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A circumbinary disc in the final stages of common envelope and the core-degenerate scenario for Type Ia supernovae

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ABSTRACT

We study the final stages of the common envelope (CE) evolution and find that a substantial fraction of the ejected mass does not reach the escape velocity. To reach this conclusion we use a self-similar solution under simplifying assumptions. Most of the gravitational energy of a companion white dwarf (WD) is released in the envelope of a massive asymptotic giant branch (AGB) or the red giant branch (RGB) star in a very short time. This rapid energy release forms a blast wave in the envelope. We follow the blast wave propagation from the centre of the AGB outwards, and show that ∼1–10 per cent of the ejected envelope remains bound to the remnant binary system. We suggest that due to angular momentum conservation and further interaction with the binary system, the fall-back material forms a circumbinary disc around the post-AGB Core and the companion WD. The interaction of the circumbinary disc with the binary system will reduce the orbital separation much more than expected of the dynamical phase (where the envelope is ejected) of the CE alone. The smaller orbital separation favours a merger at the end of the CE phase or a short time after, while the core is still hot. This is another channel for the formation of a massive WD with super-Chandrasekhar mass that might explode as a Type Ia supernova. We term this the core-degenerate (CD) scenario.

Key words: stars: AGB and post-AGB – binaries: close – supernovae: general – white dwarfs.

1 INTRODUCTION

The 35-year-old (Paczynski 1976) common envelope (CE) model is in the heart of the formation of many close binary systems (e.g. Meyer & Meyer-Hofmeister 1979; Iben & Livio 1993; Taam & Sandquist 2000; Podsiadlowski 2001; Webbink 2008; Taam & Ricker 2010). The CE is defined as a structure where two stars share the envelope. In most cases the mass of the envelope comes from a giant star as it engulfs its companion, which is either a main-sequence (MS) star or a white dwarf (WD). During the CE phase the orbital separation decreases due to gravitational drag and tidal interaction (e.g. Iben & Livio 1993). The transfer of orbital energy and angular momentum to the envelope, as well as other possible energy sources, lead to the ejection of the envelope.

One of the major unsolved questions of the CE phase is the final orbital separation. It is customary to equate the gravitational energy released by the spiralling-in binary system $E_{\rm G}$, to the binding energy of the envelope $E_{\rm bind}$. An efficiency parameter $\alpha_{\rm CE}$ is introduced such that the final orbital separation is derived from the equality $E_{\rm bind} = \alpha_{\rm CE} E_{\rm G}$. However, different studies deduced different values of $\alpha_{\rm CE}$ for different systems. Ivanova & Chaichenets (2011) suggest that the enthalpy rather than the internal energy should be included in calculating the binding energy. A better understanding of the physical processes that determined the final orbital separation is required.

Most of the energy is deposited into the envelope in a very short time, as the spiralling-in process is accelerated with a decrease in the orbital separation (e.g. Livio & Soker 1988; Rasio & Livio 1996). There are also allegedly contradicting results, which suggest that the time for energy release in the envelope is relatively long. For example, Sandquist et al. (1998) found that the energy is released mainly during a stage lasting ∼200 d (see their fig. 11). De Marco et al. (2003) found that in representative cases the entire time-scale for CE evolution is 9–18 yr. However, this time-scale was obtained by waiting for a negligible amount of material to remain in the envelope (De Marco, Farihi & Nordhaus 2009), and therefore it represents a much longer time-scale than the strong energy release time-scale. This may also suggest that the strong energy release time-scale is of the order of ∼1 yr or less. Recent numerical simulations of Passy et al. (2011) also show that the strong energy release occurs in a time-scale of 100–200 d.

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The energy that is released during the CE is usually composed of more than one burst, and it takes for the companion a few orbits to strip the asymptotic giant branch (AGB) star envelope away (e.g. Sandquist et al. 1998). If most of the energy is released within a short time deep in the envelope, i.e. shorter than the dynamical time of the outer parts of the envelope, then one can speak of a burst of energy. Such a burst is similar in some of its properties to a blast wave which propagates outwards to the stellar surface. This is the scenario we discuss in the paper.

Sandquist et al. (1998) showed that some of the mass of the envelope can remain bound to one or both of the interacting stars. They simulated a $5\,\mathrm{M_\odot}$ AGB interacting with a $0.6\,\mathrm{M_\odot}$ companion, and found that the companion unbinds $\sim 1.55\,\mathrm{M_\odot}$ ($\simeq$23 per cent) of the AGB envelope. Passy et al. (2011) present more extreme results in their simulations. They find that when the envelope is lifted away from the binary, $\gtrsim$80 per cent of the envelope remains bound to the binary. However, in their simulations rotation of the AGB was not included, so the material that remains bound is probably overestimated. They conclude that in some cases parts of the AGB envelope remain bound or marginally bound to the remnant post-AGB core. Sandquist et al. (1998) also found that a differentially rotating structure resembling a thick disc surrounds the remnant binary during an intermediate phase of the CE interaction. This stage occurs briefly before the energy deposited by the companion and remnant AGB core drives the mass away. Soker (1992, see also Soker 2004) have analytically obtained a similar thick disc structure. De Marco et al. (2011) suggest that the envelope material, which is still bound to the binary system at the end of the CE, will fall back on to the system and will form a circumbinary disc. They suggest that such a disc might have some dynamical effects on the binary period. Other simulations show a post-CE gas to be bound to the companion, e.g. Lombardi et al. (2006).

In this paper we discuss the final stages of the CE under the assumption of a rapid binary-gravitational energy release, and the formation of a circumbinary disc. We suggest that such a scenario can result in an early merger of the WD with the post-AGB core, which will explode as Type Ia supernovae (SNe Ia) – the core-degenerate (CD) scenario.

Sparks & Stecher (1974) developed a merger scenario of a WD with an AGB core. in which the result is a Type II SN. In a later study, Livio & Riess (2003) found that the merger of a WD with an AGB core leads to an SN Ia that occurs at the end of the CE phase or shortly after. An important conclusion of Livio & Riess (2003) is that for merger to occur the AGB star should be massive. In the present paper we also reach the same conclusion.

We discuss the problem of the energy budget and characteristic time-scale at the end of the CE in Section 2. In Section 3 we discuss the properties of the blast wave. For such a disc to form, some of the material needs to fall back on to the star. We follow the blast wave and show that it indeed leaves some of the envelope material bound to the binary system. We discuss the formation of a circumbinary disc and its potential consequences on the binary system and its final fate. We argue that in many cases such a disc induces a merger and discuss the implications of the formation of such an early merger (Section 4). The most interesting implication is the CD scenario we propose for an SNe Ia. In the CD scenario a merger of a WD companion with the post-AGB core occurs while the core is still hot, and later explodes as an SN Ia. In other cases the merger might collapse to a neutron star. We summarize in Section 5.

2 PROBLEM SET-UP

2.1 Energy considerations

In this section we discuss the energy considerations behind the process of expelling the envelope. We follow a stellar companion spiralling-in inside the AGB envelope, and concentrate on the orbital separation a_f where the liberated orbital gravitational energy is almost equal to the envelope binding energy. As stated above, the final stages of the spiralling-in occur fast. Even if the envelope inward to the companion reaches synchronization with the orbital motion, the spiralling-in will continue. The reason is that the envelope outer to the companion cannot be synchronized, and tidal interactions are very strong. This tidal interaction is similar, but not identical, to that of a circumbinary disc. As we will see in Section 4, such interactions are very efficient in reducing the orbital separation. Based on this discussion, we assume that an energy almost equal to the binding energy of the envelope is released in a very short time. We examine two analytical prescriptions for this energy deposition in Section 3.

We start by examining the involved energies. For our typical giant structure we approximate the AGB envelope structures from the models of Nordhaus & Blackman (2006), Tauris & Dewi (2001) and Soker (1992), which are qualitatively similar. As we are aiming at performing self-similar calculations, we approximate the envelope density profile by a power law, e.g. Soker (1992):

$$\rho = \frac{A}{r^{\omega}}, \tag{1}$$

with $\omega = 2$. The mass inward to radius r is given by

$$M(r) \simeq M_{\rm core} + M_{\rm env}(r) = M_{\rm core} + \int_{R_{\rm core}\simeq 0}^{r} 4\pi r^2 \rho \,\mathrm{d}r. \tag{2}$$

Our AGB stellar mass is $M_\star = 5\,\mathrm{M_\odot}$, its core mass is $M_{\rm core} = 0.77\,\mathrm{M_\odot}$, its envelope mass is $M_{\rm env} = 4.23\,\mathrm{M_\odot}$ and the stellar radius is $R_\star = 310\,\mathrm{R_\odot}$. For these parameters and $\omega = 2$ we find from equations (1) and (2) that $A \simeq 3.1 \times 10^{19}\,\mathrm{g\,cm^{-1}}$. The core radius is very small, few$\times$ $0.01\,\mathrm{R_\odot}$.

The binding energy of the envelope mass outside radius r is given by

$$E_{\rm bind} = \int_{r}^{R_\star} \frac{G[M(r) + M_2]}{r} 4\pi r^2 \rho \mathrm{d}r, \tag{3}$$

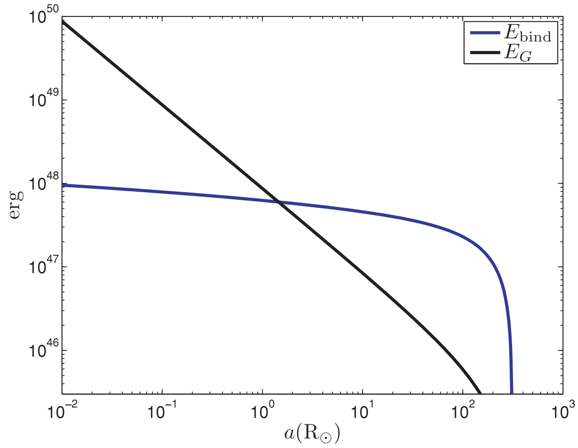

Figure 1. The binding energy of the AGB envelope mass residing at $r > a$, and the released gravitational energy, for our case of a $5\,\mathrm{M}_\odot$ AGB with envelope density $\sim r^{-2}$ and a $0.6\,\mathrm{M}_\odot$ companion, as function of the orbital separation. As the companion spirals in towards the centre, it releases more and more energy, namely is releases most of its gravitational energy close to the centre. Until it reaches the inner intersection of the binding energy of the AGB and the released gravitational energy ($a_\mathrm{f} \simeq 1.4\,\mathrm{R}_\odot$), the companion liberates $E_\mathrm{f} \simeq 6 \times 10^{47}$ erg and unbinds most of the AGB envelope.

where we have taken into account the companion mass M_2, and neglected the thermal energy of the gas. Before the companion enters the envelope its influence is mainly through tidal interaction that makes the envelope spin-up. This spin-up and the entrance of the companion to the envelope is likely to cause some loss of mass. This mass will be significant if the companion manages to bring the envelope to synchronization with the orbital motion (Bear & Soker 2010). We consider a case where, if a synchronization is achieved, it does so before the AGB reaches its maximum radius. As the AGB continues to expand, it swallows the companion. We assume that not much mass was lost before the onset of the CE. The gravitational energy released by the companion inside the envelope as it spirals in from the stellar surface to binary separation a is given by

$$E_\mathrm{G} = \frac{GM(a)M_2}{2a} - \frac{GM_\star M_2}{2R_\star}. \tag{4}$$

The binding energy and the released gravitational energy for our AGB model and for $M_2 = 0.6\,\mathrm{M}_\odot$ are plotted in Fig. 1. For $a_{\mathrm{f}1} \simeq 1.4\,\mathrm{R}_\odot$ the two energies become equal to each other and to $E_\mathrm{f} \simeq 6 \times 10^{47}$ erg. If we use the alpha prescription, $\alpha_\mathrm{CE} E_\mathrm{G}(a_{\mathrm{f}\alpha}) = E_\mathrm{bind}(a_{\mathrm{f}\alpha})$, then the ejection of the envelope occurs at an envelope separation of $a_{\mathrm{f}\alpha} \simeq \alpha_\mathrm{CE} a_{\mathrm{f}1}$.

2.2 Time-scales

In this section we examine some time-scales relevant to the final stages of the CE phase. The spiralling-in time-scale due to tidal interaction when the companion is inside the envelope is almost equal to the circularization time:

$$\begin{aligned}\tau_\mathrm{tid}(a) \simeq \tau_\mathrm{circ}(a) \simeq 6.9 &\left(\frac{L_\star}{2\times 10^4\,\mathrm{L}_\odot}\right)^{-1/3} \\ &\times \left(\frac{R_\star}{300\,\mathrm{R}_\odot}\right)^{2/3} \left(\frac{M_\mathrm{env(a)}}{4\,\mathrm{M}_\odot}\right)^{-2/3} \left(\frac{M(a)}{5\,\mathrm{M}_\odot}\right)^{-1} \\ &\times \left(\frac{M_2}{0.2M(a)}\right)^{-1} \left(1+\frac{M_2}{M(a)}\right)^{-1} \mathrm{yr},\end{aligned} \tag{5}$$

where $L_\star$ is the stellar luminosity, and the circularization time is from Verbunt & Phinney (1995), and where inside the envelope we take the radius inward to the companion, it equals the orbital separation $r = a$. In this expression only the tidal interaction with the envelope inward to the orbit is considered; however, there is also a tidal interaction with the envelope exterior to the orbit (see Section 2.1) that shortens the process.

We assume that the convection in the envelope inner to the companion radius still exists. For that, the tidal dissipation is assumed to be valid. Spiral wave dissipation exists as well (e.g. Sandquist et al. 1998), for which the characteristic time-scale is $\sim$200 d.

There are two additional time-scales to consider. These are the Keplerian time-scale

$$\tau_\mathrm{K}(a) = 2\pi \left(\frac{a^3}{GM(a)}\right)^{1/2}, \tag{6}$$

and the friction time-scale (e.g. Iben & Livio 1993)

$$\tau_\mathrm{fric}(a) = \frac{GM(a)M_2}{2aL_\mathrm{drag}(a)}, \tag{7}$$

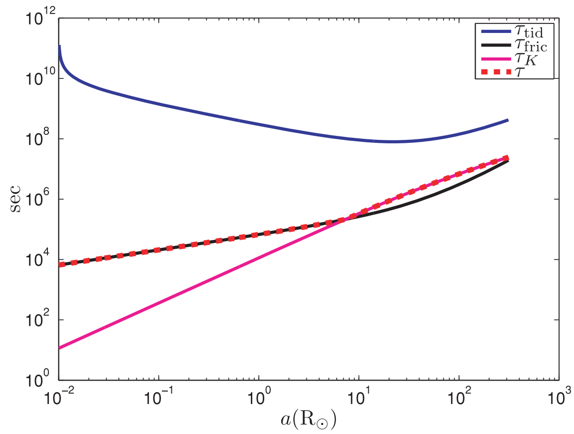

Figure 2. The characteristic time-scales for unbinding the AGB envelope, for our case of a 5 $\mathrm{M}_\odot$ AGB with envelope density $\sim r^{-2}$ and a 0.6 $\mathrm{M}_\odot$ companion. $\tau_{\rm tid}$: the spiralling-in time-scale due to tidal interaction with the envelope residing inwards to the companion orbit. $\tau_{\rm fric}$: the friction time-scale. $\tau_{\rm K}$: the Keplerian time-scale. τ: the relevant time-scale at radius r according to equation (10).

where

$$L_{\rm drag}(a) = \xi \pi R_{\rm acc}^2 \rho v_{\rm K}^3 \tag{8}$$

is the luminosity due to energy loss by drag, $v_{\rm K}(a) = [GM(a)/a]^{1/2}$ is the Keplerian velocity,

$$R_{\rm acc}(a) = \frac{2\,GM_2}{v_{\rm K}^2} \tag{9}$$

is the Bondi–Hoyle accretion radius, and we take the constant $\xi = 1$. The usage of $v_{\rm K}$ in equation (9) is an approximation. A more accurate expression takes into account the rotation of the envelope and the sound speed. Including envelope rotation reduces the relative velocity, while the sound speed increases it. Overall, the approximation used here is adequate.

The accretion rate might be less than the Bondi–Hoyle–Lyttleton value. However, in the drag equation there is another term resulting from interaction of the companion with mass that is not accreted. This term has the form of (8), but with the coefficient $\ln(R_{\rm max}/R_{\rm acc})$ instead of ξ, where $R_{\rm max}$ is the distance up to where the companion gravity influences the envelope gas (e.g. Ruderman & Spiegel 1971). This implies that even if the accretion rate is much below the Bondi–Hoyle–Lyttleton value, the term we ignored will become important and the value of the drag will be about the same as in equation (8) with $\xi \simeq 1$. Therefore, even if there is a large deviation from Bondi–Hoyle–Lyttleton accretion rate, the value of $\tau_{\rm fric}$ will only change by a small factor from the value used here.

The spiralling-in time-scale at orbital separation a is dictated by the most efficient process, but under our assumptions it cannot be shorter than the Keplerian time-scale. It is given by

$$\tau(a) = \max[\min(\tau_{\rm tid}, \tau_{\rm fric}), \tau_{\rm K}]. \tag{10}$$

The value of $\tau(a)$ is plotted in Fig. 2, for our stellar model and for $M_2 = 0.6\,\mathrm{M}_\odot$. We note that the tidal interaction is small compared with the gravitation drag inside the envelope. At the radius where the released gravitational energy equals the envelope binding energy, $a_{\rm f1}$, $\tau_{\rm f} = \tau(a_{\rm f1}) \simeq 0.95$ d. From Fig. 1 it is evident that most of the gravitational energy is released very close to $a_{\rm f1}$. For example, 90 per cent (80 per cent) of the energy is released when the companion spirals in from 14 $\mathrm{R}_\odot$ (7.1 $\mathrm{R}_\odot$) to $a_{\rm f}$, and the time-scale in that location is $\tau(14\,\mathrm{R}_\odot) \simeq 6.1$ d [$\tau(7.1\,\mathrm{R}_\odot) \simeq 2.4$ d]. Overall, most of the energy is deposited in the inner region of the envelope and in a time much shorter than the dynamical time of the outer regions. The consequences of this instantaneous energy liberation are discussed in the following sections.

We note that had we taken the spiral wave dissipation time-scale instead of the tidal time-scale (e.g. from fig. 3 in Sandquist et al. 1998), we would have gotten a shorter time-scale than $\tau_{\rm tid}$ (the blue line in Fig. 2 would have been located a little lower). But, according to equation (10) the relevant time-scale for τ would not have changed.

3 THE BLAST WAVE PROPAGATION THROUGH THE AGB ENVELOPE

We perform a self-similar calculation of a blast wave propagating from the interior of the AGB outwards. The calculation is based on the work of Sedov (1959). To facilitate the analytic solution we consider two limiting cases of the short energy deposition discussed above. In the first case we assume an instantaneous release of the binary gravitational energy (Section 3.1), while in the second case the energy is released at a constant rate over a short period (Section 3.2).

3.1 Instantaneous energy release

Here we assume that the energy is deposited instantaneously at $t = 0$. In this case (based on chapter 14.4 in Sedov 1959) we neglect the initial pressure in the AGB envelope and consider an adiabatic motion of a perfect gas. The density profile before the blast is given by equation (1),

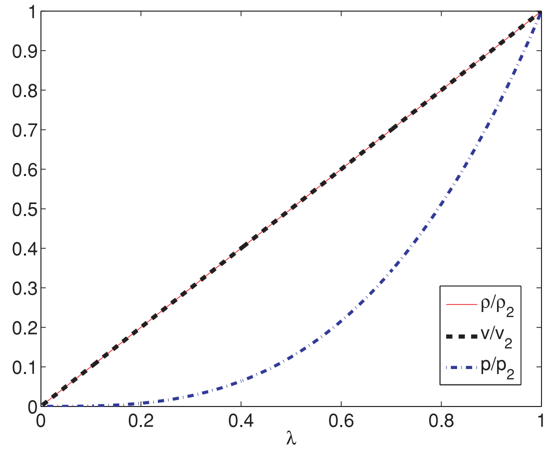

Figure 3. The asymptotic self-similar solution of an intense explosion blast wave which satisfies $\omega = (7 - \gamma)/(\gamma + 1)$, as solved by Sedov (1959).

and the envelope is static. The general solution for the blast wave is rather complicated for a general value of ω, and adiabatic exponent γ. However, when the relation $\omega = (7 - \gamma)/(\gamma + 1)$ holds, as in the case of the AGB model we use with $\omega = 2$ and $\gamma = 5/3$, there is a simple asymptotic solution for the velocity, density and pressure behind the shock, given respectively by

$$v = v_2\lambda = \frac{2}{3\gamma - 1}\frac{r}{t},$$
$$\rho = \rho_2\lambda = \frac{A(\gamma + 1)}{r^{\omega}(\gamma - 1)}\lambda^{8/(\gamma+1)},$$
$$p = p_2\lambda^3 = \frac{A}{r^{\omega-2}t^2}\frac{2(\gamma + 1)}{(3\gamma - 1)^2}\lambda^{8/(\gamma+1)}, \tag{11}$$

where the index 2 refers to the immediate post-shock values, and where λ is the self-similar variable,

$$\lambda = \left(\frac{A\alpha}{E_0}\right)^{1/(5-\omega)} r t^{-[2/(5-\omega)]} = \frac{r}{r_2}, \tag{12}$$

where r_2 is the location of the shockwave and E_0 is the energy deposited in the envelope. The constant α is determined from energy conservation. The self-similar solution for $\omega = 2$ and $\gamma = 5/3$ is shown in Fig. 3.

In addition to the energy deposited by the spiralling-in companion, the envelope itself has thermal energy $E_{\rm th}$, which we also add to that energy. We use the virial theorem $E_{\rm th} = 0.5E_{\rm bind}$ to find the energy deposited in the envelope:

$$E_0 = E_{\rm bind} + E_{\rm th} = \frac{3}{2}E_{\rm bind}. \tag{13}$$

The justification for considering the thermal energy of the envelope was recently discussed by De Marco et al. (2011). We use this energy in equation (12) to obtain the solution that is shown in Fig. 4.

We use the solution to estimate the mass that stays bound to the remnant binary system. We calculate the escape velocity $v_{\rm esc}(r) = [2G(M(r) + M_2)/r]^{1/2}$ taking the companion mass into account, as its mass contributes to the gravitational force at the time the envelope is ejected. Radiation pressure is not an important factor in expelling material outwards (e.g. Passy et al. 2011). We therefore neglect it, and set the condition for ejecting material to have its velocity larger than the escape velocity. The amount of material that does not reach the escape velocity and hence remains bound is determined from the condition at the end of the calculation $v(r, t_{\rm end}) < v_{\rm esc}(r, t_{\rm end})$, where $t_{\rm end}$ is the time when the shockwave reaches $R_\star$. This material is expected to fall back towards the centre, as the pressure decreases inwards (Fig. 4). For our case we find $t_{\rm end} = 9.5\,{\rm d} \simeq 10\tau_{\rm f}$. This shows that our assumption that the energy is released instantaneously is quite adequate here.

We find that for $M_2 = 0.6\,{\rm M}_\odot$, our characteristic case, the fall-back mass is $M_{\rm fb} \simeq 0.22\,{\rm M}_\odot$. For larger M_2 more of the envelope mass is falling back, as we show in Fig. 5. In Fig. 5 we also show the ratio between the time it takes for the blast wave to reach the AGB radius ($t_{\rm end}$) and the spiralling-in time-scale at the final orbital separation ($\tau_{\rm f}$).

The formation of accretion discs around one or two of the post-CE stars by fall-back material in the post-AGB phase was discussed before, but at much later times after the end of the CE phase (e.g. Soker 2001). Here we discuss the formation of a disc immediately after the main CE phase ends, and emphasize a disc around both stars – a circumbinary disc.

The total internal energy of the gas should include ionization energy as well, as the gas cools and recombines when it is expelled. Here we include only the thermal energy. The ionization energy might become an important source in extended AGB stars that have low binding energy per unit mass (e.g. Han et al. 2002; see Soker & Harpaz 2003 for a different view). For AGB stars the effects of ionization energy in the internal energy of the gas may be important. As we consider here massive AGB stars, it seems the ionization energy will not change much the amount of expelled mass. While the binding energy goes as $E_{\rm bind} \propto M^2$, the ionization energy goes as $E_{\rm i} \propto M$. For an envelope of

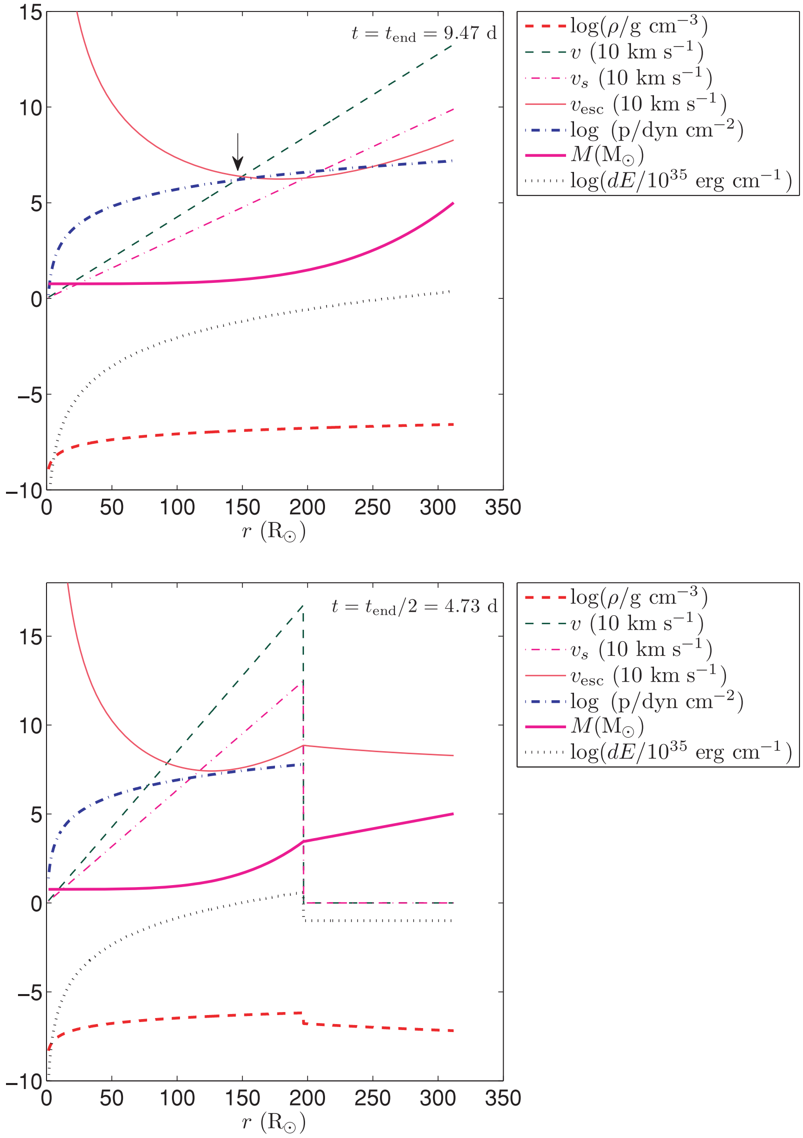

Figure 4. The solution of an intense explosion blast wave, for an instantaneous release of energy inside the AGB envelope. We use AGB envelope mass $M_{\rm env} = 4.2\,{\rm M_\odot}$ and the stellar radius of $R_\star = 310\,{\rm R_\odot}$. The companion mass is $M_2 = 0.6\,{\rm M_\odot}$. The physical quantities are plotted at the time the shock reaches the stellar surface ($t_{\rm end} = 9.47$ d, upper panel), and at half of that time ($t = t_{\rm end}/2 = 4.73$ d, upper panel). The plotted physical quantities are the density of the envelope ρ, the velocity v, the sound velocity $v_{\rm s}$, the escape velocity $v_{\rm esc}$, the pressure p, the mass M inner to a radius r (including the core mass), and the total energy dE in a shell dr. In the upper panel we find that the intersection of the post-shock velocity v with the escape velocity $v_{\rm esc}$ occurs at a radius of $\sim 150\,{\rm R_\odot}$ (marked with a small arrow). The mass inner to this point does not reach the escape velocity and falls back to the binary system. We find the fall-back mass to be $M_{\rm fb} \simeq 0.22\,{\rm M_\odot}$.

$M_{\rm env} = 4.23\,{\rm M_\odot}$ the ionization energy available is $\sim 10^{47}$ erg, while the energy deposited in our calculation is $\sim 9 \times 10^{47}$ erg. Furthermore, the bound mass resides in the inner part of the expanding envelope. The recombination radiation of the bound mass will help in part to expel the mass further out. Therefore, the ~ 11 per cent additional energy of recombination will not be efficiently deposited into the bound mass itself. It might expel an extra small amount of mass that is just barely bound.

In our analytic study we are limited to spherically symmetric geometry. In asymmetrical cases we expect that more mass will stay bound. The reasoning goes as follows. Part of the mass stays bound because a fraction of the envelope leaves with much more energy than is required to escape, hence leaving part of the envelope with a total negative energy. An asymmetric effect, either a denser envelope in the equator or more energy injection into the equatorial plane, for example, will increase the uneven energy distribution in the envelope. More energy deposition in the equatorial plane might leave more mass bound near the polar directions. The question is what is exactly the value of the specific angular momentum of the bound envelope. It is possible that interaction of the bound mass with the binary will lead to the formation of a circumbinary disc even when the initial specific angular momentum is small.

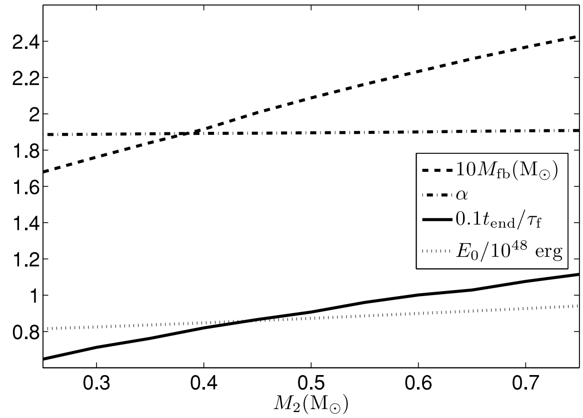

Figure 5. Physical properties as a function of the companion mass M_2: the spiralling-in time-scale at the final orbital separation (τ_{f}), the energy deposited in the envelope E_0 (equation 13), the constant α (equation 12), the fall-back mass M_{fb}, and the blast-wave propagation time to envelope expel time ratio $t_{\mathrm{end}}/\tau_{\mathrm{f}}$.

3.2 Continuous energy deposition

We find that the time-scale for binary energy deposition τ_{f} is smaller but not negligible relatively to the time for the shock to reach the stellar surface t_{end}. For that we now turn to calculate a case where the energy is deposited continuously during a finite time. In this second case we do not neglect the pressure of the envelope The self-similar calculation of the blast wave we perform in this section is based on chapter 6.6 of Sedov (1959). For this self-similar solution the energy deposition rate is $E_0 \propto t^{2(5-2\omega)/\omega} \propto t$, where the last equality is for $\omega = 2$.

As a result of the continues energy deposition and the counterpressure of the pre-shock envelope, a void is created in the inner region, and the post-shock material is contained within the blast shell, defined as the volume between the outer boundary of the void and the shockwave. The blast shell moves outwards, until it reaches the stellar surface. The problem with this description is that in our case the energy source is the companion. When the inner boundary of blast shell crosses the companion, there is no mechanism to transfer the energy from the central engine to the shell, because the companion is inside the void. We therefore limit ourselves, for treating a blast with a continuous energy release, to low radii and short times when the blast shell is close to the centre. As in our case the continuous energy release solution is relevant only to short times, we stop the energy repositioning at τ_{f}.

The self-similar variable here is

$$\lambda = r(\beta A G)^{-1/\omega} t^{-2/\omega} = \frac{r}{r_2}. \tag{14}$$

The self-similar functions are R, V, Z and $\mathcal{M}$, defined as (chapter 6.6 in Sedov 1959)

$$\begin{aligned}
\rho(r,t) &= \frac{1}{Gt^2} R(\lambda),\\
v(r,t) &= \frac{r}{t} V(\lambda),\\
p(r,t) &= \frac{r^2}{Gt^4} P(\lambda) = \frac{r^2}{\gamma G t^4} R(\lambda) Z(\lambda),\\
M(r,t) &= U \frac{r^3}{Gt^2} \mathcal{M}(\lambda).
\end{aligned} \tag{15}$$

The differential equations that describe the blast, in terms of the self-similar functions, are

$$\begin{aligned}
\frac{\mathrm{d}R}{\mathrm{d}\lambda} &= -\frac{R\left[\mathcal{M} - V - (3V-2)\left(V-\frac{2}{\omega}\right) + \frac{2Z}{\gamma} + V^2 - \frac{2Z(V+\gamma-2)}{\left(V-\frac{2}{\omega}\right)\gamma}\right]}{\lambda\left[Z - \left(V-\frac{2}{\omega}\right)^2\right]},\\
\frac{\mathrm{d}V}{\mathrm{d}\lambda} &= \frac{\left(V-\frac{2}{\omega}\right)\left[\mathcal{M} - V - (3V-2)\left(V-\frac{2}{\omega}\right) + \frac{2Z}{\gamma} + V^2 - \frac{2Z(V+\gamma-2)}{\left(V-\frac{2}{\omega}\right)\gamma}\right]}{\lambda\left[Z - \left(V-\frac{2}{\omega}\right)^2\right]} - \frac{3V-2}{\lambda},\\
\frac{\mathrm{d}Z}{\mathrm{d}\lambda} &= -\frac{(\gamma-1)\left[\mathcal{M} - V - (3V-2)\left(V-\frac{2}{\omega}\right) + \frac{2Z}{\gamma} + V^2 - \frac{2Z(V+\gamma-2)}{\left(V-\frac{2}{\omega}\right)\gamma}\right]}{\lambda\left[Z - \left(V-\frac{2}{\omega}\right)^2\right]} - \frac{Z(2V+2\gamma-4)}{\lambda\left(V-\frac{2}{\omega}\right)},\\
\frac{\mathrm{d}\mathcal{M}}{\mathrm{d}\lambda} &= -\frac{3\mathcal{M} - 4\pi R}{\lambda},
\end{aligned} \tag{16}$$

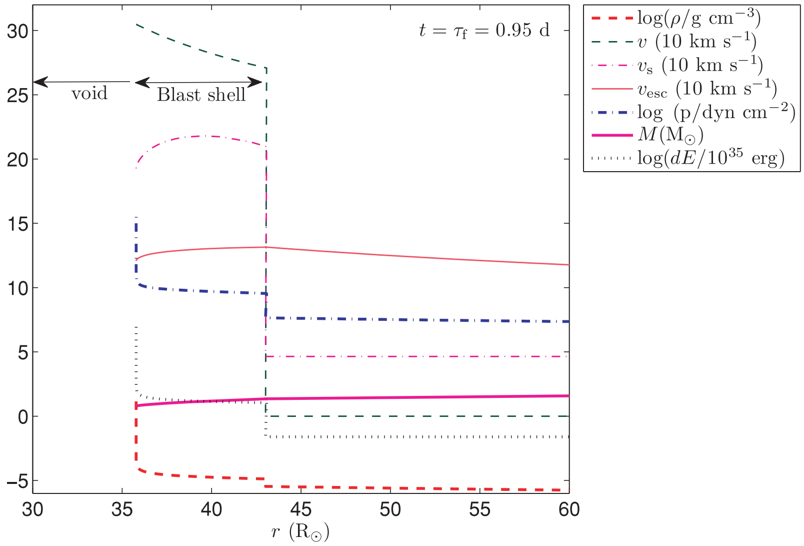

Figure 6. The solution of a blast wave with continuous energy release from $t = 0$ to $t = \tau_f \simeq 0.95$ d. A blast shell is formed, starting from the end of the void and ending at the shockwave. For explanations of the legend, see the caption to Fig. 4.

and the boundary conditions at the shock front are

$$
\begin{aligned}
R_2 &= -\frac{2q\,(\gamma+1)\,(\omega-1)\,(\omega-3)}{\pi\omega^2\gamma\,(2q+\gamma-1)},\\
V_2 &= -\frac{4\,(q-1)}{\omega\,(\gamma+1)},\\
Z_2 &= -\frac{4\,(q\,(\gamma-1)-2\gamma)\,(2q+\gamma-1)}{\omega^2\,(\gamma+1)^2},\\
\mathcal{M}_2 &= \frac{8q\,(\omega-1)}{\omega^2\gamma},
\end{aligned}
\tag{17}
$$

where

$$q = -\frac{\pi\omega^2\gamma}{2\,(\omega-1)\,(\omega-3)\,\beta}, \tag{18}$$

and β is a constant determined from energy conservation.

The solution is plotted in Fig. 6. We can see that the blast shell resides in the inner part of the envelope, and supersonically moves outwards. The energy in the blast wave is the same as that in the previous case (equation 13), such that we are back to the intense explosion case (Section 3.1). Namely, if the process does not last for a long time (see below), the two cases studied in this section lead to the same result that a non-negligible mass stays bound.

4 POSSIBLE IMPLICATIONS

4.1 The role of a circumbinary disc

The fall-back mass is expected to possess angular momentum, originally coming from the orbital angular momentum of the companion. We suggest that the fall-back mass forms a disc around the remnant binary system – the companion and the leftover core of the AGB. Even if the specific angular momentum of the fall-back mass is not sufficient to form a circumbinary disc, later interaction of this gas with the binary system might lead to the formation of a disc. Such an interaction is thought to lead to the formation of the circumbinary discs around post-AGB binary stars with an orbital separation of ∼1 au (van Winckel, Waelkens & Waters 2000; van Winckel et al. 2006; van Winckel et al. 2009). van Winckel et al. (2009) conclude that circumbinary disc can tremendously affect the evolution of the binary system at the centre, and that the formation of circumbinary gravitationally bound discs occur in a wide range of post-AGB binaries. In these binaries the typical orbital separation is ∼1 au, many of them have substantial eccentricity, and even if they avoided a CE phase, the companion had a strong interaction with the AGB envelope.

In the simulations of Passy et al. (2011) even a larger amount of material than found in our calculations remains bound to the binary system. However, they find that the orbital separations of the post-spiral-in phase are larger than the observed ones. Although Passy et al. (2011) found that there is enough orbital energy to unbind the envelope, the binary system in their simulations remains in a completely evacuated volume and the leftover-bound envelope is at ∼100 $\mathrm{R_\odot}$.

We therefore assume that the fall-back gas forms a circumbinary disc. The binary system transfers angular momentum to the circumbinary disc, and consequently the radius of the disc expands, the binary separation decreases and the eccentricity increases (e.g. Artymowicz et al. 1991). We assume that the structure of the circumbinary disc formed in our scenario resembles the structure of the disc studied in Artymowicz et al. (1991). In the calculations of Artymowicz et al. (1991) the disc extends from the nearest stable circumbinary orbit of $\sim$2.5a up to 6a, where a is the binary separation. We substitute for the binary system used in Section 3, and take the binary system to be composed of $M_{\rm core} = 0.77\,{\rm M}_{\odot}$ and $M_2 = 0.6\,{\rm M}_{\odot}$ components at a separation of $a_{\rm f} \simeq 1.4\,{\rm R}_{\odot}$, which is the final binary separation of the CE phase. We assume that the initial binary orbit is circular ($e_0 = 0$). The circumbinary disc mass is the most uncertain quantity here. We here scale it with $M_{\rm disc,0} = M_{\rm fb} \simeq 0.2\,{\rm M}_{\odot}$. Scaling the results of Artymowicz et al. (1991) by these values we find that the rate of change of the semimajor axis is

$$\frac{\dot{a}}{a} \simeq -3 \times 10^{-8} \left(\frac{q_{\rm d}}{0.16}\right) \left(\frac{\Omega_{\rm b}}{4.2 \times 10^{-4}\,{\rm s}^{-1}}\right) {\rm s}^{-1}, \quad (19)$$

and the rate of change of the eccentricity is

$$\dot{e} \simeq 1.3 \times 10^{-7} \left(\frac{q_{\rm d}}{0.16}\right) \left(\frac{\Omega_{\rm b}}{4.2 \times 10^{-4}\,{\rm s}^{-1}}\right) {\rm s}^{-1}, \quad (20)$$

where

$$\Omega_{\rm b} = \sqrt{\frac{G(M_{\rm core} + M_2)}{a_{\rm f}^3}} \simeq 4.2 \times 10^{-4}\,{\rm s}^{-1} \quad (21)$$

is the binary orbital angular velocity, and

$$q_{\rm d} = \frac{M_{\rm disc}}{M_{\rm core} + M_2} \quad (22)$$

is the disc-to-binary mass ratio. At the end of the CE, when the circumbinary disc–binary interaction starts, the disc-to-binary mass ratio is $q_{\rm d}, 0 \simeq 0.16$.

Another effect we take into account is the dissipation of the circumbinary disc. The energy released by the shrinking binary orbit is transferred to the disc. Because the total power is larger than the Eddington limit, a large fraction of the energy is carried by gas escaping from the disc. We take this mass-loss into account by equating the energy carried by the wind to that released by the shrinking binary system. As material probably escape from the entire disc, we take as an approximation the wind to leave from the middle of the disc, $R_{\rm wind}(t) = 4.25a(t)$. More likely, more mass will leave from the inner region of the disc, hence less mass can carry the same amount of energy. This will leave more mass in the disc, making the scenario discussed here even more efficient. In general, winds escape from stars and discs with a velocity almost equal to the escape velocity. Therefore, the total energy carried by the wind is the sum of the binding energy of the gas and the kinetic energy at infinity. We also consider the kinetic energy of the wind, assuming its terminal velocity to be the escape velocity from $R_{\rm wind}$. The kinetic energy introduces a factor of χ in equation (26) below.

We use equations (19) and (20) to integrate the semimajor axis decrease and eccentricity increase due to the circumbinary disc–binary interaction. We take the circumbinary disc mass to reduce according to the orbital energy released from the reduction of the binary orbit:

$${\rm d}E(t) = -\frac{GM_{\rm core}M_2}{2a_{\rm f}} + \frac{GM_{\rm core}M_2}{2a(t)}. \quad (23)$$

The binding energy of the escaping disc material per unit mass is

$$\frac{{\rm d}E_{\rm bind,disc}(t)}{{\rm d}m} = -\frac{G(M_{\rm core} + M_2)}{2R_{\rm wind}}, \quad (24)$$

and the kinetic energy is parametrized as

$$\frac{{\rm d}E_{\rm kin,disc}}{{\rm d}m} = (\chi - 1)\frac{{\rm d}E_{\rm bind,disc}}{{\rm d}m}. \quad (25)$$

Hence, the rate of change of the disc mass is

$${\rm d}M_{\rm disc}(t) = -\frac{{\rm d}E(t)}{\chi \frac{{\rm d}E_{\rm bind,disc}(t)}{{\rm d}m}}. \quad (26)$$

Our results for an initial eccentricity $e_0 = 0$ are presented in Fig. 7. The decrease in orbital separation and increase in eccentricity are clearly seen. The circumbinary disc–binary interaction terminates when one of the two things occur: either the circumbinary disc is completely depleted, or the two stars merge (occurs when $e \simeq 1$). We term the time period of the circumbinary disc–binary interaction τ_1. As can be understood from Fig. 7, taking $e_0 > 0$ would shorten τ_1, so taking $e_0 = 0$ is a conservative parameter selection. As stated above, the most uncertain parameter is the initial disc mass, which sets the value of $q_{\rm d,0}$ (equation 22). We repeat the calculation for a few values of $q_{\rm d}, 0$. For the case where the disc wind is ejected at the escape velocity (namely $\chi = 2$) we find that the circumbinary disc–binary interaction leads to a merger for $q_{\rm d,0} \gtrsim 0.12$, without requiring a following process [emission of gravitational waves (GW), discussed in Section 4.3 below].

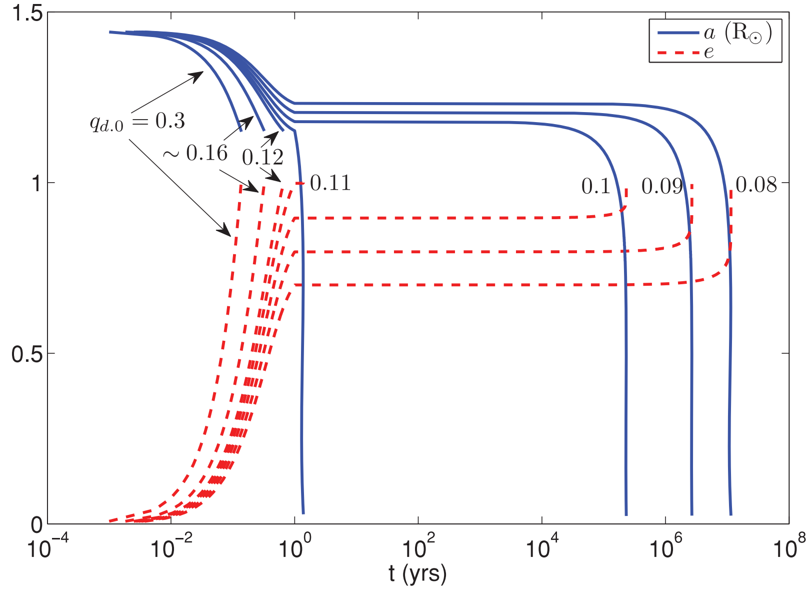

Figure 7. The variation of the semimajor axis a (in solar units) and eccentricity e as a function of time, in the core-degenerate scenario. We take $\chi = 1$ in equation (25), and an initial eccentricity $e_0 = 0$. The variation occurs in two phases. In the first phase the circumbinary disc–binary interaction reduces a and causes an increase in e. The final values of a and e in this phase are the initial values in the second phase – semimajor axis reduction and eccentricity increase due to emission of gravitational waves. This phase lasts a time interval τ_2, which strongly depends on the value of e at the beginning of this phase, and hence on τ_1. The first phase terminates when one of two things occur: the circumbinary disc is completely depleted, or when the binary system merge (occurs when $e \simeq 1$). We find that the first phase is enough to create a merger for $q_{\rm d0}, \gtrsim 0.12$. If the first phase has not resulted in a merger (because the circumbinary disc is not massive enough to result in a strong interaction with the binary), then the second phase of GW emission can reduce a to the extent of merging. We find that for $0.1 \lesssim q_{\rm d0}, \lesssim 0.12$ the second phase will result in a merger within $\tau_2 < \tau_{\rm cool} \simeq 10^5$ yr. According to the core-degenerate scenario, such an early merger will explode as an SN Ia. For $q_{\rm d} \lesssim 0.1$ a late merger is formed, and such a merger is not expected to explode as SNe Ia.

We note that post-CE circumbinary discs were suggested under other assumptions about the CE evolution. The formation of a post-CE thick circumbinary disc was invoked by Soker (1992) in a case of a more gradual spiralling-in process where part of the envelope remains in a hydrostatic equilibrium rather than falling back. A differentially rotating thick disc or torus was found in the intermediate stages of the CE evolution in the 3D numerical simulations of Sandquist et al. (1998). In these cases a rapid merging is expected as well.

Ivanova (2011) suggests another mechanism for post-CE merger, applicable to stars with non-degenerate cores. Ivanova (2011) divides the CE phase into two phases. In the first stage the envelope is expelled, and a remnant is left at the centre. This stage takes ∼1 yr. Some envelope mass is left above the core. In the second stage the leftover envelope undergoes thermal readjustment, and its outer region expands. This occurs on a time-scale of a few $\times 10^3$ yr. Ivanova (2011) concludes that in many cases the leftover envelope would expand beyond the companion, forming another CE phase that substantially increases the merger probability.

As we found above that some of the material in the envelope falls back towards the binary system, we suggest that the readjustment phase in the case studied here can be even more dramatic than in the case studied by Ivanova (2011). In the case studied here there are two possible scenarios. In the first scenario the falling-back material will be added to the remnant material and will be included in the readjustment phase. This material will be added to the outer layer of the leftover envelope, which has the potential to expand and swallow the companion. In the second scenario, where there is enough angular momentum, a circumbinary disc is formed and the binary separation decreases as discussed above. Overall, our new finding of the existence of a large amount of fall-back material supports the conclusion of Ivanova (2011) of a higher percentage of immediate post-CE mergers, although the Ivanova (2011) mechanism works for giants with non-degenerate core.

We emphasize the role played by the circumbinary disc. At early stages of the CE the spiralling-in is relatively slow and the rate of energy release by the spiralling-in companion is low. At that stage the convection in the AGB envelope efficiently transfers the energy outwards. Most (but not all) of the extra energy is radiated away at this early phase. In the final phase of the CE, when the companion is deep inside, energy is released at a very high rate, under the assumption used here. In that case energy is carried by the expelled envelope. This stops the spiralling-in process. Here we suggest that further spiralling-in can take place due to interaction with a circumbinary disc. The circumbinary disc is an efficient mechanism to absorb the binary orbital angular momentum. In addition, the circumbinary disc dissipates the binary orbital energy and radiates it by expelling mass and radiation from its surface. This phase ends either with a merger or with a very small orbital separation. A final small orbital separation will be interpreted as a small $\alpha_{\rm CE}$.

The value of $\alpha_{\rm CE}$ will be small if the release of energy is a long process, even if our assumption of a rapid release of energy at the end of the CE does not hold (e.g. ∼1 yr, Sandquist et al. 1998; ∼10 yr, De Marco et al. 2003). De Marco et al. (2011) suggest that if the spiral-in of the companion takes longer than the dynamical time, as in that case of long-time energy release, the thermal energy of the AGB will be used to help unbind the envelope. Therefore, they conclude that it is expected that the companion will spiral-in inwards to the radius predicted by taking $\alpha_{\rm CE} = 1$.

4.2 The SN Ia scenario from an early merger

Here we discuss some possible outcomes of WD–WD (double-degenerate) merger, where the two components are made of CO. The WD–WD merger is one of the scenarios that lead to SNe Ia (Iben & Tutukov 1984; Webbink 1984). Most simulations and calculations of double-degenerate merger (e.g. Yoon, Podsiadlowski & Rosswog 2007) assume the merger to occur a long time after the CE phase, when the two WDs are already cold. In the scenario we propose here, the merger occurs within the final stages of the CE, while the core is still hot. Below we speculate that some of these will become SNe Ia after a very long time dictated by angular momentum loss. The speculation is based on studies of double-degenerate mergers that we describe below. We then discuss their application to the scenario proposed here.

In analysing the outcome of the double-degenerate merger we must take into account the following. (i) In the merger process the more massive WD remains at the centre of the merger product, while the lighter WD is destructed and becomes the envelope, and possibly a thick disc around the centre. (ii) The post-AGB core is hot as it had no time to cool, while the companion WD is colder as it had the time to cool since its formation.

In general, the merger of two WDs can end in one of the three basic products (e.g. Yoon & Langer 2005): SN Ia, core collapse to a neutron star (NS), or a more massive WD. In the first two cases, where the mass of the merger is above the critical limit, the most important factor in determining whether or not there will be an explosion as an SN Ia is the nature of the carbon ignition. In the third case, the mass of the merger is below the critical limit. We now discuss the three cases.

(1) In the first case, the companion mass is larger than the post-AGB core mass, and therefore the companion will become the cold core of the merger (assuming it had enough time to cool) and the destructed hot post-AGB core will be its envelope. We note that before the merger but already in the CE phase, the centre of mass will be closer to the companion, and the envelope will arrange itself as if the companion were the giant core. In this case carbon ignition is more likely to occur during the merger processes and be off-centre (Saio & Nomoto 1985). Most of the WD merger models indicate that a non-violent off-centre ignition that occurs during the merger process, burns carbon and oxygen into oxygen, neon and magnesium. As not enough energy remains to unbind the WD when the mass reaches the critical limit, it will gravitationally collapse and form an NS rather than SN Ia (Nomoto & Iben 1985; Saio & Nomoto 1985; Mochkovitch & Livio 1990; Hillebrandt & Niemeyer 2000; Yoon & Langer 2005). In that case an NS will be formed (e.g. Yoon & Langer 2005). It is interesting to note the possibility that in such a scenario planets can form around the NS from SN fall-back material (Hansen, Shih & Currie 2009).

However, in many cases no carbon ignition will occur during the merger process (Yoon et al. 2007), and if the mass reaches the critical mass an SN Ia might occur after the WD slows down.

(2) In the second case, the companion mass is smaller than the post-AGB core mass, and therefore the centre of the product will be hot.

Yoon et al. (2007) raised the possibility that a hot remnant core is less likely to ignite carbon off-axis (in the envelope) during WD–WD merger. The reason is that a hot WD is larger, such that its potential well is shallower and the peak temperature of the accreted destructed-WD material is lower. Hence, in such a case supercritical-mass remnant is more likely to ignite carbon at the centre at a later time, leading to an SN Ia. The merger remnant becomes a rapidly rotating massive WD that can collapse only after it loses sufficient angular momentum. Yoon et al. (2007) parametrized the angular momentum loss, and found that explosion can occur as late as $\sim 10^6$ yrafter the merging occurred.

(3) The third possibility is that the merger is below the critical mass. In this case the merger will remain a WD that will continue to cool, and neither an SN Ia nor a collapse is expected.

However, there is also an exceptional case. van Kerkwijk, Chang & Justham (2010) suggest that in cases where the two CO WDs have an approximately equal mass below the critical limit, the WD merger will be fully mixed and hottest in its centre. Initially, the merger rotates differentially, but at later times some of the mass is expelled, and a uniformly rotating core near the mass-shedding limit is obtained. The structure is of a core which contains ~ 80 per cent of the mass, surrounded by a sub-Keplerian, very dense, partially degeneracy-pressure-supported disc. In the model of van Kerkwijk et al. (2010) the disc accretes at a high rate that leads to compressional heating that is likely to cause central carbon ignition. This ignition occurs at densities for which pure detonations lead to events similar to SNe Ia (van Kerkwijk et al. 2010). If such a scenario occurs in our case, then the explosion will be surrounded by the expelled common envelope.

4.3 Core-degenerate merger at the termination of the common envelope

Our new suggestion is that there are many double-degenerate merger events where one of the components, the post-AGB core, is hot. The time-scale for angular momentum loss might be very long ($>10^6$ yrafter merger) even when the remnant rotation is modest: not fast enough to slow down rapidly, but fast enough to influence the critical mass. We hence suggest that some SNe Ia might originate from a double-degenerate merger that occurs at the last phase of the CE or shortly afterwards during the planetary nebula phase, rather than a long time after. We term this scenario core-degenerate (CD), as one of the WDs has just emerged from the core of the AGB star.

The immediate implication of processes that reduce the orbital separation at the final stages of the CE (and later form an early merger) is that the effective value of $\alpha_{\rm CE}$ is considerably smaller than 1. The reduction in the orbital separation can lead to merger immediately after the CE phase, or even before the entire envelope has been lost. In some cases the final orbital separation will be very small, such that the merger will occur after the CE phase, but while the core is still hot. We now examine this case.

As mentioned in Section 4.2, Yoon et al. (2007) showed that if the merger has a hot remnant core it is less likely to ignite carbon off-centre, and therefore a massive CO WD is formed. The reason is as follows. If the post-AGB core is still hot when it merges with a lighter WD companion, the core radius is still relatively large (e.g. Koester & Schoenberner 1986; Bloecker 1995). When the post-AGB core radius is larger, then the gravitational potential well on the surface, where the companion mass is accumulated, is smaller. Therefore, when the companion WD merges with the post-AGB core, the peak temperature of the merger will be in an outer radius, and at a lower temperature. So the hotter the post-AGB core, the colder the merger peak temperature will be, for a given core mass. An increase by a factor of 1.2 in the post-AGB core radius results in a factor of ∼1.2 decrease in the merger's peak temperature. Such a decrease might be enough to prevent off-centre carbon ignition and the collapse of the merger (Yoon et al. 2007). Instead, the merger product will later produce an SN Ia.

We consider the condition for a core-degenerate merger while the core is still ∼1.2 larger than its final radius. As a post-post-AGB core cools, its radius decreases to an asymptotic value of a cold WD. More massive cores cool faster (e.g. Bloecker 1995). For an SN Ia progenitor we take a post-AGB core of $M_{\rm core} \sim 0.7$–$0.8\,{\rm M}_\odot$ with a WD companion mass of $M_2 < M_{\rm core} \sim 0.6$–$0.7\,{\rm M}_\odot$. Such cores shrink to a radius 1.2 times larger than the final radius in $\tau_{\rm cool} \sim 10^5$ yr(Bloecker 1995). If by that time the companion WD will merge with the post-AGB core then it will be less likely to ignite carbon off-centre during the merging process.

There are two possible mechanisms to further reduce the orbital separation in cases where the circumbinary disc did not reduce the separation down to merger. These are emission of gravitational waves and energy dissipation by tidal interaction. The time-scale for the energy dissipation by tidal interaction is much larger, even longer than the Hubble time (e.g. Willems, Deloye & Kalogera 2010). We therefore consider emission of gravitational waves to be the dominant mechanism.

In an eccentric orbit WD–WD binary, the reduction of the semimajor axis due to emission of gravitational waves is faster, as the eccentricity increases (Peters & Mathews 1963). The semimajor axis reduction and the eccentricity increase are [Peters & Mathews 1963; see also equations (2) and (3) in Ignatiev et al. 2001]

$$\dot{a} = -\frac{64}{5}\frac{G^3 M_{\rm core} M_2 (M_{\rm core}+M_2)}{c^5 a^3 (1-e^2)^{7/2}}\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right) \tag{27}$$

and

$$\dot{e} = \frac{304}{15}\frac{G^3 M_{\rm core} M_2 (M_{\rm core}+M_2) e}{c^5 a^4 (1-e^2)^{5/2}}\left(1+\frac{121}{304}e^2\right), \tag{28}$$

where c is the speed of light.

For the core-degenerate scenario to occur, the semimajor axis reduction due to emission of gravitational waves should result in a merger, within a period $\tau_2 < \tau_{\rm cool}$. The value of τ_2, which satisfies $\tau_2 \gg \tau_1$ (where τ_1 is the time period of the circumbinary disc–binary interaction), is very sensitive to the value of e from which the GW emission process starts. This value of e is determined by the final eccentricity in the circumbinary disc–binary interaction process, which depends on τ_1. Namely τ_2 strongly depends on τ_1. Our results are presented in Fig. 7.

We repeated our calculation (with $\chi = 2$; equation 24) for different values of $q_{\rm d,0}$, and found that for $q_{\rm d,0} \gtrsim 0.1$ the two components will merge within a short time after the CE (Fig. 7). As we take disc wind into account, we find that the process of emission of gravitational waves is required for merging to occur for $0.1 \lesssim q_{\rm d,0} \lesssim 0.12$, but due to the interaction with the disc it will occur in a considerably shorter time than it would have without it. This shorter time can fulfil the requirement that when the core-degenerate scenario will occur (namely that the companion WD will merge with the post-AGB core while it is still hot) and that there will be a central ignition that will result in an SN Ia. We have also repeated the calculation for $\chi = 1$ (Fig. 8), namely with the disc wind having a zero terminal velocity. In this case we find that a higher circumbinary disc mass (larger $q_{d,0}$) is required for the companion WD to merge with the post-AGB core while the latter is still hot.

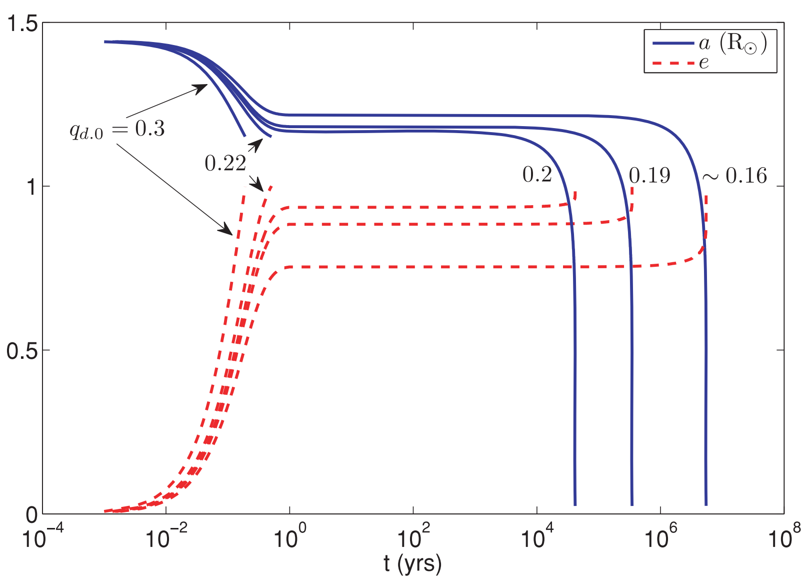

Figure 8. The same as Fig. 7, but taking $\chi = 1$ in equation (25). Namely, with the disc wind having zero terminal velocity. In this case we find that a higher circumbinary disc mass (larger $q_{d,0}$) is required for the binary stars to undergo a fast merge.

This means that stars with massive envelopes ($M_{\rm env} \gtrsim 5\,{\rm M}_{\odot}$) that engulf a WD companion, might end the CE phase with a very close WD–WD binary. This is in agreement with the results of Livio & Riess (2003), who found that for a merger to occur the AGB must be massive. The WDs might merge before the hot WD (which was the post-AGB core) had time to cool much. We note that as $\tau_{\rm GW}$ is short, the probability of observing such systems is low. But, observationally, there are indeed post-CE systems that have periods much shorter than simulations predict (De Marco, Hillwig & Smith 2008; De Marco et al. 2009). Our result that $\alpha_{\rm CE} < 1$ can account for this discrepancy.

It is in order to mention that recent studies also reach the conclusion that $\alpha_{\rm CE} < 1$. De Marco et al. (2011) perform simulations for many sets of binary parameters (see also Passy et al. 2011). They find that there is a possible negative correlation between the mass ratio of the two stars and the value of $\alpha_{\rm CE}$. As De Marco et al. (2011) include the thermal energy (which is half of the binding energy according to the virial theorem), as we too do here, they also find that the values of $\alpha_{\rm CE}$ are smaller than 1 (in cases where the thermal energy is not included, the values of $\alpha_{\rm CE}$ they obtain are larger than 1). Zorotovic et al. (2010) also come to the conclusion that $\alpha_{\rm CE} < 1$. Their statistical analysis of observed post-CE binaries give $\alpha_{\rm CE} = 0.2$–0.3, but contrary to De Marco et al. (2011) they do not find any correlation between $\alpha_{\rm CE}$ and the mass of the companion.

5 SUMMARY

We conducted an analytical study of the ejection of a CE under some simplifying assumptions. Despite the many 3D numerical hydrodynamical simulations of CE evolution, such an approach has its own merit as it points to the basic physics behind the finding that some of the envelopes remain bound to the system.

Most of the binary gravitational energy in a CE is released in a short time during the final stages of the CE phase. Here we took this time to be much shorter than the dynamical time in the outer regions of the envelope. Therefore, the energy release process resembles a blast wave propagating from the centre outwards. We discuss two types of blast waves (Section 3) that practically lead to the same result. In the first type the energy release is instantaneous, i.e. an explosion, while in the second type there is a continuous energy deposition over a short time. We used a self-similar solution to track the blast wave in each type as it propagates through the AGB envelope. We found that part of the ejected envelope stays bound to the binary system, i.e. it does not reach the escape velocity. This material is expected to fall back towards the centre. It is very likely that due to angular momentum conservation and further interaction of the fall-back gas with the binary system, a circumbinary disc will be formed. The spherically symmetric geometry of the self-similar solution does not allow the introduction of angular momentum to the solution of the blast wave.

As discussed in Section 4.1, the binary semimajor axis decreases rapidly during the short circumbinary disc–binary interaction period. During that time the power of the system greatly exceeds the Eddington limit. Most of the energy goes to blow the wind, but some goes to radiation. It is therefore possible that such systems will be observed by the transient increase in their luminosity.

We showed that interaction of the binary system with the circumbinary disc will further reduce the orbital separation. Consequently, in the alpha-prescription, we find that effectively $\alpha_{\rm CE} < 1$. In many cases a merging will occur immediately after the dynamical phase of the CE. In many cases where the companion is a WD, it will form with the remnant post-AGB core of the AGB (or RGB) core a WD–WD system with a very small separation. In cases where the mass of the circumbinary disc is large enough, we suggest that a CD is likely to occur – an early merger between the WD and the post-AGB core will take place while the post-AGB core is still hot. Such a merger is more likely to explode later as an SN Ia if the mass is above critical and the AGB's core is more massive than the WD companion (see Section 4). The supercritical-mass WD is stabilized by the rapid rotation. Only after it slows down that it will explode as an SN Ia. We speculate, based on the results of Yoon & Langer (2005), that this time can be as long as $\gtrsim$ few $\times\,10^9$ yr, and that the core-degenerate scenario is another channel leading to an SN Ia.

Our finding that effectively $\alpha_{\rm CE} < 1$ (see also Ivanova 2011) can explain the recent findings of De Marco et al. (2011). De Marco et al. (2011) find that the value of $\alpha_{\rm CE}$ they deduce from observations is much smaller than what their numerical simulations of the CE phase give (De Marco et al. 2008, 2009, 2011). We strongly encourage a numerical study of the CE phase to follow the gas and its angular momentum after ejection in order to find the amount of gas that falls back, and whether a circumbinary disc is formed.

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