Real wage rigidity and the unemployment volatility puzzle in small open economies

Tord S. Krogh

Statistics Norway and Norges Bank, Bankplassen 2, P.O. Box 1179 Sentrum, NO-0107 Oslo; e-mail: tskrogh@gmail.com

Abstract

Standard search models of the labour market feature a volatility puzzle: labour market variables move too little in response to productivity shocks. I investigate if real wage rigidity is sufficient to solve this in an open economy. Starting from a closed economy benchmark in which wage rigidity makes labour market variables sufficiently volatile, I find that the puzzle reopens in the open economy, despite a rigid real wage. This is because terms of trade movements move the wedge between the consumer and producer real wage in such a way that labour market variables respond less to productivity shocks. A quantitative exercise shows that the effect of this mechanism can be sizeable.

JEL classifications: E24, E32, F41

1. Introduction

The baseline search and matching model cannot account for why variables such as unemployment and labour market tightness are so much more volatile than labour productivity (Shimer, 2005; Costain and Reiter, 2008). This has been labelled the ‘unemployment volatility puzzle’, and there exists a large literature that attempts to deal with this issue.

One well-known suggestion is to introduce real wage rigidity, since that amplifies the effect of productivity shocks on labour market variables. It has been argued that a realistic degree of wage rigidity is enough to make unemployment sufficiently volatile (Hall, 2005). In this article, however, I show that such a conclusion may restrict to the closed economy case.

Why should it matter for the effect of real wage rigidity to move from a closed to an open economy? The reason is that a productivity shock affects not only the marginal cost of firms but also the terms of trade. Movements in the terms of trade alter the wedge between consumer and producer prices, and thus also between the consumer and producer real wage. If productivity declines and the terms of trade improves (due to lower supply of domestic goods), the producer real wage can fall even if the consumer real wage is constant. This limits the drop in hiring and the increase in unemployment when the consumer real wage is rigid.
My article is motivated by two stylized facts. First, the volatility puzzle is not just confined to the USA, although most of the literature focuses on the US economy. For a wide range of countries, including many small open economies, the volatility puzzle prevails. Second, for the same set of countries, the consumer real wage is typically less volatile than the producer real wage. Thus, if wages are rigid, it seems to apply the most to consumer real wages. It is therefore a need for studying the impact of consumer real wage rigidity in small open economy models.

The rest of the article is organized as follows. In Section 2, I present the related literature and motivate the article with two stylized facts. The model I use is presented in Section 3, and I use a quantitative version of it in Section 4. The results will confirm that there may be a vast difference between a closed and an open economy in terms of how the volatility of labour market variables is affected by real wage rigidity. However, I also point out that this result is sensitive to the value of two elasticities, namely, the elasticity of substitution between domestic and imported goods and the inter-temporal elasticity of substitution. I conclude in Section 5.

2. Literature and some stylized facts

The unemployment volatility puzzle is well known in the literature. As shown by Shimer (2005) and Costain and Reiter (2008), the standard Diamond-Mortensen-Pissarides (DMP) search and matching model is not able to replicate key labour market patterns with the conventional calibration. Hornstein et al. (2005) and Mortensen and Nagypál (2007) review the literature triggered by the volatility puzzle. One issue that has been particularly discussed is how wages are set.

In the DMP model, the wage is usually determined in a Nash bargaining game. Workers’ surplus from being employed rather than unemployed determines their reservation wage, $w_t$, whereas the expected profits earned from employing one worker gives the reservation wage of firms, $\bar{w}_t$. The Nash bargaining solution is one specific value $w_t^* \in [w_t, \bar{w}_t]$ that maximizes some objective function (often specified to be the geometric weighted average of surpluses).

Is the wage necessarily equal to the Nash solution? As Hall (2005) points out, any wage in the bargaining set $[w_t, \bar{w}_t]$ is in principle consistent with private efficiency on the part of both the firm and the worker. In that sense, the equilibrium wage rate is only set-identified. Hall goes on to analyse the equilibrium with a constant wage $c \in [w_t, \bar{w}_t]$. In a standard DMP set-up, this modification makes unemployment and labour market tightness respond much more to productivity shocks compared to the model with Nash bargaining. Although the assumption of a constant wage is extreme, the conclusion of Hall (2005) is that a realistic degree of real wage rigidity is sufficient to resolve the volatility puzzle (see, e.g., Dickens et al., 2007 for empirical evidence of real wage rigidity).

After Hall (2005), a number of papers have incorporated rigid wage setting in search models. For instance, Gertler and Trigari (2009) present a DMP model where the frequency of wage bargaining is constrained by a Calvo (1983) style lottery, leading to wage stickiness. Blanchard and Gali (2010) combine a reduced form of search model with real wage rigidity with a new Keynesian model to study how this impacts monetary policy. They show that for a calibration meant to match the European labour market, targeting the unemployment rate instead of inflation may approximate the optimal policy.

However, it is not uniformly accepted that real wage rigidity is the right solution to the volatility puzzle. An influential critic is Pissarides (2009). He shows that what matters
theoretically is whether the wage of new hires is rigid or not. He also presents empirical results indicating that the wage of new hires is not rigid. Similar data are provided by Haefke et al. (2013), whilst Gertler and Trigari (2009) argue against the evidence presented by Pissarides (2009). Hagedorn and Manovskii (2008) make a different point. Their argument is that although real wage rigidity may be a sufficient mechanism for resolving the unemployment volatility puzzle, it need not be necessary. They argue that a more appropriate calibration (in particular of workers’ bargaining power) may solve the puzzle without making wages rigid, see also Costain and Reiter (2008).

This article belongs to the literature that questions whether wage rigidity is the solution to the volatility puzzle, and it takes a novel route. I start out with the model of Hall (2005), in which a rigid real wage is sufficient to solve the volatility puzzle. Then I consider an open economy generalization and analyse how a rigid consumer real wage influences the dynamics of the model. The relevance of my analysis can be motivated by two stylized facts.

First, although the literature on the volatility puzzle so far has almost exclusively focussed on the closed economy case, it is also present in a number of OECD countries, as highlighted by Amaral and Tasci (2012). To illustrate this point, I plot the standard deviation of labour market tightness, unemployment, and labour productivity for an unbalanced panel of 18 countries in Panel A of Fig. 1; see the Data Appendix for details on the sample periods being used. Clearly, labour market tightness and unemployment vary substantially more than labour productivity across OECD countries, including many small open economies.

Second, real wage rigidity ought to imply a rigid consumer real wage. Ellingsen and Holden (1998) provide a theoretical justification, and this is also how it has been modelled in recent papers (Sheen and Wang, 2012; Rhee and Song, 2013) as well as in earlier contributions (Sachs, 1980; Branson and Rotemberg, 1980). This assumption is furthermore supported by Panel B in Fig. 1. Here, I plot the standard deviation of the producer and consumer real wage for the same sample of countries as in Panel A. The consumer wage is earnings deflated by the consumer price index, whereas the producer real wage is earnings deflated by a producer price index (see Data Appendix for more details). In all the countries, the producer real wage is more volatile than the consumer real wage. Thus, if real wages are rigid, it seems most relevant to focus on a rigid consumer real wage.

Taken together, these stylized facts imply that real wage rigidity may affect labour market dynamics differently in closed and open economies. Whereas workers care about the consumer real wage, firms’ pricing decisions depend on the producer real wage. In an open economy, the presence of imported goods can create a wedge between the two real wage concepts. To analyse unemployment dynamics in the open economy, it is then important to understand how this wedge moves in response to shocks.

1 One notable exception is Sheen and Wang (2012), who combine labour market frictions as modelled by Blanchard and Gali (2010) with the open economy model of Adolfson et al. (2007). They estimate their model using Australian data and find strong evidence in favour of both hiring costs and real wage rigidity.

2 Note that vacancy data are missing for Denmark, Canada, Korea, and New Zealand. For these countries I have nevertheless included the standard deviations for the other variables in these countries, explaining the missing bars in Panel A of Fig. 1.
Fig. 1. Volatility of selected labour market and wage variables in 18 OECD countries
Unbalanced quarterly panel. Labour market tightness is missing for Canada, Denmark, Korea, and
New Zealand. All variables are in logs and have been HP-filtered with a smoothing parameter equal to
10^5 (as in Shimer, 2005). See the Data Appendix for more details.
3. Model

In this section I give a compact presentation of the model; see the Online Appendix for a detailed outline. It is almost identical to the model in Hall (2005), except for a small open economy extension plus a more general specification of household preferences.

3.1 Labour market with search frictions

There exists a unit measure of workers and firms. At the beginning of period \( t \), \( u_t \) workers are unemployed and searching for employment. Firms post \( v_t \) vacancies, and \( m(u_t, v_t) \leq \min(u_t, v_t) \) is the number of matches. New hires become productive one period after the match takes place. \( x_t = v_t/u_t \) measures labour market tightness, and if \( m \) features constant returns to scale, then \( \phi(x_t) = m(1, x_t) \) represents the job finding rate. Furthermore, \( \rho(x_t) = \phi(x_t)/x_t \) is the vacancy-filling rate. I specify the matching technology such that job finding is given by:

\[
\phi(x_t) = ox_t^x
\]

At the end of each period, a share \( d \) of employer-employee pairs is separated, but matches created in the same period run no risk of separation. This implies the following law of motion for unemployment:

\[
\begin{align*}
    u_t &= \delta[1 - u_{t-1}] + [1 - \phi(x_{t-1})]u_{t-1} \\
    &\quad \text{(1)}
\end{align*}
\]

Let \( V^E_t \) be the value a worker assigns to the state of employment at the end of period \( t \) (after having received the consumer real wage \( w_t \)), and \( V^U_t \) be the value of being unemployed. For a stochastic discount factor \( b_t \), these value functions must satisfy:

\[
\begin{align*}
    V^E_t &= E_t\{b_{t,t+1}[(1 - \delta)(w_{t+1} + V^E_{t+1}) + \delta V^U_{t+1}]\} \\
    V^U_t &= b_t + E_t\{b_{t,t+1}[(1 - \phi(x_t))V^U_{t+1} + \phi(x_t)(w_{t+1} + V^E_{t+1})]\}
\end{align*}
\]

where \( b_t \) represents the opportunity cost of employment. Workers’ reservation wage, \( w^* \), satisfies \( w = V^U - V^E \), and can be expressed as:

\[
    w^* = b_t - [1 - \delta - \phi(x_t)]E_t\{b_{t,t+1}[w_{t+1} - V^E_{t+1}]\} \quad \text{(2)}
\]

Firms either produce, post vacancies, or exit the market. Due to linear technology, each firm can be thought of as employing one worker who produces \( z_t \) units of output. Productivity follows the process:

\[
z_t = z_0 e^{at}
\]

where \( a_t = \rho a_{t-1} + e_t \), and \( e_t \sim N(0, \sigma_e^2) \). Output is sold to consumers (potentially both domestic and foreign) at a real price \( q_t \). Firms with no workers can post vacancies. This has a real cost \( q_t k_t \), and the firm is matched with a worker with probability \( \rho(x_t) \). Exiting the market has zero value.

Due to free entry, the value of vacancies is zero in equilibrium. The value of being a producer at the end of period \( t \) (after having paid a wage \( w_t \) to the worker) is then:

\[
    V^F_t = q_t z_t + (1 - \delta)E_t\{b_{t,t+1}[V^F_{t+1} - w_{t+1}]\} \quad \text{(3)}
\]

The reservation wage of firms, \( \overline{w} \), satisfies \( \overline{w} = V^F \), and we can write it as:

\[
    \overline{w}_t = q_t z_t + (1 - \delta)E_t\{b_{t,t+1}[\overline{w}_{t+1} - w_{t+1}]\}
\]
With the definition of \( \bar{w}_t \), we can write the free-entry condition as:

\[
0 = -q_t k + \rho(x_t) E_t \left\{ \beta_{t+1} \left[ \bar{w}_{t+1} - w_{t+1} \right] \right\}
\] (4)

### 3.2 Wage determination

The wage rate \( w_t \) is determined by the wage schedule suggested by Hall (2005) that spans both Nash sharing and a constant wage:

\[
w_t = (1 - \gamma) w_t^{nash} + \gamma w_{ss}
\] (5)

Here, \( w_t^{nash} \) is the real wage consistent with Nash bargaining, whilst \( w_{ss} \) is the steady state level of the Nash wage. The parameter \( \gamma \) can be interpreted as the extent of real wage rigidity. Hall (2005) analyses extensively the case with \( \gamma = 1 \). See Blanchard and Galí (2010) for another application of such a wage schedule.

The Nash bargaining game is formulated such that \( w_t^{nash} \) maximizes the joint surplus \( \zeta \log[w_t + V^L_t - V^U_t] + (1 - \zeta)[\log V^L_t - w_t] \), with \( \zeta \) representing the relative bargaining power of workers. It can be shown that the solution is \( w_t^{nash} = (1 - \zeta) w_t + \zeta w_t \). If we combine this result with the free entry condition (4) and the expressions for the reservation wage rates eqs (2) and (3), we find that:

\[
w_t^{nash} = (1 - \zeta) b_t + \zeta [q_t z_t + q_t k x_t]
\] (6)

### 3.3 Stochastic discount factor and opportunity cost of employment

The traditional search and matching model is specified with risk-neutral agents and a constant opportunity cost. However, to get any nontrivial effects of the open economy extension in this article we need more general preferences.

As in Merz (1995) and Blanchard and Galí (2010), I use a large household assumption. There is a continuum of workers, indexed on the unit interval. Labour supply of each worker is indivisible, and within the household there is perfect risk sharing. The objective function of the household is:

\[
U_0 = \sum_{t=0}^{\infty} \beta^t \left[ c_t^{1-\sigma} \left( \frac{c_t}{\bar{c}_t} \right)^{-1-\sigma} - b \frac{n_t^{1+\phi}}{1+\phi} \right]
\]

where \( c_t \) is average consumption and \( 0 \leq n_t \leq 1 \) is the share of workers currently employed. \( \beta \) is the subjective discount factor, \( \sigma \) the inverse of the inter-temporal elasticity of substitution, and \( \phi \) is the inverse of the Frisch elasticity of labour supply. \( b \) is a free parameter. With full participation, there is a simple link between employment and unemployment:

\[
u_t = 1 - n_t
\] (7)

The household lives in a small open economy, which collapses to a closed economy in a special case described below. It has access to complete asset markets. The Euler equation implies the following stochastic discount factor:

\[
\beta_{t,t+1} = \beta E_t \left\{ \left( \frac{c_t}{\bar{c}_{t+1}} \right)^{\sigma} \right\}
\] (8)
The opportunity cost of employment for the household, $b_t$, is equal to the marginal rate of substitution between consumption and labour. This gives the definition:

$$b_t = bc_t^\sigma n_t^\phi$$  \hspace{1cm} (9)

If we assume $\sigma = \phi = 0$, then this implies risk-neutral agents and a fixed opportunity cost of employment, as in Hall (2005). For all other cases, the opportunity cost varies with the business cycle.

3.4 Output market clearing and the open economy dimension

As in Sutherland (2005), I model the small open economy in an Obstfeld-Rogoff type two-country set-up, where one country (‘Home’) is infinitely small relative to the other (‘Foreign’ or ‘rest of the world’). Agents in both countries have identical preferences, except for a home bias in consumption, represented by $1 - x$. $x$ can be interpreted as the economy’s degree of openness.

The open economy dimension leads to the introduction of two key relationships. First, due to international risk sharing, there is an equation linking domestic consumption to world output, $y^*$, and the real exchange rate, $rer_t$. The risk sharing condition reads:

$$c_t = rer_t^{1/\sigma} y^*_t$$  \hspace{1cm} (10)

Equation (10) follows from combining the consumption Euler equation for consumers in both Home and Foreign and the complete markets assumption (as well as a suitable initial condition). Second, consumption goods produced in Home are demanded by consumers from both Home and Foreign. In addition, there is also demand for domestic goods from firms that must pay the vacancy cost $k$. With preferences of a particular CES type, market clearing for domestically produced goods reads:

$$ztnt = \left(1 - x\right)\frac{c_t}{q_t^\eta} + x\frac{y^*_t}{[q_t/rer_t]^\eta} + kv_t$$  \hspace{1cm} (11)

Let the price of imported goods relative to domestic goods be denoted $s_t$. This is Home’s terms of trade, and it is possible to write the two other relative prices as functions of $s_t$. We have:

$$q_t = [(1 - x) + xs_t^{1-\eta}]^{1/\eta} = q(s_t)$$  \hspace{1cm} (12)

$$rer_t = s_tq(s_t)$$  \hspace{1cm} (13)

3.5 Equilibrium

An equilibrium allocation is a set of values for $x_t$, $u_t$, $v_t$, $w_t$, $w_{t}\chi$, $w_{t}\phi$, $q_t$, $s_t$, $rer_t$, $c_t$, $n_t$, $b_t$, and $\beta_{t+1}$ that satisfies eqs (1)–(13) and the definition $x_t = v_t/u_t$ for all $t$.

To characterize the equilibrium, start by combining eqs (3) and (4). Having inserted for $q_t$ using eq. (12), we get to the following job creation equation:

$$\frac{k}{\rho(x_t)} = Et\left\{\beta_{t+1}q(s_{t+1})\left[zt+1 + \frac{(1 - \delta)k}{\rho(x_{t+1})} - \frac{w_{t+1}}{q(s_{t+1})}\right]\right\}$$  \hspace{1cm} (14)

This equation show how labour market tightness is determined by the wage rate and terms of trade. The left-hand side is akin to an expected vacancy cost (in units of domestic
output) if labour market tightness were to remain at $x_t$ sufficiently long. The right-hand side is the benefits from finding a worker that becomes productive tomorrow. In period $t+1$ this will have, in units of domestic output, a real value $z_{t+1}$, in addition to a continuation value $(1 - \delta)k/\rho(x_{t+1})$. The cost is the producer real wage, $w_{t+1}/q(s_{t+1})$.

Next, combine eq. (5) with eqs (6), (9), (7), (10), and (13). This gives an equation for the equilibrium wage rate:

$$w_t = (1 - \gamma)q(s_t)[(1 - \xi)b (1 - \mu_t)^{\phi} + \xi[z_t + kx_t]) + \gamma w_{st}$$

(15)

The Nash wage puts a weight $\xi$ on $z_t + kx_t$, which signals how profitable it is for firms to keep employees, and a weight $(1 - \xi)$ on the marginal rate of substitution, written as a function of terms of trade, world output, and unemployment. Finally the Nash wage is proportional to the relative price of home goods, $q(s_t)$.

Finally, terms of trade is determined by the supply of domestic goods relative to world output. To see how, combine eqs (10)–(13), and use also eq. (7) to insert for $n_t$. This gives:

$$b(s_t) = \frac{z_t(1 - \mu_t) - kx_t u_t}{\gamma_r}$$

(16)

where $b(s_t) = s_t^{\sigma}[\sigma + (1 - \sigma)]s_t q(s_t)^{1/\sigma - \eta}$. $s_t$ is increasing in the right-hand side, meaning that the terms of trade improves ($s$ falls) if the relative supply of domestic goods declines.

### 3.5.1 A closed economy special case

Consider the special case of a closed economy with linear utility ($\sigma = \phi = 0$). This is basically the model in Hall (2005). In this case we have $q(s_t) = 1$ and $\beta_{t+1} = \beta$, whilst eq. (15) defines the wage, $w(x_t, z_t) = (1 - \gamma)\beta (1 - \xi)b + \xi z_t + kx_t + \gamma w_{st}$. If we use this information in eq. (14), we get:

$$\frac{k}{\rho(x_t)} = \beta E_t\left\{z_{t+1} + (1 - \delta)k - w(x_{t+1}, z_{t+1})\right\}$$

(17)

which defines equilibrium labour market tightness as a function of productivity, $x^c(z_t)$. Given this solution, all other endogenous variables follow. In particular, unemployment is defined by the law of motion eq. (1).

From eq. (17), one can infer the importance of wage dynamics for the effect of productivity shocks on labour market tightness and unemployment. Under complete wage rigidity ($\gamma = 1$), $w_{t+1}$ remains constant in the face of a productivity shock, meaning that the path for $x$ do all the adjustment to keep eq. (17) satisfied. If, on the other hand, the wage is completely flexible ($\gamma = 0$), the term $E_t\{z_{t+1} - w(x_{t+1}, z_{t+1})\}$ moves less after shocks to productivity, leading to smaller fluctuations in labour market tightness (which in turn may yield a volatility puzzle).

### 3.5.2 An open economy special case

In the general case of the open economy, we can use eq. (15) to insert for the wage, and eq. (16) to insert for the terms of trade, both as a functions of $x_t, u_t$ and $z_t$. Equilibrium labour market tightness will then follow from eqs (1) and (14), defining $x_t$ as a function $x^o(z_t, u_t)$ (unemployment is pre-determined).

The most important difference, with regards to labour market dynamics in the face of technology shocks, is seen from the last term in the job creation curve eq. (14),

...
namely, \( w_{t+1}/q(s_{t+1}) \). Consider again a fixed wage equilibrium (\( \gamma = 1 \)) and let \( \eta = \sigma = 1 \). For constant values of the terms of trade, productivity shocks will have the same impact on labour market tightness as in the closed economy. But from eqs (12) and (16) with \( \eta = \sigma = 1 \), we know that the relative price \( q_t \) is given by \( s_t^* \), with:

\[
s_t = \frac{z_t(1 - u_t) - kx_tu_t}{\gamma_t^*}
\]

Hence, when productivity falls, terms of trade improves \( (s_t \text{ falls}) \) due to the lower supply of domestic goods. This in turn implies a higher relative price of domestic goods. As the shock is expected to persist, the future producer real wage, \( w_{t+1}/q(s_{t+1}) \), also falls, despite a constant consumer real wage. As a result, firms cut hiring to a smaller extent than in the closed economy.

This mechanism is stronger the larger \( \alpha \) is, as it makes the response of \( q(s_t) \) to shocks in \( z_t \) larger. This implies that wage rigidity may have less impact on the volatility of labour market variables in open economies than in closed.

4. Quantitative results

In this section I calibrate and simulate the model to highlight the importance of the open economy dimension in understanding how real wage rigidity affects unemployment dynamics.

4.1 Calibration

The model has 14 parameters, and the frequency is monthly. My calibration targets are fairly standard long-run moments plus some labour market features of a typical small open economy.

Two key parameters are \( \gamma \), the extent of wage rigidity, and \( \alpha \), the degree of openness. I do not fix these parameters to any particular value. Instead, I vary them on the unit interval to highlight how the two features interact.

The preference parameters are \( \beta \), \( \sigma \), \( \phi \), \( \eta \), and \( b \). The discount factor \( \beta \) is set to 0.9967 to get the annual real interest rate to 4% in steady state. The Frisch elasticity is 1/\( \phi \), and I set it to unity, although estimates of it tend to vary quite a bit (Chetty et al., 2011; Reichling and Whalen, 2012).\(^4\) \( \sigma \) and \( \eta \) are for the baseline results set to unity, but the sensitivity of the results with respect to these parameters will be carefully evaluated. Given \( \sigma \) and \( \phi \), \( b \) determines the opportunity cost of employment. In DMP models with risk-neutral agents, the opportunity cost is constant and equal to \( b \). Shimer (2005) calibrates \( b \) to match the average replacement ratio of unemployment benefits in the USA, making him set \( b = 0.4 \). It has later been argued that calibrating \( b \) to match only the replacement ratio is too conservative (e.g., Hagedorn and Manovskii, 2008). Furthermore, other OECD countries typically have a higher replacement ratio than the USA (Amaral and Tasci, 2012). I therefore pick \( b \) to get a steady-state opportunity cost equal to \( 2/3 \).

\( \tau \), \( \omega \), \( \delta \), \( \zeta \), and \( k \) are the labour market parameters. \( \tau \) is the elasticity of the job finding rate with respect to labour market tightness. Views on the correct value of this parameter

\(^4\) If were to increase \( \phi \) (i.e., a lower Frisch elasticity), that would make the opportunity cost of unemployment, \( b \), more responsive to employment, and thus also the Nash wage. The volatility of labour market tightness and unemployment would as a result fall, both in the closed and open economy. This has no impact on the difference between the two versions of the model, so \( \phi \) is therefore not crucial for the results.
have differed in the literature. For example, Hall (2005) argues for $\tau = 0.765$, whereas Shimer (2005) estimates $\tau$ to be 0.28. Mortensen and Nagypál (2007) investigated the sensitivity of Shimer’s results with respect to his calibration of $\tau$ and found that it did not matter much. As a middle ground, I use the same value as Pissarides (2009), that is, $\tau = 0.5$. $\omega$ and $\delta$ are calibrated using the estimates of Hobijn and Sahin (2009). Based on the average of eight countries, $\omega$ is set to get a job finding rate of 20%, whilst $\delta$ is calibrated to 1.5%. $\zeta$ is the relative bargaining power of workers. As in most of the literature, I apply the Hosios (1990) condition for an efficient equilibrium, meaning $\zeta = 1 - \tau$. $k$ is calibrated to ensure a certain steady-state level of labour market tightness. Since the steady-state solution is homogeneous with respect to $x_{ss}$ and $k$, I normalize labour market tightness to unity.

Finally, I approximate the autoregressive process for the productivity shock by a Markov chain with 21 elements. $\rho_a$ and $\sigma_a$ are calibrated to make the autocorrelation and standard deviation of the HP-filtered productivity series generated by this approximation match an average of what I find using the same productivity data as in Section 2. $\rho_a = 0.96$ and $\sigma_a = 0.0075$.\footnote{These parameter values make the HP-filtered quarterly productivity series feature a first-order degree of autocorrelation equal to 0.85 and a standard deviation of 2% (after transforming the simulated monthly data to a quarterly frequency).}

### 4.2 Dynamic response to productivity shocks

My first exercise is to produce impulse response functions. We start with a closed economy, that is, $\alpha = 0$, and Fig. 2 shows the impact on unemployment and the job finding rate from a 1% temporary decline in productivity for two versions of the closed economy. The solid lines are the responses under wage flexibility ($\gamma = 0$), whilst the dashed lines represent an economy with fairly rigid real wages ($\gamma = 0.92$).

Figure 2 is clear evidence of how important real wage rigidity can be for the dynamics of labour market variables in the closed economy (compare also with Figs 4–5 in Hall, 2005). What happens is that since the shock is persistent, lower productivity today also makes expectations about future productivity decline. From the job creation curve, we know this leads to a drop in labour market tightness, so firms post fewer vacancies and unemployment goes up. But with flexible wages, lower productivity and labour market tightness make the real wage fall. This dampens the drop in hiring substantially, and unemployment increases less than it otherwise would have. With real wage rigidity, most of the fall in the wage is prevented from happening. Therefore, unemployment increases by 0.4 percentage point after about one year, compared to an increase of less than 0.02 percentage point in the flexible wage case. The drop in labour market tightness is also substantial, and it leads to a fall in the job finding rate of more than 1 percentage point. As will be confirmed shortly, this is more than enough to resolve the volatility puzzle in the rigid wage version of the closed economy model.

Next, let us see how the plots change if the economy is open. The results for a model with $\alpha = 0.5$ are shown in Fig. 3.

The following three effects play out in the open economy after a decline in productivity. First, as in the closed economy, lower expected future productivity pushes the job finding

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5 I pick a set of non-Eurozone small open economies: Australia, Canada, Denmark, Iceland, Norway, Sweden, Switzerland, and the UK.

6 This strategy differs from Hall (2005). His productivity shock is calibrated to make the constant wage model feature a realistic degree of labour market volatility.

7 These parameter values make the HP-filtered quarterly productivity series feature a first-order degree of autocorrelation equal to 0.85 and a standard deviation of 2% (after transforming the simulated monthly data to a quarterly frequency).
Fig. 2. Impulse response plots for a negative shock to productivity in the closed economy (based on the model with $x = 0$ and $\sigma = \phi = \eta = 1$).
Fig. 3. Impulse response plots for a negative shock to productivity in the open economy (based on the model with $\alpha = 0.5$ and $\sigma = \phi = \eta = 1$).
rate down and unemployment up. Second, as in the closed economy, the Nash real wage declines, dampening the change in unemployment and the job finding rate when the wage is flexible. Third, and unique to the open economy, the terms of trade improves. This implies that the relative price of domestic goods increase, lowering the producer real wage, \( w/q \).

The third effect acts as an additional dampening mechanism on part of unemployment and labour market tightness. Furthermore, this mechanism is active also when the real wage is rigid. The effect is to reduce the change in unemployment and labour market tightness by about two thirds compared with the closed economy rigid wage equilibrium. This is sizeable, and may imply that the volatility puzzle continues to prevail in the open economy version even though I allow for real wage rigidity.

4.3 Simulated moments
The second exercise is to compare simulated moments for different versions of the model in order to relate the results more explicitly to the volatility puzzle. To do this, I use a simplified version where unemployment behaves like a jump variable and is determined by:

\[ u_t = \frac{\delta}{\delta + \phi(x_t)} \]  

rather than following from the law of motion eq. (1); see Hall (2005) for a discussion of this assumption. With eq. (18) replacing eq. (1), we can solve for labour market tightness as a function of productivity only, \( x^\prime(z_t) \). For each version of the model, I then generate \( J = 1,000 \) data sets, each containing 600 monthly observations (plus a 100-period burn-in sample). For each data set, I take averages to obtain a quarterly frequency and then use the HP-filter with smoothing parameter equal to \( 10^5 \) (as in the stylized facts) to remove the trend. Table 1 contains the average standard deviation of each detrended variable for four different models.

The impression from Table 1 is the same one gets from the impulse-response plots in Figs 2–3. Introducing wage rigidity in the closed economy leads to a substantial increase in the volatility of labour market variables, whereas doing the same in the open economy has less effect. Comparing the simulated moments to their empirical counterparts in Fig. 1, we observe that it is only the closed economy model with rigid wages that comes close to

\[ \text{Table 1. Simulated standard deviations} \]

<table>
<thead>
<tr>
<th></th>
<th>( u )</th>
<th>( v )</th>
<th>( x )</th>
<th>( z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Closed economy, flexible wage</td>
<td>0.007</td>
<td>0.009</td>
<td>0.016</td>
<td>0.02</td>
</tr>
<tr>
<td>Closed economy, rigid wage</td>
<td>0.192</td>
<td>0.227</td>
<td>0.417</td>
<td>0.02</td>
</tr>
<tr>
<td>Open economy, flexible wage</td>
<td>0.003</td>
<td>0.004</td>
<td>0.008</td>
<td>0.02</td>
</tr>
<tr>
<td>Open economy, rigid wage</td>
<td>0.061</td>
<td>0.071</td>
<td>0.132</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Notes: This table reports the standard deviation of the cyclical component of unemployment (\( u \)), vacancies (\( v \)), labour market tightness (\( x \)), and productivity (\( z \)) with four different versions of the model. For each model, the results are based on the average standard deviation from 1,000 data sets, each containing 200 quarterly observations. The closed economy is defined by \( \alpha = 0 \), whilst for the open I have used \( \alpha = 0.5 \). The flexible wage versions feature \( \gamma = 0 \), while the rigid wage versions have \( \gamma = 0.92 \).

The impulse response functions in Figs 2–3 are based on the proper solution, but changing to the model where eq. (18) replaces eq. (1) would not change them much.

8 The impulse response functions in Figs 2–3 are based on the proper solution, but changing to the model where eq. (18) replaces eq. (1) would not change them much.
matching the data. In the open economy, introducing real wage rigidity is also helpful, but far from enough to solve the volatility puzzle.

Up to this point, the key parameters $c$ and $a$ have been held fixed at somewhat arbitrary values. To show the results for a more complete set of parameter constellations, I create an evenly spaced grid of 30 points between 0 and 1 for $a$ and between 0.5 and 1 for $c$. Next I simulate the model for each possible combination of their values. Figure 4 shows the standard deviation of labour market tightness as a function of the two parameters.9

Two features of Fig. 4 are striking. First, when $a$ is small, a sufficiently high degree of real wage rigidity is indeed able to solve the volatility puzzle, but the effect of $c$ is highly non-linear. We know from Fig. 1 that in most countries, labour market tightness has a standard deviation of at least 0.3. This is consistent with the rigid wage model in a closed economy if $c$ is above, say, 0.85. Moderate degrees of rigidity (around 0.5) is far from enough. Second, as soon as we permit $a$ above 0, the standard deviation falls quickly. The effect of openness is also very non-linear, and moving from $a = 0$ to $a = 0.2$ makes the value obtained in a constant wage equilibrium fall from far above 0.7 to 0.3. If $a$ is around 0.5,

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9 Note that for $c$ I plot only the results for $c \leq 0.92$. Larger values of $c$ make the standard deviation so large when $a$ is small that it is hard to read the rest of the figure.
one can not get the standard deviation above 0.15 even with $\gamma = 1$. For this model it is therefore not possible to resolve the volatility puzzle with real wage rigidity in an open economy.

### 4.4 Two important elasticities

For all the results so far, the elasticity of substitution between domestic and imported goods, $\eta$, and the inter-temporal elasticity of substitution, $1/\sigma$, have been restricted to unity. However, these parameter have previously been seen to be crucial for the dynamics of open economy models (De Paoli, 2009; Bodenstein, 2010; Thoenissen, 2011). In this section I therefore check how the results change if I allow $\eta$ or $1/\sigma$ to differ from unity.

The trade elasticity is important in the goods market clearing condition eq. (16). It determines the size of the terms of trade adjustment following a shock to productivity, and increasing $\eta$ makes the terms of trade response smaller. Since the terms of trade response is key to the difference between unemployment dynamics in closed and open economies in this article, $\eta < 1$ will make the discrepancy greater, whereas $\eta > 1$ makes it smaller.

What about the temporal elasticity of substitution? This parameter enters together with $\eta$ in the market clearing equation (16), so it, too, will matter for the terms of trade response. A large elasticity (small value of $\sigma$) will have the same impact on the terms of trade variation as a large $\eta$, and the effect goes through international risk sharing. When $\sigma$ is small, domestic consumption will react more to real exchange rate movements (which in turn are caused by shifts in terms of trade). When this mechanism is put into the market clearing condition, we see that a big $\sigma$ makes the terms of trade move less to productivity shocks because domestic demand is more responsive to this relative price.

To illustrate how different values of $\eta$ and $\sigma$ matter, I produce new impulse response functions for unemployment, and Fig. 5 contains the results. In both panels I plot the impulse response function for the closed economy and for the open economy with $\eta = \sigma = 1$. In Panel A, I also add the impulse response function for an open economy with $\eta = 2$ and $\eta = 5$, whilst in Panel B I add the functions for the economy with $\sigma = 1/2$ and $\sigma = 1/5$.

The message from Fig. 5 is clear. The main result of this article is not robust to allowing $\eta$ or $1/\sigma$ to be sufficiently large. What we see is that search models with real wage rigidity produce similar impulse response functions for both the closed and open economy case if the latter features rather small terms of trade movements (due to greater responsiveness to any given change in the terms of trade).

How relevant is it to consider a large value of $\eta$? In the literature, $\eta$ seems to be notoriously difficult to estimate, a problem coined the elasticity puzzle by Ruhl (2008). Support for a low elasticity is typically found in studies that use aggregate data, for example, Taylor (1993), Lubik and Schorfheide (2005), and Rabanal and Tuesta (2010). An elasticity below unity has also been used in some studies as there are several other puzzles in international macro that can be resolved if the elasticity is low (Collard and Dellas, 2007; Corsetti et al., 2008). With this literature as a point of reference, it seems most relevant to keep $\eta$ in the range between 1/2 and 3. On the other hand, the trade literature typically report much larger values, say, above 6 (see discussion in Engel and Wang, 2011), but that is often based on micro-data. With this in mind, it seems like a small elasticity is most relevant in a macro setting.

How relevant is it to consider a small value of $\sigma$? Regarding this parameter, the empirical evidence is more precise than for $\eta$. Starting with Hall (1988), studies using aggregate data have typically found very small estimates of $1/\sigma$ (below 0.3), a conclusion reiterated by Yogo (2004) using methods robust to identification failure. On the high end of
Fig. 5. Impulse response plots for unemployment following a negative shock to productivity (based on the model with $\alpha = 0.5$).

In Panel A, we have $\sigma = 1$. In Panel B, we have $\eta = 1$. 
estimates, Beaudry and van Wincoop (1996) find $1/\sigma$ to be closer to 1 with a panel of state-level data for the US, but very few studies report estimates much higher than this. Engelhardt and Kumar (2009) find a confidence interval of 0.37–1.21 using 401(k) data in the USA. I conclude that a very large value of $1/\sigma$, and therefore a small value of $\sigma$, is not relevant.

5. Conclusion

Can real wage rigidity solve the volatility puzzle? In this article I have highlighted one reason the answer might differ for closed and open economies. In the open economy, terms of trade movements move the wedge between the consumer and producer real wage in such a way that it dampens the effect of productivity shocks on labour market variables. This limits the impact of real wage rigidity on the volatility of labour market variables. By simulating a calibrated version of the model, I show that this mechanism is quantitatively important. Hence, even though taking real wage rigidity into account may solve the volatility puzzle in the closed economy, it need not do so if the economy is open.

The quantitative results are however sensitive to the values of two elasticities, the elasticity of substitution between domestic and foreign goods ($\eta$) and the inter-temporal elasticity of substitution ($1/\rho$). My main findings are not robust to allowing $\eta$ or $\sigma$ to be far above than unity. Nevertheless, a brief review of the empirical evidence suggests that this area of the parameter space is not very relevant.

My focus in the article is on the effects of domestic productivity shocks. It is of course possible that the solution to the volatility puzzle is to consider other shocks, such as world output shocks, but that is outside the scope and focus of this article. Here I have dealt with variances conditional on productivity shocks.

If the key to understanding the unemployment volatility puzzle really lies in how wages are set, then future work must continue to advance our knowledge of wage bargaining, particularly in open economies. Not only is the wedge between consumer and producer prices essential, it is also important whether real or nominal wages are rigid, as the wedge effects I have identified can potentially be less important under the latter. Furthermore, how wages are determined need not be exogenous. For instance, Larsson Seim and Zetterberg (2013) test for the impact of monetary regime on wage formation. Finally, it would be fascinating to extend the present analysis to allow for a two-sector economy as well as incomplete markets.

Supplementary material

Supplementary material is available online at the OUP website.

Acknowledgements

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10 Furthermore, the distinction between real and nominal rigidity is not always absolute. Duval and Vogel (2012) introduce real wage rigidity by combining nominal wage stickiness with indexation.
Larsson Seim and Tommy Sveen for valuable comments. The paper is a heavily revised version of Chapter 2 in my PhD thesis, Krogh (2014). I am currently working at Norges Bank, but most of the article was written when I was employed at Statistics Norway. The views expressed are those of the author and do not necessarily reflect those of Norges Bank.

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**References**


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Data appendix

The data are quarterly, and for the most part collected from the OECD (the World Economic Outlook [OUT] and Main Economic Indicators [MEI] databases), but I have also relied on a few other sources.

Productivity is defined as labour productivity in the total economy, that is, real GDP divided by total employment (in hours). For all countries except Germany this is collected from OUT. For Germany I get data from the FRED database of the St. Louis Fed.

The wage variable is hourly earnings in manufacturing and is available for all countries in the MEI. Consumer prices are measured as the total consumer price index, whilst the producer price index is for manufactured goods sold domestically. These three variables are available in the MEI for all countries. For Austria I combine a PPI from Statistics Austria for the period 1996–2000 with the data in the MEI for the consecutive period.

I collect data on unemployment in both levels and rates from OUT for all countries. For Germany I use registered unemployment from the MEI prior to 1990.

Vacancy data is from MEI for all countries except the Netherlands. Data for the Netherlands are from Statistics Netherlands and Eurostat. Note also the following deficiencies with the data: for France and Japan, I only observe new vacancies, not the total stock. In Australia, the job vacancy survey was suspended in 2008 and 2009. I have linearly interpolated to fill in for missing observations. For the UK, the OECD have estimated the values for the period 1999(2)–2001(2). Furthermore, there is a missing value in 1976(4), which I have replaced by the average of the quarter before and after. Finally, I do not have sufficiently long (at least 10 years) vacancy data or none at all for Canada, Denmark, Korea, and New Zealand.

<table>
<thead>
<tr>
<th>Table A1. Sample period used for each country</th>
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<tbody>
<tr>
<td>Sample period</td>
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<td>Australia</td>
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<td>Austria</td>
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<td>Belgium</td>
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<td>Canada</td>
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<td>Germany</td>
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<td>Denmark</td>
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<td>France</td>
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<td>Great Britain</td>
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<td>Korea</td>
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<td>Luxembourg</td>
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<td>Netherlands</td>
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<td>New Zealand</td>
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<td>Norway</td>
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<tr>
<td>Portugal</td>
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<tr>
<td>Sweden</td>
</tr>
<tr>
<td>USA</td>
</tr>
</tbody>
</table>
The sample period that is available for each variable differs within and across countries. For the standard deviations presented in Fig. 1, these have been calculated using different sample periods across countries, but for each country the time span used to compute the volatilities of all the variables is the same. Table A1 shows the sample periods I have used. In general, it is the data on vacancies and producer price indices that put a limit on the sample periods.\footnote{Vacancy data for Canada exist but are not available free of charge. The same is true for vacancy data for the USA in the period prior to 2001.}