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Metallicity, planet formation and disc lifetimes

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ABSTRACT

The lifetime of protoplanetary discs is intimately linked to the mechanism responsible for their dispersal. Since the formation of planets within a disc must operate within the time frame of disc dispersal, it is crucial to establish what is the dominant process that disperses the gaseous component of discs around young stars. Planet formation itself as well as photoevaporation by energetic radiation from the central young stellar object has been proposed as plausible dispersal mechanisms. There is, however, still no consensus as what the dominant process may be. In this paper, we use the different metallicity dependence of X-ray photoevaporation and planet formation to discriminate between these two processes. We study the effects of metallicity, Z, on the dispersal time-scale, $t_{\rm phot}$, in the context of a photoevaporation model, by means of detailed thermal calculations of a disc in hydrostatic equilibrium irradiated by extreme ultraviolet and X-ray radiation from the central source. Our models show $t_{\rm phot} \propto Z^{0.52}$ for a pure photoevaporation model. By means of analytical estimates, we derive instead a much stronger *negative* power dependence on metallicity of the disc lifetime for a dispersal model based on planet formation.

A census of disc fractions in lower metallicity regions should therefore be able to distinguish between the two models. A recent study by Yasui et al. in low-metallicity clusters of the extreme outer Galaxy ([O/H] ~ -0.7 dex and dust-to-gas ratio of $\sim$0.001) provides preliminary observational evidence for shorter disc lifetimes at lower metallicities, in agreement with the predictions of a pure photoevaporation model. While we do not exclude that planet formation may indeed be the cause of some of the observed discs with inner holes, these observational findings and the models and analysis presented in this work are consistent with X-ray photoevaporation as the dominant disc dispersal mechanism.

We finally develop an analytical framework to study the effects of metallicity-dependent photoevaporation on the formation of gas giants in the core accretion scenario. We show that accounting for this effect strengthens the conclusion that planet formation is favoured at higher metallicity. We find, however, that the metallicity dependence of photoevaporation only plays a secondary role in this scenario, with the strongest effect being the positive correlation between the rate of core formation and the density of solids in the disc.

Key words: accretion, accretion discs – circumstellar matter – planetary systems: protoplanetary discs – stars: pre-main-sequence.

1 INTRODUCTION

It is not yet established what is the dominant process that disperses the gaseous component of the discs around young stars. What is clear observationally is that evidence of gas accretion on to stars, together with infrared/submm diagnostics associated with small dust grains entrained in the gas, disappears in young stars at an average age of a few Myr (Haisch, Lada & Lada 2001). This figure needs to be interpreted as an *average* figure since clearly individual stars may lose their discs on time-scales that differ from this by at least a factor of 3 (Armitage, Clarke & Palla 2003). It is equally clear, based on the relatively few systems observed in a state of transition between disc possessing and discless status, that the time-scale for disc dispersal (specifically the time-scale over which regions of the disc are optically thin in the infrared) is much shorter than the overall disc lifetime (Skrutskie et al. 1990; Kenyon & Hartmann 1995; Duvert et al. 2000). The fact that transition discs are relatively rare (constituting around 10 per cent of the population of young stars with discs) is, however, an observational hindrance when it

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comes to establishing the mechanism for disc clearing: transition discs appear to be a rather heterogeneous class of objects and it is hard to classify their diversity when the number of well-studied objects in nearby star-forming regions is still in single figures. An important feature of many transition discs, however, is that their spectral energy distributions are best fit by inner holes in the disc (these holes are not necessarily devoid of dust, but there is a large contrast in surface density between the outer disc and inner cavity).[1]

Although an obvious mechanism for the disappearance of dust diagnostics in discs is the simple coagulation of grains into entities that are large compared with the wavelength of observation (Dullemond & Dominik 2005), this scenario does not itself explain why, in general, there is a correlation between the disappearance of dust and gas accretion diagnostics. Moreover, it is not obvious why dust coagulation should create a well-defined inner-hole structure rather than homogeneous depletion at all radii (although models combining grain growth with photophoresis – once the inner disc is optically thin to visible light in the radial direction – are promising in this regard; Krauss et al. 2007).

The two leading disc dispersal mechanisms that satisfy these considerations are the formation of giant planets and photoevaporation. Both these mechanisms create an inner hole in the disc and also disrupt the accretion flow on to the star to some extent.[2] In the case of photoevaporation, inner-hole creation is succeeded by the rapid clearing of the outer disc thereafter. In the case of planet formation, clearing of the outer disc requires further planet formation at larger radii, i.e. the relative scarcity of transition discs argues that planet formation at one radius is rather rapidly followed by a wave of successive planet formation events in the outer disc (Armitage & Hansen 1999). Some attempts have been made to classify whether individual transition discs are likely to be generated by planet formation or photoevaporation (Najita, Strom & Muzerolle 2007; Alexander & Armitage 2007; Alexander 2008; Cieza et al. 2008; Kim et al. 2009) based on estimates of the accretion rate and disc mass. These analyses are, however, based on extreme ultraviolet (EUV) photoevaporation models (Hollenbach et al. 1994; Clarke et al. 2001; Alexander, Clarke & Pringle 2006a; Alexander et al. 2006b) which predict low accretion rates and disc masses in transition objects, in contrast to many of the observed systems. More recent models based on X-ray photoevaporation (Ercolano et al. 2008b; Ercolano, Clarke & Drake 2009, hereafter ECD09; Owen et al. 2009) predict higher accretion rates and disc masses during transition and thus somewhat muddy the observational distinction between the two classes of disc dispersal mechanism. Nevertheless, it would appear clear that no single mechanism can explain all observed inner hole systems: for example, photoevaporation is incompatible with the predominantly high accretion rates in the sample of Kim et al. (2009) whereas a planet is perhaps an unlikely candidate for many of the objects contained in Cieza et al. (2008), which have very large holes and low accretion rates.

At this point, it is perhaps worth noting – despite the obvious interest of observed transition objects – that they do not *necessarily* hold the key to the process that disperses the *majority* of discs around young stars. This is because objects observed in transition can in principle be a mixture of stars that spend a significant time in this state (and must therefore be a minority of all young stars on statistical grounds) and a general population of 'typical' young stars that must pass through such a transition quickly. (The argument that some fairly abrupt end to a disc's lifetime is required is well illustrated by considering what would happen if a disc merely continued to accrete on to the central star as a result of viscous evolution – the observed accretion rates, ages and disc masses of T Tauri stars would imply that integrated forward in time they would remain optically thick for >100 Myr and then would linger for a comparable period as partially cleared systems). Therefore, regardless of what process explains the bulk of observed transition discs, we also need to decide what is the mechanism that terminates the lifetime of the majority of disc bearing young stars.

In this paper, we therefore take a different approach to discriminating between planet formation and photoevaporation as the prime disc dispersal mechanism, by instead considering the *metallicity dependence* of these two processes. Although most studies of circumstellar discs have currently been undertaken in regions with a rather low range of metallicities, the advent of the Herschel Space Observatory and later the James Webb Space Telescope raises the prospect of being able to conduct disc censuses in more distant regions whose metallicity differs considerably from that of star-forming regions in the solar neighbourhood. Such studies are already possible in the extreme outer Galaxy (EOG) with current instrumentation as demonstrated by the recent work of Yasui et al. (2009, hereafter YS09) which we will discuss in more detail later. Our focus here will not be on the nature of the (in any case rare) transition objects, but will instead centre on how the *mean disc lifetime is expected to vary with metallicity in these two scenarios*. To this end, we combine simple models for disc viscous evolution with (metallicity dependent) photoevaporation and also with semi-analytic prescriptions for planet formation (Section 2). We will show that in the photoevaporation model the disc lifetime is a mildly increasing function of metallicity, resulting from the rather higher photoevaporation rates in the case of low-metallicity gas for which opacities are lower and line cooling less efficient. On the other hand, the disc lifetime is a strongly decreasing function of metallicity in the case that disc clearing is initiated by planet formation. This is simply because at higher metallicity the assumed higher surface density of planetesimals in the disc encourages the more rapid formation of a gas giant planet, specifically because it accelerates the formation of a rocky core of the critical mass required to initiate the accretion of a gaseous envelope. This is borne out, for example, by the hydrodynamical models of gas giant formation of Pollack et al. (1996) and Hubickyj, Bodenheimer & Lissauer (2005).[3] This positive link between metallicity and planet formation has been noted by a number of authors (e.g. Ida & Lin 2004b) as a possible explanation of the observed increase in the frequency of giant planets as a function of metallicity (Santos, Israelian & Mayor 2000, 2001, 2004; Gonzalez et al. 2001; Sadakane et al. 2002; Heiter & Luck 2003; Laws et al. 2003; Fischer & Valenti 2005; Santos et al. 2002 and references therein), but has not been previously linked to the issue of disc lifetimes. In Section 3, we revisit the connection between

[1] This situation may not extend to discs around lower mass stars, as there are recent claims that discs in M stars may instead pass through an extended phase in which their dust content is homogeneously depleted at all radii (Currie et al. 2009).

[2] More or less completely in the case of photoevaporation (Clarke, Gendrin & Sotomayor 2001; Alexander, Clarke & Pringle 2006b) and to a variable extent in the case of planet formation, depending on the mass of the planet (Lubow, Seibert & Artymowicz 1999; Rice et al. 2003).

[3] Note that this conclusion is based on an assumed linear relationship between the density of planetesimals and the metallicity; this conclusion would be only strengthened if one takes into account the recent suggestion of Johansen, Youdin & MacLow (2009) that the efficiency of planetesimal formation by the streaming instability should increase steeply at higher metallicity.

the frequency of giant planets and metallicity in the case of a hybrid model where planet formation takes place in the context of a nebula that is subject to metallicity-dependent photoevaporation of the disc gas. Since photoevaporation is more effective at low metallicities and thus reduces the disc lifetime, this strengthens the conclusion that planet formation is favoured at high metallicity. Nevertheless, the additional ingredient of metallicity-dependent photoevaporation is found to be a second-order effect in determining the positive correlation between planet frequency and metallicity. We present our conclusions in Section 4.

2 THE DEPENDENCE OF DISC LIFETIME ON METALLICITY

2.1 Lifetime against photoevaporation

We consider the case where the gas in the disc (initially with mass $M_{\rm go}$) undergoes viscous accretion on to the central star. As a result of this secular evolution, the disc radius grows while the disc mass and the accretion rate on to the star decline with time. We here parametrize this process according to the similarity solutions that pertain in the case that the kinematic viscosity in the disc is a simple power-law function of radius ($\nu \propto r^p$; Lynden-Bell & Pringle 1974, Hartmann et al. 1998). This is not likely to be a good description in detail since even in a simple α disc model the viscosity is a function of both surface density and radius, depending on the dominant local opacity source; yet further complications are introduced when more realistic prescriptions for angular momentum transport – involving the action of the magnetorotational instability or of self-gravitating torques are included (e.g. Armitage, Livio & Pringle 2001; Clarke 2009; Rice & Armitage 2009; Zhu, Hartmann & Gammie 2009; Cossins, Lodato & Clarke 2009). The prescription we use here, however, has the advantage of providing a simple analytic form such that the value of p is linked to the power-law index of the surface density profile (i.e. over much of the radial range of the similarity solution, the disc's surface density is a power law $\Sigma \propto r^{-p}$). Our choice of p (which fixes the power-law exponents for the evolution of disc quantities) is therefore observationally motivated by fits to the infrared spectral/submm energy distributions of young stars, which suggest $p \sim 1$ (Andrews et al. 2009; Kitamura et al. 2002; Isella, Carpenter & Sargent 2009).

The appendix contains a heuristic derivation of the asymptotic form of the similarity solutions; according to equation (1) the accretion rate declines as

$$\dot{M} \propto M_{\rm g0} r_{\rm d0}^{1/2} t^{(-5+2p)/(4-2p)}, \qquad (1)$$

where $r_{\rm d0}$ is the initial disc scaling radius. Such a power-law decline obviously does not enable one to define a disc lifetime, since the time-scale on which disc properties change by order unity is always of the order of the present age. We therefore need to introduce a further condition for disc dispersal. In the case that we invoke (metallicity dependent) photoevaporation, at a constant rate $\dot{M} = \dot{M}_W(Z)$, then the disc disperses rapidly at the point that the accretion rate through the disc falls to $\dot{M}_W(Z)$.[4] Thus, the disc lifetime against photoevaporation can be written:

$$t_{\rm phot} \propto \dot{M}_W(Z)^{(4-2p)/(-5+2p)} M_{\rm g0}^{(4-2p)/(5-2p)} r_{\rm d0}^{(2-p)/(5-2p)}, \qquad (2)$$

At this point, it becomes necessary to specify the source of the photoevaporative flow. Until very recently, it has been widely assumed that such a flow is mainly driven by the EUV photons from the central star. However, we have recently shown that soft (0.1–1 keV) X-ray irradiation can drive powerful winds with typical mass-loss rates of 10^{-9} to $10^{-8}\,{\rm M}_\odot\,{\rm yr}^{-1}$ (Ercolano et al. 2008b; ECD09; Owen et al. 2009). In the hydrostatic equilibrium model of ECD09, X-rays from the central pre-main-sequence star ionize and heat the gas in the disc atmosphere; the photoevaporation rate is estimated by assuming that gas that becomes hotter than the local escape temperature becomes unbound and escapes at the sound speed. In reality, pressure gradients within the disc are such that a photoevaporative flow can in fact be initiated subsonically in deeper layers where the temperature is less than the escape speed. This produces even larger winds (Owen et al. 2009) which exceed photoevaporation rates from EUV radiation by two orders of magnitude. In fact, for strong X-ray emitters (i.e. with $L_{\rm X} \sim 10^{30}$ erg s $^{-1}$), these large flow rates are similar to the median accretion rates measured in T Tauri stars (Natta, Testi & Randich 2006). When one also takes into account of the dispersion in X-ray luminosities among young stars (Preibisch et al. 2005; Albacete Colombo et al. 2007) and the associated range of expected photoevaporation rates, it then becomes plausible that – in the absence of competing disc dispersal mechanisms – the lifetime of gas discs in T Tauri stars is terminated by X-ray powered photoevaporation.

With this in mind, we therefore use the metallicity dependence of the photoevaporation rate derived from our X-ray photoevaporation models. These employ the modelling strategy described by Ercolano et al. (2008b) and ECD09 to obtain the temperature structure and photoevaporation rates for gaseous discs with a range of metallicities, spanning from 0.01 solar to twice solar. Briefly, this involves the coupling of the three-dimensional photoionization and dust radiative transfer code MOCASSIN (Ercolano et al. 2003; Ercolano, Barlow & Storey 2005; Ercolano & Storey 2006; Ercolano et al. 2008a) with an iterative solution of the hydrostatic equilibrium structure of the irradiated disc. Our initial guess at the density distribution is the disc model of D'Alessio et al. (2001) for a 4000 K 0.7 ${\rm M}_\odot$ pre-main-sequence star surrounded by an optically thick flared circumstellar disc with an outer radius of 500 au and a total mass of 0.027 ${\rm M}_\odot$. This structure is irradiated by an unscreened EUV+X-ray spectrum (see description in ECD09). We refer to ECD09 for further details including the dust model and the irradiating spectrum. Our solar abundance set is that of Asplund, Grevesse & Sauval (2005) depleted according to Savage & Sembach (1996). The dust-to-gas ratio for the solar abundance case was set to 6.5×10^{-4} (see the discussion in ECD09). For models of different metallicity, the dust-to-gas ratio and the abundances of metals were multiplied by ($Z/Z_\odot$). As noted above, we estimate the photoevaporation rates by assuming that a sonic flow is launched from regions where the temperature exceeds the local escape temperature. Our preliminary hydrodynamic calculations (Owen et al. 2009) suggest that this may underestimate the total mass flow rates, although we do not expect this to have a large effect on the relative scaling with metallicity.

The left-hand panel of Fig. 1 shows the dependence of the total photoevaporation rates on metallicity, $\dot{M}_{\rm W}(Z)$, which can be approximated by a power law of index -0.77. The model data points shown in the left-hand panel of this figure were obtained using the approach described above, which involved

[4] Here, we are ignoring the fact that, as shown by Owen et al. (2009), the rapid disc dispersal begins once the accretion rate has fallen to roughly 10 times below the photoevaporation rate, causing the system to experience a period of photoevaporation-starved accretion (Drake et al. 2009) prior to the fast dispersal phase. These consideration, however, do not affect the proportionality relations and therefore the conclusions of this work remain unchanged.

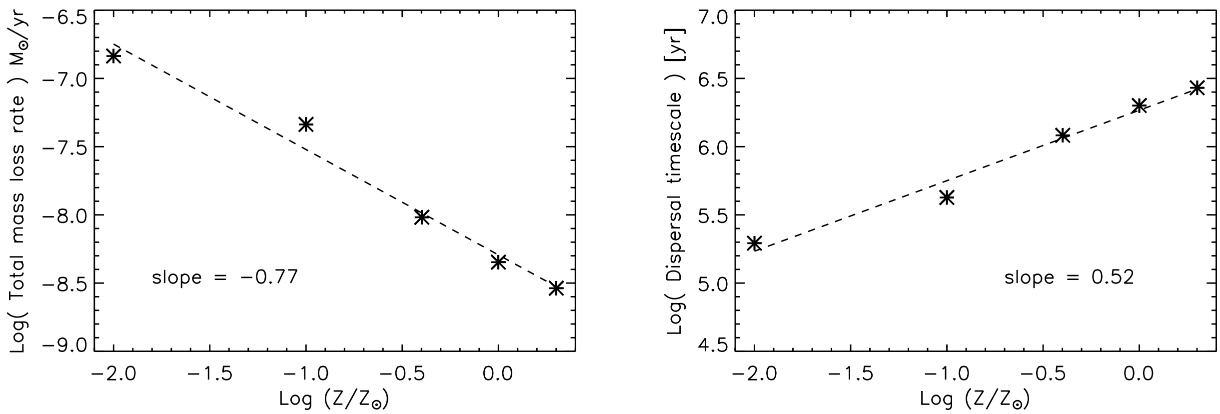

Figure 1. The metallicity dependence of mass-loss rates (left-hand panel) and dispersal time-scales (right-hand panel) of protoplanetary discs photoevaporated by X-ray and EUV from the central star. Right-hand panel: model data points (asterisks) obtained from our photoionization modelling; the dashed line is a least square fit to these data. Left-hand panel: the asterisks show the photoevapoaration time-scales obtained substituting the points on the right-hand panel into equation (A10) and normalizing to 2 Myr for solar metallicities. The dashed line shows the relation given in equation (4) for $p = 1$.

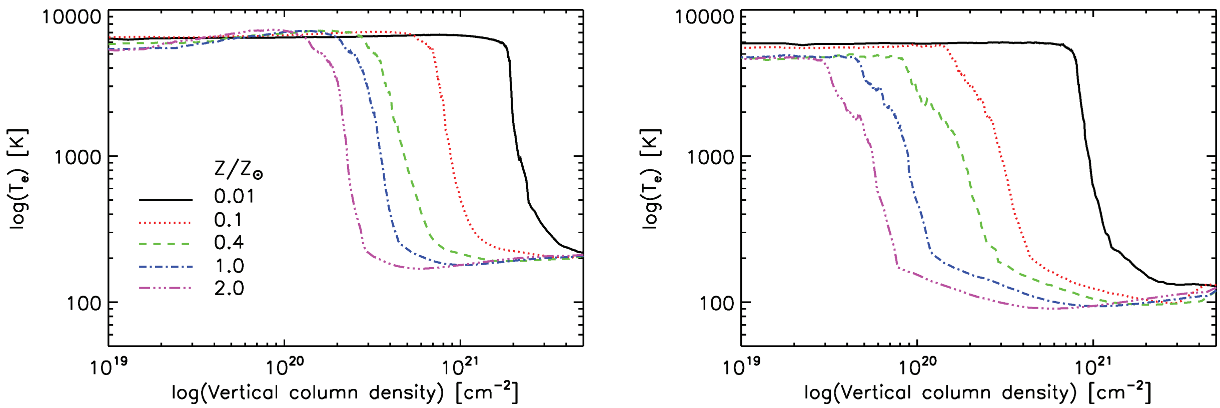

Figure 2. Gas temperatures distribution at a radial distance of 5 au (left-hand panel) and 20 au (right-hand panel).

rerunning model FS0H2Lx1 from the set described by ECD09 for the appropriate metallicities and dust-to-gas ratios. The corresponding photoevapoaration time-scales, shown in the right-hand panel, were then obtained by substituting these numerical values of $\dot{M}_{\mathrm{W}}(Z)$ in equation (A10). The resulting time-scales were then normalized such that a lifetime of 2 Myr is obtained at solar metallicities.

The increasing X-ray photoevaporation rates at lower metallicity can be readily understood by looking at the gas temperature distributions in the disc for the different-metallicity cases that are shown in Fig. 2. The reduced extinction in the low-metallicity cases allows high-density gas at larger columns to be ionized and heated to temperatures sufficiently high for the gas to be entrained into a photoevaporative flow. Other metallicity-dependent effects are also present and work in the same direction (i.e. lower metallicity = higher gas temperatures), but they play a secondary role. These effects are listed here for completeness: a low metallicity implies a reduced cooling by fine structure lines of ions and neutrals (such as [O I] and [C II]; e.g. Ercolano et al. 2008b), which leads to higher temperatures in the X-ray heated disc atmosphere, as particularly evident in the right-hand panel of Fig. 2. Also, a lower dust-to-gas ratio means that the gas experiences a reduced competition from the dust grains for the absorption of the energetic photons and that the dust gas collisional cooling term, which may provide a significant contribution at the base of the photoevaporative envelope, is reduced at lower metallicities. We emphasize, however, that by far the dominant mechanism here is the reduced opacity which increases the penetration column of the ionizing radiation. It is worth noting at this point that such a large effect is not expected to occur for gas ionized by EUV only. X-ray photons are mainly absorbed by the inner shells of the more abundant heavy elements in the gas and dust (e.g. oxygen, carbon, etc.) while for EUV photons by far the largest source of opacity is hydrogen, which is not affected by changes in metallicity (although a reduction in the dust-to-gas ratio would still reduce the total EUV opacity). We also note that a photoevaporation process based on far-ultraviolet irradiation would also follow a different dependence on metallicity (Gorti & Hollenbach 2009). Therefore, *the predicted metallicity dependence shown here pertains only to X-ray photoevaporated discs*.

When this metallicity-dependent wind mass-loss rate is combined with equation (2), one obtains the result that

$$t_{\mathrm{phot}} \propto Z^{0.77(4-2p)/(5-2p)} M_{\mathrm{g0}}^{(4-2p)/(5-2p)} r_{\mathrm{d0}}^{(2-p)/(5-2p)}. \quad (3)$$

Since we do not expect the distribution of initial disc gas properties (M_{g0} and r_{d0}) to depend on metallicity, we thus deduce that the distribution of disc lifetimes in regions of different metallicity should scale with a Z-dependent factor, i.e.

$$t_{\mathrm{phot}} \propto Z^{0.77(4-2p)/(5-2p)}. \quad (4)$$

This Z dependence is rather weak, with a power-law exponent of 0.52 (for $p = 1$) and 0.38 (for $p = 1.5$). The right-hand panel of Fig. 1 is an illustrative example of the dependence on metallicity of the mean disc lifetime of a population of discs (with $p = 1$) that is normalized such that the mean disc lifetime for discs of solar metallicity is 2 Myr. We emphasize that this normalization is set by our assumptions about the disc's secular evolution time-scale, as controlled by its equivalent viscous 'α' parameter (Shakura & Sunyaev 1973). We can readily adjust this quantity within a plausible range in order to reproduce an observationally reasonable mean disc lifetime. An increase in metallicity from solar to twice solar causes the disc lifetime to increase from ~2 to ~3.1 Myr, and a decrease from solar to −0.7 dex solar, as in Cloud 2 in the EOG observations of YS09, produces a decrease in the disc lifetime from ~2 to ~0.7 Myr, which is roughly consistent with the observed disc fractions.

2.2 Disc lifetime against gas giant planet formation

We now assume that disc dispersal is instead initiated by the formation of the first gas giant planet, as is a common interpretation of observed transition discs (Rice et al. 2003; Setiawan et al. 2007). According to the core accretion model, the formation of a gas giant involves first the accumulation of a solid core of critical mass (of the order of 10–20 earth masses) followed by the hydrodynamic accretion of gas from the surrounding disc (Pollack et al. 1996). This latter occurs on the Kelvin Helmholtz time-scale of the planet, which is ~1 Myr for a 20 earth mass planet and decreases strongly with increasing planet mass. For the purpose of our analytic estimates (following Ida & Lin 2004a, hereafter IL04), we therefore assume that the limiting time-scale in the creation of a gas giant planet (i.e. a roughly Jupiter mass planet which can clear a gap in the disc) is the time required to accumulate a core containing a critical mass of solids. In fact, hydrodynamic calculations of planet formation paint a more complex picture in which the majority of the formation process is instead spent in a phase ('Phase 2'; Hubickyj et al. 2005) where the planet accretes a mixture of gas and planetesimals and, depending on the properties of the background nebula, gas giants can form, albeit more slowly, with a core mass somewhat below the critical value mentioned above (we discuss below how such considerations affect our conclusions).

Proceeding for now with our simple analytic estimate, we evaluate the time-scale needed to achieve a fixed critical core mass (together with its scaling with metallicity), by using the analytic expression for the growth of solid core mass ($M_{\rm c}$) given in IL04:

$$\frac{{\rm d}M_{\rm c}}{{\rm d}t} \propto \Sigma_{\rm d}(r) r^{-3/5} M_{\rm c}^{2/3} \Sigma_{\rm g}^{2/5}, \tag{5}$$

where $\Sigma_{\rm d}$ is the surface density of rocky planetesimals and $\Sigma_{\rm g}$ is the instantaneous local value of the gas surface density. We follow IL04 by assuming that the planetesimal distribution is decoupled from the evolutionary processes that control the evolution of $\Sigma_{\rm g}$. We thus assume that a fixed fraction of the initial solid content of the disc forms a planetesimal disc whose radial profile follows the initial radial profile of the gas and that $\Sigma_{\rm d}$ is then constant in time (we thus neglect the depletion in $\Sigma_{\rm d}$ due to the accretion of planetesimals during core growth; this is acceptable to first order provided that the core is not close to achieving its isolation mass, an issue to which we return below.)

We however (see the appendix) differ from IL04 in that we use the viscous similarity solutions to determine the time dependence of $\Sigma_{\rm g}$ rather than adopting an ad hoc exponential reduction in the gas surface density. Solving equation (A13) and requiring that the core mass attains a fixed value for runaway gas accretion (see the appendix) then imply a formation time-scale:

$$t_{\rm form} \propto Z^{-5(2-p)/(5-3p)} M_{\rm g0}^{-7(2-p)/(5-3p)} r_{\rm d0}^{(2-p)(9-5p)/(5-3p)} \times r^{(2-p)(7p+3)/(5-3p)}. \tag{6}$$

Since this time-scale is an increasing function of radius, it follows that planet formation is first favoured at small radius. However, this effect is counteracted by two effects that favour core growth at larger radius. First, the sticking efficiency of planetesimals increases once the radius at which ice sublimes ($a_{\rm ice}$) is exceeded (Hayashi 1981; Pollack et al. 1994). Secondly, it is necessary that a critical core mass can form without consuming all the planetesimals in its 'feeding zone'. This latter is often assumed to be an annular region around the core with width related to the Hill radius of the core (i.e. of width that scales linearly with orbital radius). Thus, the radius (a_{tg}) at which a core of critical mass just consumes all the planetesimals in its feeding zone obeys a scaling of the form[5]

$$\Sigma_{\rm d} a_{tg}^2 = {\rm constant} \tag{7}$$

i.e.

$$a_{tg} \propto Z^{-1/(2-p)} r_{\rm d0} M_{\rm g0}^{-1/(2-p)}. \tag{8}$$

Therefore, if $a_{\rm ice} > a_{tg}$ (i.e. if the minimum radius for planet formation is set by ice sublimation rather than feeding zone considerations) we have

$$t_{{\rm form}_{\rm min}} \propto \left(Z^{-5} M_{\rm g0}^{-7} r_{\rm d0}^{(9-5p)}\right)^{(2-p)/(5-3p)}. \tag{9}$$

Alternatively, if $a_{tg} > a_{\rm ice}$ we substitute (8) into (6) and obtain

$$t_{{\rm form}_{\rm min}} \propto \left[Z^{(-13-2p)} M_{\rm g0}^{-17} r_{\rm d0}^{(4-2p)(6+p)}\right]^{1/(5-3p)} \tag{10}$$

We thus see that these planet formation time-scales are highly Z dependent, in the sense that planet formation is faster in discs with higher metallicity. For $p = 1$ and 1.5, equation (9) implies that the planet formation time-scale scales either with $Z^{-2.5}$ or Z^{-5}, while 10 implies scaling either with $Z^{-7.5}$ or with Z^{-32}!".[6]

This extreme level of Z dependence is almost certainly an artefact of the fact that we have assumed that the time-scale for giant planet formation is completely controlled by the time-scale for solid core growth. It is also worth mentioning that this very steep scaling stems from the fact that the prescription for core growth (IL04; equation 5 above) involves a dependence on gas density; physically, this accounts for the fact that gas drag reduces the velocity dispersion of the planetesimals and hence increases the core's accretion cross-section due to gravitational focusing (Kokubo & Ida 2002). We assume that the disc gas surface density declines as a power law due to viscous evolution and thus the core grows as a low power of time. Thus, the time required to achieve a given core mass is a strong function of the normalization of the solid mass. This is not necessarily incorrect and is in this respect probably more realistic than models of core accretion which adopt a temporally constant background gas surface density (to date such models either keep this quantity constant or introduce a linear decline to zero over 1–3 Myr; Lissauer et al. 2009).

[5] If, as discussed in IL04, one relaxes this assumption so that the core can feed from the entire stock of planetesimals interior to its orbit then one also obtains a scaling of the same form.

[6] We also note that the planet formation time-scale is highly dependent on the initial disc mass, implying that planet formation should only be possible in systems belonging to the upper centiles of the disc mass distribution. When the disc mass threshold implied by the Ida & Lin models is combined with observations of the disc mass distribution by Andrews & Williams (2005), the predicted planet frequency is consistent with those found in radial velocity surveys at solar metallicity (see the discussion in Wyatt, Clarke & Greaves 2007).

We now turn to the results of hydrodynamic models of planet formation and see to what degree the limited range of available models in fact support our estimates above. These models demonstrate two effects: increasing the surface density of solids decreases the time-scale for planet formation, in qualitative agreement with the arguments above. Another factor in the opposite direction is that at higher metallicity the opacity of the accreting gaseous envelope is increased; this slows the contraction of the envelope and thus slows down gas accretion. (Note that our simple model above only considers the time needed to acquire a critical solid core mass and does not consider the subsequent time spent in acquiring a similar mass in gas which is considerable in the hydrodynamic calculations; Hubickyj et al. 2005). Nevertheless, it is found that the acceleration of planet formation due to enhanced planetesimal surface density is a much stronger effect than the retarding effect of increased opacity. For example, Hubickyj et al. found that the time to form a gas giant increases by around a factor of 3 if the dust opacity is boosted by a factor of 50 (see also Ayliffe & Bate 2009); on the other hand, a modest increase of the planetesimals surface density by a factor of 1.6 reduces the formation time-scale by a factor of 6. Thus, the simulations support our qualitative conclusion that the net effect of increasing the metallicity is to strongly decrease the formation time-scale of giant planets.

More detailed comparison is not really warranted given the fact that the choice of simulations performed was motivated by different scientific questions than the one we ask here. We are interested in the time required to form a planet at given metallicity *anywhere* in the disc. Since we have imposed a hard requirement that the growing protoplanet should achieve a fixed core mass, we exclude planet formation in inner regions of the disc where the isolation mass is less than this critical value; in our prescription, we first form a planet at a larger radius where the formation time-scale is longer. Evidently, this slows down planet formation at low metallicity. The hydrodynamical calculations are instead motivated by creating Jupiter at a particular radial location. In low-metallicity runs, planet formation takes longer because the forming protoplanet has to 'make do' with a lower core mass and this then greatly increases the time-scale for accreting a comparable mass of gas. In a real disc, both these effects would come into play and it is not clear whether, under these circumstances, the first planet to form is at larger radius (where the core mass is higher but the core accumulation time-scale is longer) or at smaller radius where the bottleneck is instead the slow accumulation of gas on to a smaller mass rock core. What is important to our discussion is that both effects are in the same direction and imply a strong (negative) dependence of planet formation time-scale on planetesimal surface density (and thus implicitly metallicity).

If the majority of protoplanetary discs dispersed due to planet formation then a disc census in a lower metallicity region, such as Cloud 2 in the EOG, would yield much longer disc lifetimes than those derived in the solar neighbourhood. This is in contradiction with the recent observations of YS09 who find disc lifetimes shorter than 1 Myr, compared to the few Myr found in the solar neighbourhood (e.g. Haisch et al. 2001).

3 THE METALLICITY DEPENDENCE OF PLANET FORMATION IN DISCS SUBJECT TO PHOTOEVAPORATION

As the list of exoplanets discovered in the solar neighbourhood continues to grow, statistical significance is lent to the observation that solar-type stars hosting giant planets are on average more metal rich than field stars (Santos et al. 2000, 2001, 2004; Gonzalez et al. 2001; Sadakane et al. 2002; Heiter & Luck 2003; Laws et al. 2003; Fischer & Valenti 2005; Santos et al. 2002 and references therein). While the origin of this metallicity excess has been the object of much debate in the literature, with the two main scenarios being 'primordial' or 'external' enrichment, there is a growing consensus that metal-rich gaseous discs are a more favourable environment for the formation of the rocky cores required by core accretion models (e.g. IL04).

The phenomenological simulations of gas giant planet formation of IL04 (whose assumptions imply, as we showed above, a strong reduction in planet formation time-scale with metallicity) demonstrated a statistical preference for planet formation and survival in discs with a high ratio of solids to gas. If one ignores the possibility of stellar surface contamination then the solid-to-gas ratio simply scales with the metallicity of the central star. Such simulations therefore reproduce the observed positive correlation between the incidence of planets and stellar metallicity. Indeed, to zeroth order, the result results of IL04 can be understood by adopting their prescription for core growth (equation A11) and deriving the region of parameter space of initial conditions, at given Z, that lead to the creation of a critical mass core. Wyatt et al. (2007) showed that this is roughly equivalent to thresholding the disc mass distribution at a fixed mass in solids; they showed that using the *observed* disc mass distribution (Andrews & Williams 2005) one can reproduce the observed dependence of planet frequency on metallicity under this simple assumption.

As noted above, the models of IL04 omitted viscous evolution of the disc gas and instead depleted the gas on an adjustable global e-folding time-scale. Moreover, they included no hard cut-off in disc gas lifetimes as would result from photoevaporation. We here therefore revisit this issue under the assumption that the lifetime of the disc gas is terminated by (metallicity-dependent) photoevaporation. In the context of our analysis in Section 2, we can readily establish which sets of disc initial parameters should lead to giant planet formation by looking at the subset of models for which $t_{\mathrm{form_{min}}} < t_{\mathrm{phot}}$ (equations 3, 9, 10). We then obtain the criteria:

$$M_{\mathrm{g0}} > K_1 \times r_{\mathrm{d0}}^{2(2-p)^2/(9-4p)} Z^{-(5-2p)/(9-4p)} \dot{M}_W(Z)^{2(5-3p)/5(9-4p)} \tag{11}$$

if $a_{\mathrm{ice}} > a_{tg}$ and

$$M_{\mathrm{g0}} > K_2 \times \left[r_{\mathrm{d0}}^{-(2-p)(55-11p+4p^2)} Z^{-(13+2p)(5-2p)} \times \dot{M}_W(Z)^{2(2-p)(5-3p)} \right]^{(1/(105-56p+6p^2)} \tag{12}$$

if $a_{\mathrm{ice}} < a_{tg}$. K_1 and K_2 are constants.

We thus obtain the expected result that a higher threshold gas mass is required at low metallicities; therefore, if we make the reasonable assumption that the disc gas *mass* distribution should be independent of metallicity one recovers the qualitative result that the incidence of giant planets should increase with metallicity. In order to quantify this, one of course has to make specific assumptions about the form of the distributions of initial disc masses and radii (Wyatt et al. 2007). Here, however, our interest lies elsewhere: we just want to discover whether the particular aspect of metallicity-dependent photoevaporation rates is an important factor in driving this relationship. To this end, we compare the powers of Z in equations (11) and (12) that would result from the case that the photoevaporation rate was independent of Z [i.e. $\dot{M}_W(Z) =$ constant] with the power of Z in the case that we employ our X-ray derived value $\dot{M}_w(Z) \propto Z^{-0.77}$. For $p = 1$, we find that this changes the power of Z in equation (11) from -0.6 to -0.88 and in equation (12) from −0.8 to −0.83. This therefore implies that the metallicity dependence of the photoevaporation rate has a rather minor part to play in explaining the observed positive correlation between planet frequency and stellar metallicity. This can be readily understood inasmuch as we have already seen (equation A10) that the disc lifetime is not expected to be a strong function of metallicity in this case and thus this does not in itself have a large impact on the planet producing capacity of a disc.

4 CONCLUSIONS

We have shown that the study of disc lifetimes in regions of different metallicities can be used as a powerful discriminant between the two currently leading models of disc dispersal – photoevaporation and planet formation. By means of detailed thermal and photoionization calculations, we have determined that a disc's lifetime against photoevaporation is a shallow positive power of metallicity, $t_{\rm phot} \propto Z^{0.52}$. This metallicity dependence is specific to a photoevaporation mechanism driven mainly by X-ray radiation as it relies on the significant reduction of the gas opacities with metallicity. We have also shown that, on the contrary, a disc dispersal mechanism based on planet formation yields disc lifetimes that are a strong negative power of Z. Therefore, a census of disc fractions in regions of lower metallicities compared to the solar neighbourhood will be crucial to determine which is the dominant mechanism responsible for the rapid demise of protoplanetary discs.

Recent observations of embedded clusters in Cloud 2 of the EOG presented by YS09 indeed find shorter disc lifetimes for this metal-poor environment ([O/H] ~ -0.7 dex), compared to the solar neighbourhood. These results are in agreement with the predictions of an X-ray+EUV photoevaporation model,[7] and argue against planet formation as the dominant dispersal mechanism.

YS09 quote a mass detection limit of 0.1 $M_{\odot}$ for the stars in Cloud2 of the EOG using the 8.2m *Subaru* telescope, similar to values obtained for embedded clusters in the solar neighbourhood with smaller (2–4 m class) telescopes, and thus argue that a comparison of the disc fractions in embedded clusters in the EOG with those of clusters in the solar neighbourhood is justified. Further observations aimed at determining disc fractions in low-metallicity regions of various ages would be very useful to confirm these important results.

We finally show that, in the context of giant planet formation in the core accretion scenario, the effect of metallicity-dependent photoevaporation is to strengthen the conclusion that planet formation is favoured in high-metallicity environments since the lifetime of the disc against photoevaporation ($t_{\rm phot}$) is a positive function of Z. This effect, however, only plays a secondary role: the main process that favours planet formation at high metallicity is simply the faster core growth in the case of a high surface density of solids in the disc.

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[7] It is worth noting at this point that these observational results are directly comparable to photoevaporation models even though the former probe the regions of the disc at radii smaller than those where photoevaporation dominates. This is because, as shown by Owen et al. (2009), the X-ray+EUV photoevaporation model predicts an inside–out clearing, following the opening of a gap at ∼1 au and a fast draining of the inner disc material.

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APPENDIX A: ANALYTICAL DERIVATIONS

In order to evaluate the metallicity dependence of giant planet formation, we need to establish how the background nebula evolves as a function of the initial conditions, i.e. the initial total disc mass in gas, $M_{\rm g0}$, and the initial disc outer radius, $r_{\rm d0}$. We assume that the evolution of the disc gas is governed by viscous evolution provided that the accretion rate through the disc exceeds the photoevaporation rate, $\dot{M}_W(Z)$, and that at this point the disc gas is dispersed, thus ruling out subsequent gas giant planet formation. We start by providing a heuristic derivation of the asymptotic evolution of a viscous disc (see Lynden-Bell & Pringle 1974; Hartmann et al. 1998) in which the kinematic viscosity is a power-law function of radius, i.e.

$$\nu \propto r^p. \tag{A1}$$

We first note that such a disc evolves through a sequence of discs with increasing radius, $r_{\rm d}$, and decreasing gas mass, $M_{\rm g}$, such that the total disc angular momentum is to the first order conserved (since the fraction of the total disc angular momentum that is advected on to the star is small). Thus, since most of the disc's angular momentum resides at large radius, we have

$$M_{\rm g} r_{\rm d}^{1/2} \sim M_{\rm g0} r_{\rm d0}^{1/2}. \tag{A2}$$

Furthermore, the disc evolves on the viscous time-scale at $r_{\rm d}$; since this is long compared with the viscous time-scale at radii $<< r_{\rm d}$, it follows that the disc at radii $<< r_{\rm d}$ is in an approximately steady state, i.e. the accretion rate, $\dot{M}$, is independent of r. In this case, accretion disc theory (e.g. Pringle 1981) relates this (at radii well away from the disc's inner edge) to the kinematic viscosity and surface density, $\Sigma_{\rm g}(r)$, via

$$\dot{M} \sim 3\pi\Sigma_{\rm g}\nu. \tag{A3}$$

It thus follows that the asymptotic form of the viscously evolving disc over much of its radial extent is in this case given by the power law $\Sigma_{\rm g} \propto r^{-p}$. If we normalize this power law by the instantaneous values of the disc mass and radius, we then obtain

$$\Sigma_{\rm g} \propto M_{\rm g} r_{\rm d}^{-(2-p)} r^{-p}. \tag{A4}$$

Combining this with equation (A2), we can eliminate $r_{\rm d}$ and obtain

$$\Sigma_{\rm g} \propto M_{\rm g0}^{-2(2-p)} r_{\rm d0}^{-(2-p)} M_{\rm g}^{(5-2p)} r^{-p}. \tag{A5}$$

Now from equations (A1), (A3) and (A5) we can write

$$\dot{M} = -\frac{{\rm d}M}{{\rm d}t} \propto M_{\rm g0}^{-(4-2p)} r_{\rm d0}^{-(2-p)} M_{\rm g}^{(5-2p)} \tag{A6}$$

from which we obtain the asymptotic power-law scalings:

$$M_{\rm g} \propto M_{\rm g0} r_{\rm d0}^{1/2} t^{-1/(4-2p)}, \tag{A7}$$

$$\dot{M} \propto M_{\rm g0} r_{\rm d0}^{1/2} t^{(-5+2p)/(4-2p)} \tag{A8}$$

and

$$\Sigma_{\rm g} \propto M_{\rm g0} r_{\rm d0}^{1/2} r^{-p} t^{(-5+2p)/(4-2p)}. \tag{A9}$$

If we now define the disc lifetime against dispersal by photoevaporation as the disc age such that $\dot{M} = \dot{M}_W(Z)$, we have

$$t_{\rm phot} \propto \dot{M}_W(Z)^{(4-2p)/(-5+2p)} M_{\rm g0}^{(4-2p)/(5-2p)} r_{\rm d0}^{(2-p)/(5-2p)}. \tag{A10}$$

We now consider planet formation in the context of such a viscously evolving gas disc and follow IL04 by assuming that the growth of the rock core is governed by an equation of the form

$$\frac{{\rm d}M_{\rm c}}{{\rm d}t} \propto \Sigma_{\rm d}(r) r^{-3/5} M_{\rm c}^{2/3} \Sigma_{\rm g}^{2/5}. \tag{A11}$$

$\Sigma_{\rm d}$ is the surface density of rocky planetesimals. We follow IL04 by assuming that this distribution is decoupled from the evolutionary processes that control the evolution of $\Sigma_{\rm g}$. We thus assume that a fixed fraction of the initial solid content of the disc forms a disc whose radial profile follows the initial radial profile of the gas and that $\Sigma_{\rm d}$ is then constant in time (we thus neglect the depletion in $\Sigma_{\rm d}$ due to the accretion of planetesimals during core growth; this is acceptable to the first order provided that the core is not close to achieving its isolation mass, an issue that we consider further below). Thus,

$$\Sigma_{\rm d} \propto Z M_{\rm g0} r_{\rm d0}^{-(2-p)} r^{-p} \tag{A12}$$

substituting from (A.9) and (A.12) into (5) we obtain

$$\frac{{\rm d}M_{\rm c}}{{\rm d}t} \propto Z M_{\rm g0}^{7/5} r_{\rm d0}^{-(9-5p)/5} r^{-(7p+3)/5} M_{\rm c}^{2/3} t^{-(2/5)(5-2p)/(4-2p)}. \tag{A13}$$

We conclude from (A13) that the time-scale for the formation of a rocky core of critical mass for the accretion of a gaseous envelope then scales with radius and initial disc parameters according to

$$t_{\rm form} \propto Z^{-5(2-p)/(5-3p)} M_{\rm g0}^{-7(2-p)/(5-3p)} r_{\rm d0}^{(2-p)(9-5p)/(5-3p)} \times r^{(2-p)(7p+3)/(5-3p)}. \tag{A14}$$

Equation (A14) demonstrates that the formation time-scale increases with increasing radius, as expected given the lower surface density and larger orbital time-scale at larger radius. Thus, planet formation occurs first at the minimum radius that is allowed according to two further criteria. First of all, it is necessary that this radius is at least as large as the ice sublimation radius ($a_{\rm ice}$) since the sticking efficiency of planetesimals is much lower in regions devoid of solid ice. Secondly, it is necessary that a critical core mass can form without consuming all the planetesimals in its 'feeding zone'. This latter is a region whose width is related to the Hill radius of the core, and thus is a linear function of orbital radius. Thus, the radius (a_{tg}) at which a core of critical mass just consumes all the planetesimals in its feeding zone obeys a scaling of the form

$$\Sigma_{\rm d} a_{tg}^2 = \text{constant} \tag{A15}$$

i.e.

$$a_{tg} \propto Z^{-1/(2-p)} r_{\rm d0} M_{\rm g0}^{-1/(2-p)}. \tag{A16}$$

Therefore, if $a_{\rm ice} > a_{tg}$ (i.e. if the minimum radius for planet formation is set by ice sublimation rather than feeding zone considerations) we have

$$t_{\rm form_{min}} \propto [Z^{-5} M_{\rm g0}^{-7} r_{\rm d0}^{(9-5p)}]^{(2-p)/(5-3p)}. \tag{A17}$$

Alternatively, if $a_{tg} > a_{\rm ice}$ we substitute (A16) into (A14) and obtain

$$t_{\rm form_{min}} \propto \left[Z^{(-13+2p)} M_{\rm g0}^{-17} r_{\rm d0}^{(4-2p)(6+p)}\right]^{1/(5-3p)}. \tag{A18}$$

Finally, the condition that at least one gas giant planet is able to form prior to disc photoevaporation is given by the condition $t_{\rm form_{min}} < t_{\rm phot}$, i.e.

$$M_{\rm g0} > K_1 \times r_{\rm d0}^{2(2-p)^2/(9-4p)} Z^{-(5-2p)/(9-4p)} \dot{M}_W(Z)^{2(5-3p)/5(9-4p)} \tag{A19}$$

if $a_{\rm ice} > a_{tg}$ and

$$M_{\rm g0} > K_2 \times \left[r_{\rm d0}^{-(2-p)(55-11p+4p^2)} Z^{-(13+2p)(5-2p)} \times \dot{M}_W(Z)^{2(2-p)(5-3p)}\right]^{(1/(105-56p+6p^2)} \tag{A20}$$

if $a_{\rm ice} < a_{tg}$. K_1 and K_2 are constants.

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