

Influence of small-scale inhomogeneities on the cosmological consistency tests

V. C. Busti[★] and J. A. S. Lima[★]

Instituto de Astronomia, Geofísica e Ciências Atmosféricas (IAG-USP), Universidade de São Paulo, 05508-900 São Paulo, SP, Brazil

Accepted 2012 July 18. Received 2012 June 20; in original form 2012 April 4

ABSTRACT

The current cosmological dark sector (dark matter plus dark energy) is challenging our comprehension about the physical processes taking place in the Universe. Recently, some authors tried to falsify the basic underlying assumptions of such dark matter–dark energy paradigm. In this Letter, we show that oversimplifications of the measurement process may produce false positives to any consistency test based on the globally homogeneous and isotropic Λ cold dark matter (Λ CDM) model and its expansion history based on distance measurements. In particular, when local inhomogeneity effects due to clumped matter or voids are taken into account, an apparent violation of the basic assumptions (‘Copernican Principle’) seems to be present. Conversely, the amplitude of the deviations also probes the degree of reliability underlying the phenomenological Dyer–Roeder procedure by confronting its predictions with the accuracy of the weak lensing approach. Finally, a new method is devised to reconstruct the effects of the inhomogeneities in a Λ CDM model, and some suggestions of how to distinguish between clumpiness (or void) effects from different cosmologies are discussed.

Key words: gravitational lensing: weak – cosmological parameters – cosmology: observations – cosmology: theory – dark energy – large-scale structure of Universe.

1 INTRODUCTION

The dark energy mystery has inspired cosmologists to test all the assumptions of the so-called cosmic concordance model (Λ CDM). In the last few years, some methods to detect possible deviations from the Friedman–Lemaître–Robertson–Walker (FLRW) metric (Clarkson, Bassett & Lu 2008; Uzan, Clarkson & Ellis 2008) or the flat Λ CDM model (Sahni, Shafieloo & Starobinsky 2008; Zunckel & Clarkson 2008), as well as to reconstruct the dark energy equation of state $w(z)$, have been proposed (e.g. Saini et al. 2000; Sahni & Starobinsky 2006; Clarkson & Zunckel 2010). However, for a real understanding of what is being measured, it is fundamental to check whether such proposals are based on assumption-free approaches or whether such deviations are naturally mimicked when a more realistic description is considered.

On the other hand, the observed Universe must be studied in two separated spatial regimes since it is homogeneous on large scales (>100 Mpc), while a hierarchy of structures involving galaxies, filaments, clusters of galaxies and voids is seen on small scales. Such inhomogeneities can change the observed distances when radiation is used because the light rays probe the local gravitational field thereby affecting the cosmological parameters.

In principle, even assuming that the FLRW metric is adequate to describe the cosmic expansion history, the existing observations may prefer underdense lines of sight as compared to the background, and as such the distance relations need to be corrected for the realistic clumpy Universe. The basic consequence is that artefacts (false positives) will be produced in the existing tests originally proposed within the globally smooth FLRW model. Reciprocally, since the magnitude of the artefacts is heavily dependent on how light propagation is described in the clumpy Universe, such tests can also unveil the most suitable method to deal with the inhomogeneities.

The purpose of this Letter is threefold. First, we show that small-scale inhomogeneities affect the distance and produce false positives for two distinct tests, namely the $\mathcal{C}(z)$ (Clarkson et al. 2008) and $\mathcal{L}(z)$ tests (Sahni et al. 2008; Zunckel & Clarkson 2008). Secondly, a new method is proposed to reconstruct the effects of the inhomogeneities directly from observational data when a Λ CDM model is assumed, and finally it is discussed how to distinguish between the clumpiness (or void) effects from different cosmologies.

2 COSMOLOGICAL TESTS

In what follows, we restrict our attention for the two above-quoted cosmological tests [$\mathcal{C}(z)$ and $\mathcal{L}(z)$]. However, it is important to stress that any test based on distance measurements will produce the artefacts discussed in this Letter. For instance, the influence of the inhomogeneities on the reconstruction of the dark energy equation

[★]E-mail: vcbusti@astro.iag.usp.br (VCB); limajas@astro.iag.usp.br (JASL)

of state was discussed by Bolejko (2011a), and a Copernican test involving the redshift drift and distances will be affected as well (Uzan et al. 2008).

2.1 The $\mathcal{C}(z)$ test

The leitmotiv of such an approach is to test the so-called ‘Copernican Principle’ (CP) which is implicit in the homogeneous and isotropic FLRW metric (Clarkson et al. 2008). In this case, the possible redshift dependence of the curvature parameter (a CP violation signature) can be discussed based on the expression of the luminosity distance (in our units $c = 1$):

$$d_L(z) = \frac{(1+z)}{H_0\sqrt{-\Omega_k}} \sin \left[\sqrt{-\Omega_k} \int_0^z dz' \frac{H_0}{H(z')} \right], \quad (1)$$

where $H(z)$ is the expansion rate (H_0 is the Hubble constant) and Ω_k is the present-day curvature parameter. By defining $D(z) = \frac{H_0 d_L(z)}{(1+z)}$, one can differentiate the above equation and rearrange the terms in order to have

$$\Omega_k = \frac{[E(z)D'(z)]^2 - 1}{D(z)^2}, \quad (2)$$

where the prime denotes redshift differentiation and $E(z) = H(z)/H_0$. Now, it is easy to see that a differentiation of equation (2) yields

$$\mathcal{C}(z) = 1 + E^2(DD'' - D'^2) + EE'DD' \equiv 0, \quad (3)$$

since it has been assumed that Ω_k is constant for all redshifts when the globally smooth FLRW metric properly describes the background geometry. It is worth noting that deviations of the order of 10^{-5} are expected in realistic models due to perturbations related to structure formation for all redshifts (Ellis 2009).

2.2 The $\mathcal{L}(z)$ test

Unlike the $\mathcal{C}(z)$ test, the basic aim here is to identify any deviation from a flat Λ CDM model. It was independently introduced by Sahni et al. (2008) and Zunckel & Clarkson (2008), and may be interpreted as a kind of consistency check. For a flat Λ CDM model, the present value of the matter density parameter, Ω_m , can be written in terms of the observed quantities:

$$\Omega_m = \frac{[H(z)/H_0]^2 - 1}{(1+z)^3 - 1} = \frac{1 - D'(z)^2}{[(1+z)^3 - 1]D'(z)^2}. \quad (4)$$

Following the same approach of the $\mathcal{C}(z)$ test, a simple differentiation yields

$$\mathcal{L}(z) = 2[(1+z)^3 - 1]D''(z) + 3(1+z)^2D'(z)[1 - D'(z)^2], \quad (5)$$

which must also be identically null (regardless of the redshift) for all flat Λ CDM models. Since the quantities appearing in equations (3) and (5) are measurable, one may expect that both null results can be checked by the available data.

3 THE DYER–ROEDER APPROACH

In the above-discussed tests, the Universe was assumed to be homogeneous and isotropic on all scales. Therefore, the basic question now is how the background cosmological tests are affected by small-scale structures? In other words, even assuming that large-scale homogeneity is preserved, the light propagation is perturbed by small-scale inhomogeneities, potentially modifying the angular diameter and luminosity distances. Therefore, it is fundamental to quantify the unknown physical conditions along the light path.

Initially, this issue was addressed by Zel’dovich (1964) and followed by Bertotti (1966), Gunn (1967) and Kantowski (1969). But in the beginning of 1970s, Dyer & Roeder (1972, 1973) adopted the average path assumption so that the underdensities in voids are compensated by overdensities in clumps, thereby making the Universe homogeneous only on very large scales. A typical line of sight is far from the clumps, not suffering from gravitational lensing effects. In this way, the unknown physical conditions along the path, associated with the clumpiness effects, are phenomenologically described by the smoothness parameter α . Such a quantity has a straightforward physical meaning: it is the fraction of homogeneously distributed matter within a given light cone. For $\alpha = 0$ (empty beam), all matter is clumped while for $\alpha = 1$ the fully homogeneous case is recovered. Then, for a partial clumpiness, the standard interpretation (involving structures more massive than the cosmic average) is that the smoothness parameter is restricted only over the interval $[0, 1]$.

Observationally, the smoothness parameter is still poorly constrained. By using compact radio sources, no constraint over α was obtained (Alcaniz, Lima & Silva 2004; Santos & Lima 2008), whereas an analysis with Type Ia supernovae (SNe Ia) in a flat Λ CDM model constrained $\alpha \geq 0.42$ within the 95.4 per cent confidence level (2σ) (Santos, Cunha & Lima 2008). The introduction of $H(z)$ data only mildly improved the results: $\alpha \geq 0.66$ within the 95.4 per cent confidence level (Busti & Santos 2011). In the same vein, by combining the 557 SNe Ia from the Union 2 compilation (Amanullah et al. 2010) and 59 gamma-ray bursts compiled by Wei (2010), it was shown that $\alpha \geq 0.52$, i.e. a more inhomogeneous universe is compatible with current data (Busti, Santos & Lima 2012).

Several generalizations of the Dyer–Roeder approach have been proposed in the literature. The dependence of the smoothness parameter on redshift was first discussed by Linder (1988, 1998) and Santos & Lima (2008). The influence of a non-standard expansion rate was analysed by Mattsson (2010), and a connection with weak lensing was investigated by Bolejko (2011b). A comprehensive study concerning inhomogeneity effects on light propagation was recently carried out by Clarkson et al. (2011). The Dyer–Roeder approach that we have chosen to deal with inhomogeneities is not unique. There are other proposals in the literature, e.g. Kainulainen & Marra (2009), but the simplest one is the Dyer–Roeder approach.

The above discussions reveal that the small-scale inhomogeneities affect the light propagation although its modelling is far from trivial. Potentially, the inhomogeneities may play an important role, thereby masking several proposed consistency checks of Λ CDM and other dark energy models. Therefore, in order to claim a violation of the CP [$\mathcal{C}(z)$ test] or any deviation from a flat Λ CDM model [$\mathcal{L}(z)$ test], it is vital to disentangle all the potential effects.

3.1 Dyer–Roeder distance

The derivation follows from the Sachs’s optical equation (Jordan, Ehlers & Sachs 1961; Sachs 1961):

$$\sqrt{A}'' + \frac{1}{2}R_{\mu\nu}k^\mu k^\nu \sqrt{A} = 0, \quad (6)$$

where a prime denotes differentiation with respect to the affine parameter λ , A is the cross-sectional area of the light beam, $R_{\mu\nu}$ the Ricci tensor, k^μ the photon four-momentum and the shear is neglected.

Five steps are needed to achieve the luminosity distance in the Dyer–Roeder approach: (i) the assumption that the angular diameter

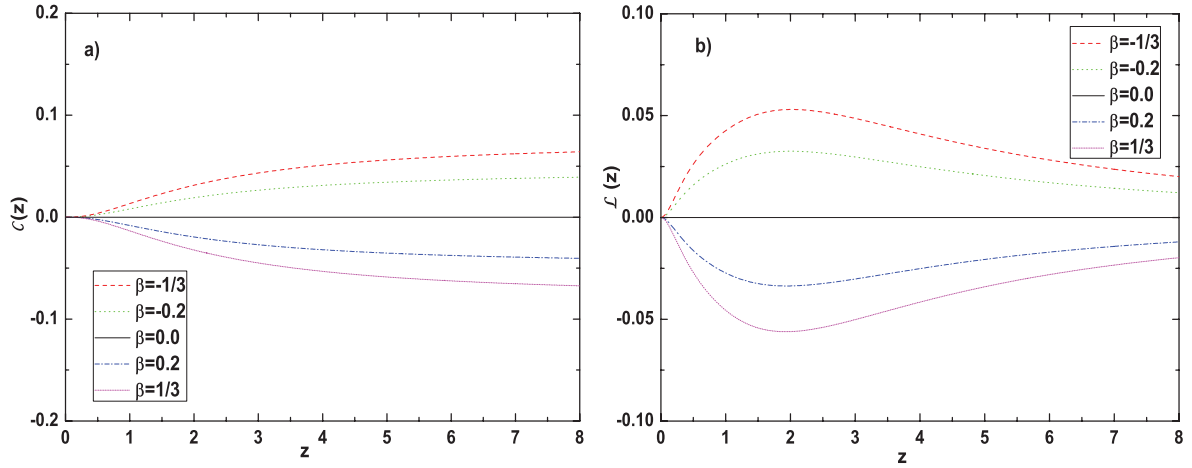


Figure 1. The β effect on the $\mathcal{C}(z)$ and $\mathcal{L}(z)$ cosmological tests. (a) $\mathcal{C}(z) \equiv 0$ (solid black line, $\beta = 0$ and $\alpha = 1$) is the prediction of the standard globally smooth Λ CDM model (see equation 9). All curves with $\beta \neq 0$ are simulating an apparent violation (false positive) of the Copernican Principle. (b) As in panel (a), $\mathcal{L}(z) = 0$ is the Λ CDM prediction with no inhomogeneous corrections. As physically expected, due to the choice of the β values, the corrections for clumps ($\beta < 0$, $\alpha < 1$) and voids ($\beta > 0$, $\alpha > 1$) are symmetric with respect to the Λ CDM prediction.

distance $D_A \propto \sqrt{A}$, (ii) the relation between the Ricci tensor and the energy–momentum tensor through the Einstein’s field equations, (iii) the relation between the affine parameter λ and the redshift z , (iv) the ansatz ρ_m goes to $\alpha\rho_m$ and (v) the validity of the duality relation between the angular diameter and luminosity distances (Etherington 1933; Bassett & Kunz 2004; Holanda, Lima & Ribeiro 2010, 2011).

For a XCDM model, one obtains the Dyer–Roeder distance ($d_L = H_0^{-1} D_L$) by solving the equation

$$\frac{3}{2} [\alpha(z)\Omega_m(1+z)^3 + \Omega_X(1+w)(1+z)^{3(1+w)}] D_L(z) + (1+z)^2 E(z) \frac{d}{dz} \left[(1+z)^2 E(z) \frac{d}{dz} \frac{D_L(z)}{(1+z)^2} \right] = 0, \quad (7)$$

where Ω_X and w are the density and equation-of-state parameters of dark energy, while the dimensionless Hubble parameter, $E(z) = H/H_0$, reads

$$E(z) = \sqrt{\Omega_m(1+z)^3 + \Omega_X(1+z)^{3(1+w)} + \Omega_k(1+z)^2}, \quad (8)$$

where $\Omega_k = (1 - \Omega_m - \Omega_X)$. The initial conditions to solve equation (7) are $D_L(0) = 0$ and $\frac{dD_L}{dz}|_0 = 1$.

4 RESULTS

4.1 Quantifying the influence of α

In order to quantify the effects of the inhomogeneities on the cosmological tests [$\mathcal{C}(z)$ and $\mathcal{L}(z)$] we plot expressions (3) and (5) by using the Dyer–Roeder distance with the following prescription for the smoothness parameter:

$$\alpha(z) = 1 + \beta a^{3\gamma} = 1 + \beta(1+z)^{-3\gamma}, \quad (9)$$

where β and γ are constant parameters, and $a \equiv (1+z)^{-1}$ is the cosmic scale factor. Since the degree of homogeneity is higher in the distant past, it follows that $\gamma \geq 0$ because the limit $\alpha \rightarrow 1$ at high redshifts must be obeyed. For a given value of γ , the β parameter quantifies the influence of the structure formation process at lower redshifts. For the sake of generality, we also consider that β (to be fixed by the data) may assume negative and positive values

in order to describe clumps and voids, respectively. The above deformation of the standard FLRW description ($\alpha = 1$) parametrizes our ignorance on the late-time structure process. It is clearly inspired by similar expressions for the $\omega(z)$ equation of state parameter of dark energy models (Linder 2003; Padmanabhan & Choudhury 2003). Interestingly, such prescription for α produces the same results as weak lensing when the parameters are tuned to $\beta \sim O(10^{-3})$ and $\gamma = 5/12$ (Bolejko 2011b).

In Fig. 1, we display the results for $\gamma = 1$ and four values of β , where we fixed a flat Λ CDM model with $\Omega_m = 0.27$. In the left-hand panel, the $\mathcal{C}(z)$ test in function of the redshift shows that when local inhomogeneities are taking into account deviations from zero are expected even with no violation of the FLRW metric. For comparison we also plotted the standard case ($\alpha = 1$). The magnitude of the effect is dependent on the chosen parameters. In this concern, we recall that deviations of the same order of magnitude as displayed in Fig. 1 were also obtained from void models constrained by SNe Ia and $H(z)$ data (February et al. 2010). In particular, when the free parameters are tuned by the weak lensing prediction, the deviation is smaller ($\sim 10^{-4}$) but higher than expected from structure formation process alone, which is 10^{-5} (Ellis 2009).

On the other hand, since the magnitude of the deviation depends on how light propagation occurs, our approach can be used in the inverted manner. More precisely, it is also suitable to probe not only the consistency of the phenomenological Dyer–Roeder approach but also the accuracy of the weak lensing formalism. In the latter case, caution with the weak lensing approximation should be signaled by higher values in the $\mathcal{C}(z)$ test.

In the right-hand panel of Fig. 1, the same analysis is performed for the $\mathcal{L}(z)$ test. It is evident that the deviations are bigger when compared to the $\mathcal{C}(z)$ test in the redshift region $[0, 1]$, so it is expected that the effect will be measurable by SNe Ia. By assuming the expected values from weak lensing, the deviations are again around 10^{-4} . One question arises whether the inhomogeneities are playing the role of curvature or a non- Λ behaviour. It is important to remark that only the distances are affected by the inhomogeneities, not the expansion rate. So a cross-check with $\mathcal{L}(z)$ involving only $H(z)$ and its derivatives is demanded in order to ascribe what is the primary cause. In principle, the amplitude of the deviations

can be used to decide which is the more realistic description of the inhomogeneities. Deviations around 10^{-4} is an indication favouring weak lensing. However, whether higher values are obtained the phenomenological Dyer–Roeder approach (or even some unknown procedure) should be preferred.

4.2 Reconstruction of α

By extending the above discussion, it is natural to investigate the possibility to obtain α directly from the data, that is without assumptions about its functional behaviour. The extra bonus is that the difference between Dyer–Roeder and weak lensing predictions can be directly inferred. In the Λ CDM model, the Dyer–Roeder equation can be rewritten as

$$\alpha(z)\Omega_m = \frac{-E(z)}{1+z} \frac{d}{dz} \left[2(1+z)^2 E(z) \frac{d}{dz} \frac{d_L}{(1+z)^2} \right] \frac{1}{d_L}. \quad (10)$$

Note also that the right-hand side of this equation depends only on observational functions, and as such one may reconstruct the smoothness parameter for a general Λ CDM model regardless of the values of the curvature parameter.

It is also worth noting that whether a smaller value of the left-hand side is measured compared to independent estimates of Ω_m , we have a constant α . But what happens if a redshift dependence is detected? In principle, two effects may be present: the α effect or an unknown deviation from Λ CDM. This is shown in Fig. 2, where in the left-hand panel the α effect is displayed for the parametrization of equation (9), and in the right-hand panel a XCDM model is considered for $\alpha = 1$ and different values for the dark energy equation of state w . We see that a XCDM model produces the same behaviour as the parametrization considered for α .

Can we distinguish the possible effects? At present, we do not have a definitive answer. For instance, if the $\mathcal{C}(z)$ test is zero and the reconstruction of α is not a constant, the true model cannot be Λ CDM. On the other hand, if the $\mathcal{C}(z)$ test is different from zero together with a departure from estimates of Ω_m , then probably the α effect is playing the basic role. Of course, both effects may be at work; then different tests combined are required in order to identify what is truly happening at low redshifts.

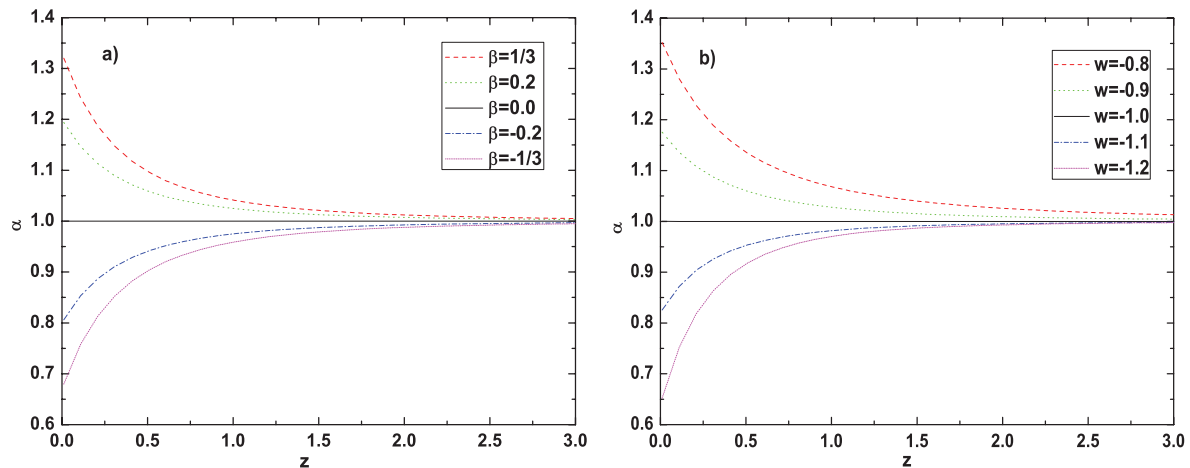


Figure 2. Λ CDM versus XCDM cosmologies. Theoretical reconstruction of the smoothness parameter. In panel (a) we show the reconstruction considering a flat Λ CDM universe with $\Omega_m = 0.27$, $\gamma = 1$ and several values for β . In panel (b) a flat XCDM model was considered with $\Omega_m = 0.27$, $\alpha = 1$ and several values of w . Note that the same result can be obtained from different assumptions. Therefore, in order to identify clearly the physical origin of the result the reconstruction should be performed in combination with other consistency checks.

4.3 Other effects?

So far we have analysed only the effects of the local inhomogeneities in the cosmological tests. But are there other effects taking place which were not accounted for? One possibility is that the Etherington principle is not valid (Etherington 1933). This effect can change the distances as well; hence, the cosmological tests will be affected. A full analysis of this effect will be published elsewhere.

5 CONCLUSIONS

In this Letter we have shown that cosmological tests originally proposed to find deviations from the FLRW metric or from a flat Λ CDM model are affected when distance measurements are used. This may happen due to the preferred lines of sight of the detected objects, e.g. SNe Ia, which results in a different distance from the standard FLRW approach. When this effect is taken into account, a new distance (sometimes called the Dyer–Roeder distance) is derived. In this approach, the effects of the local inhomogeneities are phenomenologically characterized by the smoothness parameter α . It has been shown that if such a parameter is different from the unperturbed FLRW value ($\alpha = 1$), artefacts are produced when the ‘CP’ is directly tested from observations.

It is also interesting that the fine-tuned correspondence between the Dyer–Roeder and weak lensing approaches (suggested by Bolejko 2011b) implies that the consistency tests can also be used in reverse manner, that is to probe the more realistic description of the small-scale inhomogeneities. This happens because the amplitudes of the deviations are heavily dependent on the adopted procedure (in certain sense, the parametric Dyer–Roeder description encodes more possibilities).

We have also proposed a method to reconstruct the smoothness parameter directly from the observations when a Λ CDM model is assumed. A discussion of how a different cosmology can affect the reconstruction was performed and it was recognized that different tests would be necessary in order to disentangle the cosmological model from the effects of the inhomogeneities (see Figs 2a and b). Naturally, as happens with the $\mathcal{C}(z)$ and $\mathcal{L}(z)$ tests, the reconstruction itself can also be used to identify the more realistic approach for describing the late-time clumpiness effects.

ACKNOWLEDGMENTS

The authors are grateful to F. A. Oliveira, R. C. Santos and F. Andrade-Santos for helpful discussions. VCB is supported by CNPq and JASL is partially supported by CNPq and FAPESP (Thematic Project 04/13668-0).

REFERENCES

- Alcaniz J. S., Lima J. A. S., Silva R., 2004, *Int. J. Mod. Phys. D*, 13, 1309
 Amanullah R. et al., 2010, *ApJ*, 716, 712
 Bassett B. A., Kunz M., 2004, *Phys. Rev. D*, 69, 101305
 Bertotti B., 1966, *Proc. R. Soc. Lond. A*, 294, 195
 Bolejko K., 2011a, *A&A*, 525, 49
 Bolejko K., 2011b, *MNRAS*, 412, 1937
 Busti V. C., Santos R. C., 2011, *Res. Astron. Astrophys.*, 11, 637
 Busti V. C., Santos R. C., Lima J. A. S., 2012, *Phys. Rev. D*, 85, 103503
 Clarkson C., Zunckel C., 2010, *Phys. Rev. Lett.*, 104, 211301
 Clarkson C., Bassett B., Lu T., 2008, *Phys. Rev. Lett.*, 101, 011301
 Clarkson C. et al., 2011, preprint (arXiv:1109.2484)
 Dyer C. C., Roeder R. C., 1972, *ApJ*, 174, L115
 Dyer C. C., Roeder R. C., 1973, *ApJ*, 180, L31
 Ellis G. F. R., 2009, *J. Phys. Conf. Ser.*, 189, 012011
 Etherington I. M. H., 1933, *Philos. Mag.*, 15, 761
 February S., Larena J., Smith M., Clarkson C., 2010, *MNRAS*, 405, 2231
 Gunn J. E., 1967, *ApJ*, 150, 737
 Holanda R. F. L., Lima J. A. S., Ribeiro M. B., 2010, *ApJ*, 722, L233
 Holanda R. F. L., Lima J. A. S., Ribeiro M. B., 2011, *A&A*, 528, L14
 Jordan P., Ehlers J., Sachs R. K., 1961, *Akad. Wiss. Mainz*, 1, 1
 Kainulainen K., Marra V., 2009, *Phys. Rev. D*, 80, 123020
 Kantowski R., 1969, *ApJ*, 155, 89
 Linder E. V., 1988, *A&A*, 206, 190
 Linder E. V., 1998, *ApJ*, 497, 28
 Linder E. V., 2003, *Phys. Rev. Lett.*, 90, 091301
 Mattsson T., 2010, *Gen. Relativ. Gravitation*, 42, 567
 Padmanabhan T., Choudhury T. R., 2003, *MNRAS*, 344, 823
 Sachs P. R. K., 1961, *Proc. R. Soc. Lond. A*, 264, 309
 Sahni V., Shafieloo A., Starobinsky A. A., 2008, *Phys. Rev. D*, 78, 103502
 Sahni V., Starobinsky A. A., 2006, *Int. J. Mod. Phys. D*, 15, 2105
 Saini T. D., Raychaudhury S., Sahni V., Starobinsky A. A., 2000, *Phys. Rev. Lett.*, 85, 1162
 Santos R. C., Lima J. A. S., 2008, *Phys. Rev. D*, 77, 083505
 Santos R. C., Cunha J. V., Lima J. A. S., 2008, *Phys. Rev. D*, 77, 023519
 Uzan J.-P., Clarkson C., Ellis G. F. R., 2008, *Phys. Rev. Lett.*, 100, 191303
 Wei H., 2010, *J. Cosmol. Astropart. Phys.*, 1008, 020
 Zel'dovich Ya. B., 1964, *Sov. Astron.*, 8, 13
 Zunckel C., Clarkson C., 2008, *Phys. Rev. Lett.*, 101, 181301

This paper has been typeset from a $\text{\TeX}/\text{\LaTeX}$ file prepared by the author.