. This kind of correction to the surface-brightness relation should be valid for relatively large LL galaxies, where $n$ is not too close to $-3$ and where the BBS analysis indicates a systematic spin dependence. For smaller dwarfs, the actual relation may be better approximated by equation (27) with constant $a$ and $\lambda$, namely $\mu_\ast \propto M_\ast^{0.6}$.

The scatter about the mean scaling relations is expected to partly reflect the random scatter about the mean $a$ and $\lambda$. Based on equation (27), the scatter about the mean relation $\mu_\ast (M_\ast)$ is expected to be significant, dominated by the scatter in $\lambda$, whereas the scatter in $V$ and $Z$ is expected to be smaller. The residuals in these different relations are expected to be correlated. For a given $M_\ast$, galaxies that lie at the bottom of the $\mu_\ast$ distribution are expected to be of relatively high $a$ (late formation time) and high $\lambda$. In turn, based on equations (26) and (27), these galaxies – compared to the average for that $M_\ast$ – are expected to be of low $V$ (though high $M_\ast$, given the high $a$) and low $Z$.

6 MODEL VERSUS LOCAL GROUP DWARFS

The success of the simple feedback model for LL galaxies, as described in Section 5, can be evaluated by comparing the predicted scaling relations, equation (28), to the observed scaling relations for LLS in the SDSS and in the Local Group. Given the idealized nature of the straightforward model, one might hope only for a crude qualitative fit.

The match of the predicted characteristic scale for supernova feedback, $V \approx 100$ km s$^{-1}$, with the observed transition at $M_\ast \approx 3 \times 10^{10} M_\odot$ is already remarkable; it indicates that this transition may be indeed associated with the onset of supernova-feedback effects.

Fig. 1 shows the central surface brightness $\mu_\ast$ versus stellar mass $M_\ast$ for the Local Group dwarfs (from WD). The galaxies are either of the two major types, dI and dE, or transition cases marked Tr. The data fit very well by the predicted scaling relation $\mu_\ast \propto M_\ast^{0.6}$ throughout the whole LL range, spanning five decades in $M_\ast$. We do not attempt to normalize the predicted relation and therefore the model line in the figure is normalized artificially to provide the best fit for the predicted slope of 0.6. The correlation is relatively tight, with a Pearson’s correlation coefficient for the logs of $r = 0.88$. The model slope is also a good fit to the SDSS data in the LL range (Kauffmann et al. 2003b); even the predicted slight steepening to $\mu_\ast \propto M_\ast^{0.7}$ or so can be seen at the bright end of the LL range. The SDSS data refers to the surface brightness within the half-light radius, which, for an exponential profile, is a factor of $\sim 3$ smaller than the central value. With this relative normalization, the bright end of the Local Group dwarfs lies along the upper 68 per cent contour of the SDSS distribution (fig. 7a of Kauffmann et al. 2003b).
The predicted metallicity relation, $Z \propto M_*^{0.4}$, is a very good fit to the Local Group dwarfs. The correlation is tight, with a correlation coefficient $r = 0.92$. The preliminary SDSS data indicate a similar and perhaps slightly steeper scaling relation in general, the dIs do tend to lie somewhat below the relation at the bright end, $Z \propto M_*^{0.2}$. The halo radius is determined from $M_*$ given by $\lambda \propto M_*/M_\odot$. When dividing the galaxies into three relatively narrow bins of $M_*$ values, we see that there is indeed a systematic trend within each bin, though slightly flatter than the expected $\mu_* \propto \lambda^{-2}$.

Although the surface brightnesses of dEs and dIs follow a similar scaling relation in general, the dEs below $3 \times 10^7 M_\odot$ have a primary role in determining the gross features of the galaxy properties in the LL regime.

The data from the SDSS also allow a quantitative evaluation of the distribution of galaxies about the mean relation in the $\mu_* - M_*$ plane (Kauffmann et al. 2003b). In the LL regime, the spread in $\mu_*$ at a given $M_*$ is roughly consistent with the spread in spin parameter $\lambda$ for haloes of a given mass, as measured in N-body simulations of $\Lambda$CDM. This indicates that $\lambda$ indeed can serve as the main secondary parameter for the LL family.

The idealized theory predicts a dependence of $\mu_* \propto \lambda^{-2}$ for a given $M_*$ [equation (6)]. In Fig. 4, we test the self-consistency of this prediction for the Local Group dwarfs. We display $\mu_*$ versus an estimated ‘spin parameter’ given by $\lambda \propto R_*/R$, the ratio of stellar radius to halo radius. The stellar radius is determined from $M_*$, and the central surface brightness $\mu_*$, $R_* \propto (M_*/\mu_*)^{1/2}$. The halo radius is the virial radius corresponding to virial velocity $V$, where $V = \max (V_{\text{esc}}, \sqrt{3} \sigma_t)$ as described in WD. When dividing the galaxies into three relatively narrow bins of $M_*$ values, we see that there is indeed a systematic trend within each bin, though slightly flatter than the expected $\mu_* \propto \lambda^{-2}$.

Although the surface brightnesses of dEs and dIs follow a similar scaling relation in general, the dEs do tend to lie somewhat below the best-fitting line. This is consistent with the finding in the SDSS that at a fixed $M_*$, the galaxies with lower bulge-to-disc ratio and younger stellar populations tend to have a lower surface brightness. These trends are qualitatively consistent with the $a$ and $\lambda$ dependences predicted in equation (27).

Although the predicted $V \propto M_*^{0.2}$ is a good fit across the whole dwarf range, a more detailed investigation of Fig. 3 reveals very tight, with $r = 0.89$. We see that the idealized theory for supernova feedback provides a surprisingly good fit to the characteristic scale and to the three independent scaling relations valid across the whole LL range. Indeed, this indicates that the supernova-feedback effects, via the parameter $\eta$, have a primary role in determining the gross features of the galaxy properties in the LL regime.
interesting secondary features. First, there is an apparent lower bound for galaxies at $V \simeq 10$ km s$^{-1}$. Secondly, there is an apparent transition at $M_* \simeq 3 \times 10^7$ M$_\odot$. The fainter galaxies can actually be well fit by $V \simeq$ const.. The dwarfs brighter than $3 \times 10^7$ M$_\odot$ are then fit by a line which could be as steep as $V \propto M_*^{0.4}$. Because these velocities are measured in the inner regions of the haloes, they can be regarded as being lower bounds to the actual dispersion velocities of the haloes. If the velocities of the inner de haloes are actually larger than these lower estimates (as argued by Stoehr et al. 2002), then the difference between the TF relation in the two regimes, below and above $3 \times 10^7$ M$_\odot$, could become even more significant. We note that the faint part is dominated by de's, whereas the brighter part is comprised mostly of dls. These are clues for the origin of the distinction between these two types of dwarf galaxies, which we address in the following section.

7 RADIATIVE FEEDBACK: de VERSUS dl

After demonstrating the encouraging success of supernova feedback in explaining the basic systematic trends in the LL family as a whole, we now attempt to consider the possible role of another feedback mechanism and, in particular, how it may differentiate between de's and dl's within the LL family.

7.1 Radiative feedback

Cosmological reionization of hydrogen is complete by $z_{\text{ion}} \sim 6$—7 (see a review by Barkana & Loeb 2001, hereafter BL; also Loeb & Barkana 2001). The flux of UV radiation that is generated by the first stars or active galactic nuclei (AGN)s heats and photoionizes the gas in the IGM and in virialized haloes (except perhaps for the inner regions, which can become shielded). As long as the ionizing flux persists, the gas is kept at a fixed temperature of $T_{\text{ion}} \simeq (1-2) \times 10^4$ K. This can be regarded as another feedback mechanism; it can suppress star formation and clean haloes from gas in two ways. First, by photoevaporation of gas already in haloes. Barkana & Loeb (1999) estimated that haloes of $V < 10$ km s$^{-1}$ would lose most of their gas. Their analysis provides an estimate of the gas loss during the first dynamical time after $z_{\text{ion}}$. However, if the gas is kept ionized until $z \sim 1-2$, a dynamical calculation of evaporation by a continuous wind reveals that photo-evaporation would remove the gas from somewhat larger haloes, up to $V_{\text{evap}} \simeq 20$ km s$^{-1}$ (Shaviv & Dekel 2003). Secondly, based on simulations and computations of Jeans mass, the pressure of the hot IGM shuts off gas infall into even more massive haloes — those with velocities up to $V_{\text{Jeans}} \simeq 30$ km s$^{-1}$ (see BL Section 6.5 and references therein). Gas could presumably falling into small haloes after $z \sim 1-2$ when the UV background flux declined sufficiently (Babul & Rees 1992), but only haloes of $V > 20-25$ km s$^{-1}$ can form molecular hydrogen by $z \sim 1$ and then cool further to make stars (Kepner, Babul & Spergel 1997).

In the presence of a halo potential well characterized by a velocity $V$, the fraction of gas of temperature $T_{\text{ion}}$ that is bound to the halo can be estimated by the Boltzmann distribution,

$$f_{\text{bound}} \propto 1 - e^{-V^2/(2T_{\text{ion}})}.$$

The velocity corresponding to $T_{\text{ion}}$ is of the order of $V_{\text{evap}} \simeq 20$ km s$^{-1}$ mentioned above. Note that in the limit $V < V_{\text{evap}}$, equation (32) predicts $f_{\text{bound}} \propto V^2$ (as pointed out by Ostriker, private communication). This reminds us of the energy relation for supernovae, $\eta \propto V^2$, which led to the global scaling relations of LLs in Section 5. Could radiative feedback (rather than supernova feedback) be the actual mechanism responsible for the global scaling relations of LLs? First, it is unlikely that a mechanism whose characteristic scale is $\sim 20-30$ km s$^{-1}$ can be dominant in determining the observed critical scale of $\sim 100$ km s$^{-1}$ and the properties of bright LLs not much below this scale. Secondly, $f_{\text{bound}}$ in the radiative case refers to the sum of bound mass in stars and in gas, whereas $\eta$ in the supernova case refers to the stellar mass only. Whereas $M_*/M \propto V^2$ is consistent with the observed scaling relations for LLs, a similar relation for the gas-to-mass ratio does not seem to be in agreement with the observed trend — especially not in the large, gas-rich LLs. There are indications that as one moves from bright to fainter galaxies, the ratio of gas to stellar mass increases until it reaches a maximum at some intermediate scale typical to dls before it starts decreasing towards the de regime (e.g. McGaugh & de Blok 1997). We interpret this as another piece of evidence against radiative feedback being the dominant mechanism in determining the global properties of galaxies in the upper LL regime. However, radiative feedback should have an important effect in the small, gas-poor de's, and possibly a complementary effect to the supernova feedback in the larger, gas-rich dl's and LLs.

In haloes of $V < V_{\text{evap}}$, stars can form only before the reionization epoch (and possibly much later, at $z < 1-2$). If $V_{\text{evap}} < V < V_{\text{Jeans}}$, gas that cooled and collapsed before the reionization epoch can also turn into stars at a slow rate at later times, but new gas cannot be accreted. These effects could lead to the gas-poor de's. On the other hand, in haloes of $V > V_{\text{Jeans}}$, there is not much radiative gas loss. Galaxies that form in such haloes can retain some gas that has not been blown away by supernova winds, or has come back after such blowout, and thus give rise to gas-rich dl's. Thus we propose that the main role of radiative feedback is to clean up the de's from their gas and to help regulate star formation in dls.

7.2 Dwarf elliptical galaxies

Fig. 5 shows the same data as in Fig. 3, but with separate fits below and above $M_* = 3 \times 10^7$ M$_\odot$ in the ranges dominated by the dwarf spheroidals of the Local Group and by dl's, respectively. In
30 km s$^{-1}$ collapse and quantities such as the time available for star formation between the halo because the gas that remains in dI haloes (or falls back in later on) smaller for the dIs and the rest of the LL family (Fig. 3). This is $V_{\star}$ star formation. The gas in haloes of range of halo velocities, nes all the dEs to a narrow form stars at any early epoch. This con $\propto M_z^{0.37}$. is large for the dEs and much $\propto M_z^{0.37}$, which applies throughout the LL range). If radiative feedback is indeed important in the formation of dEs, we expect for $\lambda$ a stronger dependence than $\eta \propto V^2$. If the metals were assumed to be uniformly distributed throughout the gas as assumed for larger LLs, then we might have expected a steeper dependence of $Z \propto M_z/M \propto M_z$ within the dE range (rather than the global $Z \propto M_z^{0.4}$ which applies throughout the LL range). However, since some of the gas is expected to photoevaporate or be kept away from the halo even before it cools and falls into the halo centre, we expect the metals to enrich a smaller fraction of the initial gas in fainter dEs, which should lead to a weaker dependence of $Z$ on $M_\star$, perhaps as flat as $Z \propto M_\star^{0.4}$. Given the limited width of the dE range, we can probably tolerate some deviation from the global $Z \propto M_\star^{0.4}$ there.

As for the surface brightness in dEs, the similarity in $V$ between all the dE haloes would lead to the predictions $R_\star \propto \lambda R \propto \lambda a^{1/2}$ and therefore $M_\star \propto R_\star/\lambda R \propto \lambda^{-2}a^{-1}M_\star$ (compared to the global $M_\star \propto M_\star^{4/3}$). If indeed the sequence of $M_\star$ in dEs represents variations in formation time, then the combination of the $a$ and $\lambda$ factors should be responsible for the flattening of the $M_\star$ dependence of $\mu_*$ to the observed $\mu_* \propto M_\star^{0.6}$. Whereas $a$ is expected to be smaller for larger $M_\star$, the baryonic spin parameter is expected to be smaller for dEs of smaller $M_\star$, those that formed later and closer to $z_{\text{int}}$. This is because in those only the gas from the inner halo managed to form stars before the reionization time, and this inner gas is naturally expected to be of lower than average spin (see Bullock et al. 2001b). The observed relation of roughly $\mu_* \propto M_\star^{0.6}$ tells us that the required trend should be roughly $R_\star \propto \lambda a^{1/2} \propto M_\star^{4/3}$. If $a$ is indeed anticorrelated with $M_\star$ for dEs, we expect for $\lambda$ a stronger dependence than $\lambda \propto M_\star^{2/3}$. Such a spin gradient could also explain why the dSph dwarfs at the faint end are low-spin spheroidals, whereas the brighter dwarfs tend to be centrifugally supported discs. As a consistency check, Fig. 6 shows the stellar radius $R_\star$ versus stellar mass $M_\star$ for the dEs in the range $M_\star \leq 3 \times 10^7 M_\odot$. There is indeed an apparent trend, best fit by $R_\star \propto M_\star^{4/3}$ and reasonably consistent with the required $R_\star \propto \lambda a^{1/2} \propto M_\star^{4/3}$. Again, given the limited width of the dE range, we can probably tolerate there a certain deviation from the global relation of $\mu_* \propto M_\star^{0.6}$.

8 DISCUSSION

We identify four basic characteristic scales in the theory of galaxy formation, each originating from a different physical process and each having a different imprint on the galaxy population, as follows.

(i) The upper limit for bright galaxies separating them from clusters of galaxies, at $M_\star \sim 10^{12} M_\odot$, is where radiative cooling occurs on a dynamical time scale (Rees & Ostriker 1977; Silk 1977; White...
8.1 HH galaxies

Therefore, the cooling upper limit and the supernova scale limit the stellar masses of bright galaxies to the range $3 \times 10^{10} < M_\ast < 10^{12} M_{\odot}$. A significant fraction of the gas is assumed to have turned into stars in these galaxies, such that $\eta$ is not significantly correlated with the halo properties. Then the tight TF relation, the high surface brightness and metallicity, and the weak correlation between the last two and the stellar mass all follow naturally from the simplest possible assumptions, as described in Section 3 (see also, for example, Blumenthal et al. 1984). In summary, the assumptions made are as follows.

(i) The halo is in virial equilibrium after spherical collapse from a cosmological background.

(ii) The epoch of galaxy formation is only weakly correlated with halo mass, consistent with the $\Lambda$CDM cosmology where the power index of density fluctuations is $n \lesssim -2$ in the range corresponding to HH galaxies.

(iii) The stellar mass is proportional to the total mass, $M_\ast \propto M$, such that $\eta$, the ratio of stellar to initial-gas mass, is uncorrelated with the halo mass. This assumption distinguishes the HHS from the LLs.

(iv) The size of the stellar system is related to the halo virial radius by conservation of angular momentum via the spin parameter $\lambda$, which is uncorrelated with the halo mass.

The idealized picture is disc formation via gas contraction in preformed haloes, unperturbed by recent strong galaxy interactions. This should be especially valid for the fragile LL discs discussed later, where it is supported by a weak spatial correlation observed between LSBs and other galaxies, especially below a pair separation of $2 h^{-1}$ Mpc (Mo, McGaugh & Bothun 1994).

It is not hard to understand why the galaxies at the bright end – mostly large ellipticals – are dominated by a spheroidal stellar component with little gas and a low current SFR. The high density of the cooled gas in the early progenitors of these haloes allows the formation of molecules which provide efficient cooling even after the gas has cooled to below $10^4$ K. This explains the high SFR in discs early on. Mergers of discs lead to bulges and elliptical galaxies, which therefore tend to be those galaxies that dominate the high $M_\ast$ end. The decrease in number of objects as a result of mergers may partly explain the low scatter in radius and surface brightness at a given $M_\ast$ in the HH regime, as indicated by KO3. The mergers provide an additional trigger for a high early SFR. The associated high gas consumption in these early epochs results in gas-poor systems with low SFR today. Since the supernova-feedback energy is weak compared to the depth of the potential wells in HHs, it has a negligible effect on the SFR.

A potential caveat in the picture that assumes no gas loss from HHs may arise from the indications that the baryonic fraction in these systems may in fact be lower than the universal fraction by a factor of 2 or more (e.g. Klypin, Zhao & Somerville 2002, based on semi-analytic modelling of the Milky Way within the $\Lambda$CDM scenario, and references therein). Such gas loss may be expected if the HHs result from a hierarchical merger process, where the gas is lost at early stages from the small building blocks by supernova and radiative feedback. Another possibility is that supernova feedback is actually stronger than assumed, either because of microscopic effects such as porosity in a multiphase ISM or because of hypernova from very massive stars (Silk 2003). Alternatively, there might be an even stronger feedback mechanism at work in big galaxies and in clusters. Hints from the SDSS for a correlation between HHs and AGN activity (Kauffmann et al., in preparation), together with the established presence of massive black holes in early-type galaxies and the known energetic radio jets associated with AGNs, may provide a clue for the required energetic feedback process.

8.2 LL and dwarf galaxies

Most of the galaxies and most of the mass belong to the LL and dwarf family below the transition scale: $M_\ast < 10^{10} M_{\odot}$. Their halo velocities are below the critical supernova scale of $V_{SN} \sim 100$ km s$^{-1}$ and they are therefore subject to supernova-feedback effects which can determine their characteristic scaling relations, as argued based on the simplest possible model in Section 5. The energy fed into the gas leads to a lower stellar mass fraction $M_*/M$ and therefore lower surface brightness and metallicity in haloes of lower $V$. Some of the gas may be blown out and some may be retained or may fall in at a later time. This gas is kept hot and possibly turbulent such that the SFR is regulated by supernova feedback, as well as by radiative feedback at the lower part of the dwarf sequence. Note that for the scaling relations to be valid in the LL regime, the gas does not have to be blown away – it should just be prevented from forming stars too efficiently. Our feedback model predicts $M_*/M_g \propto V^2$, where $M_g$ is the mass of the gas affected by feedback and prevented from forming stars. In LLs and dwarf irregulars, a significant fraction of this gas must have been retained in the galaxy rather than been blown away. Indeed, in this case, the model predicts an increasing gas-to-star ratio for decreasing halo mass, as observed.

Our key assumption for supernova feedback is that $E_{SN} \propto M_\ast$. It is crudely justified also in the presence of significant radiative cooling, when the gas is at $T \sim 10^5$ K, based on the analysis of supernova remnants by DS. The second assumption is the straightforward energy requirement for affecting most of the original gas, $E_{SN} \propto M_g V^2$. Together they yield that the effectiveness of feedback varies along the LL sequence as $M_*/M \propto V^2$. (33)


Then the scaling relations for LLs are obtained using the same standard assumptions as used for HHs, namely virial equilibrium after
spherical collapse and angular momentum conservation, noting that the correlation of formation time with halo mass is even weaker for dwarfs where \( n \rightarrow -3 \). Our basic energy condition is based clearly on a simplistic model for feedback, which was expected to provide rough estimates at best. The fact that this model recovers the observed scaling relations so well is partly a matter of lucky coincidences and it should not be taken too literally. However, our main moral from the remarkable success of the crude model is that supernova feedback can be the primary physical process determining the fundamental line of LL/dwarf galaxies. One may, in fact, reverse the logic and infer the feedback energy relation, \( E_{SN} \propto M_* \propto M_g V^2 \), from the observed scaling relations, via the other standard assumptions of virial equilibrium and spherical collapse in ΛCDM cosmology. Therefore our toy analysis provides the basis and the motivation for detailed future studies of the supernova-feedback effects, using more sophisticated modelling and simulations. The inferred energy relation should serve as a useful constraint which must be obeyed by these models. It may be a non-trivial challenge for such realistic models to achieve a match with observations as good as the match achieved by the naive toy model.

The actual supernova-feedback process is likely to be much more complex than assumed in our toy model. For example, supernovae exploding in a disc would affect the disc gas and the halo gas in different ways and in an aspherical configuration, with fountains punching out the ISM and the IGM in a non-uniform and possibly porous manner (e.g. MacLow & Ferrara 1999; Scannapieco, Thacker & Davis 2001; Mori, Ferrara & Madau 2002; Scannapieco, Ferrara & Madau 2002; Silk 2003). This would affect the way the ISM and IGM are enriched with metals (e.g. Madau, Ferrara & Rees 2001; Thacker, Scannapieco & Davis 2002). Another complication is that the supernova energy can be transferred to the gas in either bulk kinetic energy or thermal energy, but it can also be kept in reservoirs of other forms such as turbulence, which would amplify the feedback effects on the gas (e.g. Efstathiou 1999; Dalcanton et al. 1997; Mo et al. 1998; Firmani et al. 2002). Is this enough for explaining the observed correlation without a contribution from feedback effects, namely that the cold-gas mass is \( \propto M_* \)? Consider an SFR which depends on gas surface density as in the Kennicutt–Schmidt law, \( \dot{M}_* \propto \mu_{gas}^2 \), namely \( M_*/M \propto \mu_{gas}^{-1} \). From the virial relation we have from equations (11) and (9) \( \mu_{gas} \propto \lambda^{-2} M^3 \propto \lambda^{-2} V^3 \) with \( 0 \leq \tau \leq 1/3 \) (for \( -2 \geq n \geq -3 \)). Therefore we obtain \( M_*/M \propto V^{6(\tau-1)} \). If we assume \( M_* \propto M_* \), as indicated for the SDSS LLs and the Local Group dSphs, we get the required scaling relation \( M_*/M \propto V^2 \) for \( 3\tau(N-1) = 2 \). With \( \tau \leq 1/3 \), we obtain \( N \geq 3 \). This is a stronger dependence of SFR on surface gas density than measured for star-forming galaxies, \( N = 1.4 \pm 0.15 \) (Kennicutt 1998). It indicates that star formation efficiency is not a natural primary driver for the LL scaling relations, though it may have an important role.

8.3 Related work on the role of feedback

Our model for the additional role of radiative feedback in distinguishing dwarf spheroidals from dwarf irregulars can be regarded as a qualitative speculation, to be investigated in more detail in future work.

The trend of increasing total-mass-to-light ratio with decreasing mass, which results naturally from supernova feedback and reproduces the fundamental line, may not be enough for fully resolving the mystery of missing dwarf galaxies (Klypin et al. 1999; Moore et al. 1999; Springel et al. 2001). The discrepancy is not only between the faint-end slopes of the predicted mass function and the observed luminosity function, but it also involves the distribution of internal velocities. The halo masses inferred straightforwardly from the observed velocities in the LG dwarf galaxies, many of which are on the order of \( \sim 10 \, \text{km s}^{-1} \), are too small compared with the CDM model predictions. One possibility is that the relevant halo velocities are severely underestimated because the sampling by stars in dwarfs is biased towards the very inner halo regions, where the rotation and dispersion velocities may be significantly lower than the maximum or virial velocity relevant for mass estimation (Stoehr et al. 2002). This possibility is unlikely to provide the full answer because the velocities measured from H I gas, which samples more extended radii typically by a factor of 2–3, are still similarly low, \( \sim 10 \, \text{km s}^{-1} \) (Blitz, private communication). Thus the discrepancy between the observed velocity function and that predicted by the CDM model seems to indicate the presence of some barren haloes, which are completely dark and show no trace of luminous stars in
them (e.g. Kochanek 2001). Attempts have been made to explain the barren haloes by the radiative feedback effects discussed in Section 7.2, which are indeed expected to ‘squish’ the formation of stars in haloes that form after cosmological reionization at z ∼ 7 (e.g. Bullock, Kravtsov & Weinberg 2000; Somerville 2002; Tully et al. 2002). Such barren dark haloes may alternatively be explained by the destructive effect of energetic outflows from one galaxy on neighbouring forming protogalaxies via ram pressure brushing aside the tenuously held gas (e.g. Scannapieco, Ferrara & Broadhurst 2000; Scannapieco & Broadhurst 2001; Scannapieco et al. 2001; Thacker et al. 2002). These processes are yet to be studied with more realistic simulations, in an attempt to find out whether they can indeed explain the absence of luminous components in massive enough haloes.

A scale similar to the supernova scale, originating in a different way from the features of the cooling curve, is associated with another transition between different behaviours, as discussed in Bimboim & Dekel (2003). They found, using analytic arguments supported by simulations with a spherically symmetric Lagrangian hydro-dynamical code, that in haloes less massive than ∼ 3 × 10^{11} M⊙, the gas falling into the halo does not cross a virial shock until it hits the ‘disc’ itself. The ‘standard’ virial shock develops only in more massive haloes, hosting large galaxies and clusters, where the shock expands quickly to near the virial radius. Then, as is commonly assumed, infalling gas is heated behind the shock to the halo virial temperature and is kept pressure-supported in the halo until it cools radiatively and slowly contracts into the disc. In less massive haloes, where the virial temperature is below a few × 10^5 K, the shock that tries to develop loses energy very efficiently via radiation that is dominated by helium recombination and oxygen lines. This prevents the shock from ever expanding into the halo. A possible implication of this result is that early star formation becomes more efficient in haloes of M < 3 × 10^{11} M⊙, in shocks produced by the cold infalling gas when it hits the cold gaseous disc, giving rise to the burst which heats much of the remaining gas and produces an LSB galaxy. Further infalling gas may prevent blowout and keep the hot gas in the galaxy, giving rise to gas-rich dwarf irregulars.

The low baryonic fraction observed in V ∼ 60 km s^{-1} LSBs (by BBS), together with the prediction of q ∝ V^2 for the feedback effect, implies that V_{SN} ∼ 80 km s^{-1}, which is quite consistent with the DS estimate of V_{SN} ∼ 100 km s^{-1} and with the observed transition scale at a stellar mass of M_{∗} ≳ 10^{10} M⊙. Furthermore, as argued by Maller & Dekel (2002), feedback can help to solve the apparent angular momentum problem within the CDM scenario, where the baryons in cosmological simulations seem to lose most of their angular momentum and fail to form large discs, as observed (Navarro & Steinmetz 2000). Maller, Dekel & Somerville (2002) and Maller & Dekel (2002) modelled the properties of the LSB galaxies observed by BBS based on spin build-up from the orbital angular momenta along the halo merger history, combined with gas blowout from small merging satellites and the associated baryonic spin increase. They found that a value of V_{SN} ∼ 90 km s^{-1} can indeed explain the higher spin observed in the LSB galaxies.

We note that gas blowout in small haloes may also help resolve a third problem of the CDM scenario, the cusp/core problem of halo density profiles, where simulated haloes show a steep inner cusp although observations indicate that at least some galaxies have flat-density cores. Although feedback cannot affect the dark-matter distribution significantly in big galaxies, an impulsive blowout may reduce the core densities in dwarf satellites by a few factors (Gnedin & Zhao 2002). When these puffed-up satellites merge to build up bigger haloes, they get tidally disrupted before they manage to penetrate the inner regions and turn the cores into cusps. In this indirect way, the feedback can help the survival of cores even in relatively big galaxies (Dekel, Devor & Hetzroni 2003; Dekel et al. 2003). However, working out a feedback model within the CDM scenario that will explain the possible existence of cores in giant galaxies and clusters of galaxies could be challenging; it will require a feedback mechanism more energetic than simple supernova feedback, perhaps by hypernovae from massive stars or by radio jets from AGNs.

We conclude that feedback effects seem to be able to provide the cure to all three major problems facing galaxy formation theory within the CDM scenario. Understanding the details of the feedback processes is therefore a major goal of galaxy formation studies.

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