Expansion history with decaying vacuum: a complete cosmological scenario

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Abstract

We propose a novel cosmological scenario with the space–time emerging from a pure initial de Sitter stage and subsequently evolving into the radiation, matter and dark energy dominated epochs, thereby avoiding the initial singularity and providing a complete description of the expansion history and a natural solution to the horizon problem. The model is based on a dynamical vacuum energy density which evolves as a power series of the Hubble rate. The transit from the inflation into the standard radiation epoch is universal, giving a clue for a successful description of the graceful exit. Since the resulting late time cosmic history is very close to the concordance ΛCDM dark matter model, the new unified framework embodies a more complete past cosmic evolution than the standard cosmology.

Key words: cosmology: dark energy – cosmology: theory.

1 INTRODUCTION

In the current view of the cosmological history it is believed that matter and space–time emerged from a singularity and evolved through four different eras: early inflation, radiation, dark matter (DM) and dark energy (DE) dominated eras. During the radiation and DM dominated stages, the expansion of the Universe slows down while in the inflationary and DE eras it speeds up. So far there is no clear cut connection between the inflationary period and the normal Friedmann expansion. Moreover the current DE phase is also a mystery.

Over the past decade, studies of the available high quality cosmological data (supernovae Type Ia, CMB, galaxy clustering, etc.) have converged towards a cosmic expansion history that involves a spatially flat geometry and a recent accelerating period of the Universe (cf. Spergel et al. 2007; Amanullah et al. 2010; Komatsu et al. 2011, and references therein). This faster expansion phase has been attributed to the DE component with negative pressure. The simplest type of DE corresponds to the cosmological constant (hereafter CC) (Weinberg 1989; Peebles & Ratra 2003; Padmanabhan 2003; Lima 2004). The so-called concordance model [or ΛCDM model (Peebles 1993)], which contains CDM to explain clustering, flat spatial geometry and a CC, Λ, fits accurately the current observational data and thus it is an excellent candidate to be the model that describes the observed Universe. However, the ΛCDM suffers from, among others, two fundamental problems: (a) the ‘old’ CC problem (or fine tuning problem), i.e. the fact that the observed value of the vacuum energy density is many orders of magnitude below the value suggested in quantum field theory (QFT) (Weinberg 1989; Peebles & Ratra 2003; Padmanabhan 2003; Lima 2004), and (b) the coincidence problem (see Steinhardt 1997), i.e. the fact that the (decreasing) matter energy density and the (constant) vacuum energy density happen to be of the same order just prior to the present epoch.

One of the main attempts to solve or at least to alleviate such theoretical problems is based on the idea that the vacuum energy density is a time-dependent quantity, i.e. $\Lambda = \Lambda(t)$. There are a number of interesting $\Lambda$-variable models in the old literature (Ozer & Taha 1986, 1987; Bertolami 1986; Freese et al. 1987; Peebles & Ratra 1988; Carvalho, Lima & Waga 1992; Lima & Maia 1994; Lima 1996; Lima & Trodden 1996; Overduin & Cooperstock 1998) and even more recently (Shapiro & Solá 2000, 2002, 2009; Alcaniz & Maia 2003; Opher & Pelsson 2004; Bauer 2005; Carneiro & Lima 2005; Alcaniz & Lima 2005; Barrow & Clifton 2006; Solá & Šefančíč 2005, 2006; Montenegro & Carneiro 2007; Basilakos 2009; Solá 2011 and references therein). The functional form of $\Lambda(t)$ in most of them has usually been proposed on phenomenological grounds, as it occurs with the vast majority of DE models (Carvalho et al. 2006). With the exception of the papers by Lima & Maia (1994) and Lima & Trodden (1996), their basic motivation was to understand the present-day smallness of the vacuum energy density, that is, with no attempt to find a clear correlation between the values of $\Lambda$ during inflation and at recent times. This occurs because the functional form of $\Lambda(t)$ is chosen just by extrapolating backwards in time the present available cosmological data, including...
a divergent upper bound for \( \Lambda \). However, since \( \Lambda_0 \) is most likely a remnant from inflation, a more realistic phenomenological decaying law should describe all cosmic history, embodying the primordial inflation itself driven by a finite \( \Lambda \). Naturally, a more fundamental approach for the decaying vacuum models based on QFT or at least partially justified from first principles should be desirable in physical grounds. A proposal along these lines in which the presently observed \( \Lambda \) appears as a quantum relic from inflation has been discussed in the literature (Solà 2008). A related work but in the context of a five-dimensional non-compact Kaluza–Klein gravity model can be found in Abanitache, Aguilar & Bellini (2006).

In the present paper we propose a new complete cosmological history based on decaying vacuum models. Our basic aim is to understand the possible relationship between the late time magnitude of the \( \Lambda \)-term and its upper bound during the inflationary epoch. As it will be seen, some constraints from QFT in curved space–time are naturally incorporated in our scenario, specially those coming from recent developments inspired in the renormalization group approach (Shapiro & Solà 2002; Solà 2008; Shapiro & Solà 2009) which may have testable phenomenological consequences for the current universe (Basilakos, Plionis & Solà 2009; Grande et al. 2011).

The scenario proposed here can be viewed as a further step in the direction of trying to find a cosmological framework based on the fundamental principles of physics, and capable to link the dynamics of the early universe with that of our late universe. The scenario is termed complete in the sense that the model starts from a non-singular inflationary stage which has a natural (universal) ending into the radiation phase (thereby solving the horizon and graceful exit problems), and, finally, the small current value of the vacuum energy density can be conceived as a result of the massive disintegration of the vacuum into matter during the primordial stages. In brief, our model evolves between two extreme de Sitter phases. At early times the de Sitter phase is unstable and drives the model continuously to a late time de Sitter state driven by the remnant vacuum energy density. As we shall see, the leitmotiv of our approach is related to the structure of the effective action of QFT in curved space–time, which suggests that at early times only even powers of the Hubble parameter can contribute to a time varying \( \Lambda \)-term (see e.g. Shapiro & Solà 2009).

The structure of the paper is as follows. In Section 2, we briefly discuss the background cosmological equations. In Section 3, we discuss the basic ideas underlying the time varying vacuum in an expanding Universe and set up the basic equations whose solutions describe the complete evolution of our model. In Sections 4 and 5 we present the solutions in the early and late universe, respectively, while in Section 6 we discuss our results and at the same time we present some ideas towards extending the current vacuum model. Finally, the main conclusions are summarized in Section 7.

## 2 COSMOLOGY WITH A TIME-DEPENDENT VACUUM

Let us recall that the CC contribution to the curvature of space–time is represented by the \( \Lambda g_{\mu \nu} \) term on the l.h.s. of Einstein’s equations. The latter can be absorbed on the r.h.s. of these equations

\[
R_{\mu \nu} - \frac{1}{2} g_{\mu \nu} R = 8 \pi G \, \tilde{T}_{\mu \nu} ,
\]

where the modified \( \tilde{T}_{\mu \nu} \) is given by \( \tilde{T}_{\mu \nu} = T_{\mu \nu} + \frac{\rho_\Lambda}{\Lambda} g_{\mu \nu} \). Here \( \rho_\Lambda = \Lambda/(8 \pi G) \) is the vacuum energy density associated with the presence of \( \Lambda \) (with pressure \( p_\Lambda = -\rho_\Lambda \)), and \( T_{\mu \nu} \) is the ordinary energy-momentum tensor of matter and radiation. Modelling the expanding universe as a perfect fluid with velocity four-vector field \( U_\mu \), we have

\[
T_{\mu \nu} = -p_m g_{\mu \nu} + (\rho_m + p_m) U_\mu U_\nu ,
\]

where \( \rho_m \) is the density of matter-radiation and \( p_m = \partial \rho_m/\partial U_\mu \) is the corresponding pressure.

Clearly, the modified \( \tilde{T}_{\mu \nu} \) defined above takes the same form as \( T_{\mu \nu} \) with \( \rho_\Lambda = \rho_m + \rho_\Lambda \) and \( p_\Lambda = p_m + p_\Lambda = p_m - \rho_\Lambda \), that is \( \tilde{T}_{\mu \nu} = -\rho_\Lambda g_{\mu \nu} + (\rho_m + p_m) U_\mu U_\nu \) or, explicitly,

\[
\tilde{T}_{\mu \nu} = (\rho_\Lambda - \rho_m) g_{\mu \nu} + (\rho_m + p_m) U_\mu U_\nu .
\]

By assuming this generalized energy-momentum tensor and a spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) metric, the independent gravitational field equations reduce to (Carvalho et al. 1992; Lima & Maia 1994; Basilakos 2009; Solà 2011)

\[
8 \pi G \rho_\Lambda \equiv 8 \pi G \rho_m + \Lambda = 3 H^2 ,
\]

\[
8 \pi G p_\Lambda \equiv 8 \pi G p_m - \Delta = -2 \dot{H} - 3 H^2 ,
\]

where the overdot denotes derivative with respect to cosmic time \( t \).

Now let us discuss a bit more the possibility that the vacuum energy density is a function of the cosmic time. This is allowed by the Cosmological Principle embodied in the FLRW metric. The Bianchi identities (which ensure the covariance of the theory) then imply \( \tilde{T}^\mu_{\nu \rho} \dot{\epsilon}^\rho = 0 \). With the help of the FLRW metric, the previous identity amounts to the following generalized local conservation law:

\[
\dot{\rho}_m + \rho_\Lambda + 3H(\rho_m + p_m + \rho_\Lambda + p_\Lambda) = 0 .
\]

Notice that we keep \( G \) strictly constant, and therefore the assumption \( \rho_\Lambda \neq 0 \) necessarily requires some energy exchange between matter and vacuum, e.g. through vacuum decay into matter, or vice versa. 1

Let us remark that the equation of state of the vacuum energy density maintains the usual form \( p_\Lambda(t) = -\rho_\Lambda(t) = -\Lambda(t)/(8 \pi G) \) despite the fact that \( \Lambda(t) \) evolves with time. Inserting \( p_\Lambda = -\rho_\Lambda \) and \( p_m = \omega_m \rho_m \) into equation (5) the latter equation leads to the following energy exchanging balance between matter and vacuum:

\[
\dot{\rho}_m + 3(1 + \omega_m)H \rho_m = -\rho_\Lambda .
\]

Finally, combining equations (3) and (6), we find

\[
\dot{H} + \frac{3}{2} (1 + \omega_m) H^2 = 4 \pi G(1 + \omega_m) \rho_\Lambda = \frac{(1 + \omega_m) \Lambda}{2}. \tag{7}
\]

## 3 RUNNING VACUUM MODELS EVOLVING AS A POWER SERIES OF \( H \)

In what follows we investigate the cosmic expansion within a class of time evolving vacuum models along these lines (Solà 2011), and at the same time involving ingredients capable of yielding a smooth transition from an early de Sitter stage to proper radiation and matter epochs (Lima & Maia 1994; Lima & Trodden 1996). Consider the class of time evolving vacuum models following an even power series of the Hubble rate:

\[
\Lambda(H) = c_0 + c_2 H^2 + c_4 H^4 + c_6 H^6 \ldots \tag{8}
\]

with \( \rho_\Lambda(H) = \Lambda(H)/(8 \pi G) \) the corresponding vacuum energy density. The constant \( c_0 \) in (8) represents the dominant term at

1 There exists also the possibility that the vacuum is time evolving and nevertheless non-interacting with matter. In this case, however, either the DE has another component apart from \( \Lambda \) – see the \( \Lambda \)CDM framework of Grande, Solà & Šefćíč (2006) – or Newton’s coupling is also time varying, i.e. \( G \neq 0 \) (Solà 2008).
low energies (i.e. when $H$ is near the current value $H_0$). The $H^2(k \geq 1)$ powers represent small corrections to the dominant term and provide a time evolving behaviour to the vacuum energy density. The expression (8) has to be understood as a general ansatz for the vacuum energy in an expanding universe. The reason for selecting even powers of $H$ is because of the general covariance of the effective action of QFT in a curved background. Although we cannot presently quote its precise structure, we know it can only contain even powers which can emerge from the contractions of the metric tensor with the derivatives of the scalar factor (Shapiro & Solà 2009). Interestingly, these dynamical vacuum models have recently been linked with a potential variation of the so-called fundamental constants of Nature (Fritzsche & Solà 2012). The power like behaviour with $H$ has also been seriously considered as potentially emerging from a modification of Friedmann equation from e.g. infinite-volume extra dimensions, and as an intriguing and testable option for describing DE (Dvali & Turner 2003).

The novelty in the present approach is that we extend the domain of applicability of these models, namely we encompass here in a single unified framework both the inflationary and the current DE epochs. In a minimal model we expect that only a few powers of $H$ should matter beyond the additive term, $c_0$, which is essential to ensure a good ΛCDM limit for any model of the sort (8). While the higher order powers $H^2(k > 1)$ are completely negligible at present, they can acquire a great relevance in the early universe. This leads to the following canonical realization of equation (8) for describing the basic features of both the early and late cosmos:

\[
\Lambda(H) = c_0 + 3vH^2 + 3\alpha \frac{H^4}{H_0^2}.
\]

(9)

Here $v$ and $\alpha$ are dimensionless parameters and the scale $H_1$ can be interpreted as the inflationary expansion rate (see below). With only two additional parameters we need to choose between $v = 0$ and the existence of an early de Sitter phase connected with $\alpha \neq 0$. The former choice means that at late times the model becomes indistinguishable from ΛCDM at low redshifts. So, in order to have the initial de Sitter phase and the possibility to confront our late time model with ΛCDM we need at least three parameters. The third independent parameter has conveniently been written as the ratio $\alpha/H_1^2$.

Inserting equation (9) into equation (7) one can easily write

\[
H + \frac{3}{2}(1 + \omega_m)H^2\left[1 - v - \frac{c_0}{3H^2} - \frac{\alpha}{3} \frac{H^2}{H_0^2}\right] = 0.
\]

(10)

Remarkably, there is the constant value solution $H^2 = (1 - v)H_1^2/\alpha$ of this equation for the very early universe (where we can safely neglect $c_0/H^2 \ll 1$). It signals the presence of an inflationary epoch.

We shall present below the various phases of the decaying vacuum cosmology (9), starting from an unstable inflationary phase $[a(t) \propto e^{Ht}]$ powered by the huge value $H_1$ presumably connected to the scale of a Grand Unified Theory (GUT) or even the Planck scale $M_P$, then it deflates (with a massive production of relativistic particles), and subsequently evolves into the standard radiation and matter-dominated eras. Finally, it effectively appears today as a slowly dynamical DE slightly correcting the standard ΛCDM model.

4 FROM AN EARLY DE SITTER STAGE TO THE STANDARD RADIATION PHASE

The Hubble function and scale factor of this model in the early universe (when $c_0$ can be neglected) follow from direct integration of equation (10):

\[
H(a) = \left(\frac{1 - v}{\alpha}\right)^{1/2} \frac{H_1}{\sqrt{D a^{3(1-v)(1+\omega_m)} + 1}},
\]

(11)

\[
\int_{a_*}^a \frac{da}{a} \sqrt{D a^{3(1-v)(1+\omega_m)} + 1} = \frac{1 - v}{\alpha} H_1 \Delta t
\]

where $D > 0$ is a constant. Notice that $a_*$ is the scale factor at the transition time ($t_*$) when the inflationary period ceases, and $\Delta t = t - t_*$ is the cosmic time elapsed since then. Using equation (11) and the Einstein equations (3)–(4) we may also obtain the corresponding energy densities:

\[
\rho_m(a) = \rho_l(a) = \frac{\Lambda(a)}{8\pi G} = \frac{1 - v}{\alpha} \frac{c_0}{\rho_\Lambda}\left[\frac{D a^{3(1-v)(1+\omega_m)} + 1}{D a^{3(1-v)(1+\omega_m)} + 1}\right]^{3/2},
\]

(13)

\[
\rho_\Lambda(a) = \frac{\Lambda(a)}{8\pi G} = \frac{1 - v}{\alpha} \frac{c_0}{\rho_\Lambda}\left[\frac{D a^{3(1-v)(1+\omega_m)} + 1}{D a^{3(1-v)(1+\omega_m)} + 1}\right]^{3/2},
\]

(14)

where $\rho_l = 3H^2/8\pi G$ is the primordial critical energy density associated with the initial de Sitter stage.

Obviously, if $D a^{3(1-v)(1+\omega_m)} \ll 1$ (i.e. $t \ll t_*$) then equation (11) boils down to the particular solution mentioned above, in which $H = \sqrt{(1 - v)/\alpha} H_1$ is constant, and $\rho_m \approx 0$, $\rho_\Lambda \propto \rho_l$ (i.e. no matter and huge vacuum energy density). From (12) it is obvious that $a(t) \sim e^{c_0^{1/3}(1-v)/\alpha H_1 \Delta t}$ and the universe then inflates. For $D a^{3(1-v)(1+\omega_m)} \gg 1$ (i.e. $t \gg t_*$), instead, equations (13) tell us that both $\rho_m(a)$ and $\rho_\Lambda(a)$ decay as $\sim a^{-3(1-v)(1+\omega_m)}$ while at the same time the ratio $[\rho_\Lambda(a)/\rho_m(a)]$ remains very small, viz. of order $|v| \leq \mathcal{O}(10^{-3})$ (Basilakos et al. 2009; Grande et al. 2011). Furthermore, since the vacuum presumably decayed mostly into relativistic particles ($\omega_m = 1/3$), we find from equation (12) that in the post-inflationary regime $a \sim t^{2(1+v)/3}$ (i.e. we reach essentially the standard radiation epoch – confirmed, in addition, by the fact that the relativistic matter (and vacuum) energy densities $\rho_m(a)$ and $\rho_\Lambda(a)$ decay as $\sim a^{-4}$ in this period. The universe thus evolves continuously from inflation towards a standard (FLRW) radiation dominated stage, as shown in the inner plot of Fig. 1. In between these two eras, we see from the first equation (13) that we can have either huge relativistic particle production $\rho_l = \rho_m \propto a^4$ in the deflation period (namely around $D a^4 \geq 1$) or standard dilution $\rho_l = \rho_m \propto a^{-4}$ well in the radiation era ($Da^4 > 1$).

Naturally, due to the initial de Sitter phase, the model is free of particle horizons. A light pulse beginning at $t = -\infty$ will have travelled by the cosmic time $t$ a physical distance $d_L(t) = a(t) \left(\int_{-\infty}^t \frac{dt'}{a(t')}\right)$, which diverges thereby implying the absence of particle horizons: the local interactions may homogenize the whole Universe. It should be clear that our accessible part of the universe is only for times $t > t_*$, i.e. $\Delta t > 0$. After inflation has occurred and the FLRW (radiation dominated) phase has been causally prepared, the cosmic time $\Delta t$ can just be called $t$ and it is this one that parametrizes all cosmological equations in physical cosmology (Peebless 1993).

Although the motivation of the present model has a root in the general structure of the effective action of QFT in curved spacetime, we cannot provide the latter at this point. However we can...
mimic it through a scalar field (φ) model for the interacting DE (Maia & Lima 2002; Costa, Alcaniz & Maia 2008). This can be useful for the usual phenomenological descriptions of the DE, and can be obtained from the usual correspondences ρ_{tot} \rightarrow ρ_0 = \dot{φ}^2/2 + V(φ) and \rho_{m} \rightarrow ρ_0 = \dot{φ}^2/2 - V(φ) in Friedmann’s equations (3) and (4). We find 4πG\dot{φ}^2 = -\dot{H} and 
\begin{align}
V &= \frac{3\dot{H}^2}{8\pi G} \left(1 + \frac{H}{3H^2}\right) = \frac{3\dot{H}^2}{8\pi G} \left(1 + \frac{1}{3} \frac{d \ln H}{d \ln a}\right).
\end{align}
We can readily work out the effective potential for our model (9) in the early universe as a function of the scale factor. Neglecting small O(ν) corrections, which we have seen are not important in the early stages, we arrive at 
\begin{align}
V(a) = \frac{\rho_0}{a} \frac{1 + Da^2/3}{(1 + Da)^2}.
\end{align}
From this expression it becomes clear that the potential energy density remains large and constant while a \ll D^{-1/4} (i.e. before the transition from inflation to the deflationary regime). Afterwards (when a \gg D^{-1/4}) it decreases steadily as V(a) \sim a^{-2}, hence as radiation. This confirms, in the effective scalar field language, the previously described decay of the vacuum energy into relativistic matter in our original framework.

5 FROM THE RADIATION-MATTER ERA TO THE RESIDUAL DYNAMICAL DARK ENERGY AT PRESENT

Let us finally discuss the cosmic evolution after the inflationary period is left well behind, i.e. when H \ll H_1. The cosmic fluid will be first in the radiation dominated epoch (\omega_m = 1/3) and later on in the cold matter dominated epoch (\omega_m = 0). In this case (8) reduces to
\begin{align}
\Lambda(H) = \Lambda_0 + 3\nu (H^2 - H_0^2),
\end{align}
where \Lambda_0 = \rho_0 + 3\nu H_0^2 is the current value of the CC. Obviously, c_0 plays an essential role to determine its value, whereas the H^2 dependence gives some remnant dynamics even today, which we can use to fit the parameter ν to observations. Using a joint likelihood analysis of the recent supernovae Type Ia data, the CMB shift parameter, and the Baryonic Acoustic Oscillations one finds that the best-fitting parameters for a flat universe are: \Omega_m^0 \approx 0.27 - 0.28 and |ν| = O(10^{-3}) (see Basilakos et al. 2009; Grande et al. 2011; Basilakos et al. 2012). It is remarkable that the fitted value of ν is within the theoretical expectations when this parameter plays the role of β-function of the running CC (Shapiro & Solà 2002, 2009; Solà & Štefančič 2005, 2006). In specific frameworks one typically finds ν = 10^{-5} to 10^{-3} (Solà 2008).

Despite the dynamical character of the vacuum energy (17) near our time, it is important to understand that a model of this kind would not work for c_0 = 0, i.e. with only pure H-dependent terms. This has been shown in Basilakos et al. (2009) and recently confirmed in Xu et al. (2011), as the models of this sort fail to reproduce the observed CMB and matter power spectrum. Furthermore, for them it is impossible to have a transition from deceleration to acceleration (Basilakos et al. 2009). At the root of all these problems is the fact that the c_0 = 0 models have no ΛCDM limit.

The evolution equation for the Hubble function (10) in the post-inflationary epoch reads:
\begin{align}
H + \frac{3}{2} (1 + \omega_m) (1 - ν) H^2 - \frac{1 + \omega_m}{2} c_0 = 0.
\end{align}
Note that c_0 cannot be neglected deep in the cold matter dominated period and specially near the present time. Trading the cosmic time for the scale factor and using the redshift variable 1 + z = 1/α with the boundary condition H(z = 0) = H_0, one finds the solution of equation (18) for the late stages (α_m = 0):^2
\begin{align}
H^2(z) &= \frac{H_0^2}{1 - ν} \left[(1 - \Omega_m^0)(1 + z)^{3(1 - ν)} + \Omega_m^0 - ν\right],
\end{align}
where \Omega_m^0 = \Lambda_0/3H_0^2 = 8πG ρ_m^0/3H_0^2. In a similar way we can obtain the matter and vacuum energy densities as a function of the redshift:
\begin{align}
ρ_m(z) &= ρ_m^0 (1 + z)^{3(1 - ν)},
\end{align}
\begin{align}
ρ_λ(z) &= ρ_λ^0 + ν ρ_m^0 \frac{1 - ν}{1 - ν} \left[(1 + z)^{3(1 - ν)} - 1\right].
\end{align}
From these equations it is clear that for ν = 0 we recover exactly the ΛCDM expansion regime, the standard scaling law for non-relativistic matter and a strictly constant vacuum energy density ρ_λ = ρ_λ^0 (hence Λ = Λ_0). Recalling that |ν| is found to be rather small when the model is confronted to the cosmological data, |ν| ≤ O(10^{-3}) (Basilakos et al. 2009; Grande et al. 2011), it is obvious that at the present time this vacuum model is almost indistinguishable from the concordance ΛCDM model, except for its mild dynamical behaviour which leads to an effective equation of state for the vacuum energy that can mimic quintessence or phantom energy (Solà 2011). At very late time we get an effective CC-dominated era, H ≃ H_0 \sqrt{(\Omega_m^0 - ν)/(1 - ν)}, that implies a pure De Sitter phase of the scale factor.

6 DISCUSSION AND FUTURE WORK

Let us emphasize that in our model we need not specify the microscopic nature of the matter fields involved, but in a more detailed formulation baryons should obviously be conserved deep in the matter dominated epoch and later in the vacuum phase. Thus we assume that the vacuum always decays in the (material) dominant component at each phase. This means that at late times the baryon-antibaryon decay is negligible, and, therefore, we have an interaction in the dark sector, that is, the vacuum decays mainly in CDM (cf. Fritzsch & Solà 2012). In particular, this means that the vacuum should not exceedingly decay into photons after last scattering, as it could trigger an observable distortion of the CMB spectrum. Fortunately, this does not happen if ν is as small as 10^{-3}. This has been studied in Opher & Pelinson (2004, 2005) for models of this sort, and is consistent with Basilakos et al. (2009) results (see also Grande et al. 2011). It ensures this framework does not get in contradiction with basic facts of high-precision cosmology.

In Fig. 1 we display, in addition to the mentioned details of the early stages of the cosmic evolution (inner plot), also the numerical analysis describing the transition of the energy densities
from the radiation epoch into the matter-dominated period, leading finally to the asymptotic de Sitter phase beyond our time (outer plot). The equality time of matter and radiation, $\rho_m(z_{eq}) = \rho_\gamma(z_{eq})$ (Komatsu et al. 2011), is also marked in the plot. It corresponds to $z_{eq} \approx 3200$, thus with no essential change with respect to the LCDM. On the other hand for $z \leq 10$ (or $a \geq 0.1$) the vacuum energy density appears as effectively frozen to its nominal value, $\rho_\Lambda(z) \approx \rho_\Lambda^0$, but still displaying a slow cosmic evolution as in equation (21). The considered model therefore provides a description of the cosmological vacuum as playing a prominent role in the early (inflationary) universe and then evolving similarly to the matter densities until it finally surfaces again at the present time.

We may wonder if the virtues of the cosmological picture under consideration are confined to the peculiarities of the specific model (9). Remarkably, the latter is just the minimal or ‘canonical’ implementation of the general class of models (8) based on the even powers of $H$ which are favoured from the point of view of the general covariance. It turns out that many other models of this class can still do the job. Interestingly, as it will be shown elsewhere, all the vacuum models of the form (assuming $n$ integer)

$$\Lambda(H) = c_0 + 3 v H^2 + 3 \alpha \frac{H^{2n}}{H_0^{2n-2}} \quad (n > 1)$$

(of which the model under study is just the particular case $n = 2$) are consistent with general covariance and perform automatically a successful (‘graceful exit’) transition from an unstable inflationary phase (deflation) into the standard FLRW radiation dominated era, irrespective of the differences in other details.

Another important aspect of our model is related to the ratio between the early and current values of the vacuum energy density. Understanding this ratio is the basic difficulty defining the so-called CC problem (Weinberg 1989). For all values of the free parameter $n$ appearing in the above equation, we see that the value of $\Lambda$ at the very early de Sitter phase is $\Lambda_1 \sim H_1^2$ while at present it reads $\Lambda_0 \sim H_0^2$. Therefore, the ratio between the two extreme vacuum energy densities reads

$$\frac{\rho_\Lambda}{\rho_\Lambda} \sim \frac{\Lambda_1}{\Lambda_0} \sim \frac{H_1^2}{H_0^2}.$$  \hspace{1cm} (23)

Notice that although $H_1$ (the energy scale of the primeval de Sitter state) is not given by the model, it can be estimated, for instance, from the equilibrium temperature associated with the event horizon of an arbitrary de Sitter state (Gibbons & Hawking 1977). In the present case, it takes the following form: $k_B T_1 = h(\Lambda_1/2\pi^2)^{1/2}$ ($k_B$ and $h$ are the Boltzmann and Planck constants). Therefore, using $H_1 = \sqrt{T_1/3}$ we see that the initial temperature of our scenario is $T_1 = hH_1/(2\pi k_B)$. Now, by choosing $H_1^{-1}$ to be of the order of the Planck time, $t_p = (hG/c^3)^{1/2}$, it is easy to check that the initial temperature of the universe is $T_1 = T_p/(2\pi)$, i.e. of the order of the Planck temperature, $T_p = (h c^3/G k_B^2)^{1/2}$. More important, for $H^{-1}$ of the order of the Planck time, the model essentially predicts that the above ratio is $\rho_\Lambda/\rho_\Lambda \sim c^4/\sqrt{\Lambda k_B^2}$ which, when $M_P = (hcG)^{1/2}$ is the Planck mass. In natural units this simply reads $\rho_\Lambda/\rho_\Lambda \sim M_P^2 H_0^2 \sim 10^{122}$, where $M_P \sim 10^{19}$ GeV and $H_0 \sim 10^{-42}$ GeV in such units.

The above result is consistent with the standard theoretical expectation for the ratio between the present-day value of $\Lambda$ and its uppermost value at the very early universe (Weinberg 1989). In our model, such a result i.e. the present smallness of the CC (\$\Lambda_0 \sim 10^{-12} \Lambda_1\$) can be seen as the basic consequence of a decaying vacuum process in an aged Universe. It is worth notice that the above ratio was earlier obtained for $n = 1$ based on a similar approach, however, not taking into account that the action should be expressed in terms of even powers of the effective action (Lima & Maia 1994; Lima & Trodden 1996). It should also be stressed that the semi-classical value of $H_1$ provided by the analogy with Gibbons–Hawking temperature relation holds regardless of the value of $n$, and, in principle, must be derived from a more fundamental QFT approach for a decaying vacuum energy density. In addition, one may also think that the resulting concordance with the so-called $\Delta$-problem is not just a coincidence, and, as such, it would be signalizing for a deeper connection between the deflationary scenario and the quantum gravity regime.

It should be also stressed that the higher order $H^4$ term of our unified ansatz (9), namely the term which is driving the early universe dynamics, was motivated from the covariance of the effective action. Let us remark that our scenario is quite different from the holographic–entropic models of inflation (Easson, Frampton & Smoot 2012), in which the thermal radiation from the temperature of the horizon is associated with an energy density that varies as $\rho \sim T^4$, where $T$ is the aforementioned Gibbons & Hawking temperature. In particular, we observe that at the very early times, i.e. when $H = H_1$, there is no radiation fluid component in our scenario. The universe starts its evolution from a pure de Sitter state uniquely supported by the vacuum energy density.

On the other hand, the existence of an early isothermal de Sitter phase suggests that thermal fluctuations may be the causal origin of the primeval seeds that ultimately will form the galaxies. Such an origin indicated by the present deflationary scenario is quite different from the adiabatic fluctuations usually adopted in several
variants of the cosmic inflationary paradigm. In principle, one may expect that the footprints of the primeval scale \( H \) might be extracted from CMB observations through the influence of the initial power spectrum of density fluctuations on the pattern of polarization and temperature anisotropies at the last scattering surface. Such a possibility and its consequences for the structure formation problem deserve a closer scrutiny and are clearly out of the scope of the present paper.

It is also interesting that the \( H^2 \) behaviour at low energy, complemented here by the contribution of the true vacuum constant term \( c_0 \), permits the smooth transition from a deceleration to an acceleration phase as required from supernovae observations either in the dynamics or in the kinematic descriptions (Turner & Riess 2002; Cunha & Lima 2008; Guimarães, Cunha & Lima 2009; Guimarães & Lima 2011). As explained in detail by Basilakos et al. (2012), the necessary transition is not possible in the decaying vacuum models driven only by the dynamical \( H^2 \) term (or its variant \( \dot{H} \)) without introducing the additive constant \( c_0 \), such as e.g. in the entropic-force model of the current accelerated expansion (Easson, Frampton & Smoot 2011). Quite in contrast, for our unified model as given by (9), as well as for the entire class of models generated by the ansatz (22), we obtain a consistent description both at high energies, i.e. for the dynamics of the early inflationary universe, and at low energies, i.e. for the physics of the present universe. In this sense, our model can be considered as an effective unified framework for a practical description of the complete history of our cosmos. Somehow it is the kind of framework that is awaiting for a more fundamental theory whose effective behaviour is of the sort described here, since then all of the discussed cosmological problems could be accounted for in an efficient way. In view of the fact that our unified ansatz (9) has a structure which is consistent with the general covariance of the effective action both at low and high energies, we expect that this feature should pave the way for a smooth connection of our framework with that fundamental theory.

7 CONCLUSIONS

In this work we have put forward a new class of global cosmological scenarios which provides a consistent and rather complete account of the expansion history of the Universe. Although only partially justified from a more fundamental approach, it can be conceived as providing the simplest effective dynamical structure that the vacuum energy should inherit from some future fundamental theory in order to automatically solve these problems in an economical way. As we have emphasized throughout the paper, the present dynamical vacuum framework based on the \( H^2 \) and \( H^4 \) terms is compatible with the general form of the effective action of QFT in curved space–time.

We have unveiled the existence of a large class of generalized models of this sort (see equation 22), all of them sharing a similar phenomenological behaviour, namely: (i) the Universe starts from an inflationary non-singular state, thus overcoming the horizon problem; (ii) the early inflationary regime has a natural (universal) ending into the radiation phase; and (iii) the small current value of the vacuum energy density can be conceived as a result of the massive disintegration of the vacuum into matter in the primordial stages. The upshot is a new unified vacuum picture of the cosmic evolution spanning from the early inflation period to the late DE era and deviating only very mildly from the observed \( \Lambda \)CDM behaviour. The amplitude of the expected deviations (\( |\nu| \sim 10^{-3} \)) can decide which is the more realistic description of the vacuum at late stages, and, potentially, it may provide an indication favouring the complete decaying vacuum scenario proposed here. The extended scenario will be discussed in detail in a forthcoming communication.

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NOTE ADDED IN PRESS

After the completion of this work, a thermodynamic analysis of the present scenario was carried out by Mimoso and Pavón (2013) based on the generalized second law of thermodynamics. These authors concluded that the present \( \Lambda(H) \) cosmology is thermodynamically consistent even when the horizon entropy during the extreme de Sitter phases are taken into account.

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