Radiative Corrections for Compton Scattering.

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As reported before, the "self-consistent" subtraction method has been applied to the elastic scattering of an electron by a fixed centre of force and it has turned out that, in spite of the defective formalism of the present quantum field theory, one can still obtain a reasonable result without ambiguity if one suitably re-interprets the infinite self-energy and vacuum polarization. (An essentially same conclusion has been published by Lewis and Epstein recently.) It will be of importance, therefore, to examine whether our prescription is valid in the case of collisions between elementary particles, too.

We have thus calculated $\alpha$-corrections to the Klein-Nishina formula for the Compton scattering by a straightforward application of the perturbation method, and investigated what modifications we have to introduce into the Hamiltonian function in order to get rid of the divergence difficulty. We pertain, therefore, only to the diverging terms of the corrections in high frequency region. Our main conclusions are as follows:

1) There are two types of diverging terms; the one is related to the polarization of vacuum and the other to the self-energy of the electron.

2) Vacuum Polarization. As examples of processes which yield this type of divergence, we cite the following scheme:

\[
p \rightarrow q \rightarrow q, (-r+q)^+, \quad \cdots
\]

Here we refer to the system of the centre of gravity, and denote the incident and scattered electron resp., $p = q$. $O$ and $r$ are
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Intermediate electrons with momentum zero and \( \mathbf{r} \) (arbitrary). Symbols \( \sim \) and \( + \) indicate a photon and a positron resp. The contribution of such processes to the cross section diverges quadratically and logarithmically. In order to cancel out these infinite terms, however, it is sufficient to add to the Hamiltonian function two new terms of the form

\[
\left( \frac{e^2}{3\pi^3} \right) \int r dr \int A^2 du \quad \text{and} \\
+ \epsilon \int (\phi^* (X) \phi (X), \delta A (X)) du
\]

with \( \delta A (X) = \left( \frac{e^2}{3\pi^3} \right) \int dr / r A (X) \)

3) Self-Energy. Examples of these processes are;

\[
\psi, -\bar{\psi} \rightarrow 0, -f, f \\
\rightarrow 0, 0, (\mathbf{f})^*, f \\
\rightarrow 0, -\mathbf{q}
\]

where \( f \) means a virtual photon with arbitrary momentum. The second connection in the above scheme is half forbidden (i.e. when both \( O \)’s have parallel spins) and so gives negative contribution to the cross section (as deviation from the “vacuum”). Summing up similar terms (including those due to Coulomb interaction) one obtains a logarithmic divergence, which can be eliminated by introducing a new term \(-2m \int \phi^* \phi du\) into the interaction Hamiltonian, where \( 2m = (3e^2/3\pi^3) \int dr / r \cdot m \) is the self-energy of an electron.

4) The additive term \(-2m \int \phi^* \phi du\) can be derived in a plausible way as a “counter-term” which compensates the change in the free field Hamiltonian (\( m \rightarrow m + 2m \)) and conserves the total Hamiltonian unaltered. One of the vacuum polarization terms \(+ \epsilon \int (\phi^* \phi, \delta A) du\) could be also treated in a similar way, while the other term \( \left( \frac{e^2}{3\pi^3} \right) \int r dr \int A^2 du \) could not be foisted into the theory without radically changing the Maxwell equation for the free electromagnetic field. This point is now under investigation.

5) As an alternative method, the hypothesis of the cohesive force field\(^{(5)}\) can eliminate the self-energy type divergences, but not ones of the vacuum polarization type.

A fuller account will appear in a later issue of this journal, where more general discussions will be made.

(1) Z. Kobe and S. Tomonaga, Prog. Theor. Phys. in press; Preliminary report, ibid


(4) S. T. Epstein, Phys. Rev. 73 (1948) 177.

(5) The difficulty of the divergence in the low frequency region was discussed in detail by Jost; S. Jost, Phys. Rev. 72 (1947) 815.


5) As an alternative method, the hypothesis of the cohesive force field\(^{(5)}\) can eliminate the self-energy type divergences, but not ones of

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