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Light-Curve Diagnosis of a Hot Spot for Accretion-Disk Models

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Abstract

Light curves of a hot spot rotating in a relativistic Keplerian disk were found to be periodic with typically two peaks, originating from a gravitational focusing effect and a Doppler boost. On the other hand, we found that light curves of a hot spot spirally infalling in a sub-Keplerian advective disk are aperiodic with typically a single peak, originating from a gravitational focusing effect or a Doppler boost. Such a difference in the light curves of a hot spot can discriminate background disk models.

Key words: accretion, accretion disks — black hole physics — galaxies: active — Galaxy: center — X-rays: stars

1. Introduction

As a main engine in the center of active celestial objects, acceretion disks have been widely accepted and extensively studied. Several types of disk models have been proposed (Kato et al. 1998 for a review). In the standard disk (Shakura, Sunyaev 1973; Novikov, Thorne 1973), the disk gas is Keplerian, rotating in an almost circular orbit. In an advection-dominated disk for an optically thick version (Abramowicz et al. 1988) and for an optically thin version (Narayan, Yi 1994), the disk gas is rotating with a sub-Keplerian speed as well as infalling with a similar speed.

Besides the global structure, the emergent spectra, and the time variation of accretion disks, the light curves of emitting hot spots comoving with disks have been investigated by several researchers. Such a hot spot can be easily created by flares, internal instabilities, asteroid or stellar impacts, initial irregularities, and so on. Observationally, there exist many types of time variabilities. Hence, it is very interesting and important to examine the light variation produced by irregularities in accretion disks.

Asaoka (1989) first calculated light curves observed at infinity from an emitting bright spot comoving with a Keplerian disk. She showed only limited cases, but found the essential properties of apparent light curves of a hot spot orbiting around a black hole. There exist two peaks within one period of the light curve: the first narrow higher peak is formed by the gravitational lensing effect when the hot spot is at the far side of the hole from the observer, while the second broad plateau is caused by a Doppler effect when the hot spot is in the approaching regime.

Karas and Bao (1992) examined the influence of a self-eclipse due to the thickness of the disk on the light curve of an X-ray emitting hot spot located on the disk surface. They showed that the eclipsed light curve has a narrow cusp.

Light curves of corotating hot spots in disks were investigated more generally by others (e.g., Bao 1992; Karas et al. 1992; Karas 1996).

In these studies, the background disk is usually assumed to be a relativistic Keplerian disk with circular motion. However, there exists another type of disk with advective motion, where the radial infall speed is of the order of the rotational speed.

Hence, in this paper we consider light curves of a hot spot in such disks with advective motion. Since the behavior of light curves of a hot spot is expected to be quite different between the Keplerian disk and non-Keplerian one, a light-curve analysis can discriminate the background model.

In the next section we first show typical light curves of a hot spot comoving with the Keplerian disk around a black hole. In section 3 typical light curves of a hot spot spirally infalling with the non-Keplerian disk. The final section is devoted to concluding remarks.

2. Keplerian Case

In previous studies, a hot spot was usually assumed to be a point-like source on the disk, while in this paper we consider a hot spot to be a small hot area on the disk; *a hot circular area*. Hence, in this section we recalculate light curves of a hot spot comoving with a standard accretion disk.

In the standard disk around a Schwarzschild black hole (e.g., Shakura, Sunyaev 1973; Novikov, Thorne 1973; Kato et al. 1998), the velocity profile is relativistic Keplerian with no radial infall, and the temperature distribution is also given by the relativistic model (e.g., Page, Thorne 1974). The surface temperature distribution rises inward and has a maximum in the inner region; the maximum temperature is set to be $10^4\,$K in this paper. Due to the general relativistic effect, there is an inner edge of the disk at the marginally stable radius $r_{\rm ms}$ $(= 3\,r_{\rm g} = 6GM/c^2)$, inside which the disk gas falls freely onto the central black hole, where $r_{\rm g}$ is the Schwarzschild radius of the central black hole with mass M.

On such a background disk, a hot spot (circular area) rotates at $r_{\rm s}$ from the center. A radius $a_{\rm s}$ of a hot spot is fixed to be $1\,r_{\rm g}$ for simplicity. Strictly speaking, a circle should be defined in terms of the proper distance. However, we here use the coordinate r, since the resulting light curve is not very sensitive to the exact shape of the hot spot. The temperature $T_{\rm s}$ of a hot spot

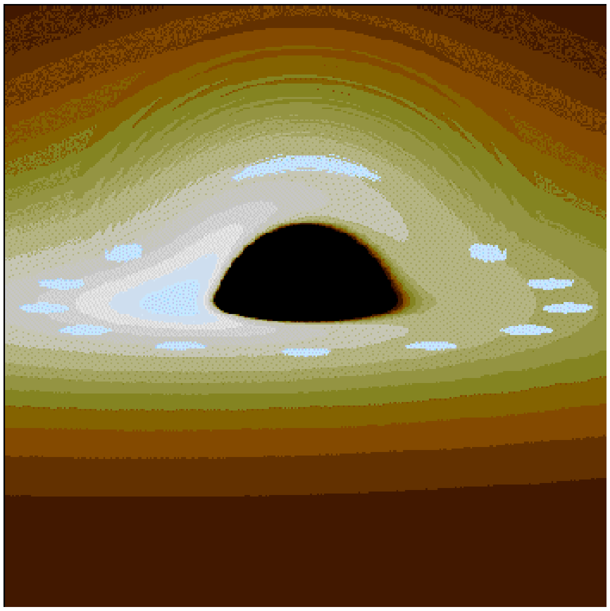


Fig. 1. Sequence of hot spots rotating with a relativistic Keplerian disk around a Schwarzschild black hole at different epoch. The maximum temperature of the disk is set to be 10000 K, while the temperature of the spot is 20000 K. In this example the rotation radius of the spot is $10 r_{\rm g}$ and the inclination angle is 80°.

is also fixed at 20000 K. The parameters are then the rotation radius $r_{\rm s}$ and the inclination angle i.

For relativistic circular motion, the angular speed Ω with respect to the stationary observer, measured at infinity, is

$$\frac{d\varphi}{dt} \equiv \Omega = \Omega_{\rm K} = \sqrt{\frac{GM}{r^3}}. \tag{1}$$

The relation between the azimuthal angle and the coordinate time then becomes

$$d\varphi = \frac{1}{\sqrt{2r^3}} dt, \tag{2}$$

where the radius and time are measured in units of $r_{\rm g}$ and $r_{\rm g}/c$, respectively.

The procedure to calculate a light curve is the ray-tracing method (cf. Luminet 1979; Fukue, Yokoyama 1988; Fukue 2003). The surface of the disk is assumed to radiate a blackbody spectrum of temperature $T(r)$ at radius r. The observer is located at a sufficiently large distance with inclination angle i from the rotation axis. The photon trajectory is traced from the photographer (observer) to the point where it originates based on Fermat's principle. In the calculation, all of the relativistic effects, such as the gravitational light bending, the gravitational redshift, and the Doppler effect associated with rotation, are incorporated. The observed bolometric flux and temperature are then determined. The blackbody spectrum having this observed temperature is integrated within the optical wavelength to give the observed brightness at each point on the disk. Finally, the summing up all of the observed brightness gives the observed flux. A bright hot spot (area) is located at an adequate position with the passage of time.

In figure 1 an apparent view of several hot spots at different epoch on the standard disk is shown for the convenience of readers. In this example the rotation radius of the hot spot is $10 r_{\rm g}$ and the inclination angle is 80°. The central black area means the inner edge of the disk at a marginally stable radius, $3 r_{\rm g}$. As is easily seen in figure 1, the gravitational focusing effect is prominent for this parameter set, while the Doppler boost effect is dominant in some cases, as shown below.

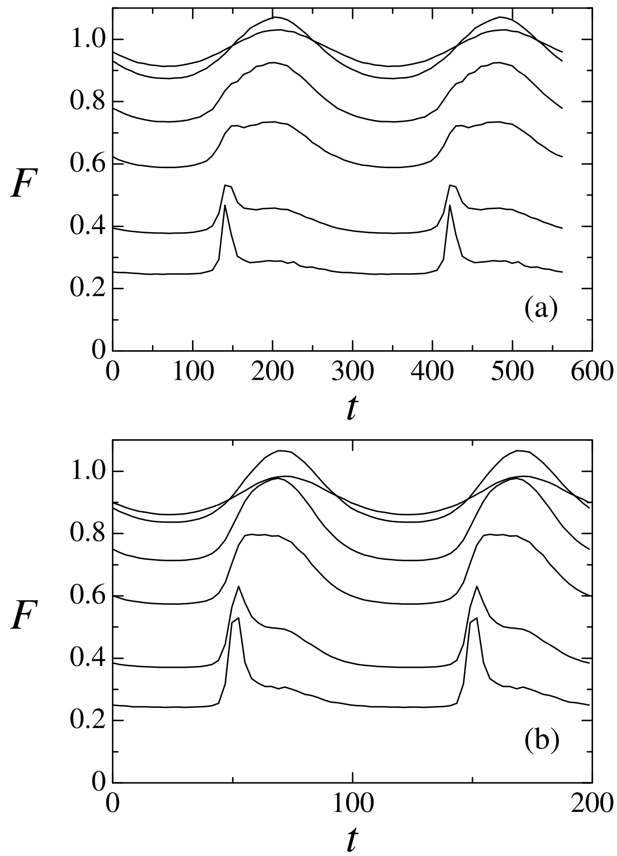


Fig. 2. Light curves of a hot spot on a relativistic Keplerian disk for several radii and inclination angles. The rotation radius $r_{\rm s}$ of a hot spot is $10 r_{\rm g}$ in (a) and $5 r_{\rm g}$ in (b), whereas the inclination angle i is 20°, 40°, 60°, 70°, 80°, and 85° from top to bottom.

In figure 2 light curves of a hot spot on a relativistic Keplerian disk are shown for several radii and inclination angles. The radial distance $r_{\rm s}$ of a hot spot is $10 r_{\rm g}$ and $5 r_{\rm g}$, while the inclination angle i is 20°, 40°, 60°, 70°, 80°, and 85°.

In figure 2 the abscissa is the non-dimensional time in units of $r_{\rm g}/c$, while the ordinate is the flux. The mean flux is higher for a small inclination angle mainly due to the projection effect of the disk.

As can be seen from figure 2, the light curve of a hot spot for a Keplerian case is *periodic* with the rotation period. Furthermore, as Asaoka (1989) pointed out, there exist two types of peaks. One is a broad sinusoidal variation, which originates from a Doppler effect; we call this *a Doppler variation*. The other is a narrow higher peak, which originates from a gravitational focusing effect; we call this *a lensing peak*. When the inclination angle is small ($0° \le i \le 60°$), the former Doppler variation dominates. On the other hand, when the inclination angle is large ($i \geq 80°$), the latter lensing peak is prominent.

3. Non-Keplerian Case

Now, we examine the light curves of a hot spot on a non-Keplerian disk, where the radial infall velocity is not negligible, but of the order of the azimuthal rotational velocity. As a simple model for such a non-Keplerian disk, we suppose an inviscidly infalling disk (cf. Fukue 2003), where the optically-thick luminous disk extends from a large radial distance down to the event horizon at $1\,r_\mathrm{g}$, and the entire disk gas infalls with a constant specific angular momentum L_0 and a constant specific energy E_0. Of these, the specific angular momentum is arbitrary given, while the specific energy is set to be $E_0 = c^2$. The radial velocity profile is then automatically determined. The temperature distribution is assumed to be a power-law form with $T(r) \propto r^{-2}$, similar to the supercritical disk (Abramowicz et al. 1988; Fukue 2000; Watarai et al. 2000; Mineshige et al. 2000), and the maximum inner temperature is set to be 10^4 K. It is emphasized that the disk model does not influence the qualitative behavior of the light curves, as long as the radial infall velocity is of the order of the azimuthal rotational velocity.

On such a background disk, a hot spot (circular area) spirally infalls from the initial position at $(r_{\mathrm{s}0}, \varphi_{\mathrm{s}0})$. The radius a_s of a hot spot is fixed to be $1\,r_\mathrm{g}$ for simplicity. The temperature T_s of a hot spot is also fixed at 20000 K. The parameters are then the initial position $(r_{\mathrm{s}0}, \varphi_{\mathrm{s}0})$ and the inclination angle i.

For the present disk with a constant specific angular momentum L_0 and a constant specific energy E_0 $(= c^2)$,

$$cu^r = \frac{\gamma}{\sqrt{g_{00}}}\frac{dr}{dt}, \tag{3}$$

$$L_0 = r^2 \frac{\gamma}{\sqrt{g_{00}}}\frac{d\varphi}{dt}, \tag{4}$$

and

$$u^r u^r = \frac{E_0^2}{c^4} - g_{00}\left(1 + \frac{L_0^2}{c^2 r^2}\right) = 1 - g_{00}\left(1 + \frac{L_0^2}{c^2 r^2}\right), \tag{5}$$

where

$$g_{00} = 1 - \frac{r_\mathrm{g}}{r}, \tag{6}$$

$$\gamma^2 = \frac{E_0^2}{c^4} + \frac{r_\mathrm{g}}{r}\left(1 + \frac{L_0^2}{c^2 r^2}\right) = 1 + \frac{r_\mathrm{g}}{r}\left(1 + \frac{L_0^2}{c^2 r^2}\right). \tag{7}$$

Hence, we have

$$d\varphi = \frac{\sqrt{g_{00}}}{\gamma}\frac{L_0}{r^2}\,dt, \tag{8}$$

$$dr = \frac{\sqrt{g_{00}}}{\gamma}\sqrt{1 - g_{00}\left(1 + \frac{L_0^2}{r^2}\right)}\,dt, \tag{9}$$

where the radius and time are measured in units of r_g and r_g/c, respectively.

Using these relations, we can calculate the loci of a hot spot, starting from $(r_{\mathrm{s}0}, \varphi_{\mathrm{s}0})$ and spirally infalling into the black hole. Several examples of loci of a hot spot are shown in figure 3.

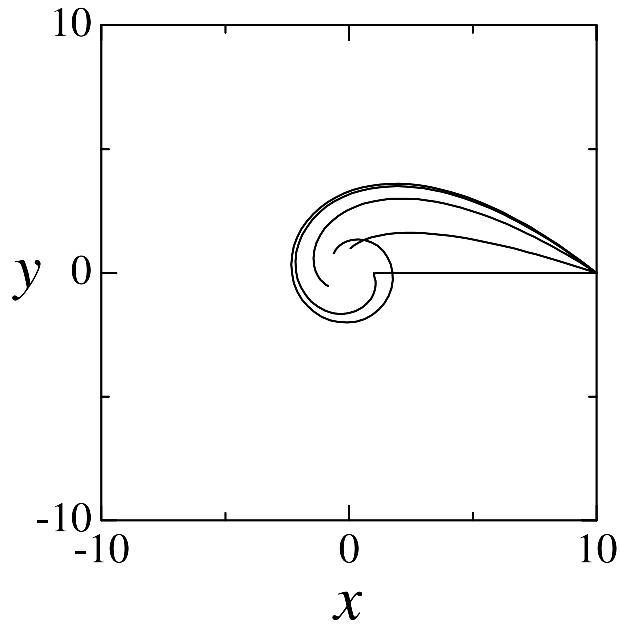


Fig. 3. Loci of a hot spot spirally infalling in the disk with a constant specific angular momentum and a constant specific energy. The initial position of a hot spot is $(10\,r_\mathrm{g}, 90°)$, and the specific angular momentum L_0 in units of $r_\mathrm{g}c$ is 0, 1, $\sqrt{3}$, 1.95, and 1.99.

In figure 3, the initial radius $r_{\mathrm{s}0}$ of a hot spot is $10\,r_\mathrm{g}$, and the specific angular momentum L_0 in units of $r_\mathrm{g}c$ is 0, 1, $\sqrt{3}$, 1.95, and 1.99. As can be seen in figure 3, when the radial infall velocity is of the order of the azimuthal rotational velocity, a hot spot spirally infalls onto the central black hole within a dynamical time, even if the value of the specific angular momentum is close to that of the marginally bound orbit.

In figure 4 an apparent view of several hot spots at different epoch on the infalling disk is shown for the convenience of readers. In this example the specific angular momentum L_0 of the disk is $1.99\,r_\mathrm{g}c$, the initial position of the hot spot is $(10\,r_\mathrm{g}, 90°)$, and the inclination angle is 80°. The central black area is smaller than that in figure 1, since the disk exists down to $1\,r_\mathrm{g}$ in this case (Fukue 2003). As can be easily seen in figure 4, the hot spot does not rotate, but spirally infalls within a dynamical time.

In figure 5, light curves of a hot spot on a non-Keplerian disk are shown for several specific angular momenta and inclination angles. The specific angular momentum L_0 in units of $r_\mathrm{g}c$ is $\sqrt{3}$ in (a), 1.95 in (b), and 1.99 in (c), while the inclination angle i is 20° (dotted curve), 40° (dashed one), 60° (chain-dotted one), 80° (solid one), and 85° (thick solid one). The initial position of the hot spot is $(10\,r_\mathrm{g}, 90°)$.

In figure 5 the abscissa is the non-dimensional time in units of r_g/c, while the ordinate is the flux. In this case, the mean flux does not decrease, but increases as the inclination angle becomes large ($0° \leq i \leq 60°$). This is because the disk extends down to $1\,r_\mathrm{g}$ in this case, and the Doppler boost is important for large inclination angles.

As can be seen from figure 5, the light curve of a hot spot for a non-Keplerian case is generally *aperiodic* with a single

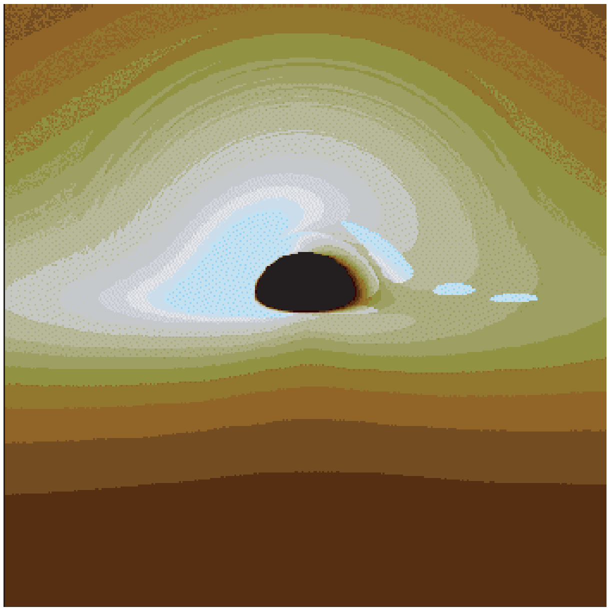


Fig. 4. Sequence of hot spots spirally infalling in a non-Keplerian disk around a Schwarzschild black hole at different epoch. The maximum temperature of the disk is set to be 10000 K, while the temperature of the spot is 20000 K. In this example the specific angular momentum is $1.99 r_{\rm g}c$, the initial position of the hot spot is ($10 r_{\rm g}$, 90°), and the inclination angle is 80°.

peak. This is quite different from the Keplerian case, where the light curve is periodic. In some cases, like figure 5c, where the specific angular momentum is large, a secondary peak appears. However, the height of the secondary peak is lower than that of the first peak, since the location of the spot shifts inwards.

Similar to the Keplerian case, when the inclination angle is small ($0^\circ \le i \le 60^\circ$), the Doppler variation dominates. On the other hand, when the inclination angle is large ($i \ge 80^\circ$), the lensing peak is prominent. However, in the case of small inclination angles, the flux level just after the Doppler variation is lower than that before the Doppler variation. This is also due to the radial shift of the spot: the radial distance of the spot decreases with time, and therefore, the amount of the gravitational redshift increases with time in the case of small inclination angles. Hence, in the pole-on case of $i = 0^\circ$, the flux originated from the spot decreases with time, if the spot spirally infalls onto the black hole.

In figure 6 the dependence on the initial position, in particular, on the initial azimuthal angle $\varphi_{\rm s0}$, is shown. That is, $\varphi_{\rm s0} = 0^\circ$ (dashed curve), 90° (solid one), 180° (chain-dotted one), and 270° (dotted one) in a counterclockwise, while $r_{\rm s0} = 10 r_{\rm g}$, $L_0 = 1.95 r_{\rm g}c$, and $i = 80^\circ$. The abscissa in figure 6 is the non-dimensional time in units of $r_{\rm g}/c$ (figure 6a) and the azimuthal angle φ (figure 6b). Only in the case of $\varphi_{\rm s0} = 180^\circ$, the Doppler variation as well as the lensing peak are seen, while the lensing peak is prominent in other cases for this large inclination angle.

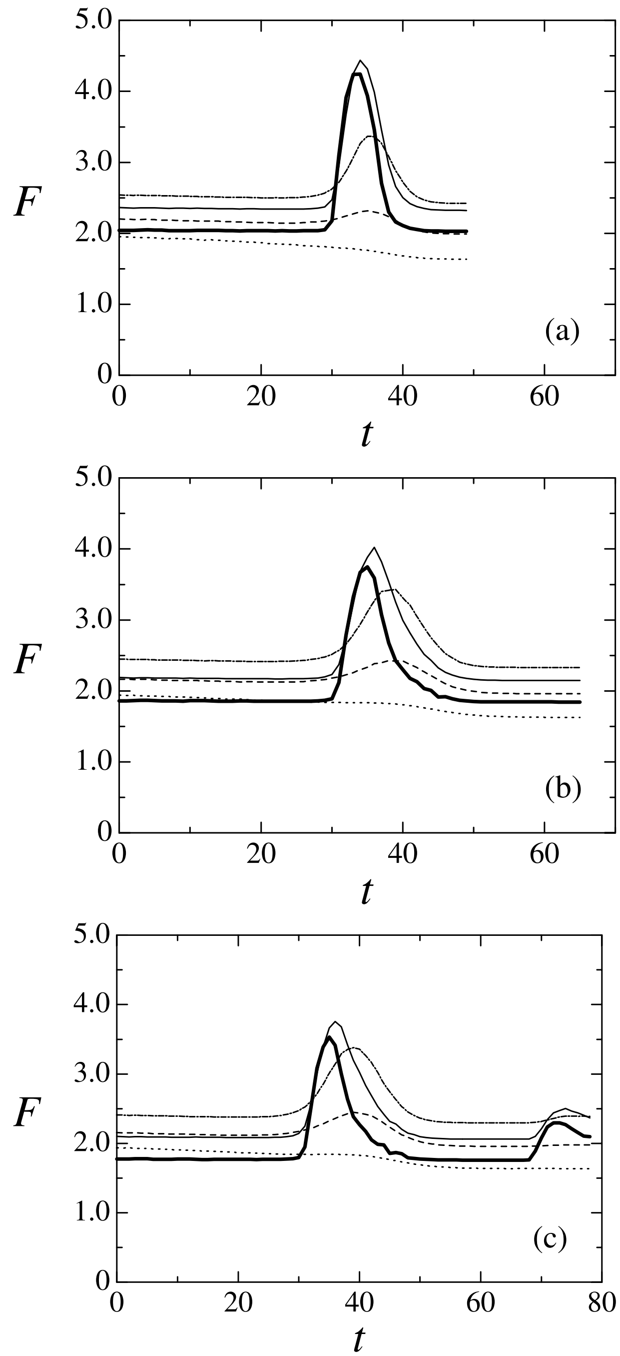
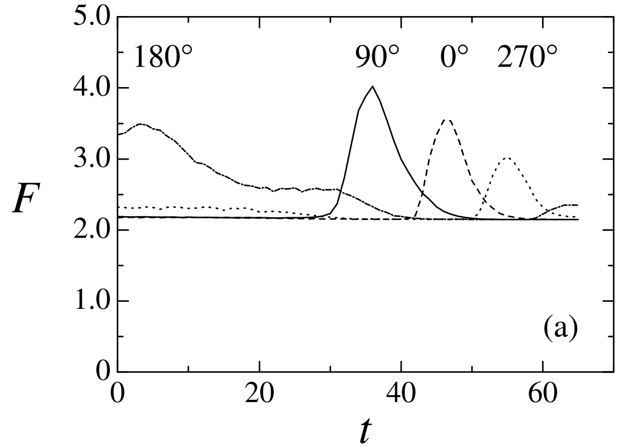


Fig. 5. Light curves of a hot spot on a non-Keplerian disk for several specific angular momenta and inclination angles. The specific angular momentum L_0 in units of $r_{\rm g}c$ is $\sqrt{3}$ in (a), 1.95 in (b), and 1.99 in (c), whereas the inclination angle i is 20° (dotted curve), 40° (dashed one), 60° (chain-dotted one), 80° (solid one), and 85° (thick solid one). The initial position of the hot spot is ($10 r_{\rm g}$, 90°).

4. Concluding Remarks

In this paper we discuss our calculations of the light curves of a hot spot on a relativistic disk around a Schwarzschild black hole. The light curves of a hot spot rotating in a Keplerian disk were periodic with typically two peaks, originating from a gravitational focusing effect and a Doppler boost. On the other



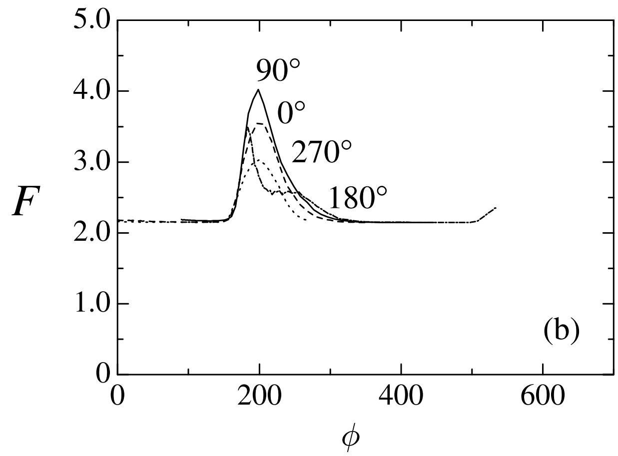


Fig. 6. Light curves of a hot spot on a non-Keplerian disk for several initial azimuthal angles φ_{s0}; $\varphi_{s0} = 0°$ (dashed curve), 90° (solid one), 180° (chain-dotted one), and 270° (dotted one) in a counterclockwise. Other parameters are $r_{s0} = 10 r_g$, $L_0 = 1.95 r_g c$, and $i = 80°$. The abscissa is (a) the non-dimensional time in units of r_g/c and (b) the azimuthal angle φ.

hand, we found that light curves of a hot spot spirally infalling in a non-Keplerian advective disk are aperiodic with typically a single peak, originating from a gravitational focusing effect or a Doppler boost. Such a difference in light curves of a hot spot can discriminate background disk models.

We have examined the behavior of emitting hot spots in the inner accretion disk ($\sim 10 r_g$), where the relativistic effect is especially important. If a hot spot is created at a sufficiently large radius and spirally infalls toward the center, there seem to appear multiple peaks with different peak heights and intervals (cf. figure 5c). The peak height initially increases, and then decreases quickly as the hot spot infalls. The peak interval decreases as the hot spot infalls. The rotating hot spot, however, would be destroyed by differential rotation.

We further fixed the size and shape of the spot, a circular area with radius r_g. As the spot circularly rotates or spirally infalls, the size and shape would be changed and sheared. These effects are also of interest, and are left as future work.

It should be noted that even for a circularly rotating hot spot, differential rotation would destroy it. If so, the light curves of a hot spot in a Keplerian disk may gradually decay. However, the peak interval maintains its periodicity with the rotation period. Such a decaying light curve was shown in Karas, Vokrouhlický, and Polnarev (1992).

In addition, instead of a single hot spot, there can appear multiple hot spots on the disk at each given moment of time, and light curves may be a superposition of individual curves. In such a case, light curves become complicated, and a periodicity analysis is required. For a Keplerian disk, such a case of multiple hot spots and superposition of light curves was discussed by Karas, Vokrouhlický, and Polnarev (1992) (cf. Martocchia et al. 2000).

Furthermore, several other phenomena, such as a flare on the disk, will produce a single peak light curve. However, it is expected that the spectral feature is different between a thermal hot spot and a non-thermal flare.

Instead of a hot spot, a wave pattern on the disk is expected in some cases (cf. Kato et al. 1998). In the case of a Keplerian disk, Karas, Martocchia, and Šubr (2001) discussed variable line profiles and light curves due to non-axisymmetric wave patterns (cf. Sanbuichi et al. 1994). Light curves of such a wave pattern under the relativistic effect are also of interest and would be used to search for an oscillating disk.

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